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Effective Field Theory phenomenology for an extended Dark Sector

Chao Zhang, Dieter Horns
Universität Hamburg, SFB-676
28/09/2017

DESY
Theory workshop 2017



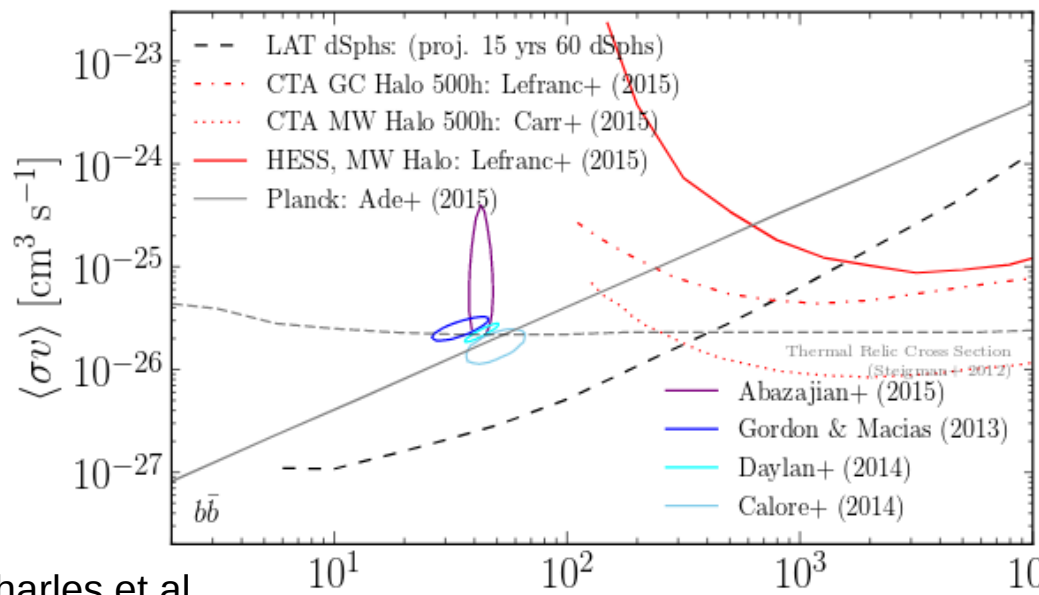
Outline

- Introduction and motivation
- Effective field theory of annihilation and co-annihilation
- Dynamic solution of the Boltzmann equation
- Conclusion and discussion

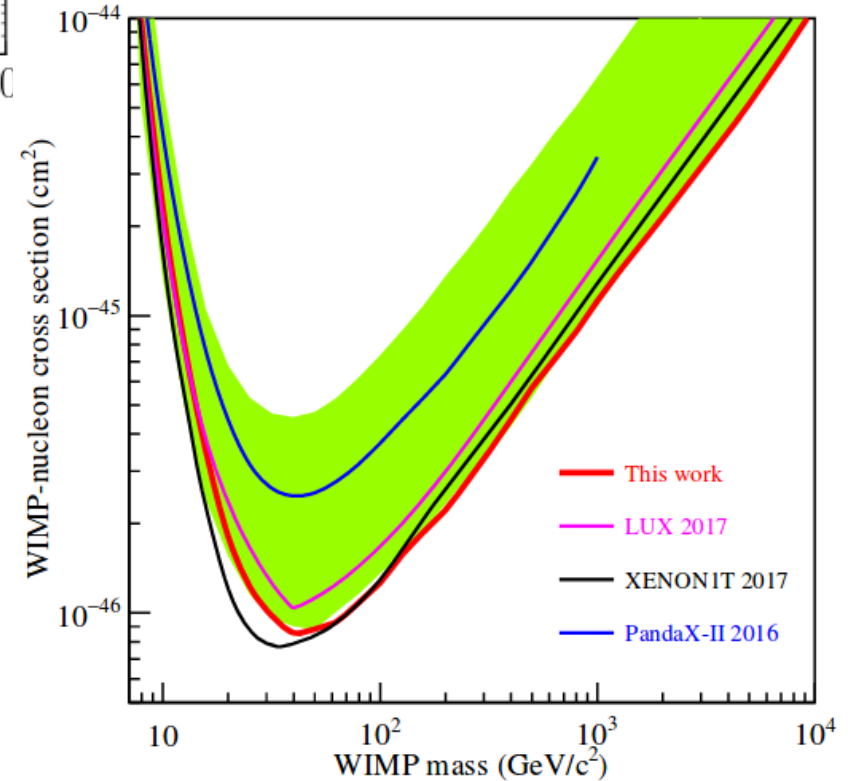
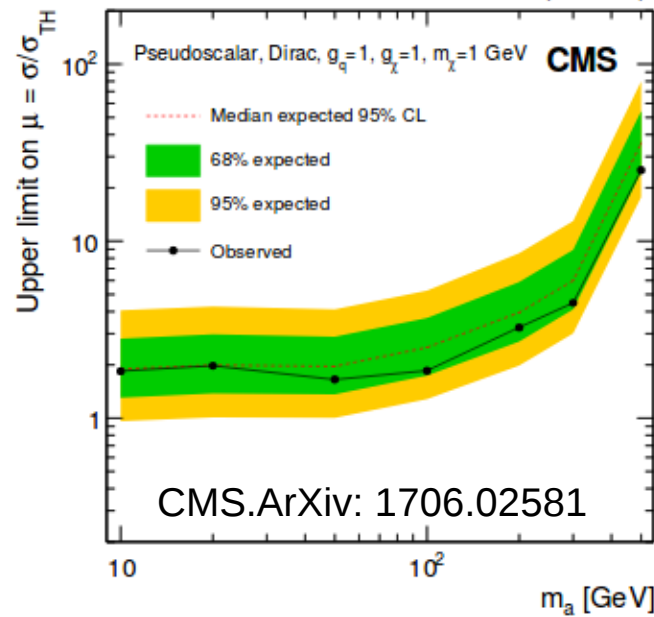
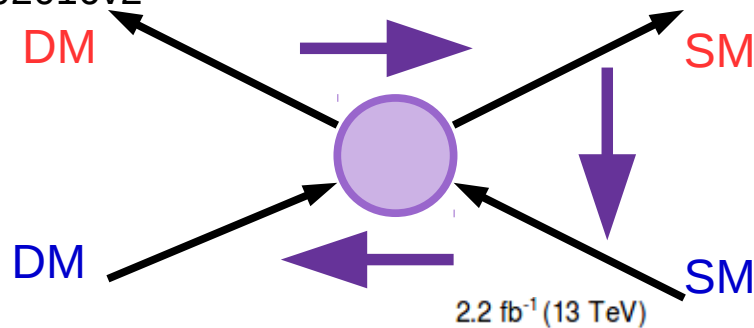
1.Introduction

$$\Omega_{\chi} h^2 = 0.1198 \pm 0.0015$$

PLANCK
arXiv:1507.02704



Charles et al.
ArXiv:1605.02016v2



PandaX. arXiv:1708.06917

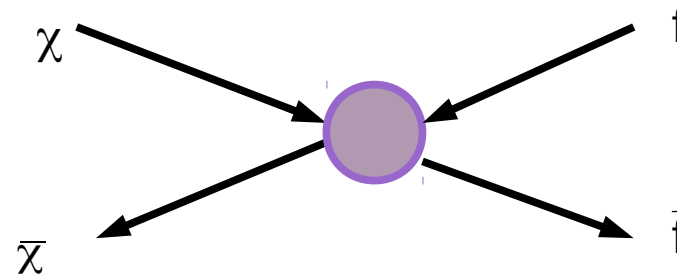
2. Effective Field Theory of annihilation and co-annihilation

- consider co-annihilation of dark sector particles in the first 10^{-14} - 10^{-12} second after the Big Bang.

- Effective operator:

$$L_{12 \rightarrow 34} = \sum_{f_1 f_2} \frac{1}{\Lambda_{1,2}^2} (\bar{\chi}_1 \Gamma_1 \chi_2) (\bar{f}_1 \Gamma_2 f_2)$$

- Transition matrix:



$$\sum_s |M|^2 = G_f^2 \cdot (k_1 m_f m_{\bar{f}} (p_1 \cdot p_2) + k_2 m_{\chi_1} m_{\chi_2} (p_3 \cdot p_4) + k_3 (p_1 \cdot p_4) (p_2 \cdot p_3) + k_4 (p_1 \cdot p_2) (p_3 \cdot p_4) + k_5 m_f m_{\bar{f}} m_{\chi_1} m_{\chi_2})$$

Annihilation	Coannihilation
$\frac{dn}{dt} = -3Hn - \sum_{i,j=1}^N \langle \sigma v \rangle (n^2 - n_{eq}^2)$	$\frac{dn}{dt} = -3Hn - \sum_{i,j=1}^N \langle \sigma_{ij} v \rangle (n_i n_j - n_i^{eq} n_j^{eq})$
$\langle \sigma v \rangle = a' + 6b'/x_f + 60c'/x_f^2$	$\sigma_{eff} = \sum_{ij} \sigma_{ij} \frac{g_i g_j}{g_{eff}^2} (1 + \Delta_i)^{3/2} (1 + \Delta_j)^{3/2} \exp[-x(\Delta_i + \Delta_j)]$

k_i are the coefficients

$\Delta_i = (m_i - m_1)/m_1$ The K. Griest method, 1991

List of (co)annihilation channels

Fermion

Scalar	23
PseudoScalar	24
Vector	25
Axial Vector	26
FV-Ch	27
FVr	28
FtS Sc	29
FtS Ps	30
FtSr Sc	31
FtSc PS	32
FtV Vec	33
FtV Ax	34
FtVr Vec	35
FtVr Ax	36
FtV Ch	37
FtVr	38

Scalar

channel	
SS Scalar	55
SS Pseudoscalar	56
SV Vector	57
SV AxialVector	58
SV Chiral	59
SF Scalar	60
SF Pseudoscalar	61
SFr Scalar	62
SFr Pseudoscalar	63

Vector

VS Scalar	73
VS Pseudoscalar	74
VV Vector	75
VV Axial	76
VV Chiral	77
VF Vector	78
VF AxialVector	79
VFr Vector	80
VFr AxialVector	81

Example:

Vector interaction:

$$\sigma v = \frac{G^2}{4\pi s} \left(\frac{y}{x}\right)^{1/2} (m_3 m_4 p_1 p_2 + m_1 m_2 p_3 p_4 + 2p_1 p_4 p_2 p_3 + 2m_1 m_2 m_3 m_4) \cdot v$$

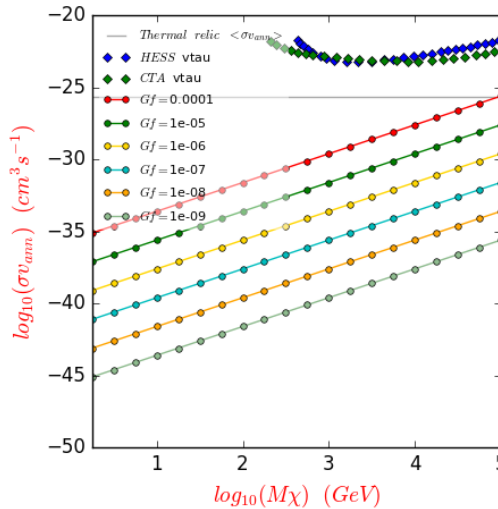
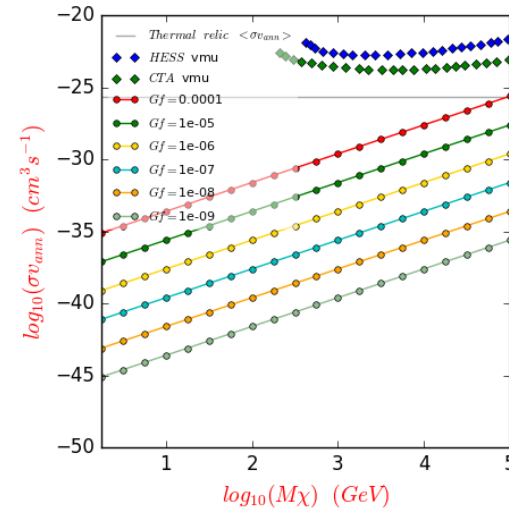
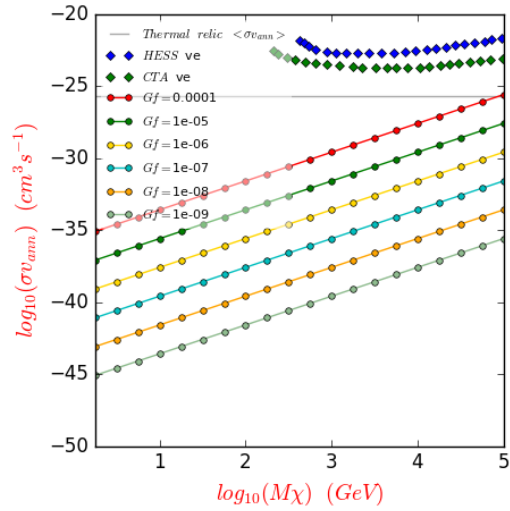
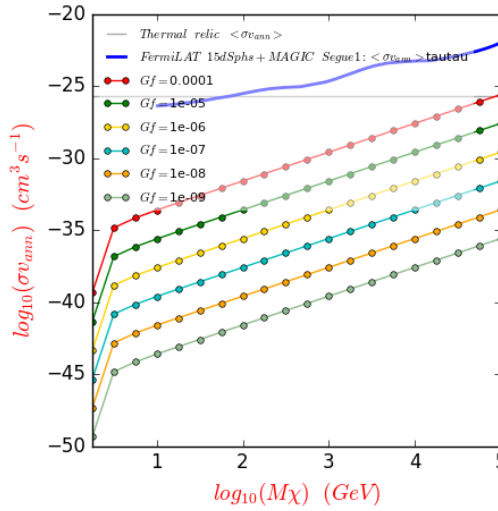
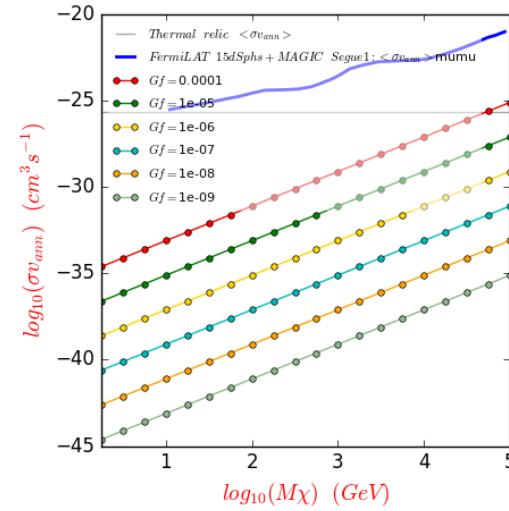
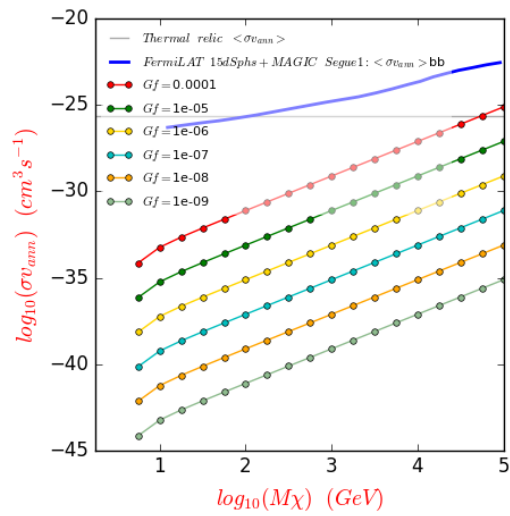
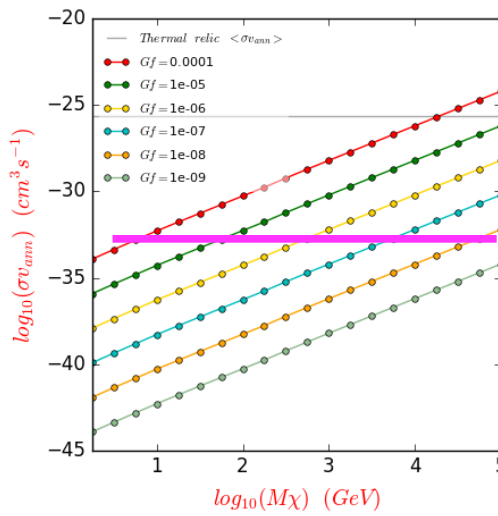
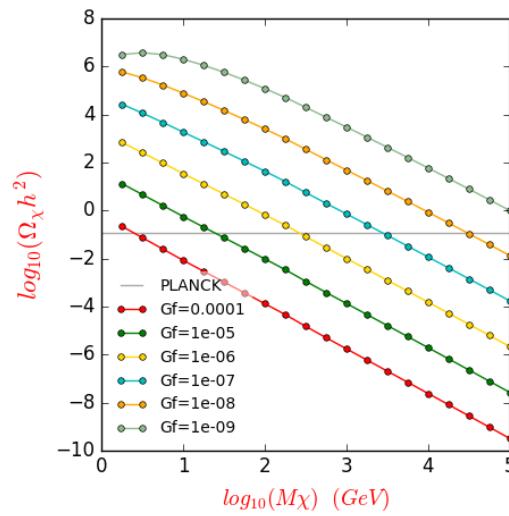
J.M.Zhang et al. arXiv:1012.2023v3
H.Dreiner et al. arXiv: 1211.2254v1

2.1 case of only 1 dark sector particle

with static solution to the Boltzmann equation

Dwarfs
GC
10km/s

Data from
PLANCK 2015
HESS [1605.08788]
FermiLat and Magic:
[jcap 022016039]
CTA:[1508.06128]



WIMP annihilation
in Scalar interaction
with universal coupling

Fig. 1.1: $\log_{10}(\Omega_\chi h^2)$ VS $\log_{10}(M_\chi)$

Fig. 1.2: $\log_{10}(\sigma v_{ann})_{total}$ VS $\log_{10}(M_\chi)$

Fig. 1.3: $\log_{10}(\sigma v_{ann})_{bb}$ VS $\log_{10}(M_\chi)$

Fig. 2.1: $\log_{10}(\sigma v_{ann})_{\mu+\mu-}$ VS $\log_{10}(M_\chi)$

Fig. 2.2: $\log_{10}(\sigma v_{ann})_{\tau+\tau-}$ VS $\log_{10}(M_\chi)$

Fig. 2.3: $\log_{10}(\sigma v_{ann})_{\nu_e \bar{\nu}_e}$ VS $\log_{10}(M_\chi)$

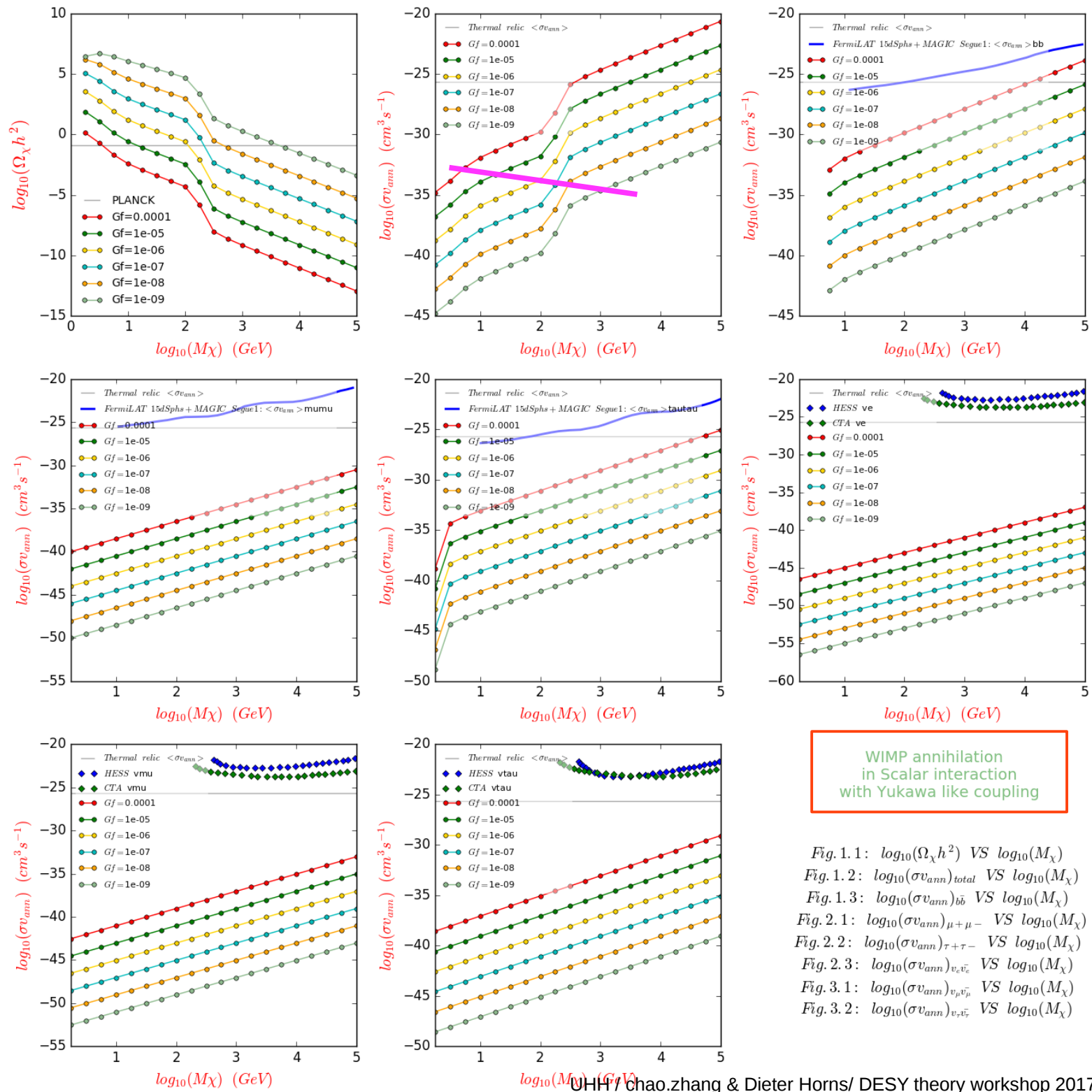
Fig. 3.1: $\log_{10}(\sigma v_{ann})_{\nu_\mu \bar{\nu}_\mu}$ VS $\log_{10}(M_\chi)$

Fig. 3.2: $\log_{10}(\sigma v_{ann})_{\nu_\tau \bar{\nu}_\tau}$ VS $\log_{10}(M_\chi)$

2.1 case of only 1 dark sector particle

with static solution to the Boltzmann equation

Dwarfs
GC
10km/s



WIMP annihilation
in Scalar interaction
with Yukawa like coupling

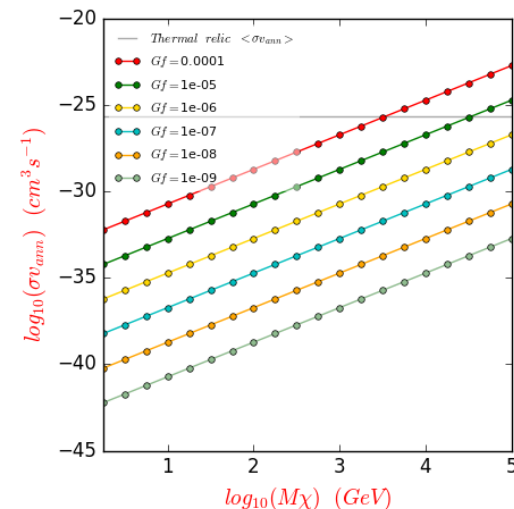
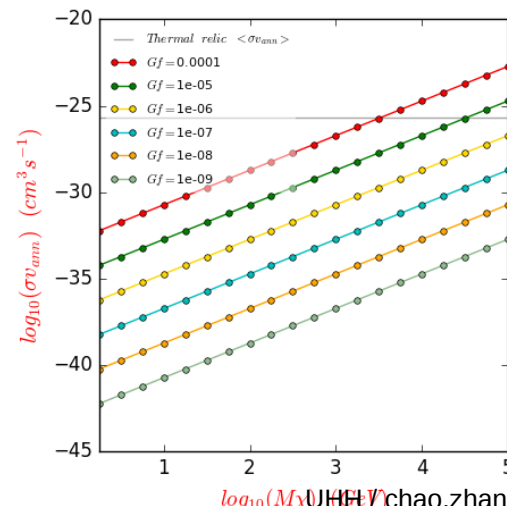
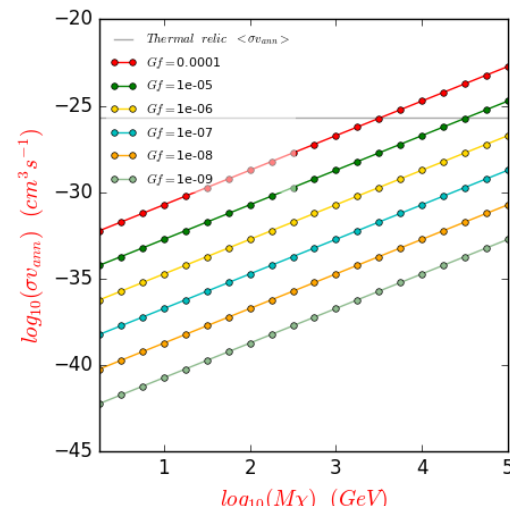
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Data from
PLANCK 2015
HESS [1605.08788]
FermiLat and magic:
[jcap 022016039]
CTA:[1508.06128]

with static
solution to
the
Boltzmann
equation

Halo
200km/s
[1603.03797]

Data from
PLANCK 2015
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CTA:[1508.06128]



WIMP annihilation
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 Fig. 3.2: $\log_{10}(\sigma v_{ann})_{\nu_e\nu_\tau}$ VS $\log_{10}(M_\chi)$

2.1 case of only 1 dark sector particle

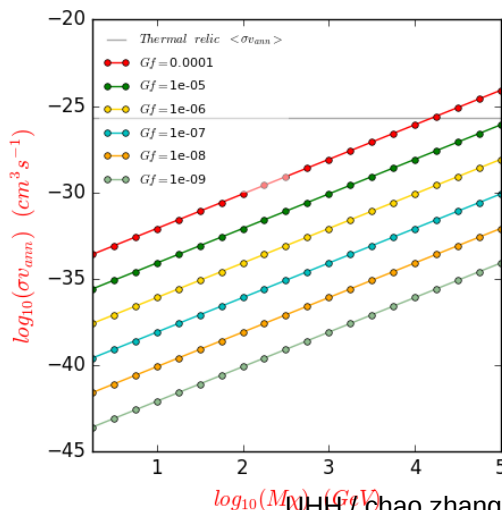
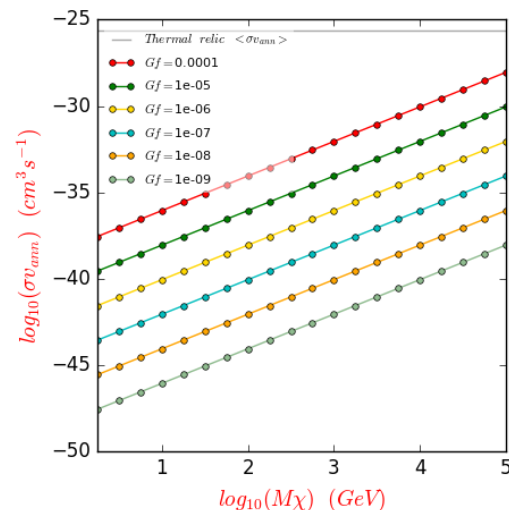
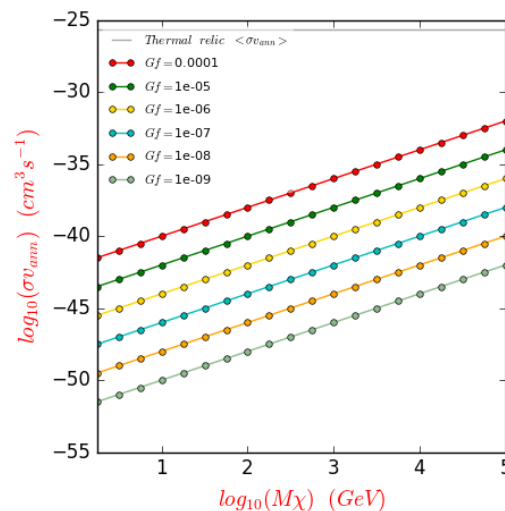
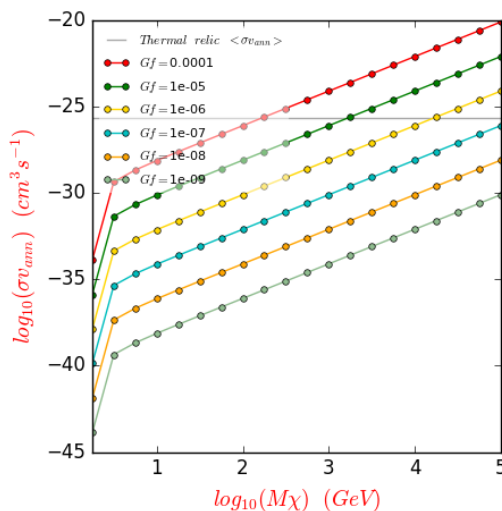
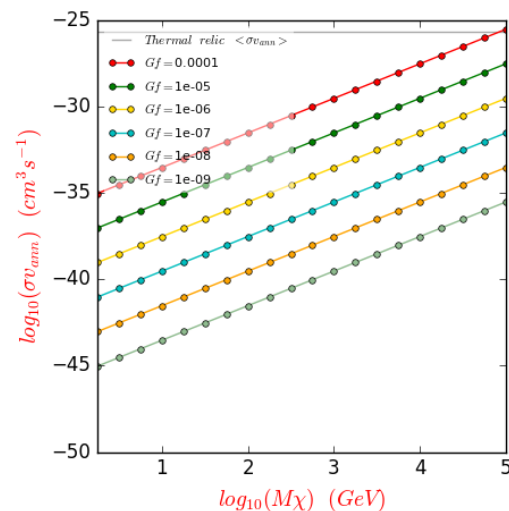
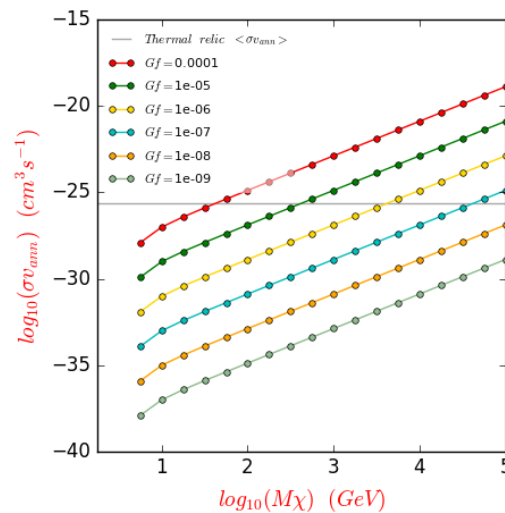
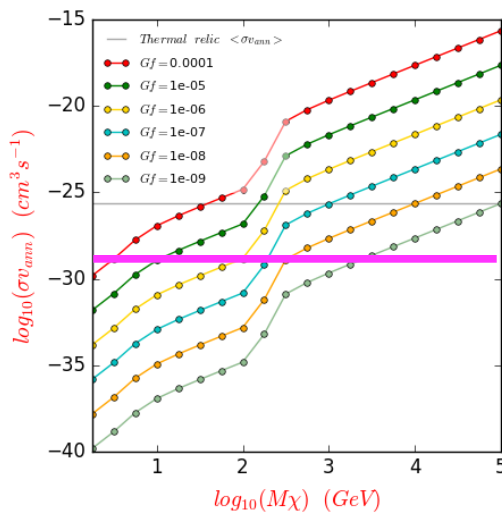
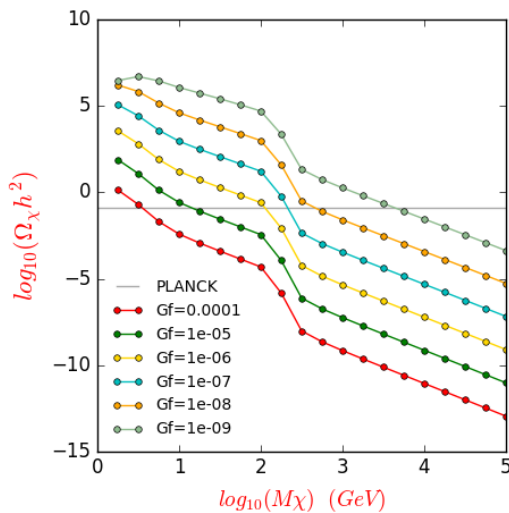
with static solution to the Boltzmann equation

Cluster
1E0657-56

3100km/s

[C.Mastropietro 2007]

No observation.



WIMP annihilation
in Scalar interaction
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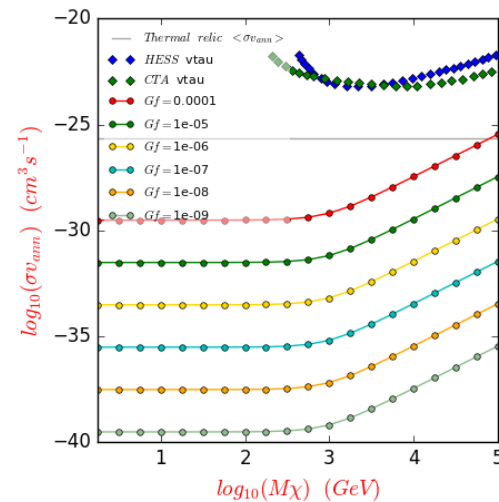
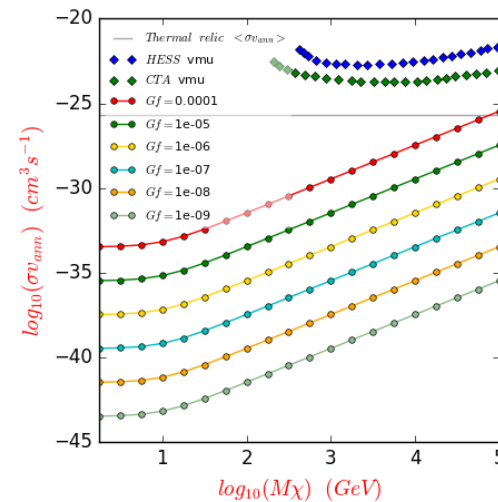
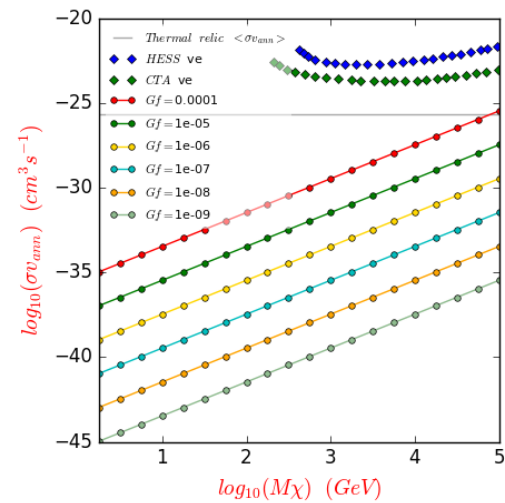
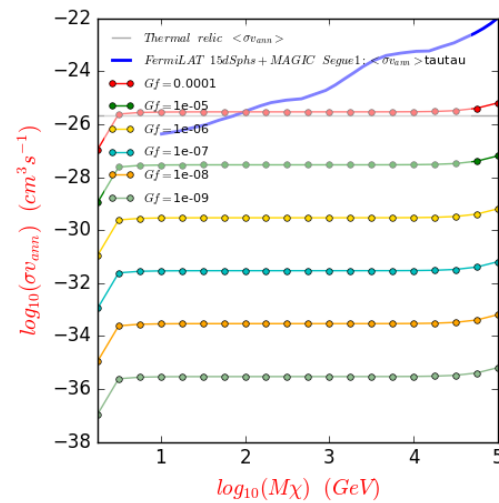
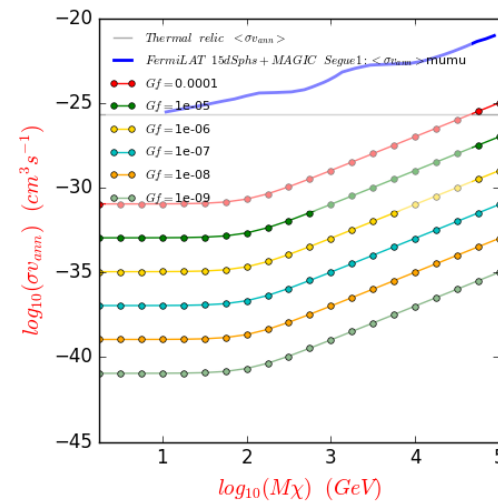
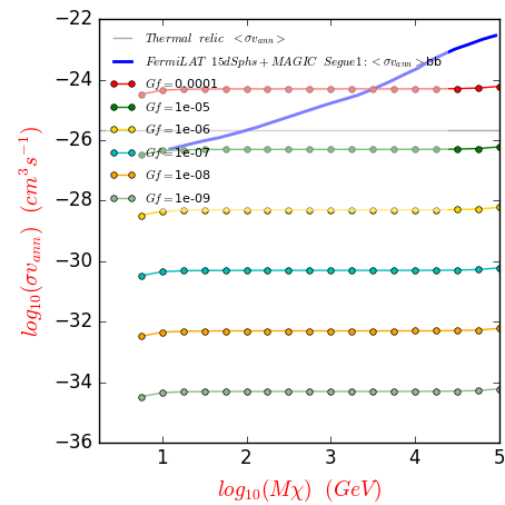
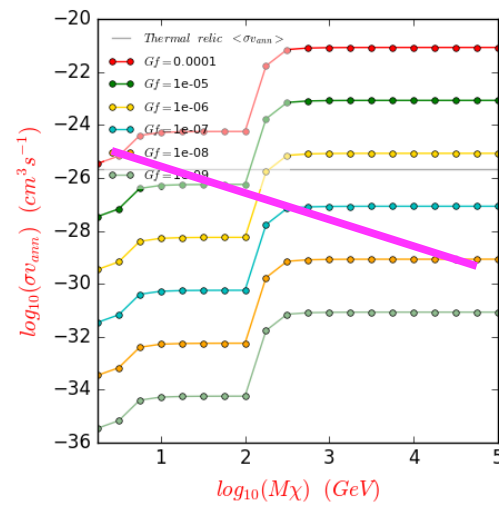
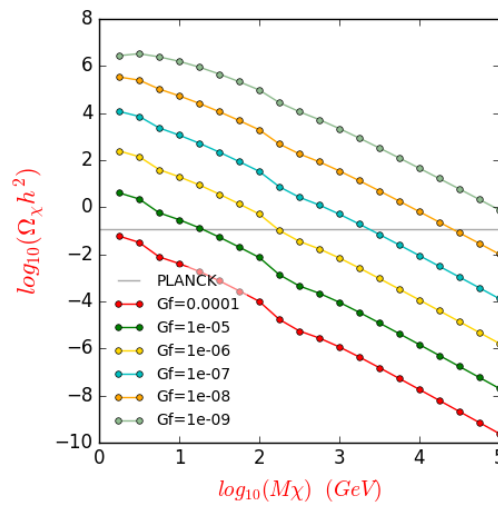
Fig. 3.2: $\log_{10}(\sigma v_{ann})_{\nu_\tau \bar{\nu}_\tau}$ VS $\log_{10}(M_\chi)$

2.1 case of only 1 dark sector particle

with static solution to the Boltzmann equation

Dwarfs
GC
10km/s

Data from
PLANCK 2015
HESS [1605.08788]
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WIMP annihilation
in Axial Vector interaction
with universal coupling

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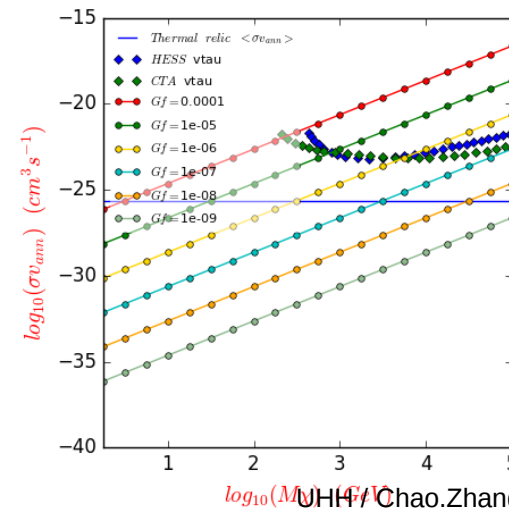
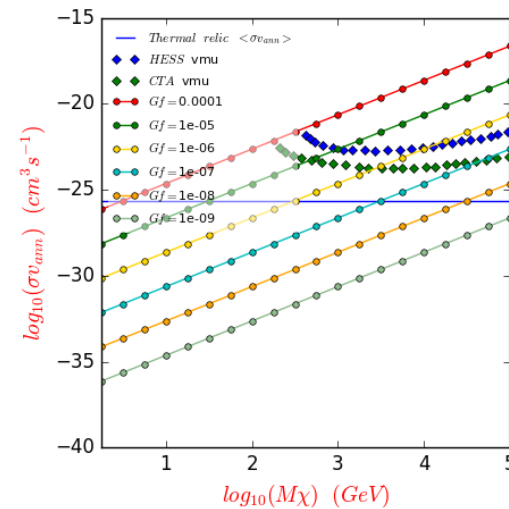
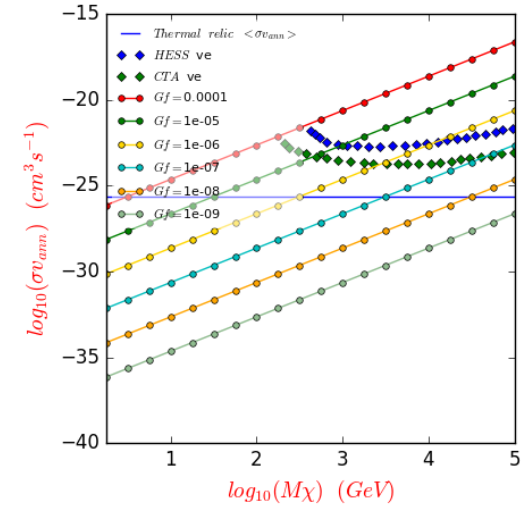
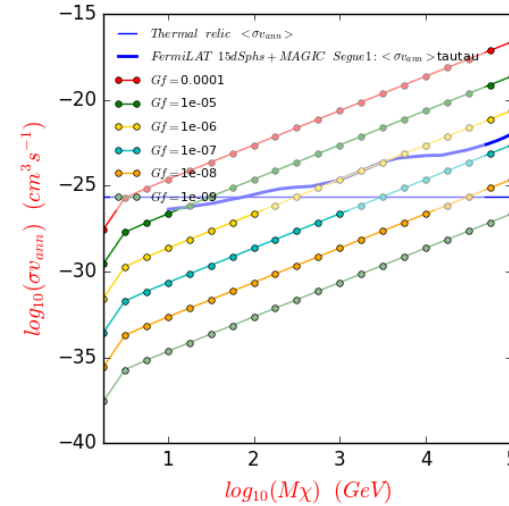
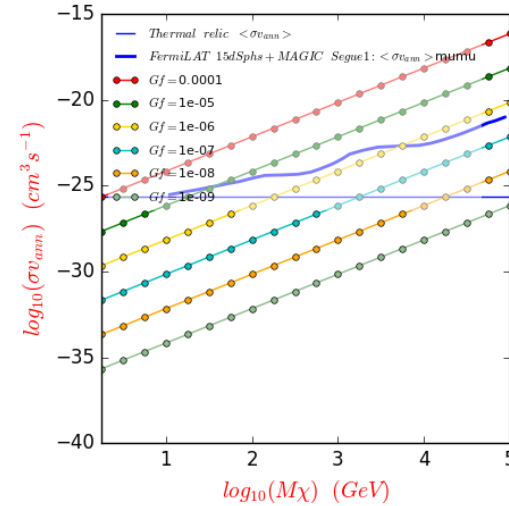
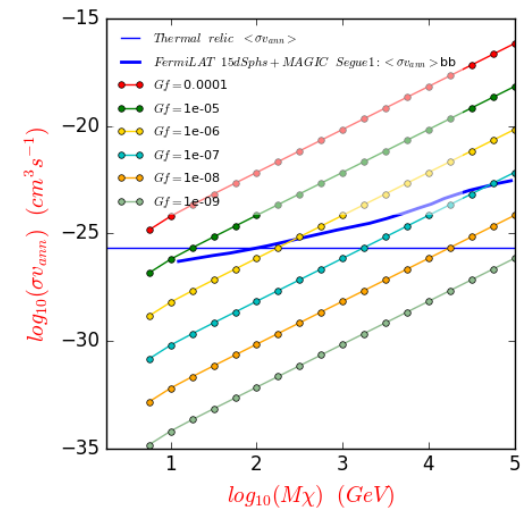
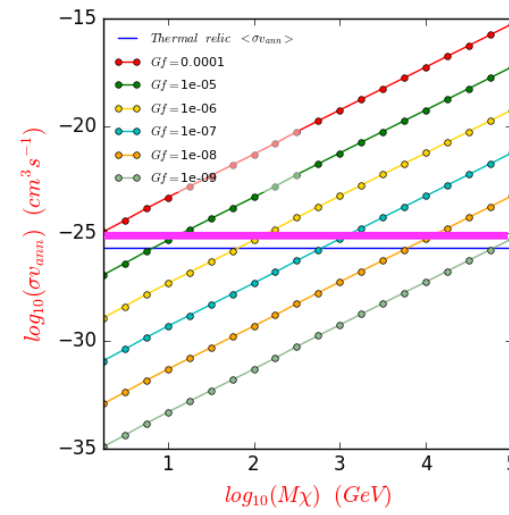
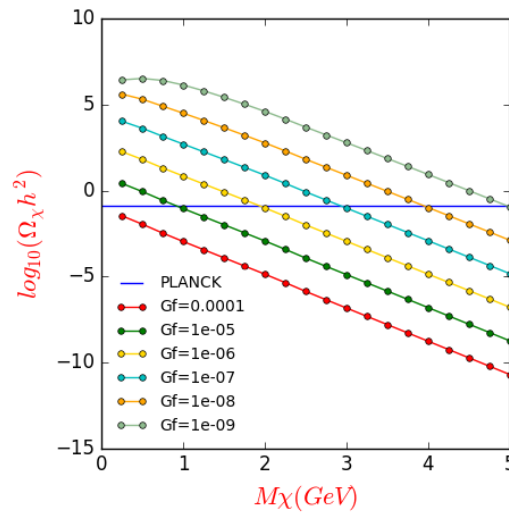
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2.1 case of only 1 dark sector particle

with static solution to the Boltzmann equation

Dwarfs
GC
10km/s



WIMP annihilation
in FtV Chiral interaction
with universal coupling

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Data from
PLANCK 2015
HESS [1605.08788]
FermiLat and magic:
[jcap 022016039]
CTA:[1508.06128]

- for fermion dark matter:

1. relic density given by annihilation through scalar or scalar-pseudoscalar interaction can not be found by indirect detection in the targets with low dark matter velocity. (invisible even if the theory is correct!)
2. axial-vector interaction. specific behavior on mass dependence .

- for scalar dark matter:

vector, axial vector, chiral interactions are invisible in targets with low dm velocity.

- for vector dark matter:

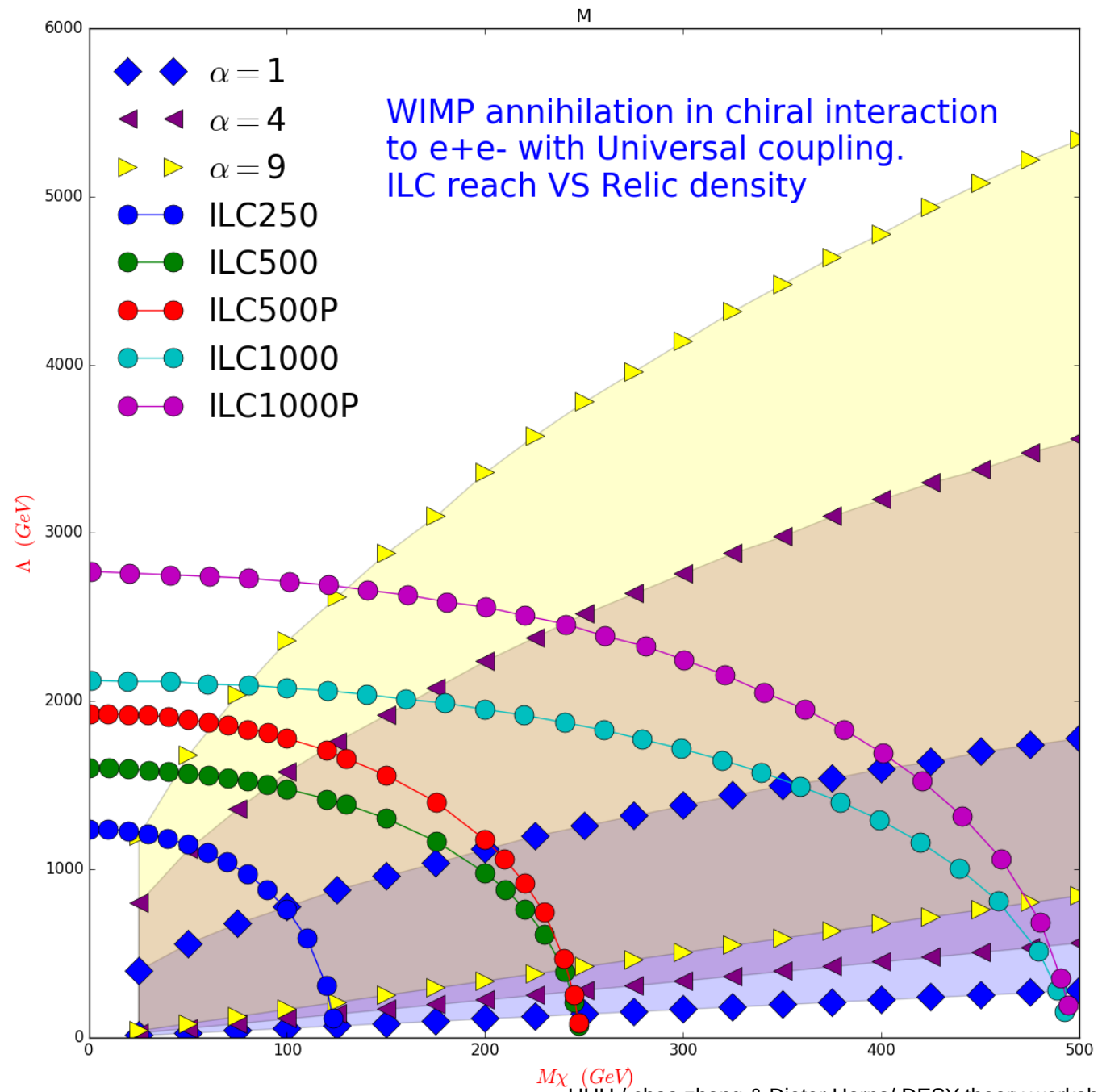
vector, axial vector, chiral, alternative vector, alternative vector-axial vector interactions are invisible in targets with low dm velocity.

for the other channels, please contact us for more details, the code will be on line in the near future.

chao.zhang@desy.de and dieter.horns@desy.de

2.2 case with only 1 dark sector particle with static solution to the Boltzmann equation

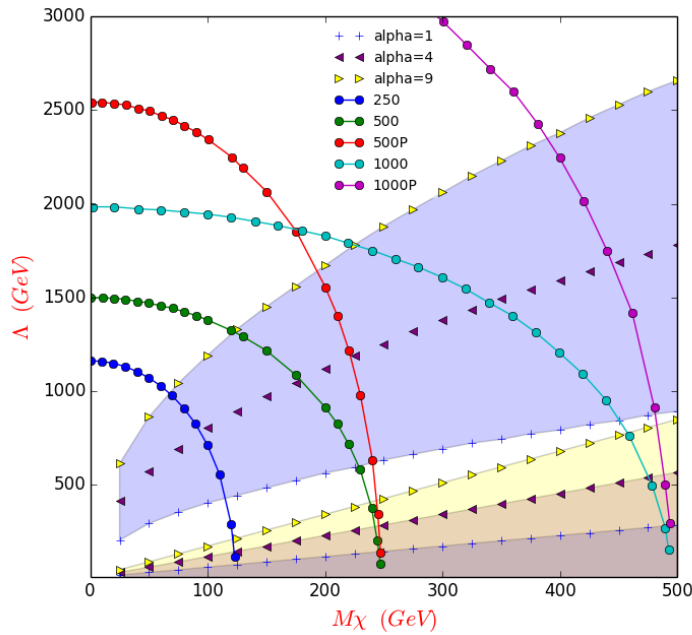
Compare with
ILC sensitivity



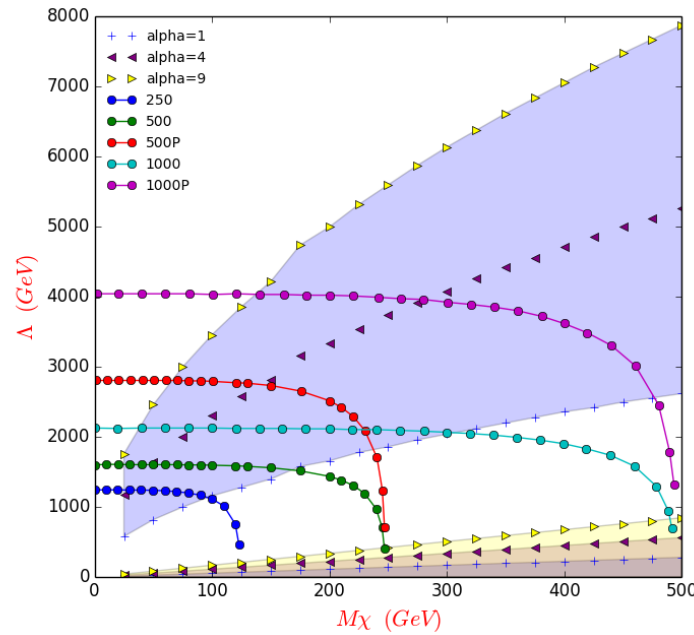
Collaboration with
Moritz Habermehl
List Jenny

2.2 case with only 1 dark sector particle with static solution to the Boltzmann equation

WIMP annihilation in FS/Sc interaction with universal coupling

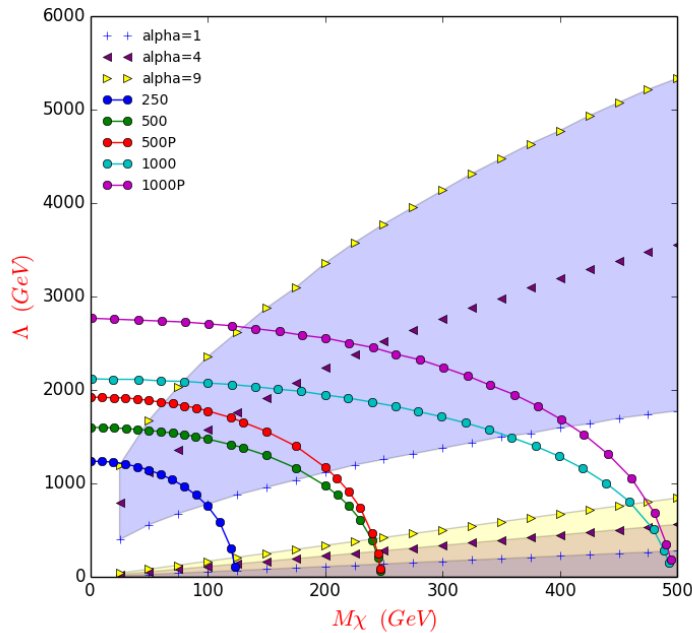


WIMP annihilation in FV/Vec interaction with universal coupling

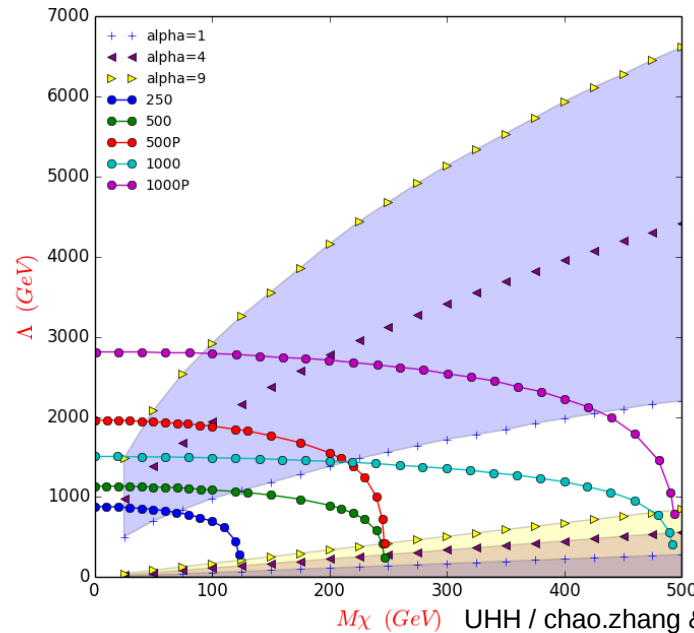


$$\frac{G_f}{\sqrt{2}} = \frac{\alpha}{\Lambda^2}$$

WIMP annihilation in FV/Ch interaction with universal coupling

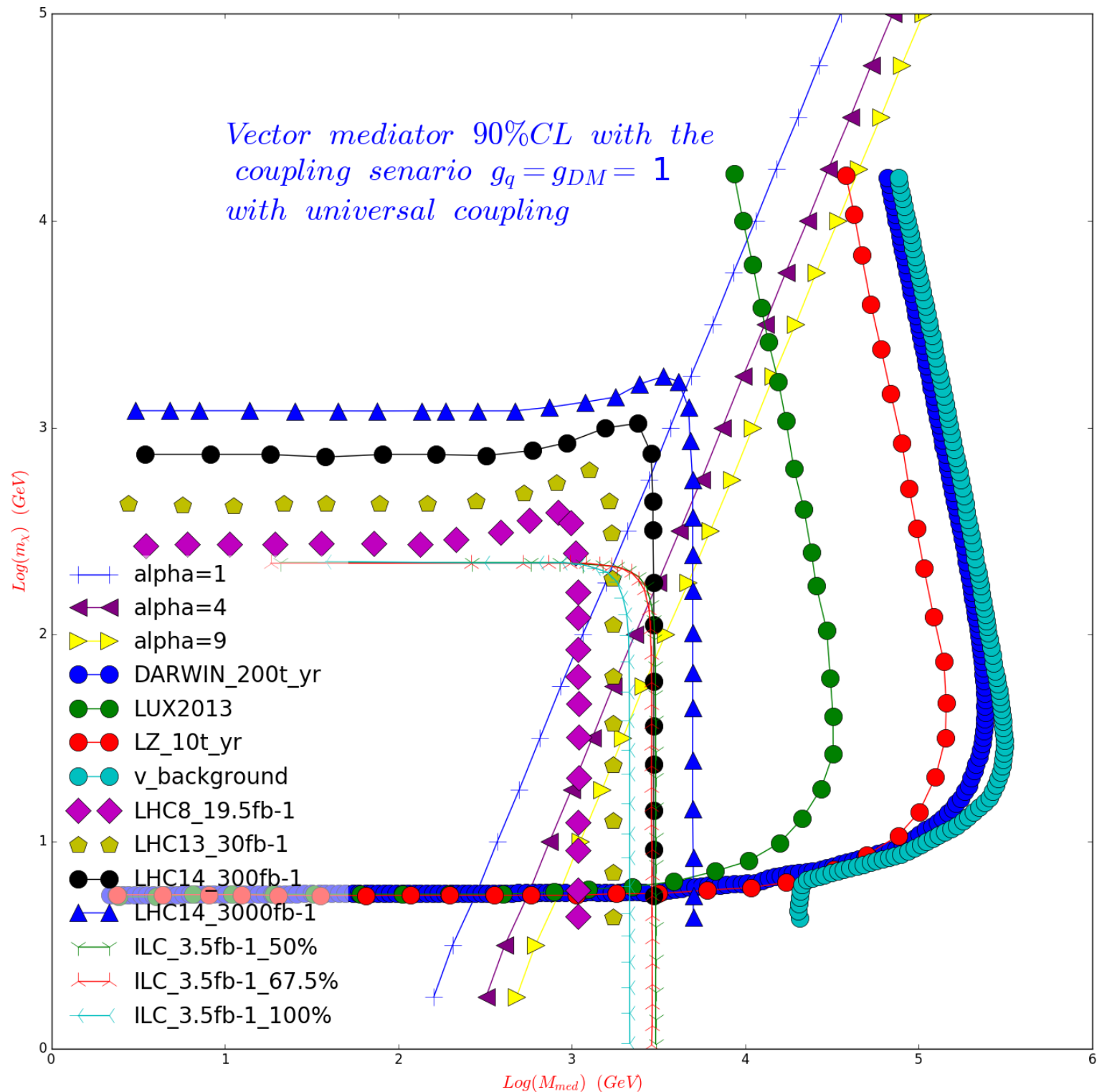


WIMP annihilation in FS/Ps interaction with universal coupling

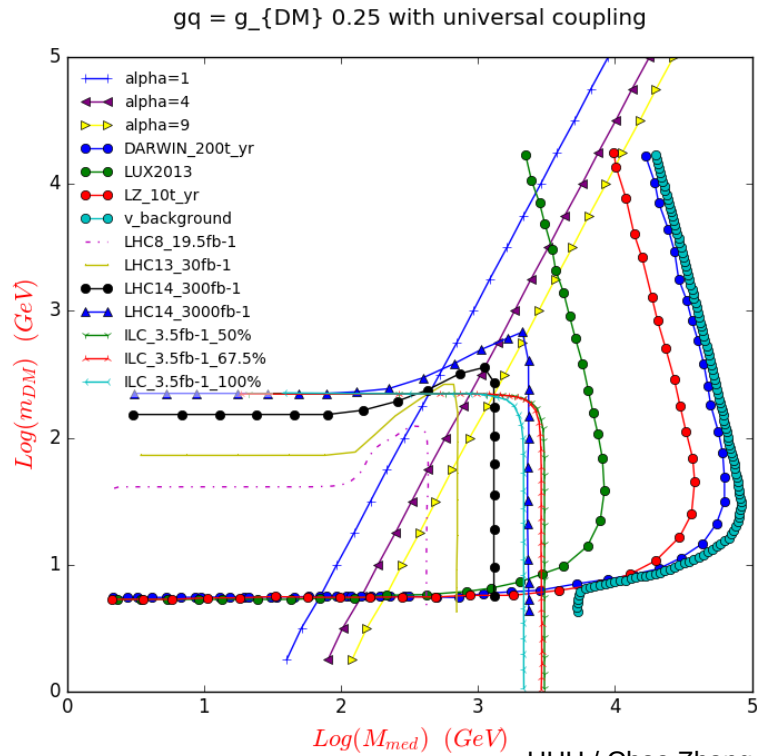
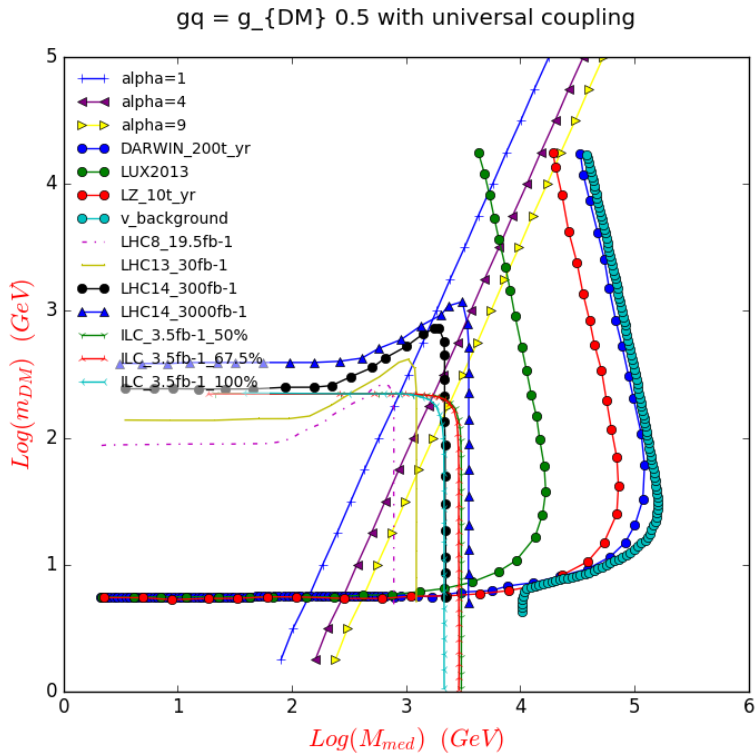
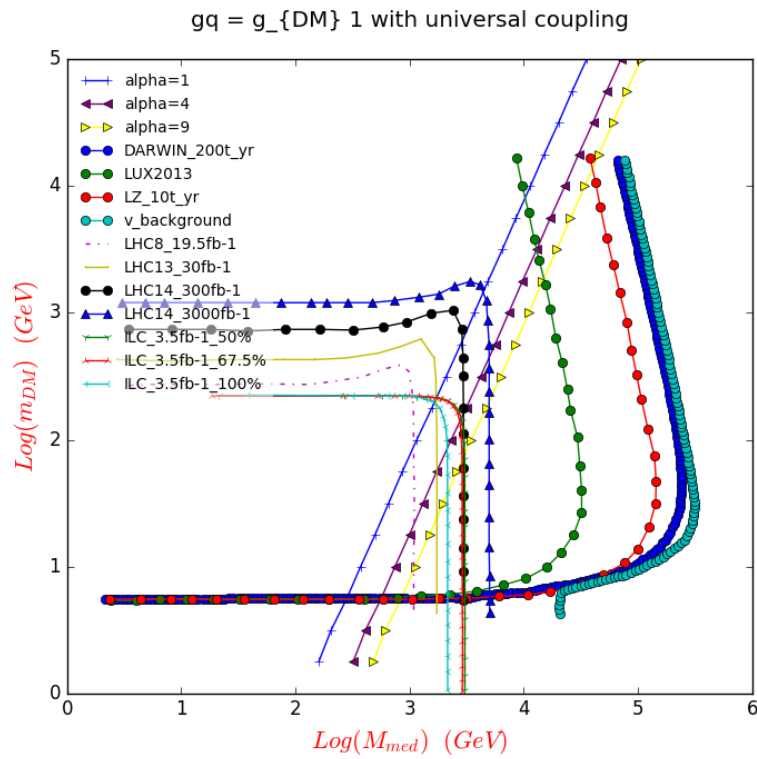
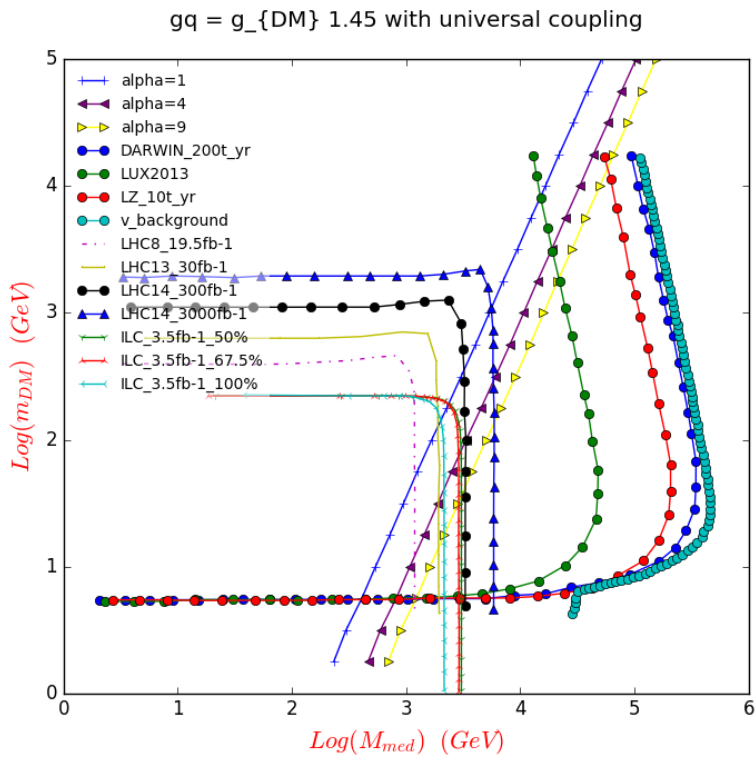


Collaboration with
Moritz Habermehl
List Jenny

2.3 case with only 1 dark sector particle with static solution to the Boltzmann equation



Collabroration with
Moritz Habermehl
List Jenny
Data from Y. Chae
arXiv:1211.4008v1

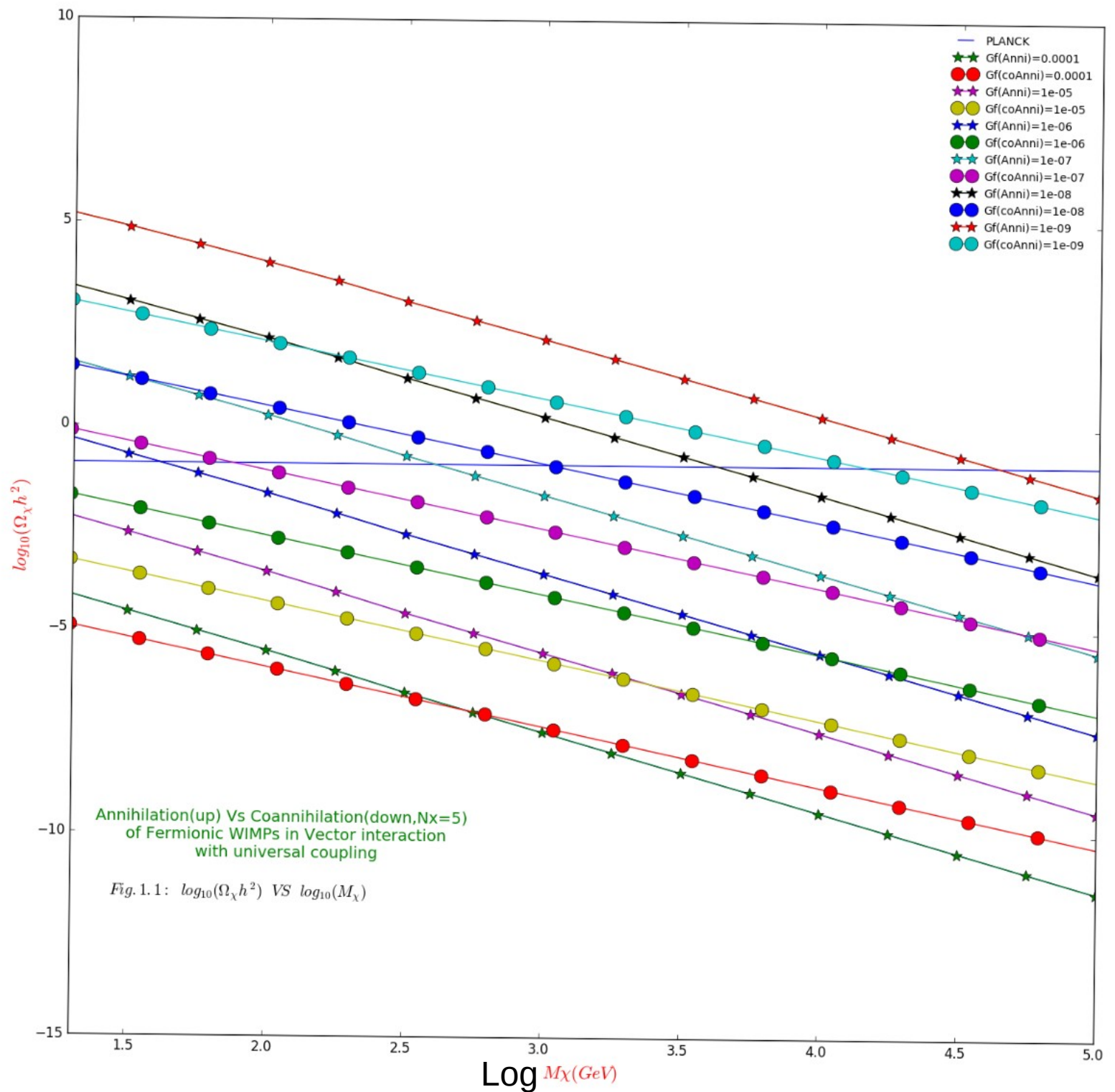


Collaboration with
Moritz Habermehl
List Jenny
Data from Y. Chae
arXiv:1211.4008v1

Co-annihilation
example

2.4 case of
multiple dark
sector
particles

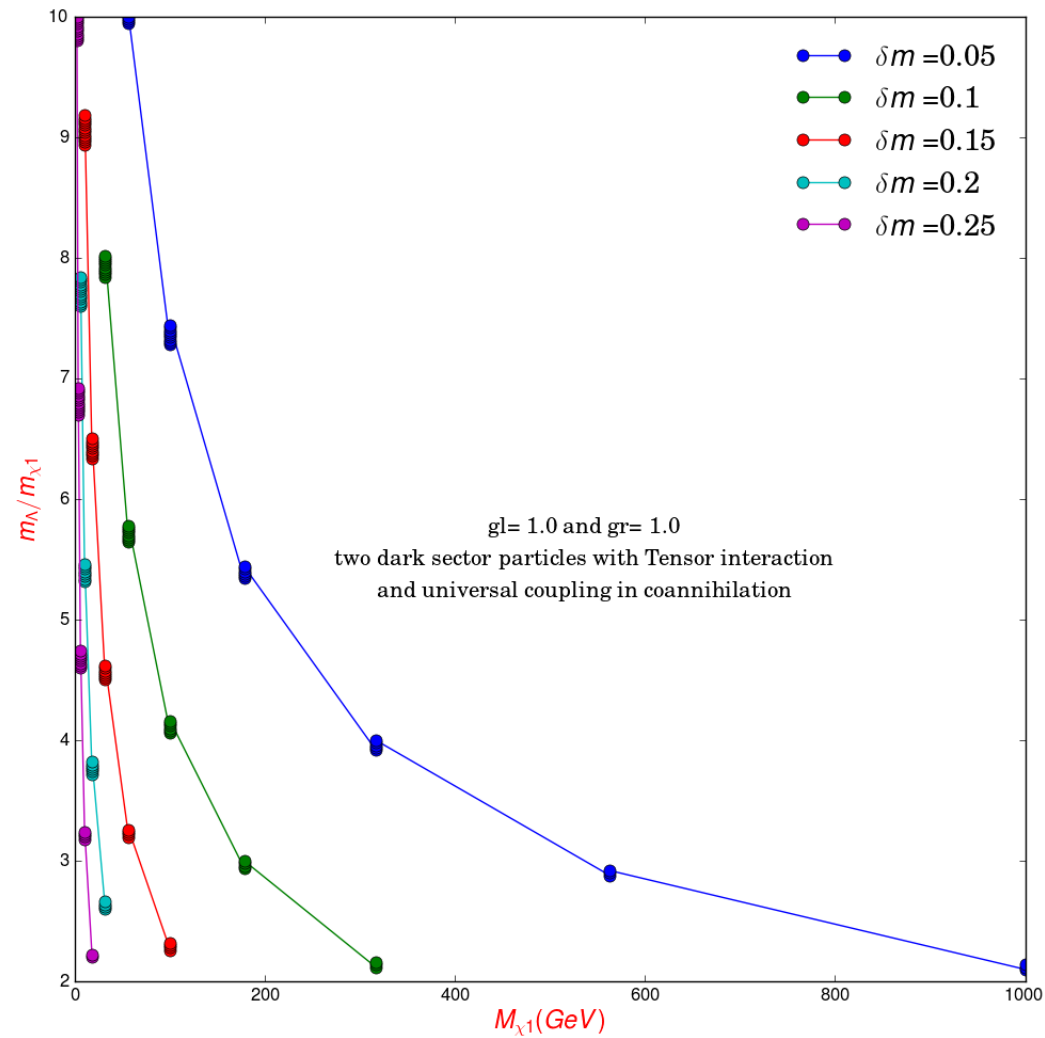
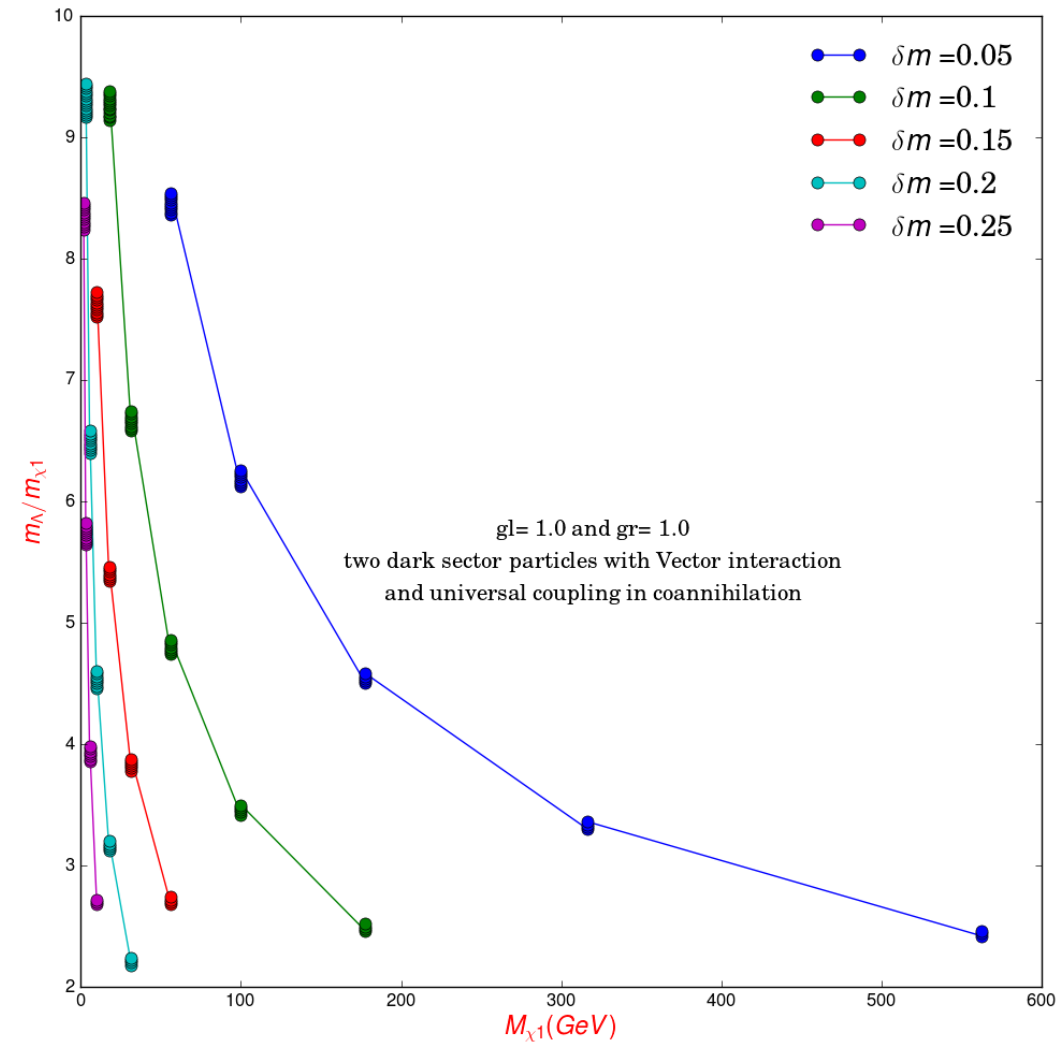
with static
solution to
the
Boltzmann
equation



2.5. case of 2 dark sector particles with static solution to the Boltzmann equation

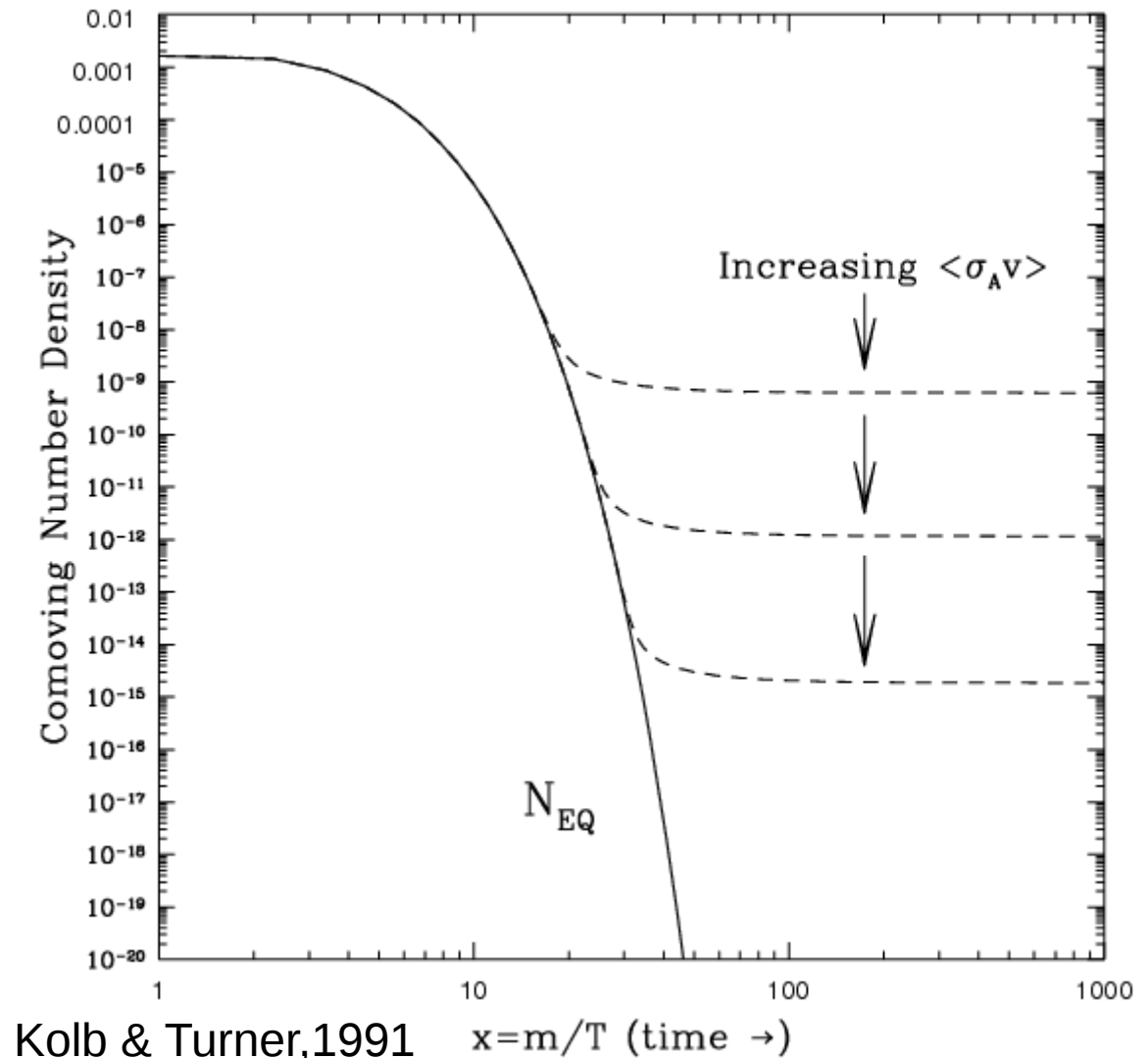
$$\delta m \text{ VS } m_\chi$$

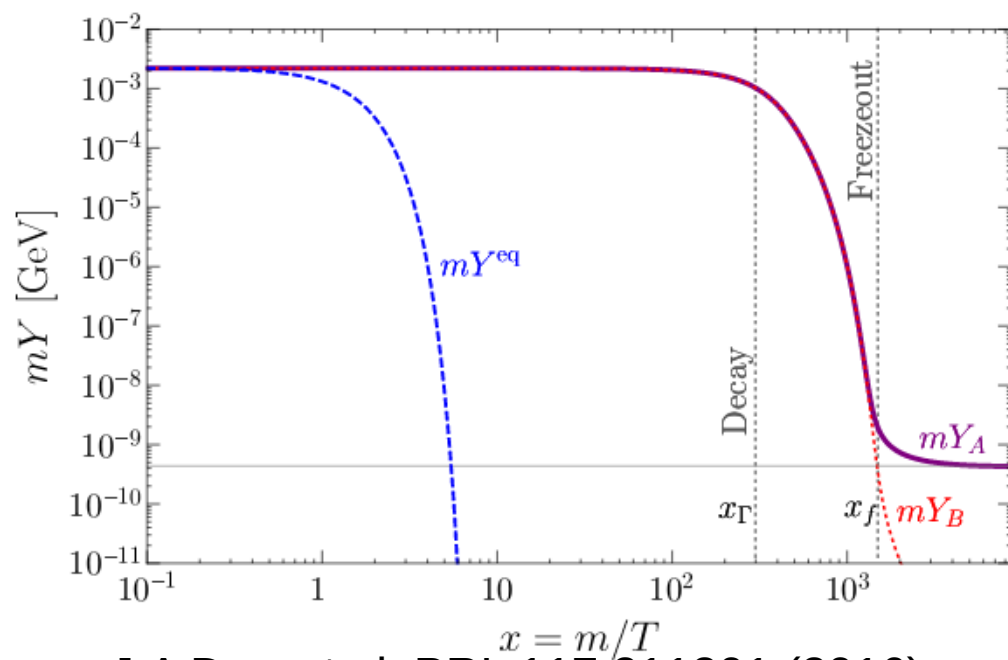
Required by the correct relic density.



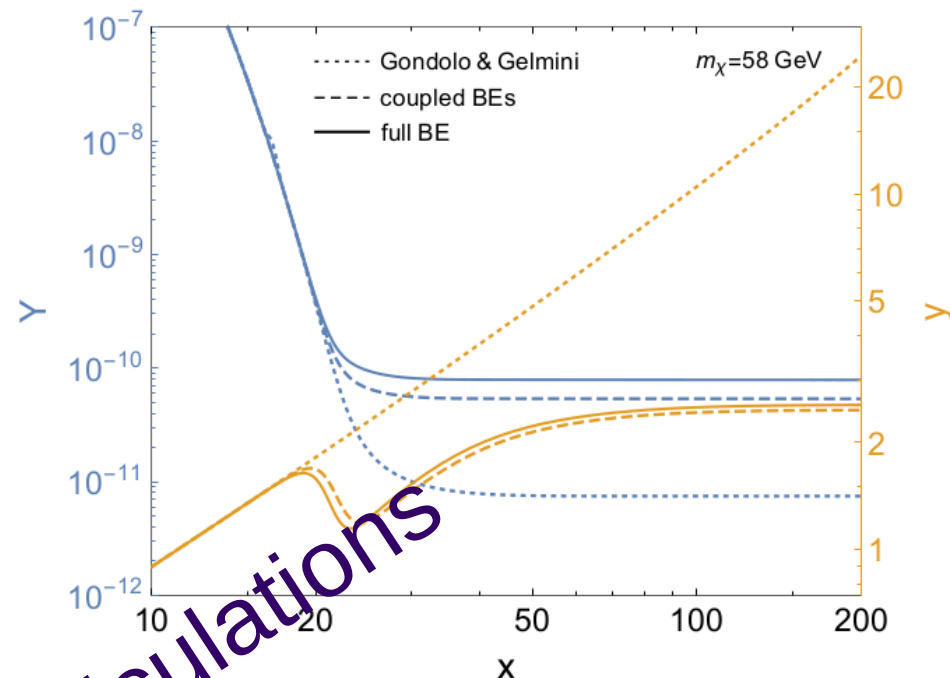
What if more particles in the dark sector? WIMP miracle still available?

Consider mathematical calculation of WIMP miracle including previously neglected effects.

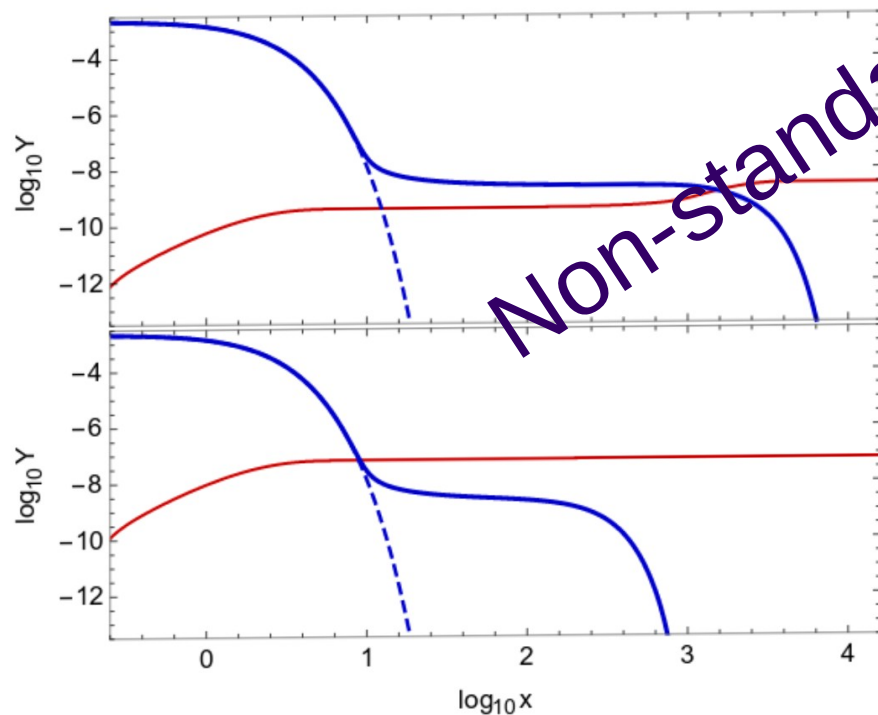




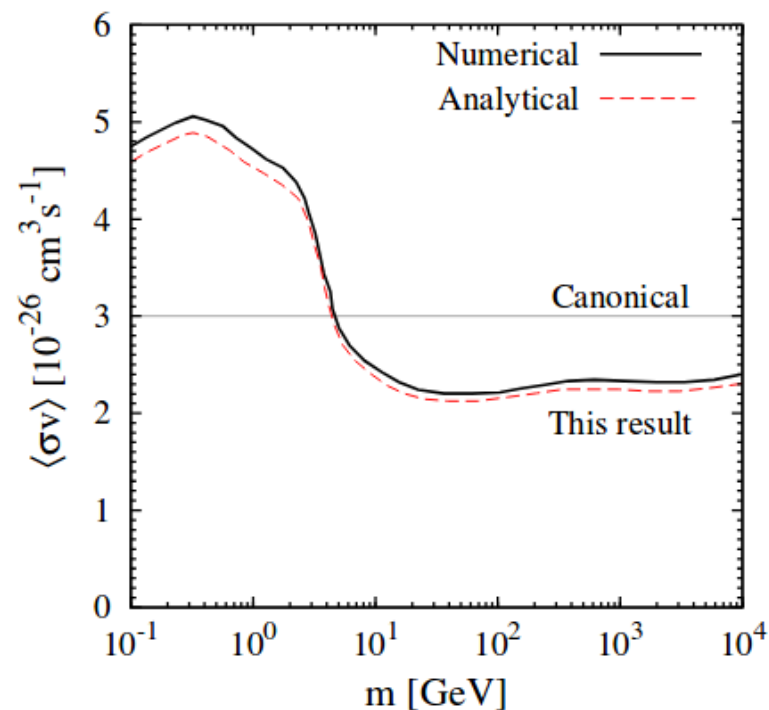
J.A Dror et al. PRL 117,211801 (2016)



G. Binder, T. Bringmann 1706.07433



M. Heikinheimo et al. 1604.02401



Gary Steigman et al. 1204.3622

for $k=0$

$$\frac{dn_\chi}{dt} + 3 H n_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{eq}^2)$$

$$n \equiv \frac{g}{2\pi^2} T^3 \int_0^\infty d\varepsilon \frac{\varepsilon^2}{\exp(\sqrt{x^2 + \varepsilon^2}) \pm 1}$$

$$\varepsilon = \frac{p}{T}$$

$$s = \frac{2}{45} g_{*s} T^3 \pi^2$$

- typical static solutions (2 methods)

1: $Y=n/s$

$$\frac{dY}{dx} = \frac{-x \langle \sigma v \rangle s}{H(m)} (Y^2 - Y_{eq}^2)$$

$$x_f = \ln \frac{0.038 g m_{pl} m_x \langle \sigma v \rangle}{g_*^{1/2} x_f^{1/2}}$$

Kolb & Turner, 1990

Kim Griest, 1991

$$\Omega h^2 = 1.07 \cdot 10^9 \frac{x_f}{g_*^{1/2} m_{pl} (GeV) \langle a + 3b/x_f \rangle}$$

2: $Y=n/T^3$

$$\frac{dY}{dx} = -T^3 \langle \sigma v \rangle (Y^2 - Y_{eq}^2) \frac{1}{Hx}$$

$$x_f \approx \ln \left[\sqrt{\frac{45}{4\pi^5}} \frac{g}{\sqrt{g_*}} \frac{m}{\sqrt{8\pi G}} \langle \sigma v \rangle_0 \right] - \left(n + \frac{1}{2} \right) \ln^2 [\dots]$$

Dodelson

$$\Omega_\chi = \sqrt{\frac{4\pi G g_*(m) \pi^3}{45}} \frac{x_f T_0^3}{30 \langle \sigma v \rangle \rho_{cr}}$$

3. Dynamic solution of the Boltzmann Equation

- the standard solution of the Boltzmann Equation of dark matter particle is based on numbers of approximations and assumptions.
- typical simplifications:

Constant g_{*S} (or small deviative of g^*s)

$Y = Y_{eq}$ for all the small x . ($x=m/T$)

Neglect Y or Y_{eq} during the calculation.

Using only s wave.

Small x not been considered.

No connection between $T \ll m$ and $T \gg m$.

$Y = \lambda y$ with $\lambda = \frac{s(m)\langle\sigma\nu\rangle}{H(m)x^{2+n}}$ as a constant.

Successful
But static.

and the others...

for $k=0$

$$\frac{dn_\chi}{dt} + 3 H n_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{eq}^2)$$

$$n \equiv \frac{g}{2\pi^2} T^3 \int_0^\infty d\varepsilon \frac{\varepsilon^2}{\exp(\sqrt{x^2 + \varepsilon^2}) \pm 1}$$

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Dodelson

$$\Omega_\chi = \sqrt{\frac{4\pi G g_*(m) \pi^3}{45}} \frac{x_f T_0^3}{30 \langle \sigma v \rangle \rho_{cr}}$$

Standard calculation with constant g_{*S} :

$$t = \int \left(\frac{3 M_{\text{P}}^2}{8\pi \rho} \right)^{1/2} \frac{dR}{R}$$
$$= - \int \left(\frac{45 M_{\text{P}}^2}{4\pi^3} \right)^{1/2} g_{\rho}^{-1/2} \left(1 + \frac{1}{3} \frac{d \ln g_{s1}}{d \ln T} \right) \frac{dT}{T^3}$$

$$t = \left(\frac{3 M_{\text{P}}^2}{32\pi \rho} \right)^{1/2} = 2.42 g_{\rho}^{-1/2} \left(\frac{T}{\text{MeV}} \right)^{-2} \text{ sec}$$

Subir Sarkar. arXiv:hep-ph/9602260v2

What if $g_{*S} = f_1(t) = f_2(T)$?

Remark:

“The value of g_{*S} decreases whenever the temperature of the universe drops below the mass of a particle species and it becomes non-relativistic.”

--- DANIEL D. BAUMANN, lecture “Thermal History” , damtp

What and how will g_{*S} change for an extended dark sector, ex: in the heavy mass scale?

3. Dynamic solution of the Boltzmann Equation

- with out any of the previous assumptions and approximations in the static solution.

(Reference here: Mark Srednicki. Richard Watkins, K.A,olive, Paolo Gondolo, Graciela Gelmini, and etc.)

- final expression with $g_{*s}=f_1(t)=f_2(T)$:

1. $Y=n/s$



$$\frac{dY}{dx} = \frac{s}{x^2} \frac{m}{\dot{T}} \langle \sigma v \rangle (Y^2 - Y_{eq}^2) = -\frac{s}{\dot{x}} \langle \sigma v \rangle (Y^2 - Y_{eq}^2)$$



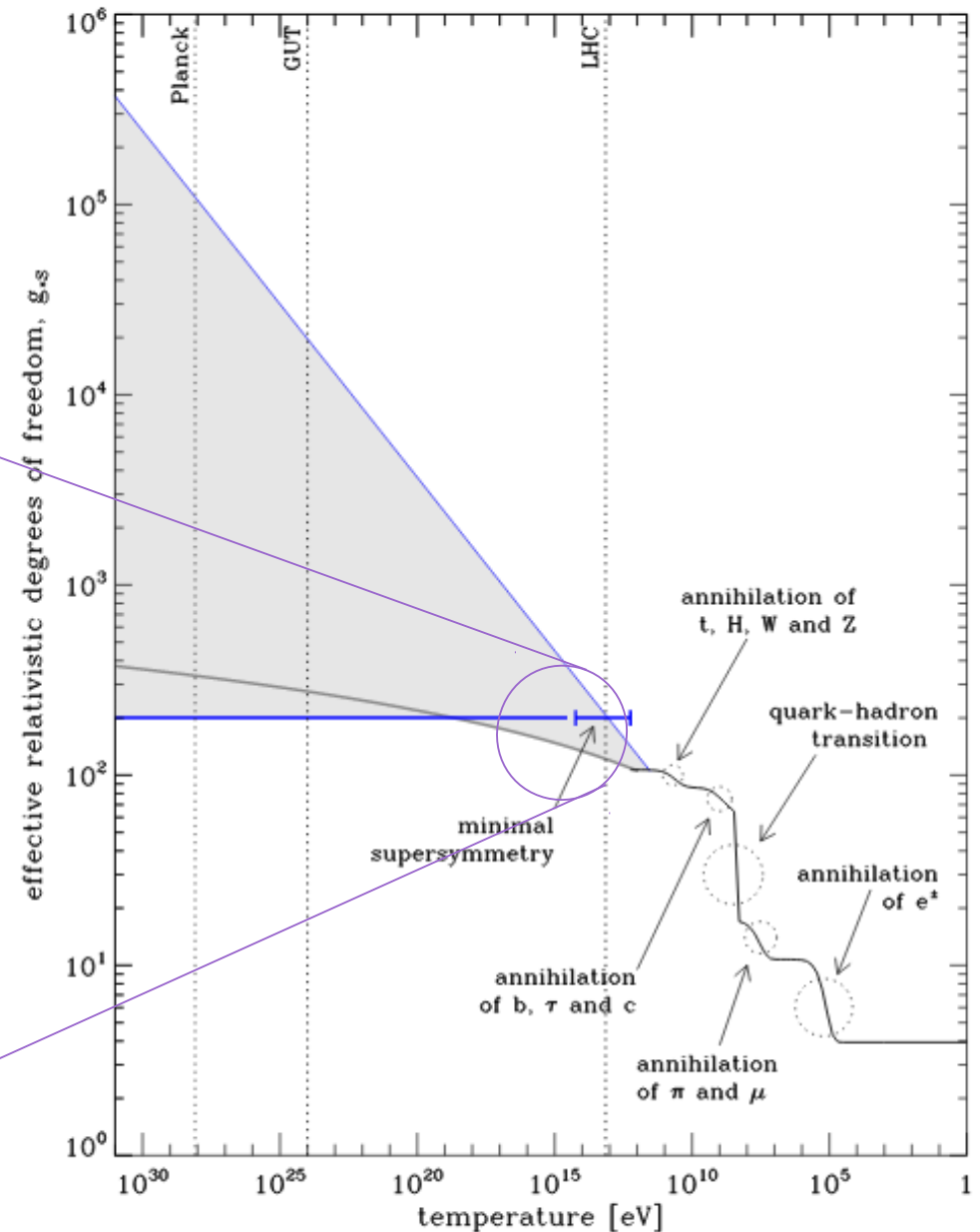
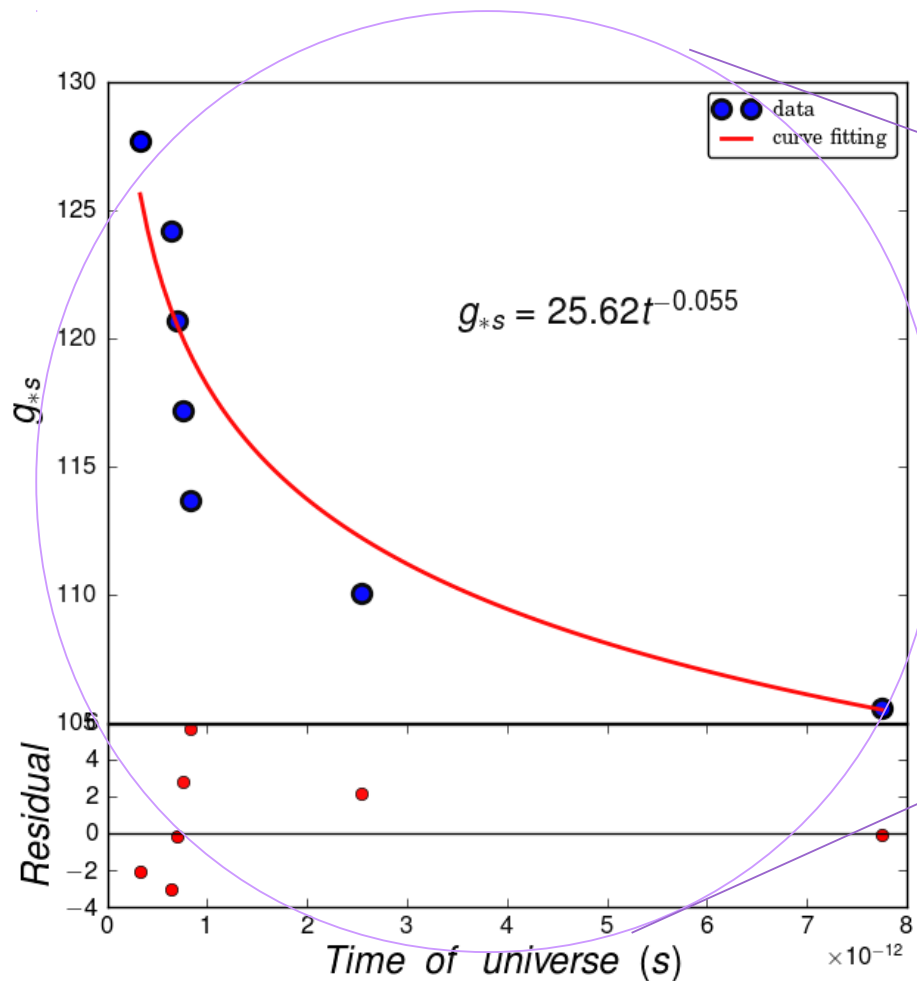
No improvement

2. $Y=n/T^3$

$$\frac{dY}{dx} = -\frac{1}{x} \frac{T}{\dot{T}} \left(Y \frac{\dot{g}_{*s}}{g_{*s}} - \langle \sigma v \rangle T^3 (Y^2 - Y_{eq}^2) \right)$$

The simplest fit by:

$$g_{*s} = c_1 T^{c_2} = c_3 t^{c_4}$$



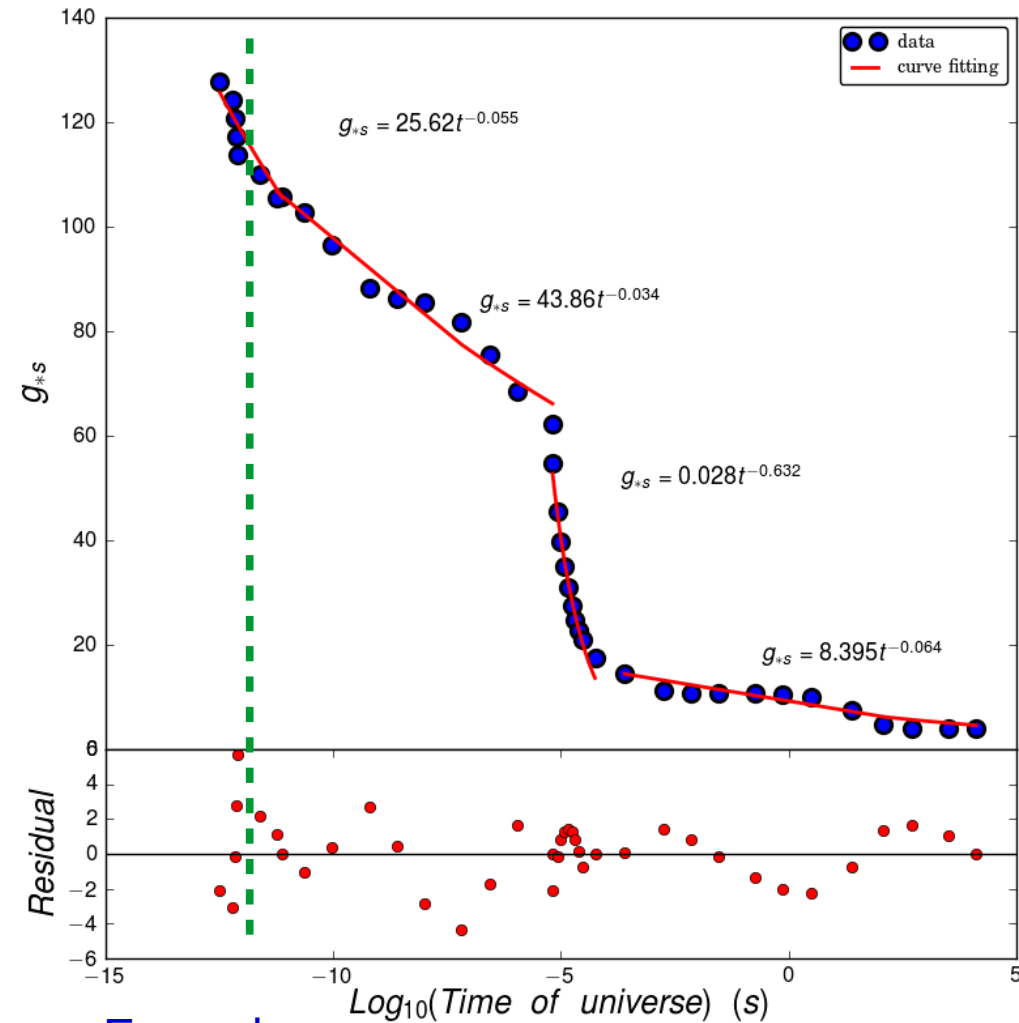
Example in this work:

$m_x(1:N_x) = (/300.,520.,540.,560.,580.,800./)$ GeV
 $g_x(1:N_x) = (/4,4,4,4,4,4/)$

Chas A. Egan, Charles H. Lineweaver.
 arXiv:0909.3983

26

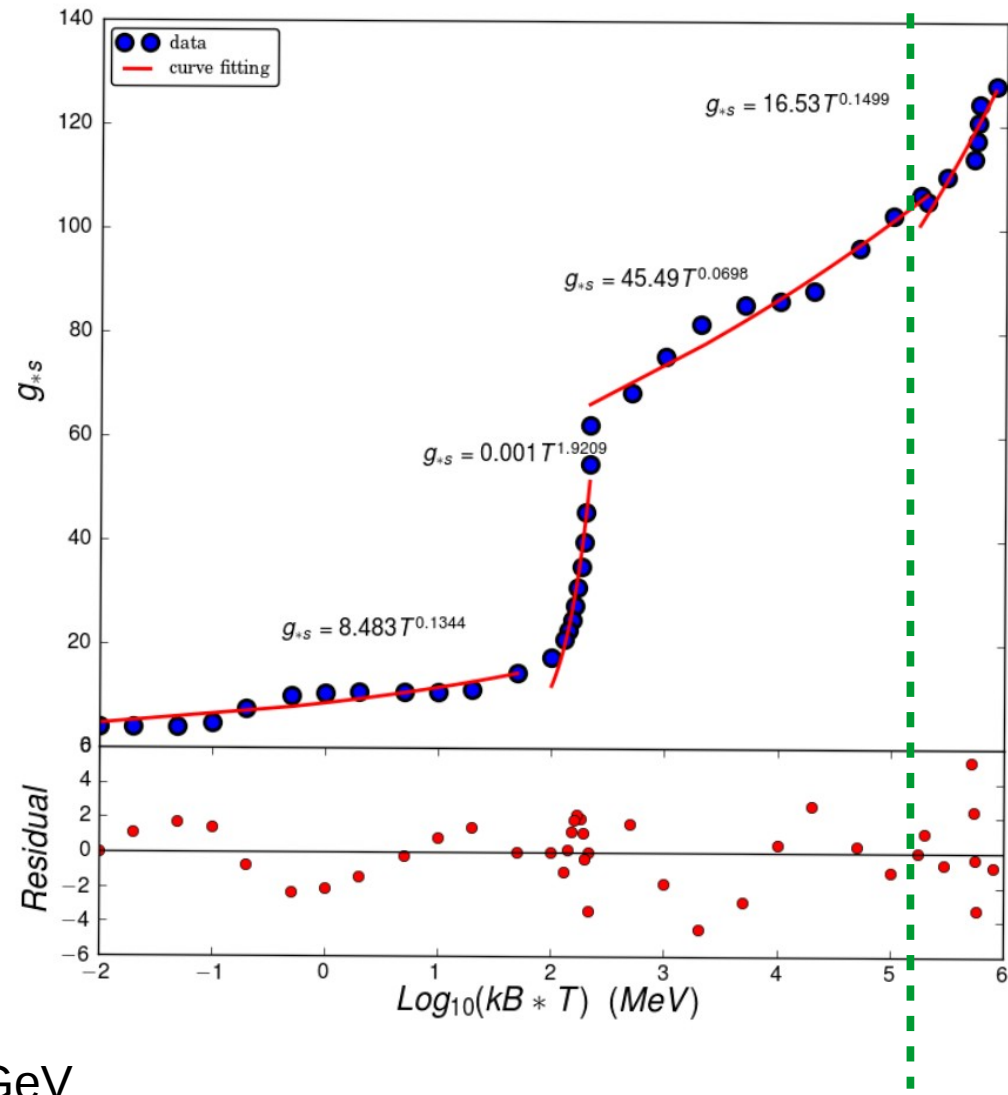
3.1 Fitting example of g_{*S}



Example:

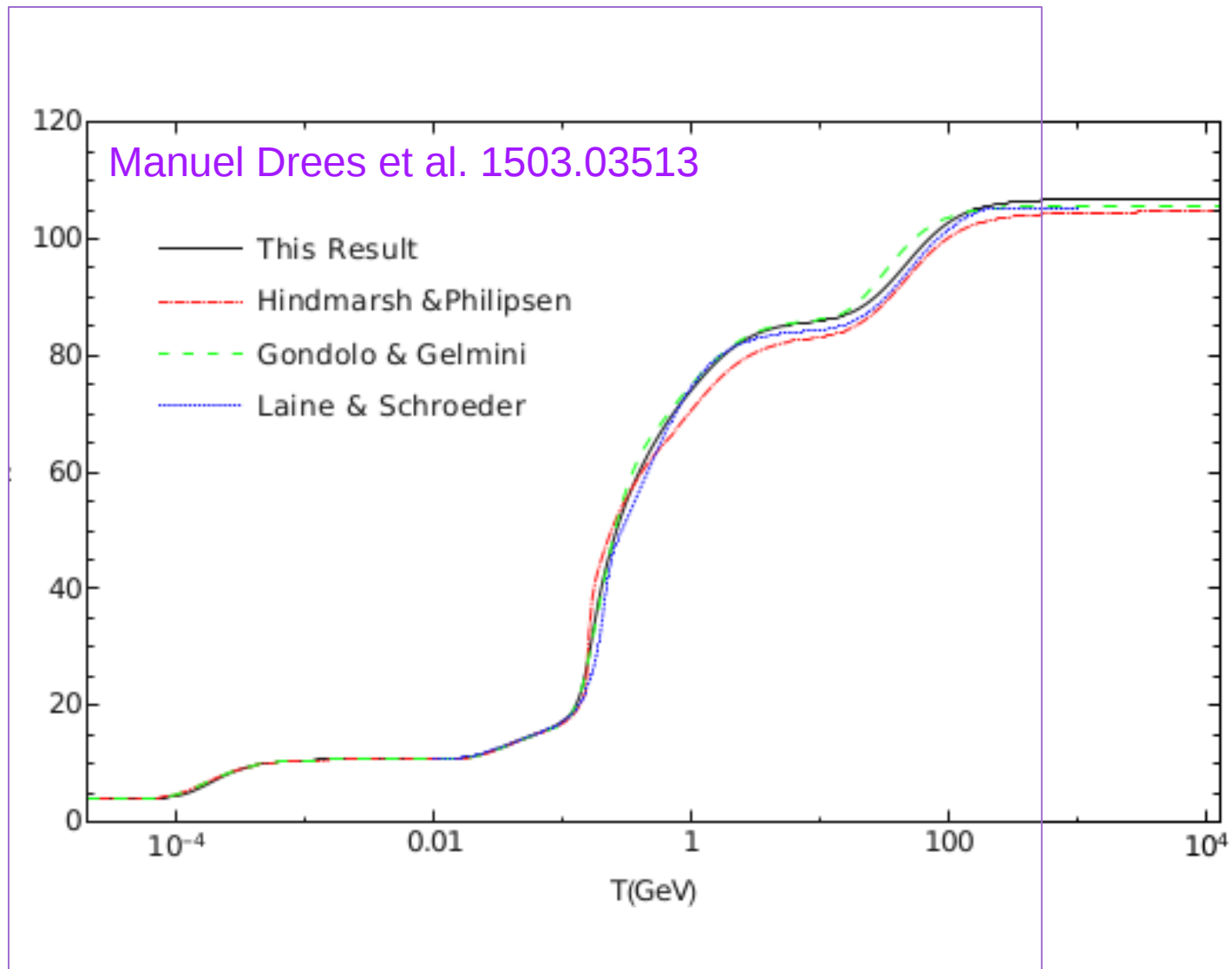
$m_x(1:N_x) = (/300.,520.,540.,560.,580.,800./) \text{ GeV}$

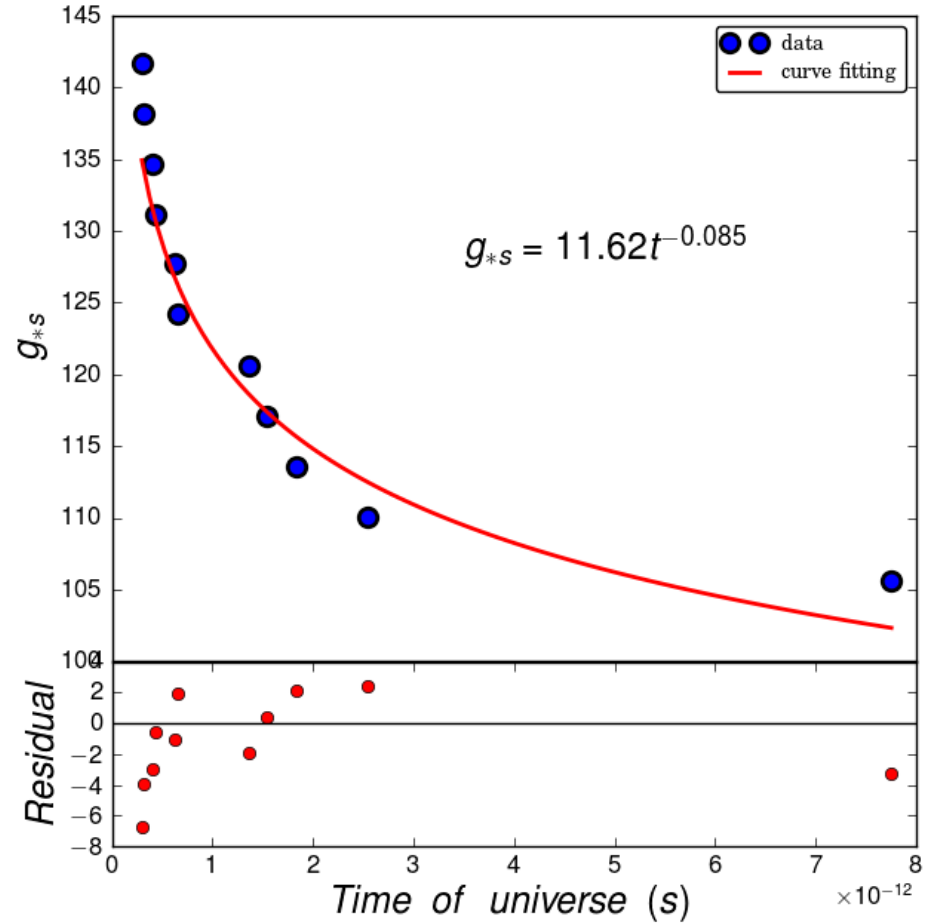
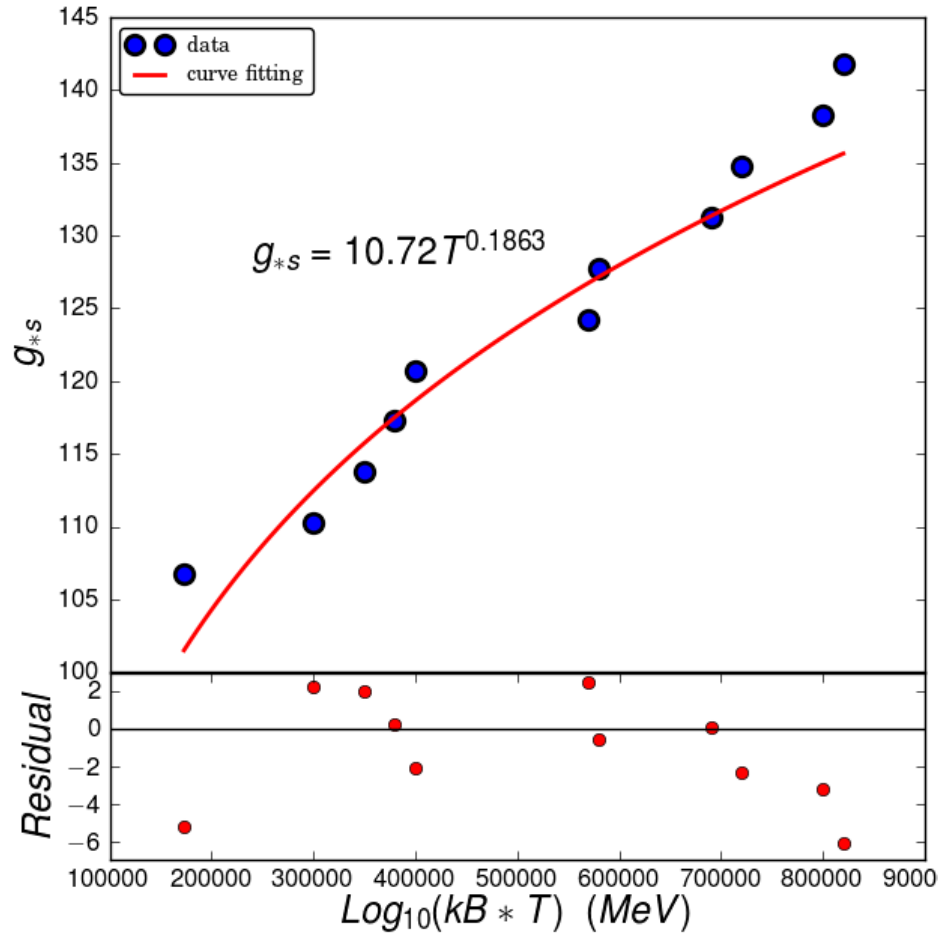
$g_x(1:N_x) = (/4,4,4,4,4,4/)$



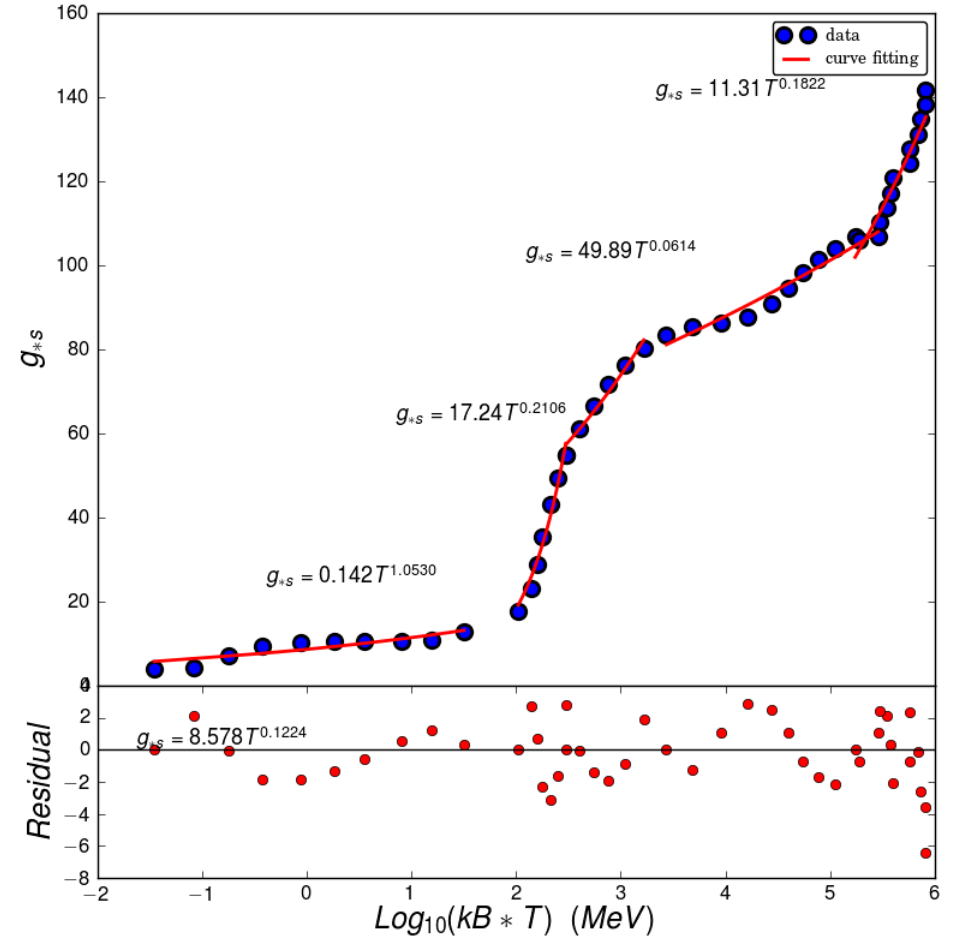
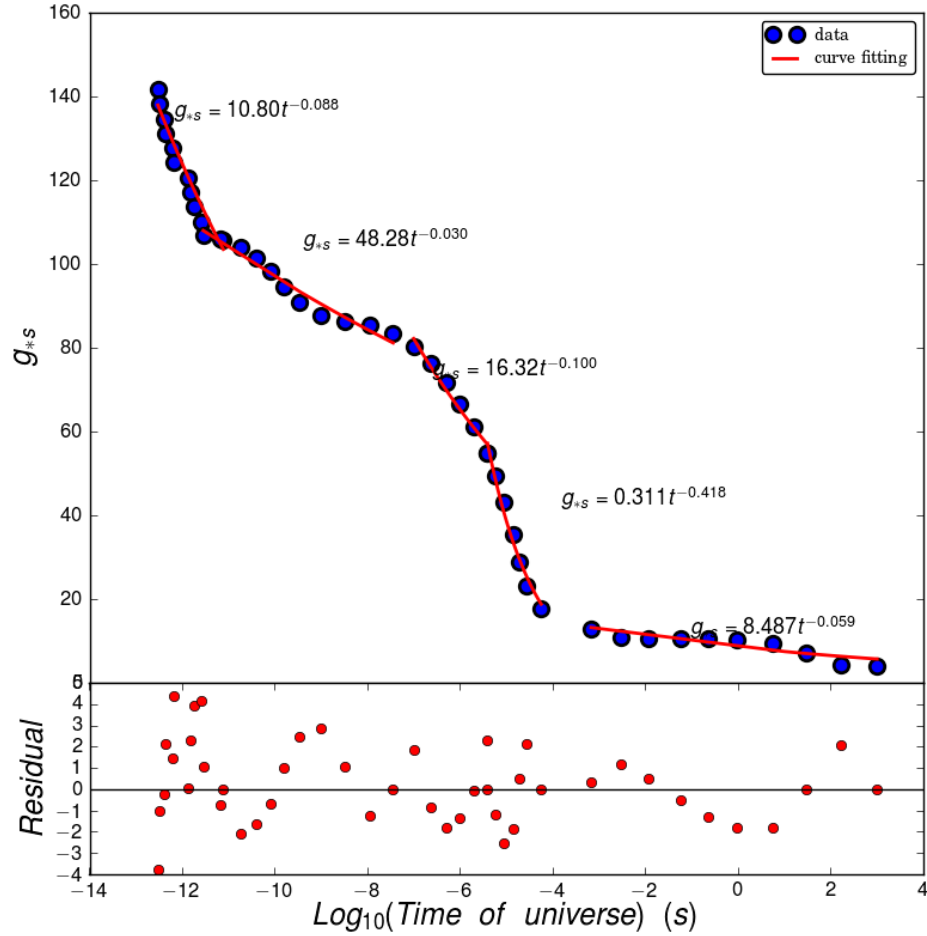
Data below 200 GeV from
Lars Husdal, arXiv:1609.04979₂₇

The Standard Model part

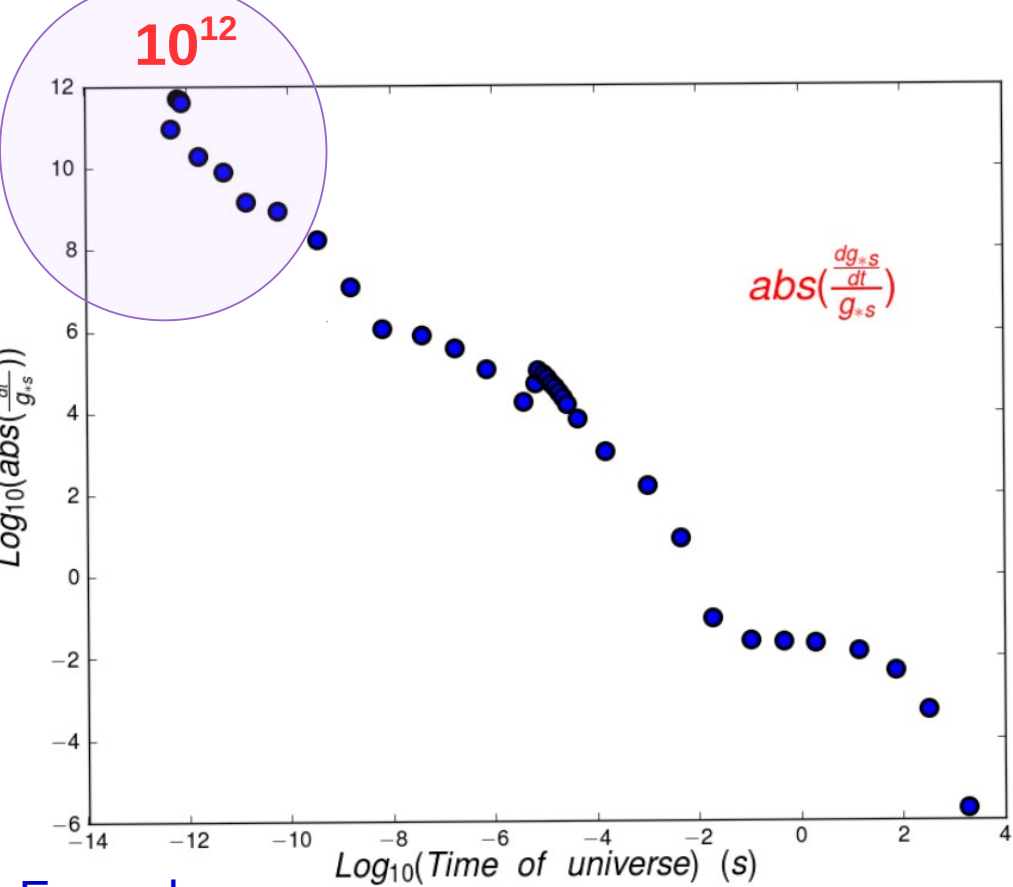




Example with 10 Dark particles



Data of the SM part from:
Manuel Drees et al. arXiv: 1503.03513



The deviation of g_{*S} over time.

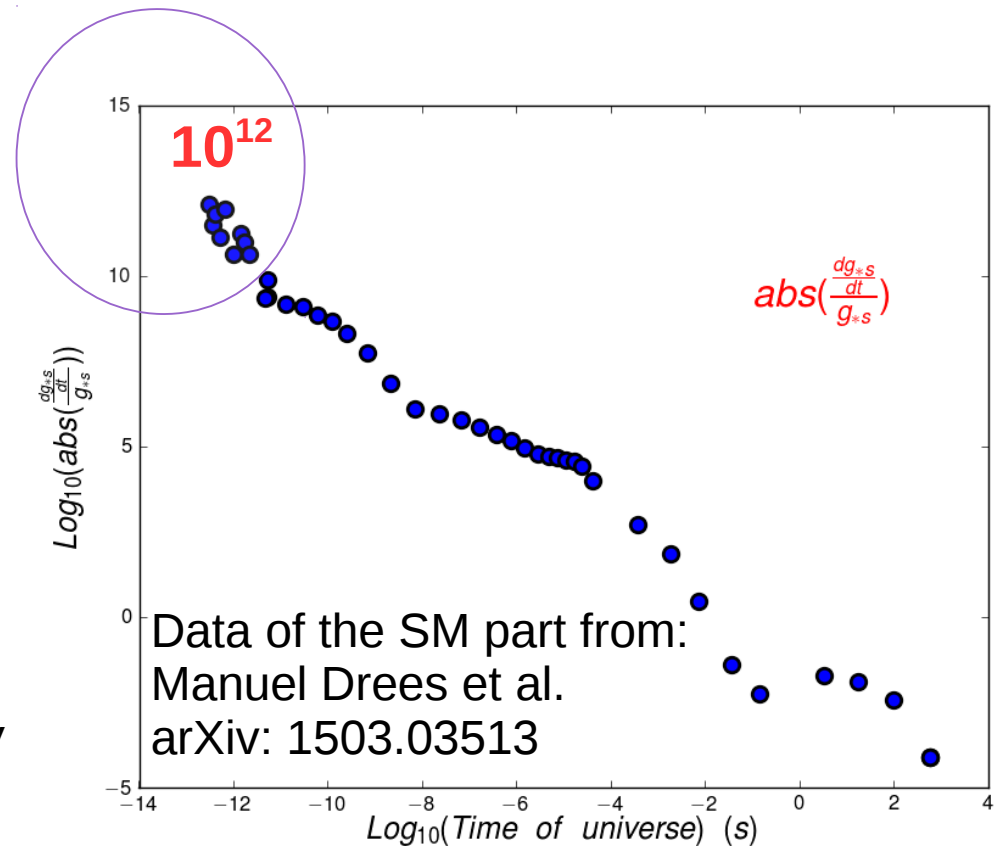
Example:

$m_x(1:Nx) = (/300., 520., 540., 560., 580., 800./)$
GeV

$gx(1:Nx) = (/4, 4, 4, 4, 4, 4/)$

$m_x(1:Nx) = (/300., 350., 380., 400., 570., 580., 690., 720., 800., 820./)$ GeV

$gx(1:Nx) = (/4, 4, 4, 4, 4, 4, 4, 4, 4, 4/)$



3.2 Time scale and Hubble Rate

$$t = \frac{1}{H} \frac{1}{2 - \frac{c_2}{6+2c_2}} = \frac{1}{2 - \frac{c_2}{6+2c_2}} \sqrt{\frac{45}{4\pi^3 G}} \frac{T^{-2}}{\sqrt{g_{*s}}} = \frac{1}{2 - \frac{c_2}{6+2c_2}} \frac{4.84}{\sqrt{g_{*s}}} \left(\frac{T}{\text{MeV}}\right)^{-2}$$

$$H > \frac{1}{2t}$$

$C_2 > 0$, the expansion rate is larger due to variable g_{*s} .



more difficult or even impossible
to stay in equilibrium.

3.3 Dynamic solution for $Y=n/T^3$

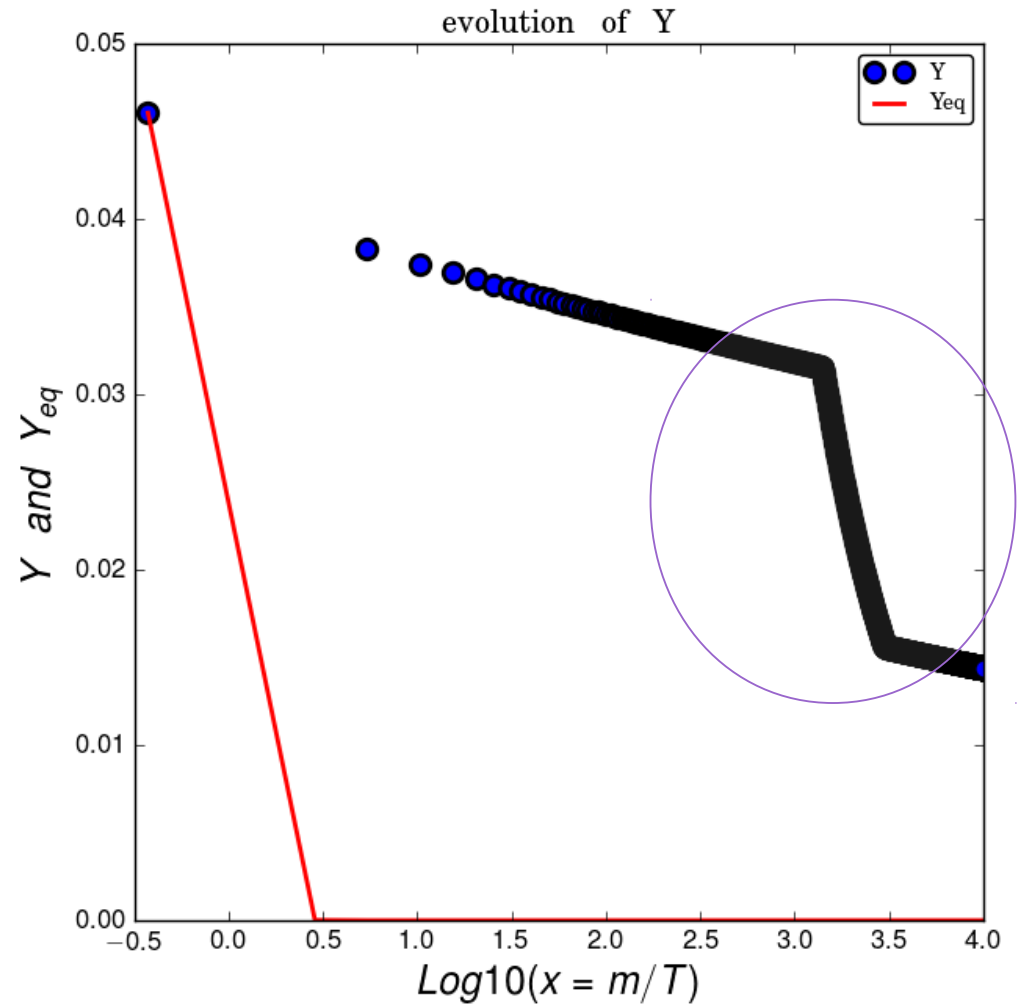
starting point :

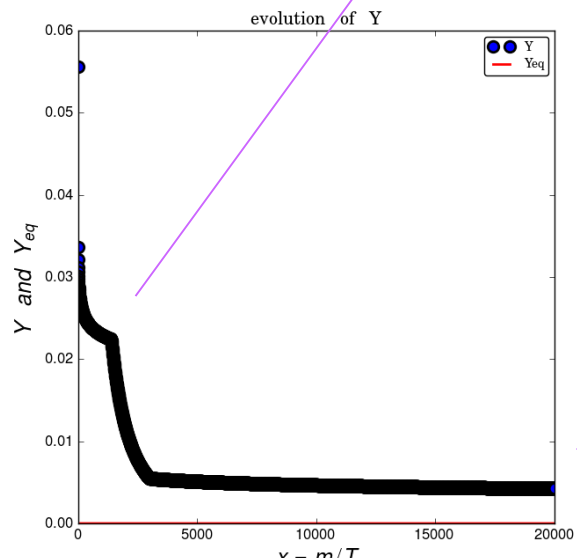
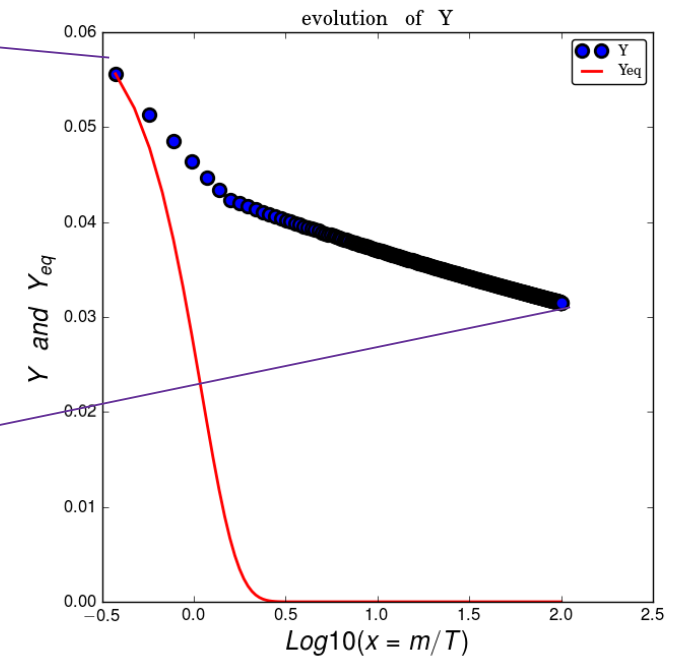
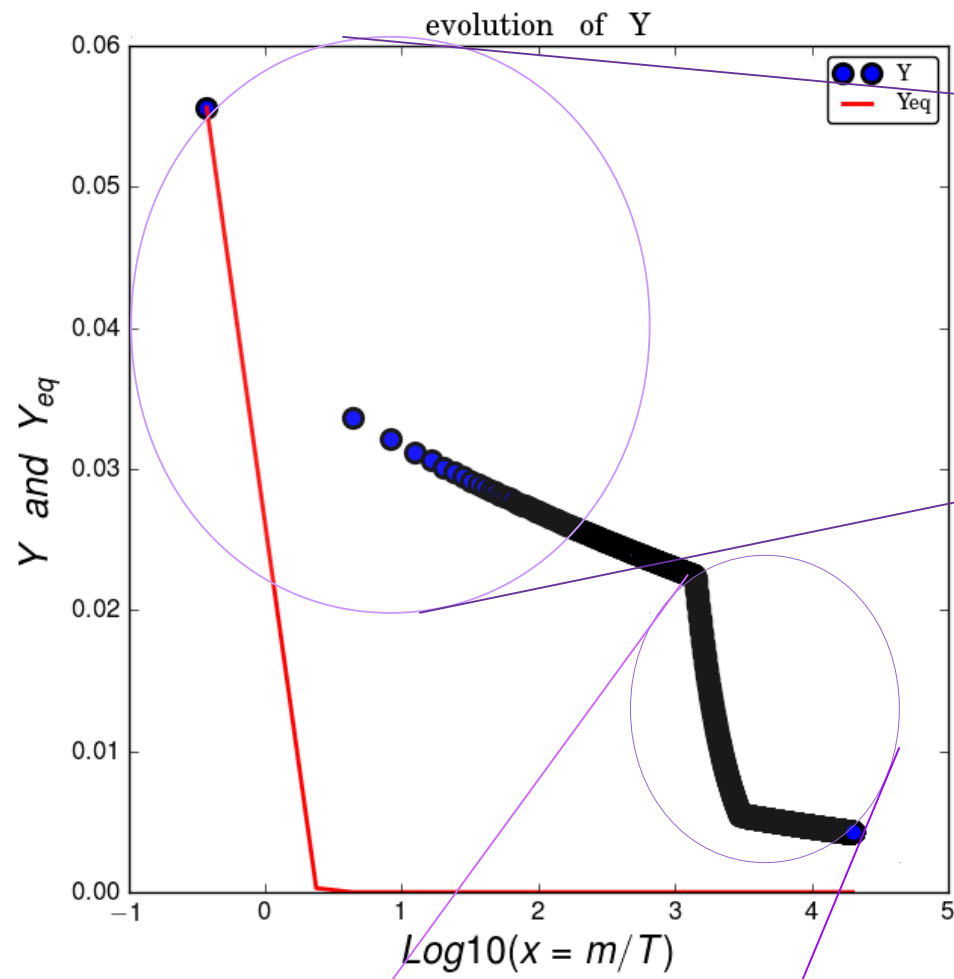
$$Y(x(1)) = Y_{\text{eq}}(x(1))$$

with

$$n = \int f(\mathbf{k}) d^3\mathbf{k} = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2} E dE}{e^{\frac{E-\mu}{T}} \pm 1}$$

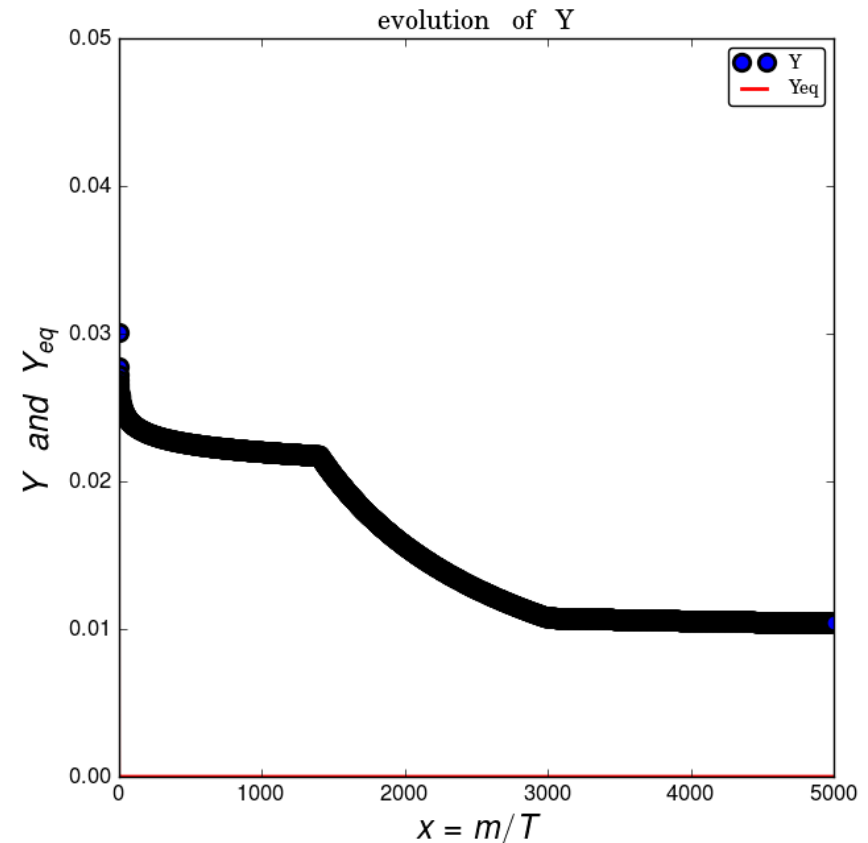
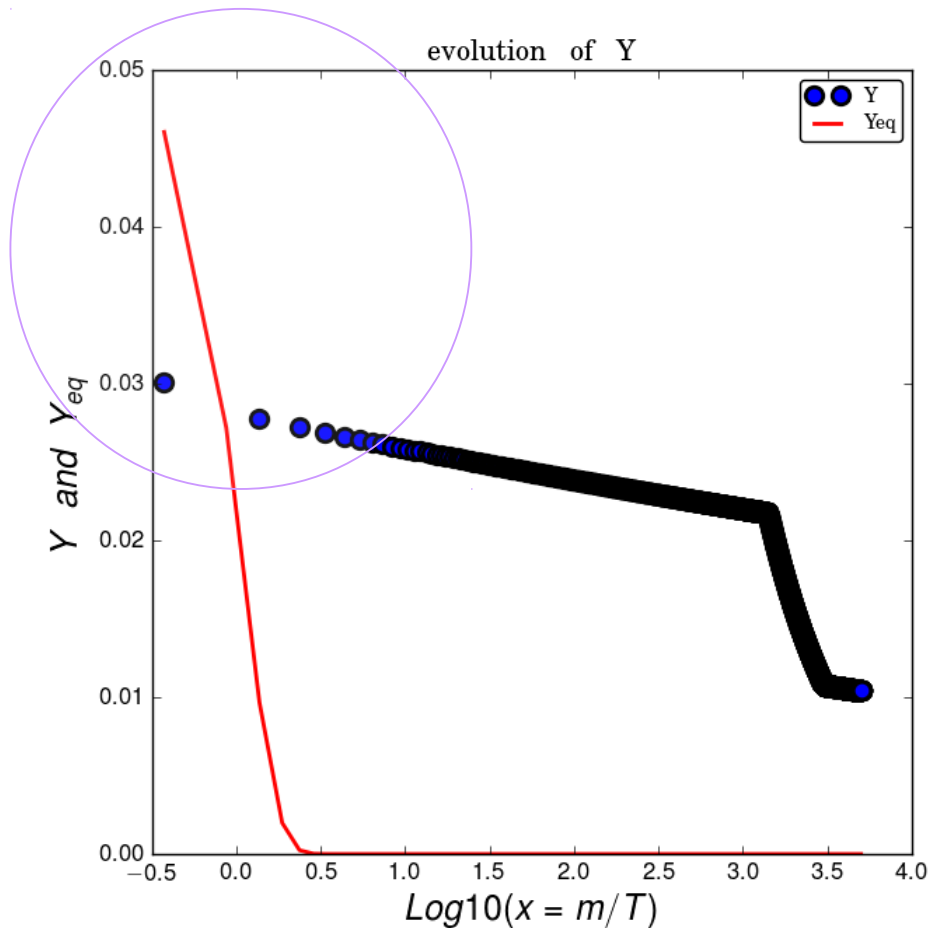
no zeta function.





Another example

it depends on the starting point.



1. The freeze-out given by $Y=n/T^3$ is not standard.
2. the starting part has been slowed down by the g^* s term.
3. It reveals the two-step freeze-out mechanism.

- more than required relic density of dark matter for small x ?

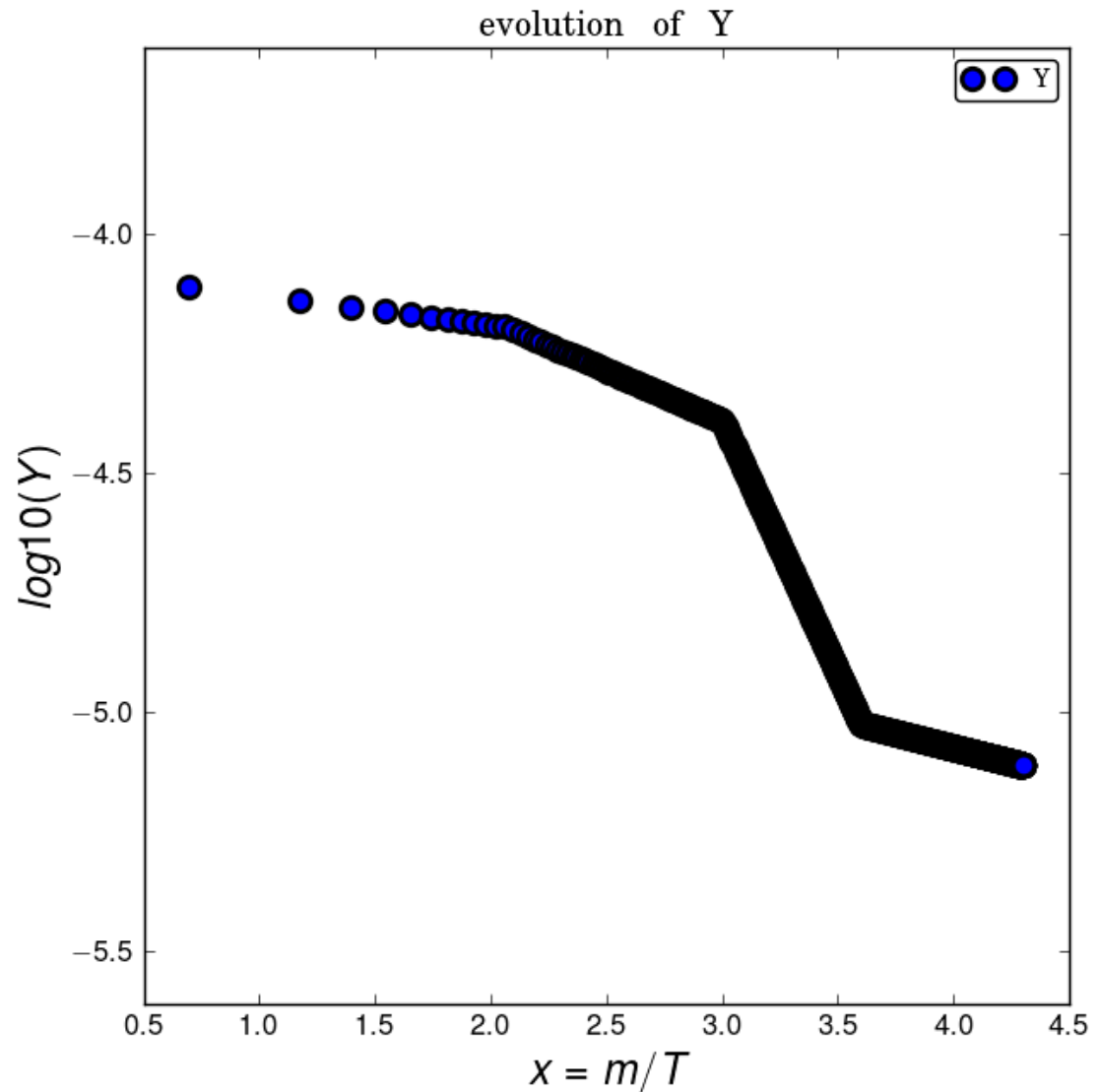
$m_x(1:N_x) =$
 (/300.,350.,380.,400.,570.,580.,690.,720.,800.,820./) GeV

$g_x(1:N_x) =$
 (/4,4,4,4,4,4,4,4,4,4/)

With $x(1) = 4$

$$\langle \sigma v \rangle = 1.0 \text{e-}24 \text{ cm}^2$$

$$\Omega h^2 = 1.4 \text{e-}6 \ll 0.1198$$



3.4 Preliminary Results

1. for small x , the g_{*s} term has slowed down the decreasing speed of Y a lot, it always produces much higher relic density.
for x too small ($x < 1$):

If higher annihilation cross section, relic density too low.

If lower annihilation cross section, enormous relic density...

2. for large x , the relic density value 0.1198 can be got in the two-step mechanism.

Example:

For 10 particle in [2000, 2180] GeV, if the decoupling starts at $x=4$, then $\langle \sigma v \rangle \approx 3.0 \times 10^{-26} \text{ cm}^2$ will still give the relic density.

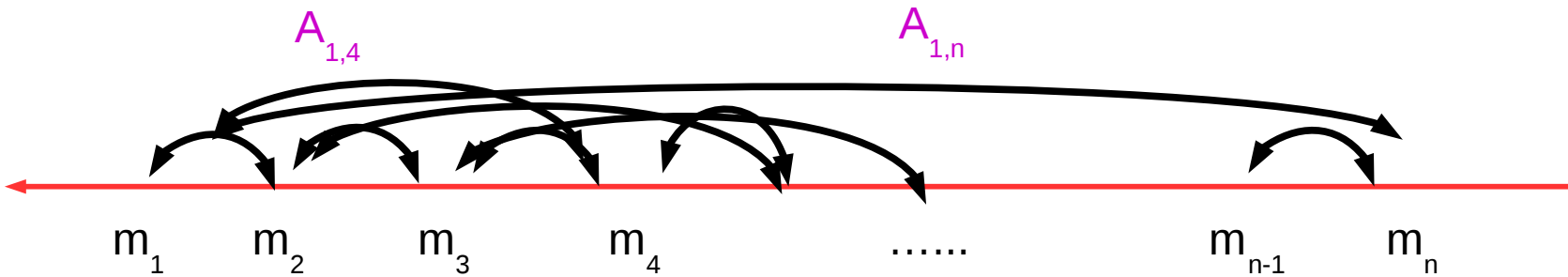
Scanning work is going on.

Conclusion and discussion

- 1. work on eft shows strong constraints. powerful! Scanning tool will be open soon.
- 2. the targets with slow dark matter velocity are invisible for some interactions in indirect detection!
- 3. the variable g_{*s} shows a two-step freeze-out mechanism. Y does not decrease as fast as expected, but the later crash still make it possible to give the required relic density. (g_{*s} is more important than expected, should no more be considered as a constant.)
- 4. the small range of x should be taken into account, the $Y=Y_{eq}$ assumption need to be used carefully for a dark sector with multiple massive particles.
- 5. the n dimension scanning is going on. (worthy or not?)
- 6. sommerfeld enhancement, decay and co-decay, self-interaction, etc. still a lot to do...

Thank you very much for your attention!

Particle coupling matrix



		SM	m_1	m_2	...	m_{n-1}	m_n
	SM	Known					
	m_1		$A_{1,1}$	$A_{1,2}$...	$A_{1,n-1}$	$A_{1,n}$
	m_2		$A_{2,1}$	$A_{2,2}$...	$A_{2,n-1}$	$A_{2,n}$

	m_{n-1}		$A_{n-1,1}$	$A_{n-1,2}$...	$A_{n-1,n-1}$	$A_{n-1,n}$
	m_n		$A_{n,1}$	$A_{n,2}$...	$A_{n,n-1}$	$A_{n,n}$

- $\langle \sigma v \rangle$ is not **constant**.

- $\langle \sigma v \rangle_{non Rel} = a' + b' v^2 + c' v^4 = a' + 6b'/x + 60c'/x^2$

the Kim Griest method (1991):

$$\langle (a' + b' v^2 + c' v^4) v_2 \rangle \simeq (1 - z^2)^{\frac{1}{2}} \times \left\{ \begin{aligned} & \left\{ a' \left(1 + \frac{3z^2}{4x(1-z^2)} - \frac{15z^4}{32x^2(1-z^2)^2} + \frac{105z^6}{128x^3(1-z^2)^3} \right) \right. \\ & + \frac{6b'}{x} \left(1 + \frac{5z^2}{4x(1-z^2)} - \frac{35z^4}{32x^2(1-z^2)^2} + \frac{315z^6}{128x^3(1-z^2)^3} \right) \\ & \left. + \frac{60c'}{x^2} \left(1 + \frac{7z^2}{4x(1-z^2)} - \frac{63z^4}{32x^2(1-z^2)^2} + \frac{693z^6}{128x^3(1-z^2)^3} \right) \right\} \end{aligned} \right\}$$

Small terms matter a lot for small x
and large z . ($z = m_2/m_1 < 1$)

The small terms are more important
during the decoupling than today.

$$\langle E_K \rangle = \frac{3k_B T}{2} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$

$$x = \frac{mc^2}{k_B T} \quad v = c \sqrt{1 - \frac{1}{\left(1 + \frac{1}{x}\right)^2}}$$