The Sommerfeld Effect at Finite Temperature

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DESY Workshop

September 27, 2017

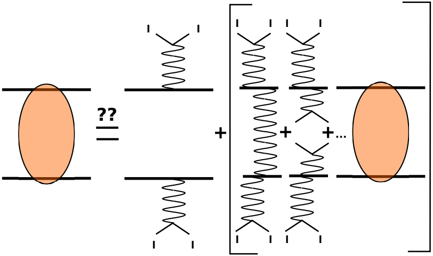
elusives
neutrinos, dark matter & dark energy physics

GK1493
DFG

$[q,p]=i\hbar$

Sommerfeld enhanced annihilation at the freeze-out.

$$\mathscr{L} \supset g\bar{\chi}\gamma^{\mu}\chi A_{\mu} + g_l \bar{l}\gamma^{\mu} l A_{\mu}$$



Impact of a hot and dense plasma environment on DM long-range self-interactions

- Conceptional question

  No formal description available in the community beyond equilibrium linear response theory estimates

- Refinement of relic abundance prediction

  Planck precision era:

  $\Omega_{\chi} h^2 = 0.1198 \pm 0.0015$ **(!)**

Thermal field theory in a nutshell

Two-point correlation function at finite temperature:

$$\begin{aligned} G_O(x,y) &\equiv \langle \hat{\rho}\, T[O(x)O^\dagger(y)] \rangle \\ &= \langle T_{\mathscr{C}}[O(x)O^\dagger(y)] \rangle \\ &= \begin{pmatrix} G_O^{++}(x,y) & G_O^{+-}(x,y) \\ G_O^{-+}(x,y) & G_O^{--}(x,y) \end{pmatrix} \end{aligned}$$

$\langle \ldots \rangle$ denotes sum over all particle states weighted by density matrix $\hat{\rho}$.

- Closed-Time-Path formalism: QFT of non-equilibrium states.
- Flattening of time contour at finite temperature not possible

  → **LSZ reduction formula not applicable, cross section does not exist!**

- Computation of rates derived from EoM of components $G^{++}, G^{+-}, \ldots$

Effective action

$$S_{\text{NR}} \simeq \int_{\mathscr{C}} \left[\eta^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) \eta + \xi^\dagger \left(i\partial_t - \frac{\nabla^2}{2M} \right) \xi \right]$$
$$+ i\frac{g^2}{2} \int_{x,y\in\mathscr{C}} J(x)D(x,y)J(y)$$
$$+ i \int_{x,y\in\mathscr{C}} O^\dagger(x)\Gamma(x,y)O(y),$$

where η, ξ NR fields, $J \equiv \eta^\dagger\eta + \xi^\dagger\xi$, $O \equiv \xi^\dagger\eta$.

- D contains thermal corrections arising from l interaction.

  D independent of η, ξ due to Boltzmann suppression.

$$D_{\mu\nu} = D^0_{\mu\nu} + g_l^2 \int D^0_{\mu\alpha} \text{Tr}\, [\gamma^\alpha S_l^0 \gamma^\beta S_l^0] D_{\beta\nu}$$

- Γ contains hard ($\sim M$) annihilation processes, e.g. obtained from cutting 'thermal box diagram'.

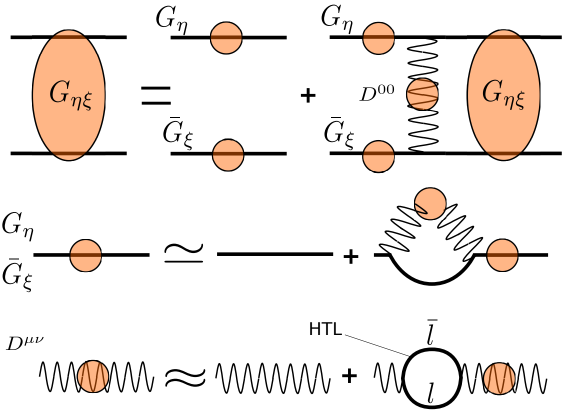
  $\rightarrow$ thermal corrections (typically $\lesssim T$) can be neglected in Γ computation for thermal freeze-out.

From η, ξ EoM we derive

$$\dot{n}_\eta + 3Hn_\eta = -(\sigma v_{\text{rel}})^{\text{s-wave}}_{T=0} G^{++--}_{\eta\xi}(x,x,x,x),$$
$$\dot{n}_\xi + 3Hn_\xi = -(\sigma v_{\text{rel}})^{\text{s-wave}}_{T=0} G^{++--}_{\eta\xi}(x,x,x,x),$$

where $G^{++--}_{\eta\xi}$ is a component of the 4 by 4 matrix

$$G_{\eta\xi}(x,y,z,w) \equiv \left\langle T_{\mathscr{C}}\,\eta(x)\xi^\dagger(y)\xi(w)\eta^\dagger(z)\right\rangle.$$



$$G_{\eta\xi}^{++--} = G_\eta^{+-}\bar{G}_\xi^{+-} + g^2\int\Big[G_\eta^{++}\bar{G}_\xi^{++}D^{++}G_{\eta\xi}^{++--} - G_\eta^{+-}\bar{G}_\xi^{++}D^{-+}G_{\eta\xi}^{-+--} - G_\eta^{++}\bar{G}_\xi^{+-}D^{+-}G_{\eta\xi}^{+---} + G_\eta^{+-}\bar{G}_\xi^{+-}D^{--}G_{\eta\xi}^{----}\Big] \tag{1}$$

$$G_{\eta\xi}^{-+--} = G_\eta^{--}\bar{G}_\xi^{+-} + g^2\int\Big[G_\eta^{-+}\bar{G}_\xi^{++}D^{++}G_{\eta\xi}^{++--} - G_\eta^{--}\bar{G}_\xi^{++}D^{-+}G_{\eta\xi}^{-+--} - G_\eta^{-+}\bar{G}_\xi^{+-}D^{+-}G_{\eta\xi}^{+---} + G_\eta^{--}\bar{G}_\xi^{+-}D^{--}G_{\eta\xi}^{----}\Big] \tag{2}$$

$$G_{\eta\xi}^{+---} = G_\eta^{+-}\bar{G}_\xi^{--} + g^2\int\Big[G_\eta^{++}\bar{G}_\xi^{-+}D^{++}G_{\eta\xi}^{++--} - G_\eta^{+-}\bar{G}_\xi^{-+}D^{-+}G_{\eta\xi}^{-+--} - G_\eta^{++}\bar{G}_\xi^{--}D^{+-}G_{\eta\xi}^{+---} + G_\eta^{+-}\bar{G}_\xi^{--}D^{--}G_{\eta\xi}^{----}\Big] \tag{3}$$

$$G_{\eta\xi}^{----} = G_\eta^{--}\bar{G}_\xi^{--} + g^2\int\Big[G_\eta^{-+}\bar{G}_\xi^{-+}D^{++}G_{\eta\xi}^{++--} - G_\eta^{--}\bar{G}_\xi^{-+}D^{-+}G_{\eta\xi}^{-+--} - G_\eta^{-+}\bar{G}_\xi^{--}D^{+-}G_{\eta\xi}^{+---} + G_\eta^{--}\bar{G}_\xi^{--}D^{--}G_{\eta\xi}^{----}\Big] \tag{4}$$

We obtain analytic (formal) solutions for $G_{\eta\xi}^{++--}$ in the cases:

- Standard SE
  Approximations: DM dilute limit, free correlators, zero temperature and static D.
- Kinetic equilibrium
  Approximations: KMS condition, DM dilute limit. Further taking chemical equilibrium limit we recover similar results derived from linear response theory (M. Laine et al. 2017).
- General out of equilibrium
  Approximations: DM dilute limit, static D.

Effective in-medium potential

In equilibrium case we find $S \propto \Im G^{++++}_{\eta\xi}(0,0;E)$, solution of

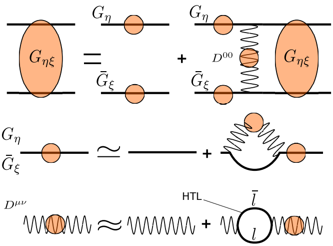
$$\left[\left(-\frac{\Delta^2_{\mathbf{r}'}}{M} + V_{\text{eff}}(\mathbf{r}')\right) - E\right] G^{++++}_{\eta\xi}(\mathbf{r}',\mathbf{r}'';E) = i\delta^3(\mathbf{r}'-\mathbf{r}''),$$

where the effective potential V_{eff} consists of temperature corrections from dressed single particle propagators AND dressed photon propagator:

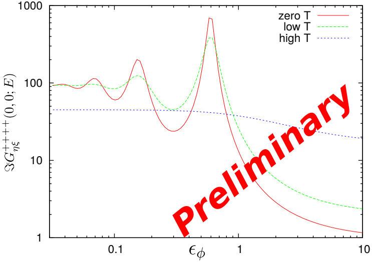
$$V_{\text{eff}}(r) \equiv \Sigma^R_\eta + \bar{\Sigma}^R_\xi - ig^2 D^{++}(\mathbf{r})$$

$$\underbrace{=}_{\text{HTL}} \underbrace{-\alpha\sqrt{m_\phi^2 + m_D^2}}_{\text{gain in kinetic energy}} \underbrace{-\frac{\alpha}{r} e^{-\sqrt{m_\phi^2+m_D^2}\,r}}_{\text{screened Yukawa potential}} \underbrace{-i\alpha T \frac{m_D}{\sqrt{m_\phi^2+m_D^2}} \Phi(\sqrt{m_\phi^2+m_D^2}\,r)}_{\text{thermal width}}$$

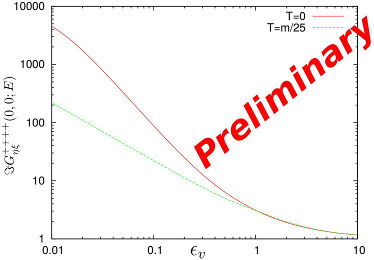
where $m_D \sim \mathcal{O}(g_l T)$ electric Debye screening mass and $\Phi(0) = 0$, $\Phi(\infty) = 1$.



Numerical Results



Enhancement factor vs. ϵ_ϕ is shown for fixed $\epsilon_v = 10^{-1.5}$. Different lines correspond to different temperatures $\epsilon_D = [0, 0.1, 20]$ (r,g,b) , where $\epsilon_T = 20\epsilon_D$ is fixed everywhere.



Enhancement factor vs. ϵ_v is shown for fixed $\epsilon_\phi = 6/\pi^2$ (First on-resonance peak). Red: Sommerfeld enhancement in vacuum. Green: Sommerfeld enhancement at DM freeze-out temperature $T = m_\chi/25$.

Dimensionless parameters: $\epsilon_\phi \equiv \frac{m_\phi}{\alpha_\chi m_\chi}$, $\epsilon_v \equiv \frac{v}{\alpha_\chi}$, $\epsilon_D \equiv \frac{m_D}{m_\chi v}$, $\epsilon_T \equiv \frac{T}{m_\chi v}$.

Summary

- Developed non-equilibrium description of SE at finite temperature for simplified $U(1)$ DM model.

  applicable to DM freeze-out, extending existing linear response theory results.

- Two independent (checks!) numerical strategies to obtain SE:

  1.) T-matrix approach: integral equation (boundary condition independent).

  2.) $G_{\eta\xi}^{++++}$ approach: differential equation, finite temp. boundary conditions.

- Leading real part corrections in V_{eff}: $\mathcal{O}(\alpha m_D)$.
- Leading imaginary part corrections in V_{eff}: $\lesssim \mathcal{O}(\alpha T)$.

  Large contribution due to resummation of soft plasma interactions in HTL approximation.

- Thermal effects able to modify SE significantly.
- Close future: Temperature dependent phenomenology and impact on the relic density.

Kim, S. and Laine, M. (2017). On thermal corrections to near-threshold annihilation. JCAP, 1701:013.