



Ultralight Particles and Black Hole Superradiance

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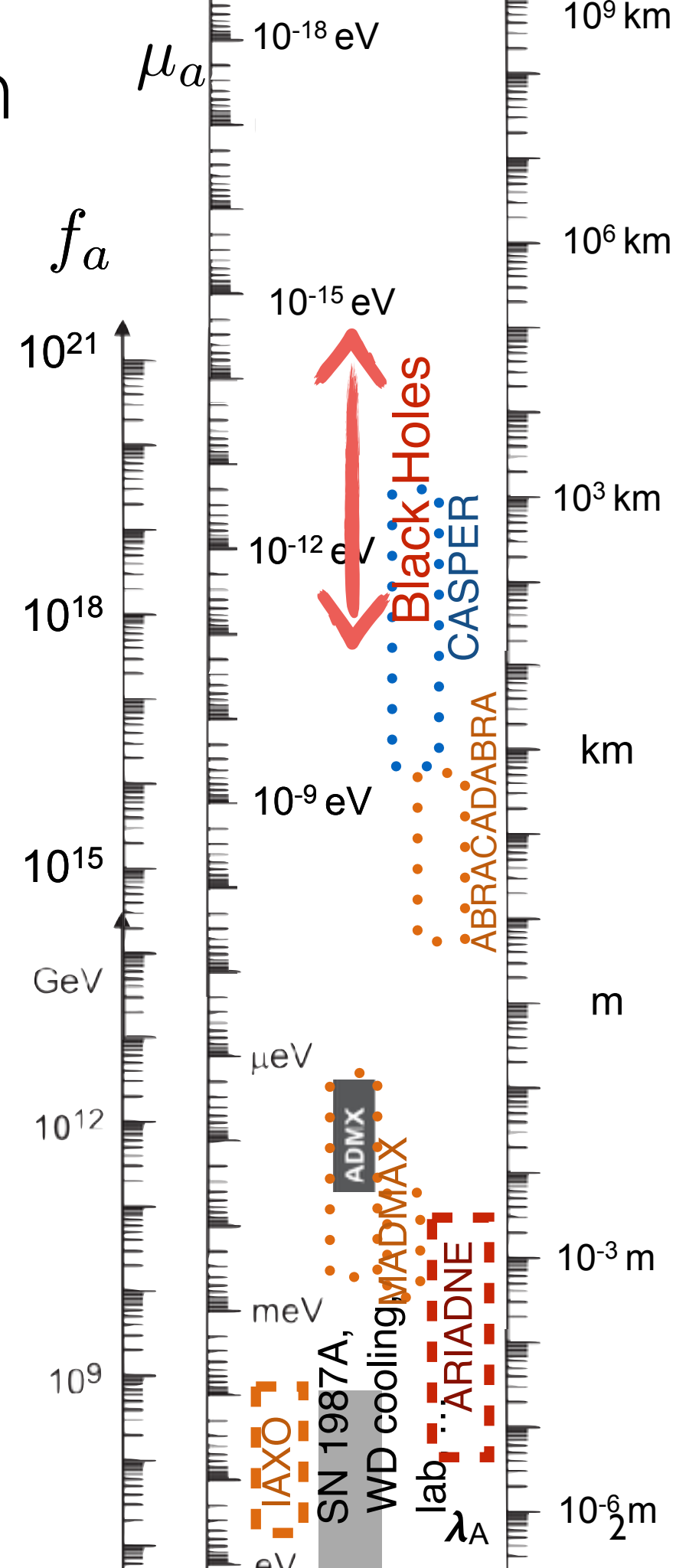
A. Arvanitaki, MB, X. Huang
A. Arvanitaki, MB, S. Dimopoulos, S. Dubovsky, R. Lasenby
MB, R. Lasenby, M. Teo

Searching for the QCD axion

- Best limits on axions come from astrophysics
- Current limit: $f_a > 10^9$ GeV from astrophysics and $f_a \sim 10^{11}$ GeV from lab experiment (if DM)
- A lot of unexplored parameter space: the lighter, the more weakly interacting

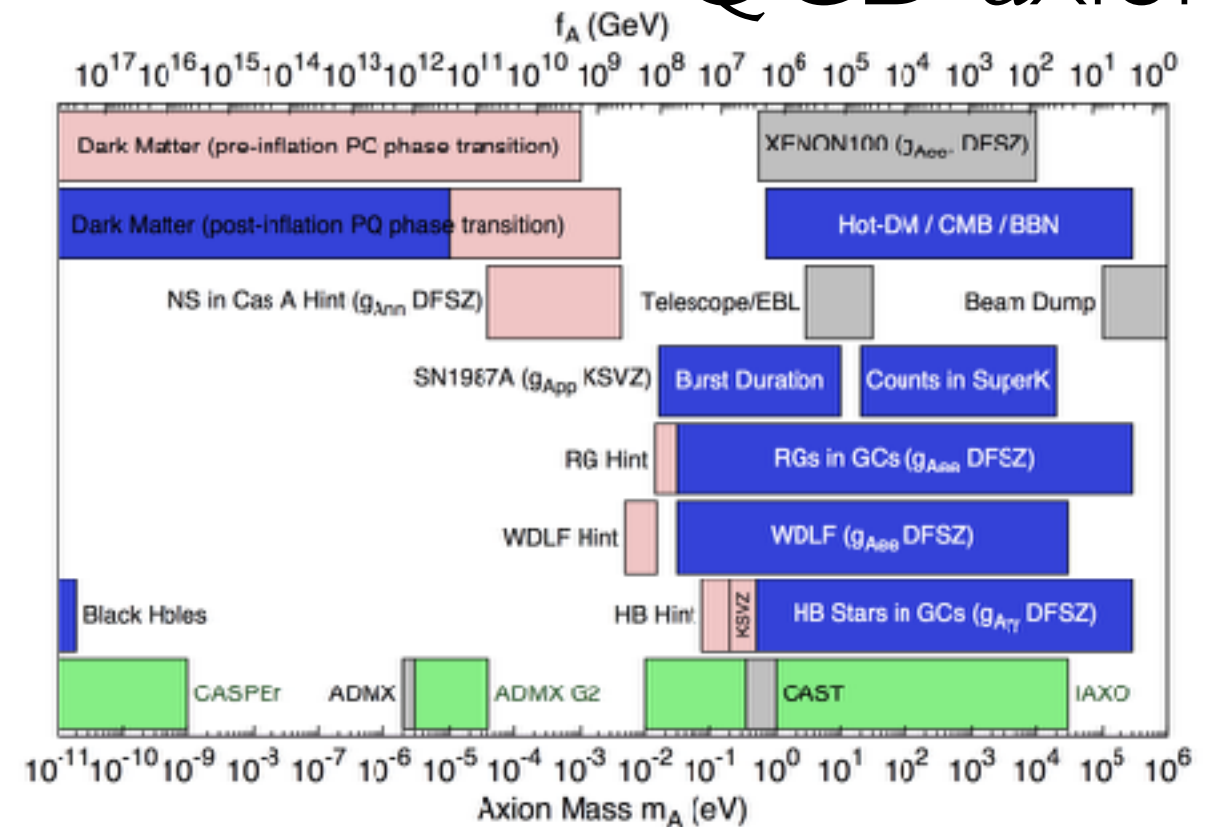
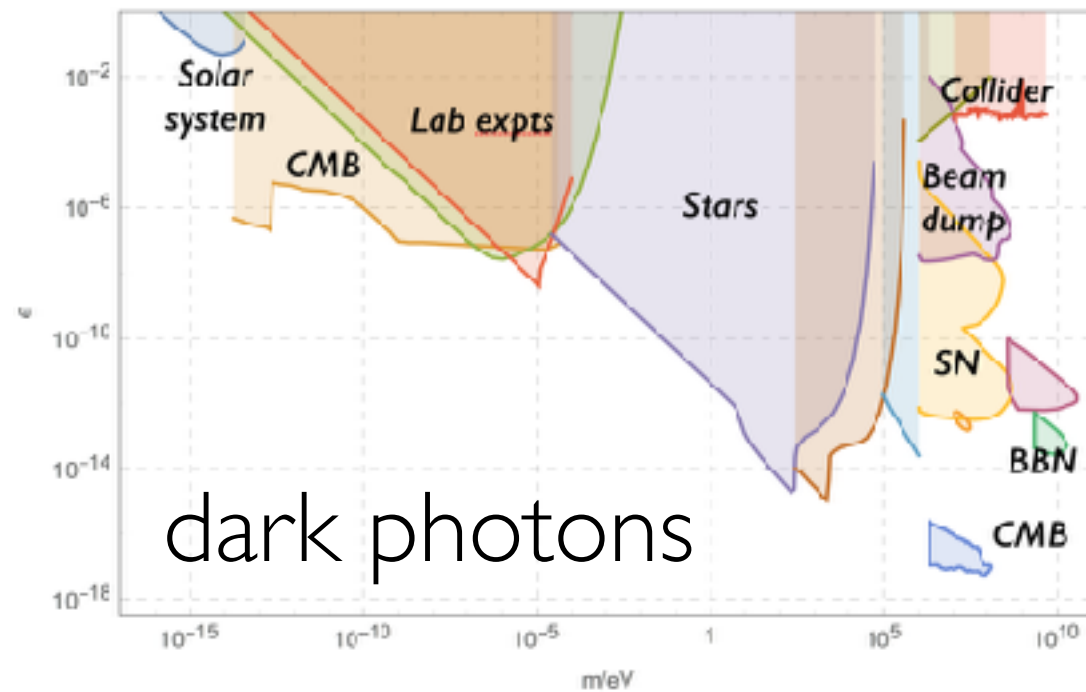
$$\mu_a \sim 6 \times 10^{-11} \text{ eV} \frac{10^{17} \text{ GeV}}{f_a}$$

$$\lambda_a \sim 3 \text{ km} \frac{6 \times 10^{-11} \text{ eV}}{\mu_a}$$



Searching for Ultralight particles

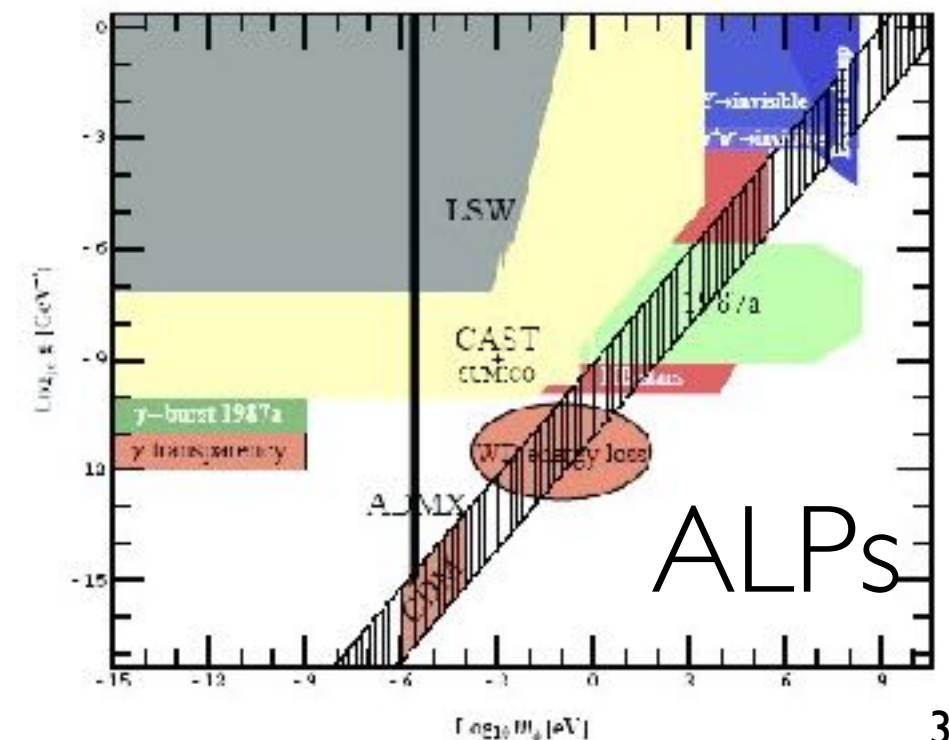
QCD axion



Spin 0 and spin 1, weakly interacting

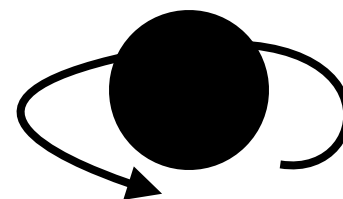
$$\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \mu_a^2 \phi^2$$

$$- \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} \mu_V^2 A_\mu'^2$$



Outline

- Black Hole Superradiance
- Spinning Black Holes
- Gravitational Wave Signals

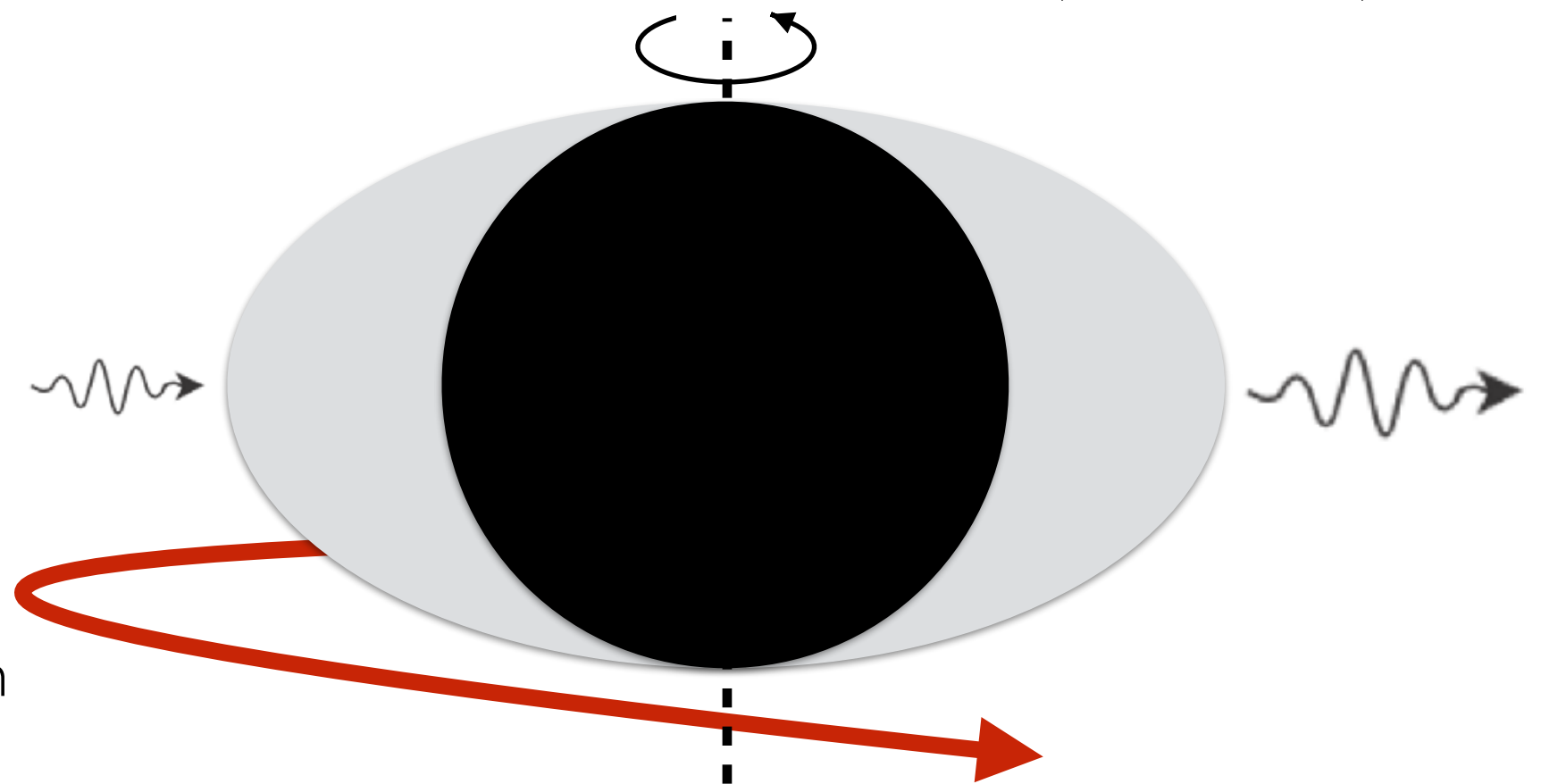


Superradiance

Zel'dovich; Starobinskii; Misner

A wave scattering off a rotating object can increase in amplitude by extracting angular momentum and energy.

Growth proportional to probability of absorption when rotating object is at rest: dissipation necessary to change the wave amplitude



Superradiance condition:

Angular velocity of wave slower than angular velocity of BH horizon,

$$\Omega_a < \Omega_{BH}$$

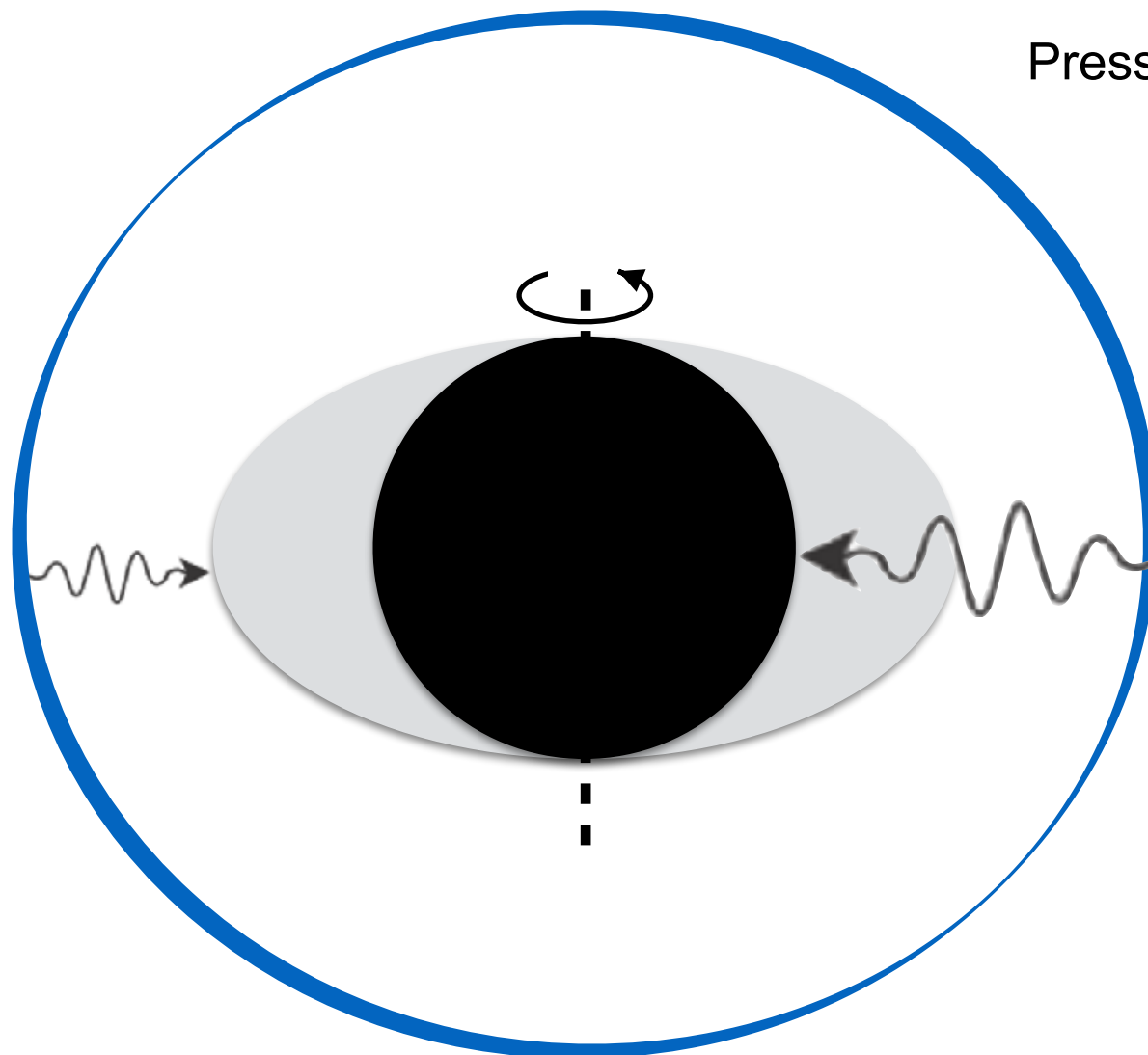
Superradiance

Particles/waves trapped in orbit around the BH repeat this process continuously

Press & Teukosky

“Black hole bomb”:
exponential instability when
surround BH by a mirror

Kinematic, not resonant
condition



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Angular velocity of wave slower than angular velocity of BH horizon,

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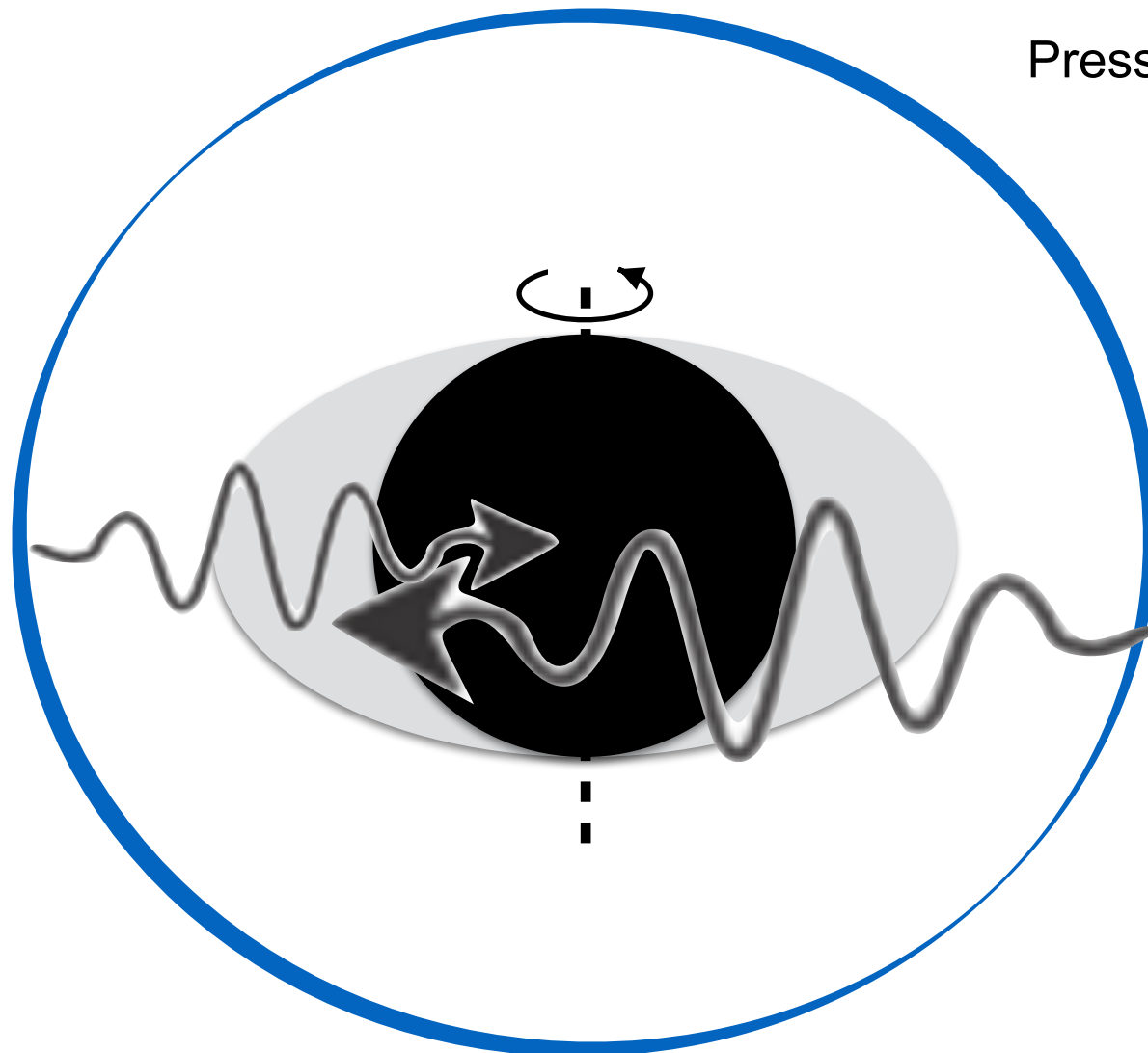
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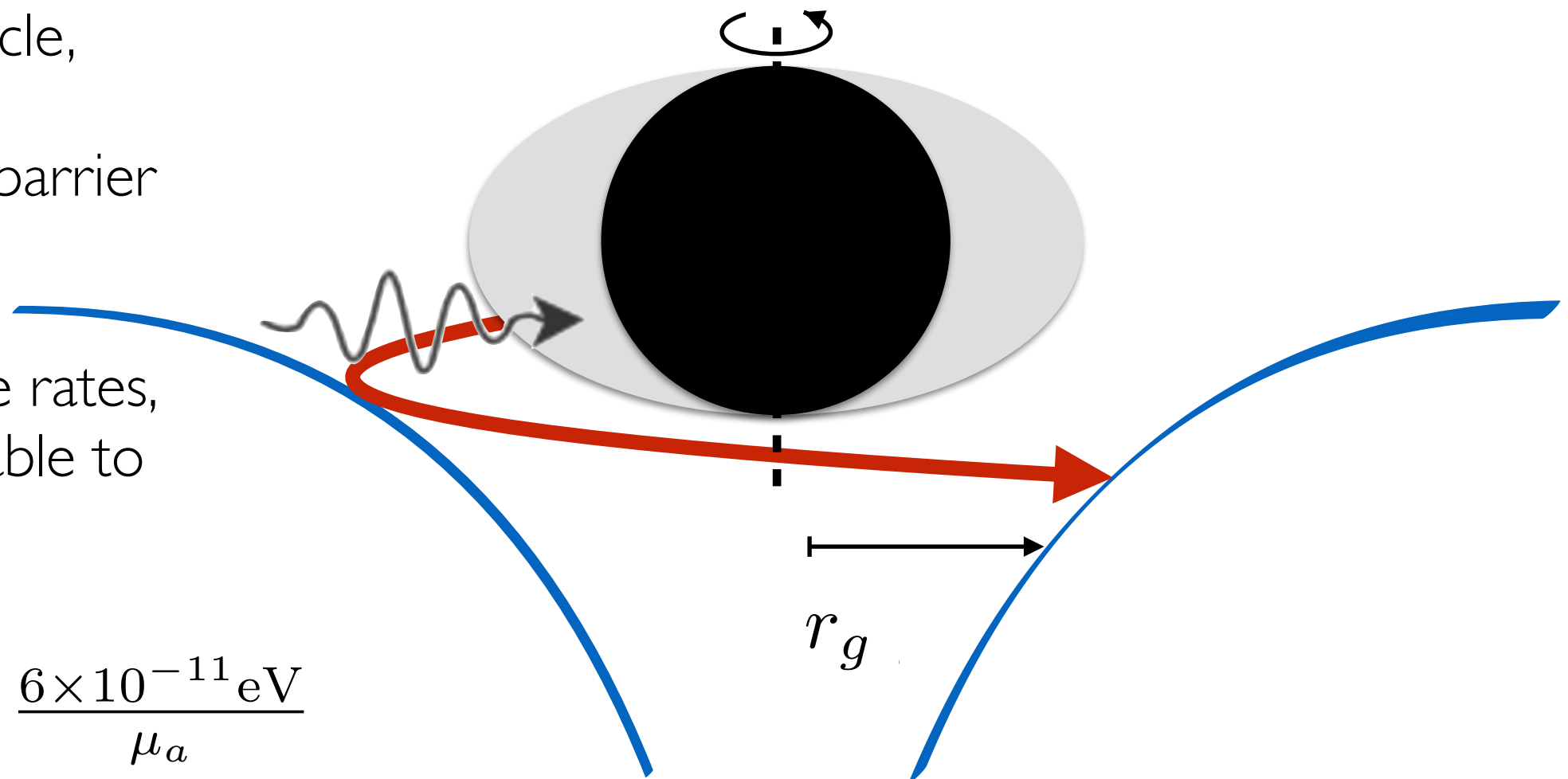
Superradiance

$$V(r) = -\frac{G_N M_{BH} \mu_a}{r}$$

For a massive particle,
e.g. axion,
gravitational potential barrier
acts as “mirror”

For high superradiance rates,
“mirror” size comparable to
BH size:

$$r_g \lesssim \mu_a^{-1} \sim 3 \text{ km} \frac{6 \times 10^{-11} \text{ eV}}{\mu_a}$$



[Zouros & Eardley'79; Damour et al '76; Detweiler'80; Gaina et al '78]

[Arvanitaki, Dimopoulos, Dubovsky, Kaloper, March-Russell 2009; Arvanitaki, Dubovsky 2010]

Superradiance: Gravitational Atoms

$$V(r) = -\frac{G_N M_{BH} \mu_a}{r}$$

Hydrogen atoms

Gravitational ‘atoms’

‘Fine structure constant’

α_{em}

$$\alpha = G_N M_{BH} \mu_a = r_g \mu_a$$

Radius

$$r_B = \frac{n^2}{\alpha_{em} m_e}$$

$$r_c \sim \frac{n^2}{\alpha \mu_a} \sim 4 - 400 r_g$$

Occupation number

$$N = 1$$

$$N \sim 10^{70} - 10^{80}$$

— classical field

Boundary conditions

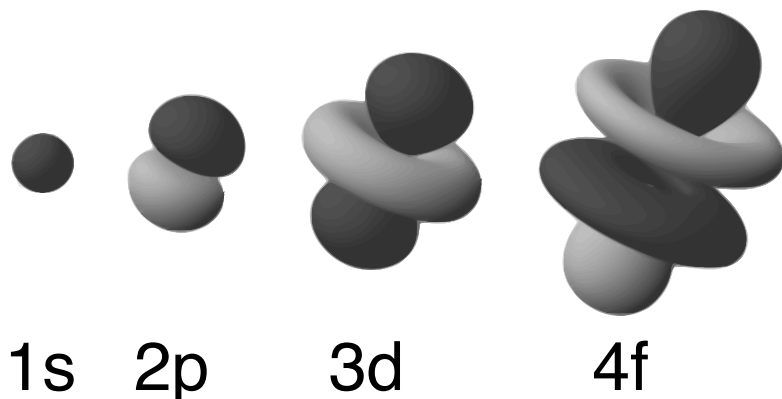
regular at origin

ingoing at horizon

Energy levels

$$m_e \left(1 - \frac{\alpha_{em}^2}{2n^2} \right)$$

$$\mu_a \left(1 - \frac{\alpha^2}{2n^2} + i\Gamma_{sr} \right)$$



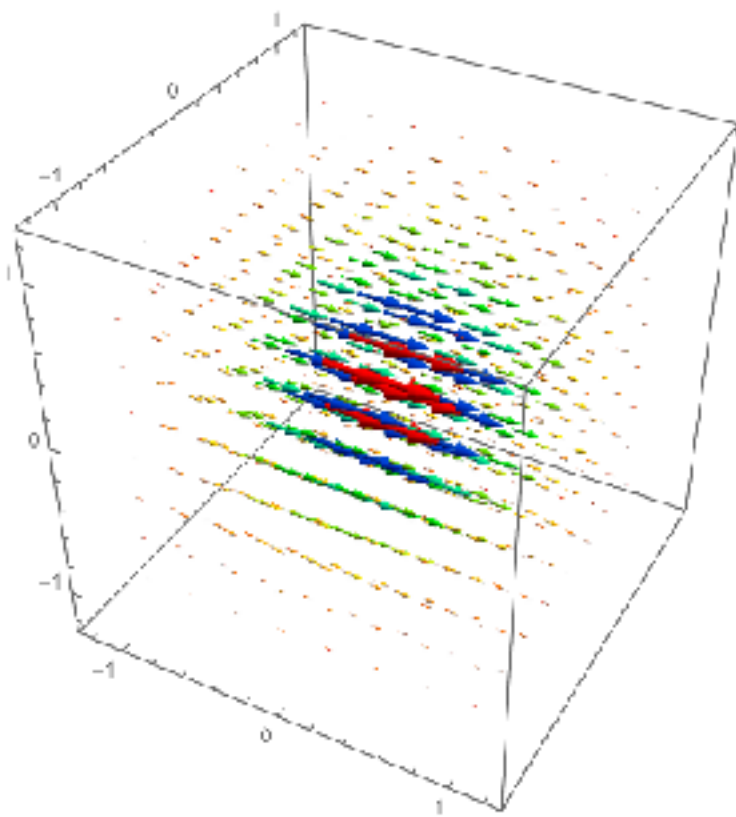
Superradiance condition:

$$\frac{\omega_a}{m} < \Omega_{BH}$$

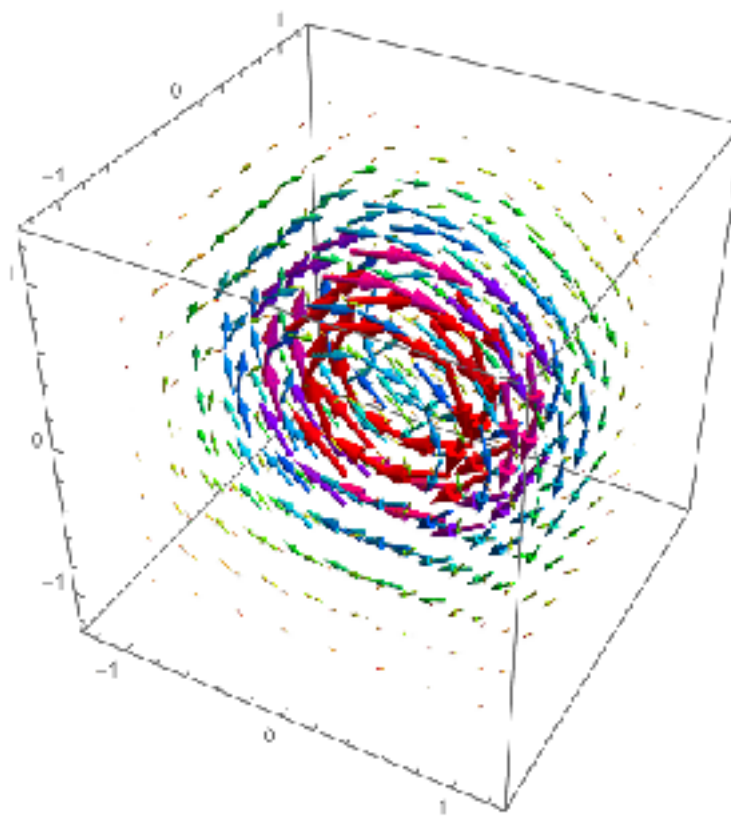
(m = magnetic quantum number)

Superradiance: Vector Gravitational Atoms

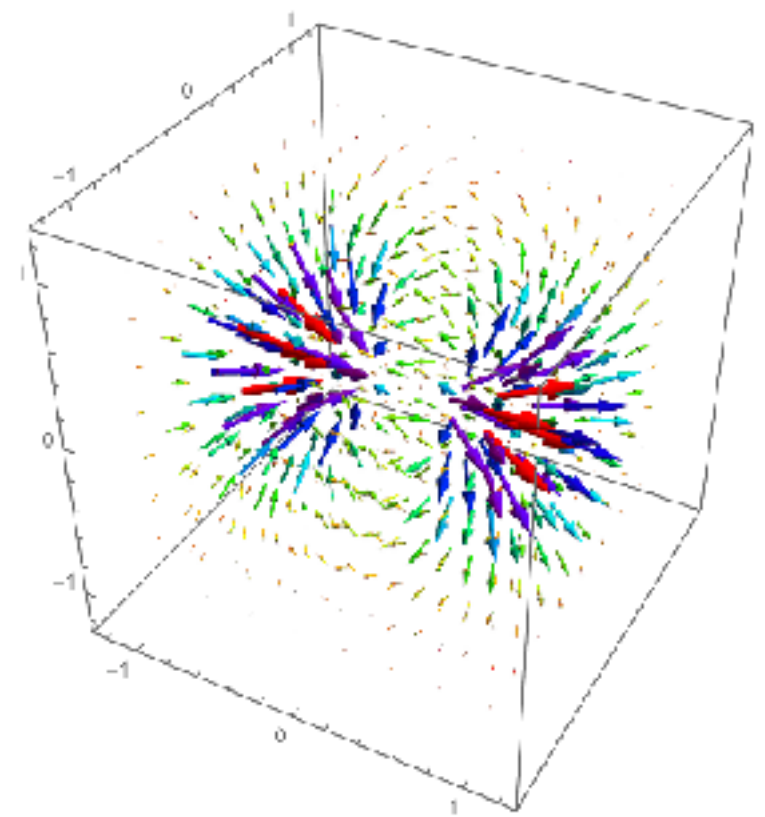
Hydrogen-like radial profile, vector spherical harmonic angular



$$j = 1, l = 0$$



$$j = 1, l = 1$$



$$j = 1, l = 2$$

Analytic superradiance rates

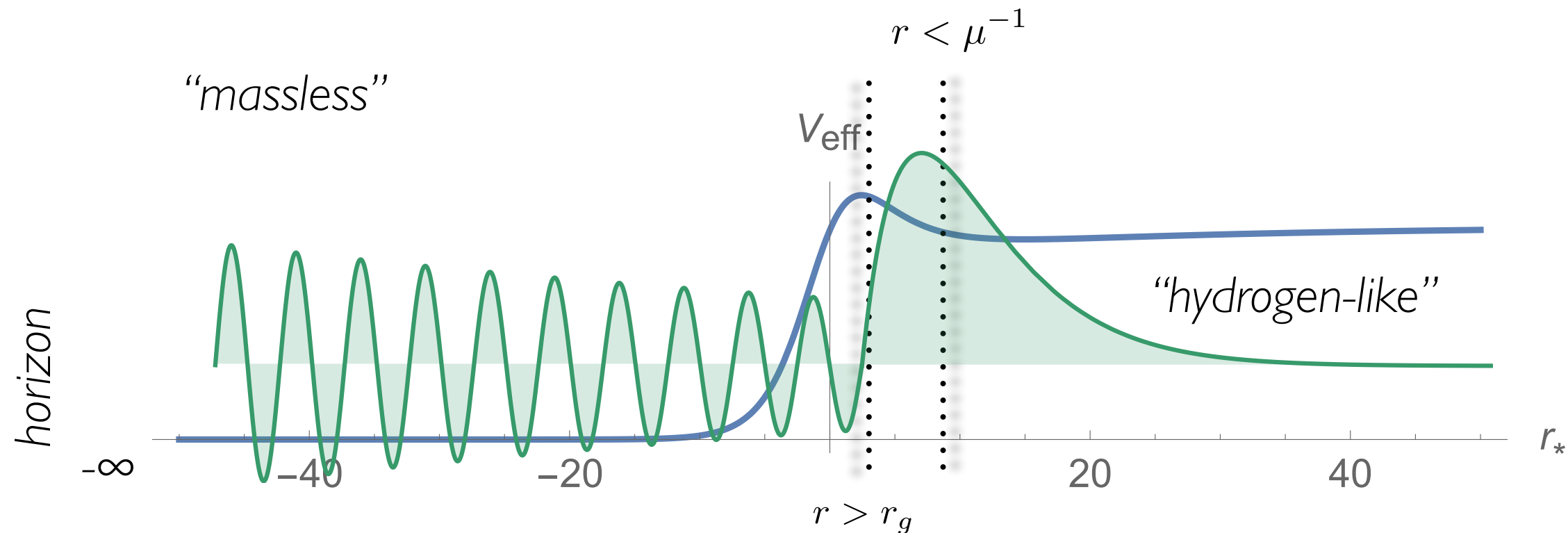
$$\mu_a \left(1 - \frac{\alpha^2}{2n^2} + i\Gamma_{sr} \right)$$

- Bound states: enough to take $1/r$ potential at leading order
- Superradiance rates: include near-horizon effects by definition
- Scalars in Kerr: equations of motion separable, match hydrogen and near-horizon wavefunctions

$$\square\Phi + \mu^2\Phi = 0 \quad \longrightarrow \quad \Phi(x^\mu) = e^{im\phi} e^{-i\omega t} S_{lm}(\theta) R_{lm}(r).$$

Analytic superradiance rates

- Match wavefunctions:



e.g. scalar in Schwarzschild:
$$V_{\text{eff}}(r) = \left(1 - \frac{2r_g}{r}\right) \left(\mu^2 + \frac{l(l+1)}{r^2} + 2\frac{r_g}{r^3}\right)$$

Analytic superradiance rates

$$\mu_a \left(1 - \frac{\alpha^2}{2n^2} + i\Gamma_{sr} \right)$$

- Bound states: enough to take $1/r$ potential at leading order
- Superradiance rates: include near-horizon effects by definition
- Scalars in Kerr: equations of motion separable, match hydrogen and near-horizon wavefunctions

$$\square\Phi + \mu^2\Phi = 0 \quad \longrightarrow \quad \Phi(x^\mu) = e^{im\phi} e^{-i\omega t} S_{lm}(\theta) R_{lm}(r).$$

- Massive vectors,

$$\nabla_\sigma F^{\sigma\nu} - \mu^2 A^\nu = 0 \quad \longrightarrow \quad \text{equations not separable}$$

- Match on to massless scalar + vector behavior: possible at small α , match far and near wavefunctions. Know superradiant scattering of massless bosons.

Superradiance Timescales

$$\alpha = G_{\text{N}} M_{\text{BH}} \mu_a = r_g \mu_a \lesssim \frac{m}{2} a_*$$

BH lightcrossing time

$$r_g$$

Particle wavelength

$$\mu^{-1} = \frac{r_g}{\alpha}$$

Cloud size

$$r_c \sim \frac{n}{\alpha^2} r_g$$

Superradiance time

Annihilation time

Superradiance Timescales

$$\alpha = G_{\text{N}} M_{\text{BH}} \mu_a = r_g \mu_a \lesssim \frac{m}{2} a_*$$

BH lightcrossing time

$$r_g$$

Particle wavelength

$$\mu^{-1} = \frac{r_g}{\alpha}$$

Cloud size

$$r_c \sim \frac{n}{\alpha^2} r_g$$

Superradiance time

$$\tau_{\text{sr}} \propto \frac{1}{\alpha^{2\ell+2j+4}} \frac{1}{(m\Omega_{\text{BH}} - \mu)r_+} r_g$$

Annihilation time

Flux into horizon: $\Gamma_{\text{sr}}^{\text{scalar}} \sim \int_{r=r_g} \psi^* \psi \cdot dA$

Superradiance Timescales

$$\alpha = G_N M_{\text{BH}} \mu_a = r_g \mu_a \lesssim \frac{m}{2} a_*$$

BH lightcrossing time	r_g
Particle wavelength	$\mu^{-1} = \frac{r_g}{\alpha}$
Cloud size	$r_c \sim \frac{n}{\alpha^2} r_g$
Superradiance time	$\tau_{\text{sr}} \propto \frac{1}{\alpha^{2\ell+2j+4}} \frac{1}{(m\Omega_{\text{BH}} - \mu)r_+} r_g$
Annihilation time	$\tau_{\text{ann}} \propto \frac{1}{\alpha^{4\ell+11}} r_g$

Gravitational Wave Power: $P_{\text{GW}} \sim G_N \omega^2 \bar{T}_{ij}(\omega, k) \bar{T}_{ij}^*(\omega, k)$

Superradiance Timescales

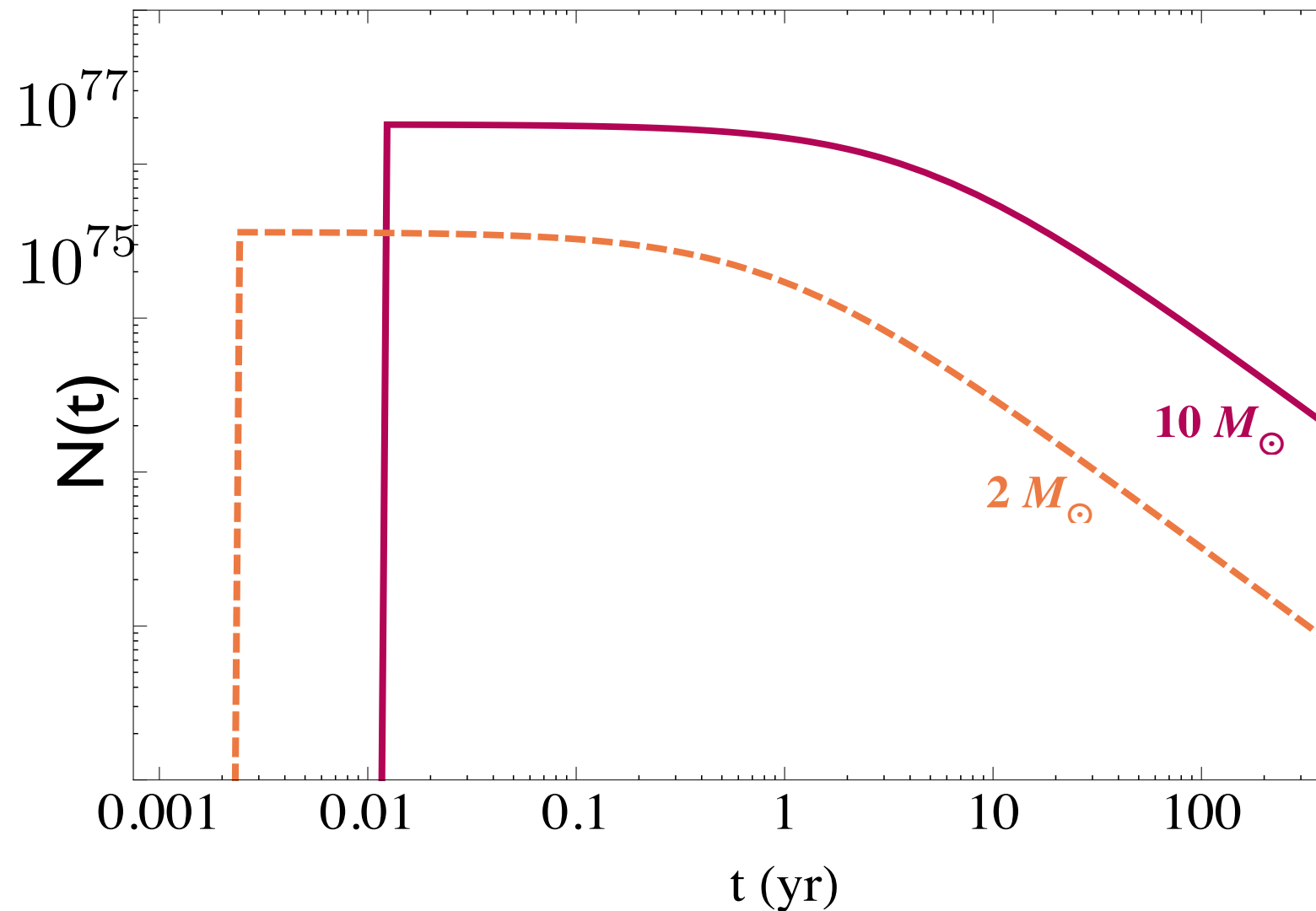
BH lightcrossing time

Particle wavelength

Cloud size

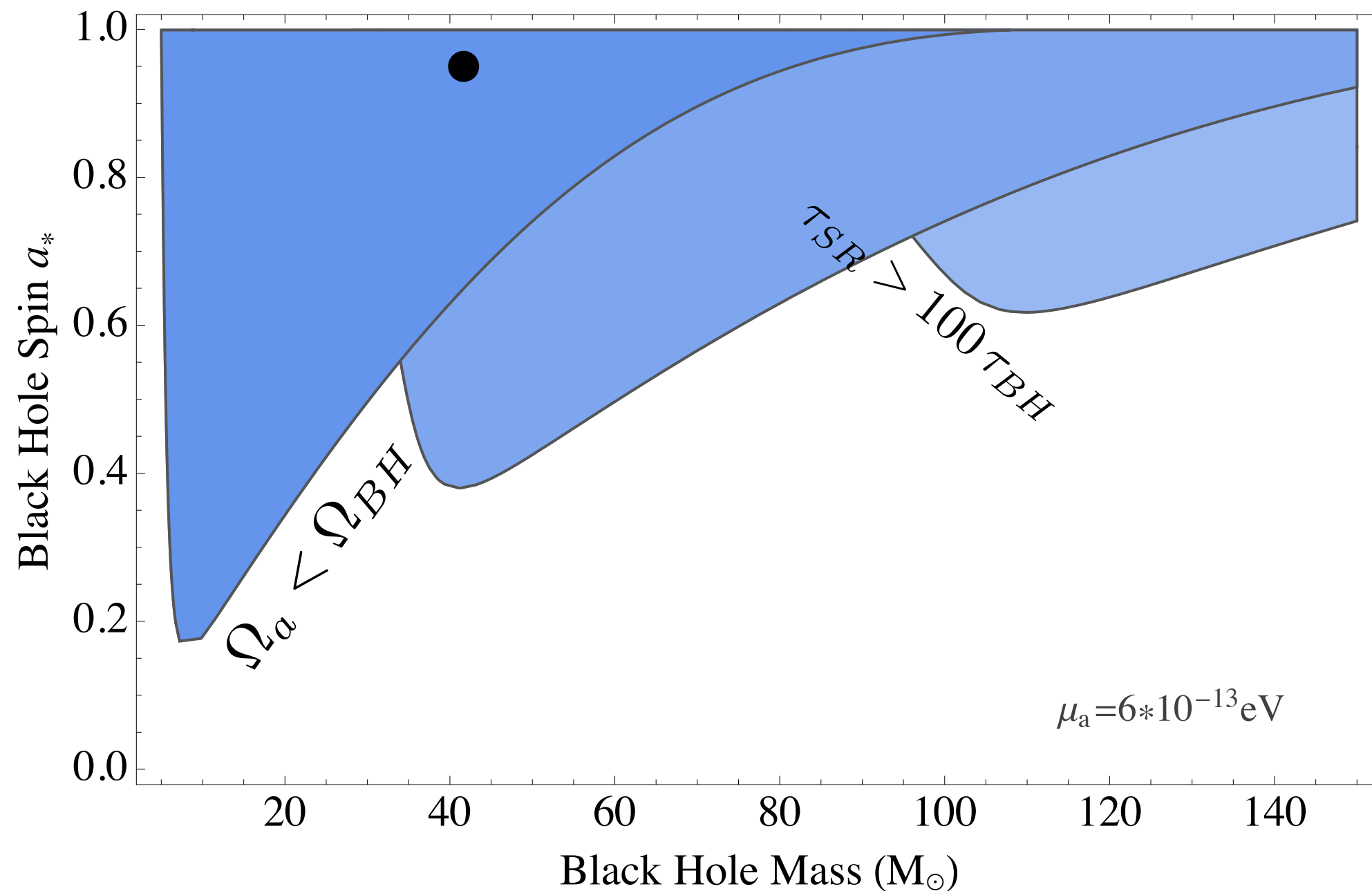
Superradiance time

Annihilation time

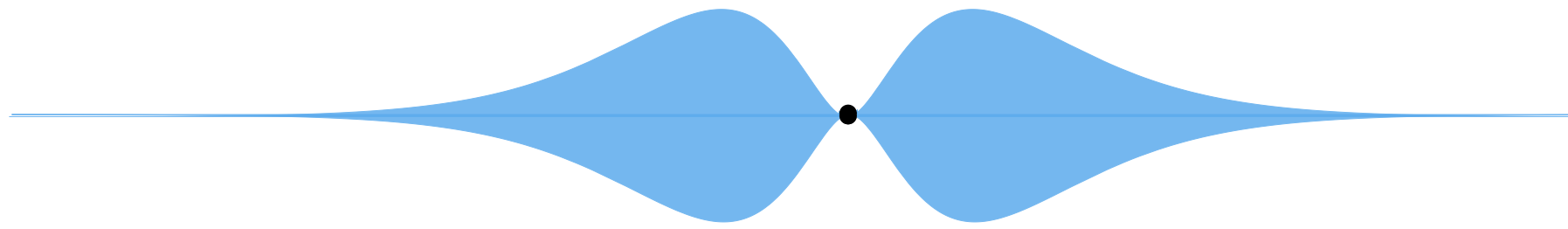


Superradiance: a stellar black hole history

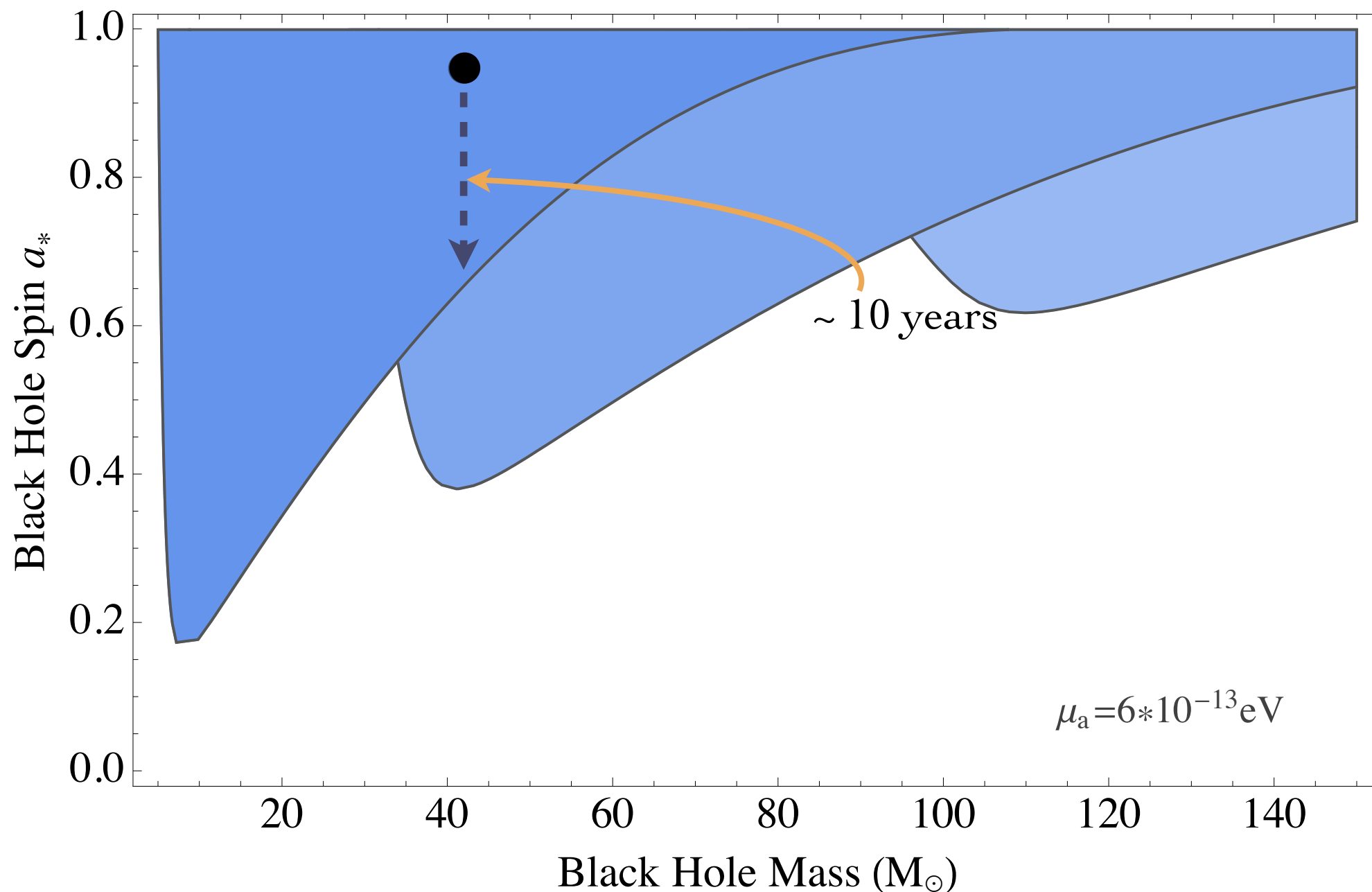
A black hole is born with spin $a^* = 0.95$, $M = 40 M_\odot$.



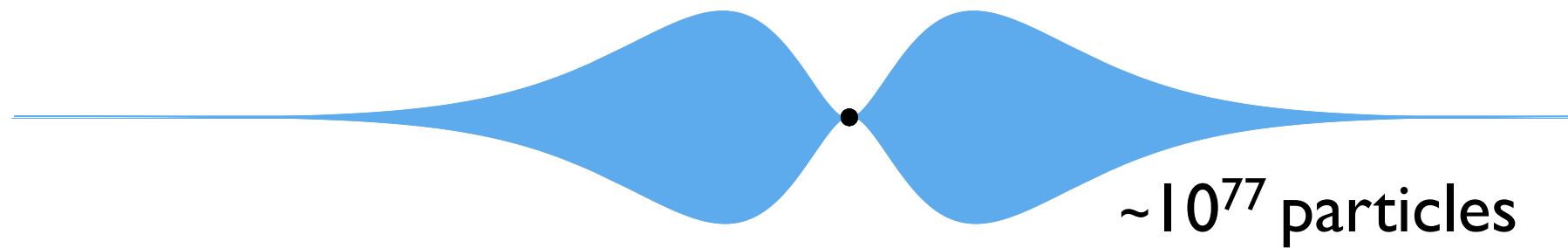
Superradiance: a stellar black hole history



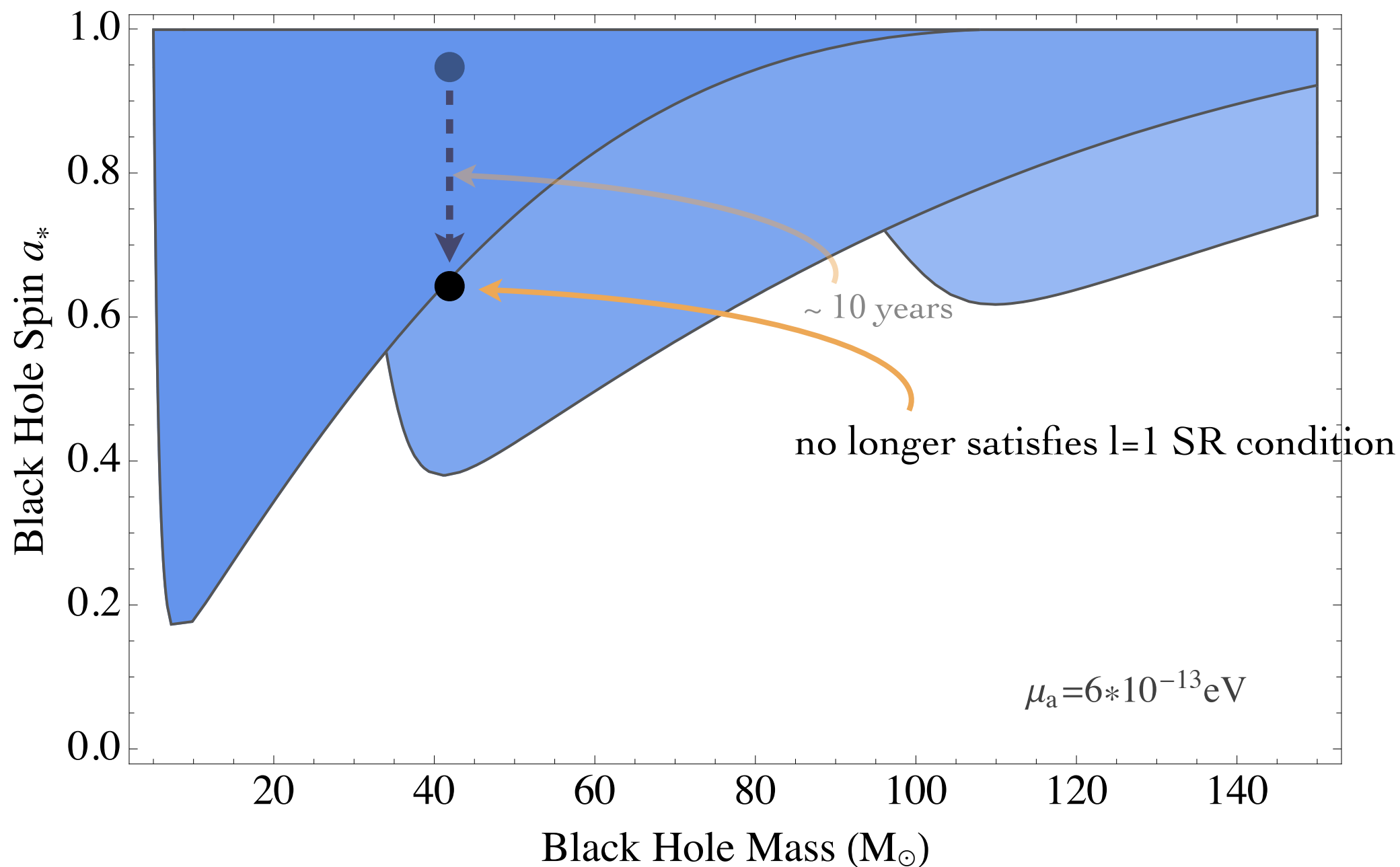
BH spins down and *fastest-growing* level is formed



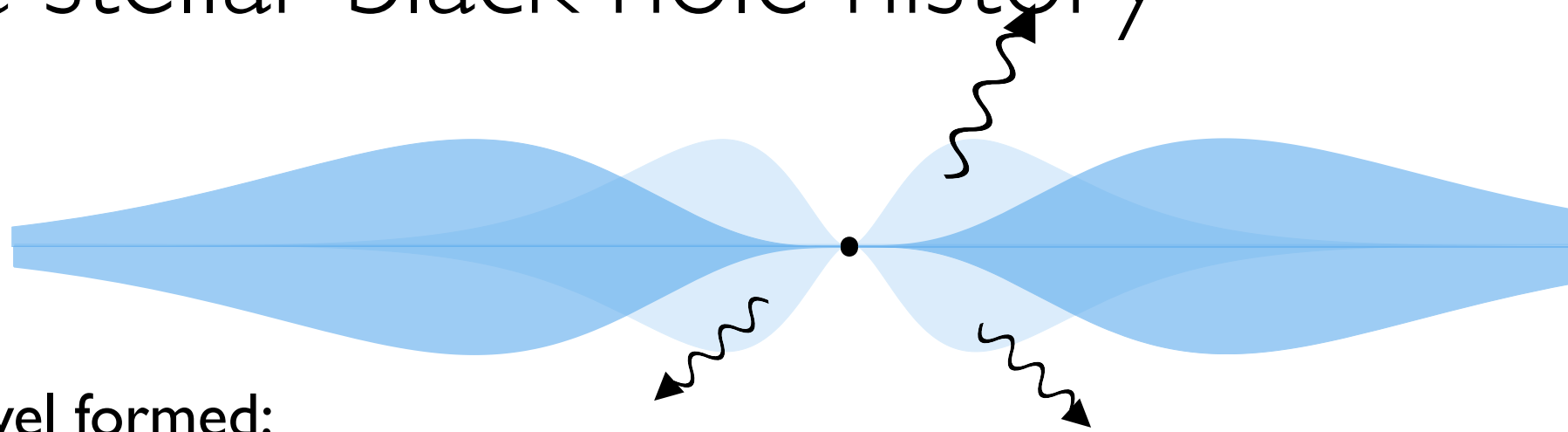
Superradiance: a stellar black hole history



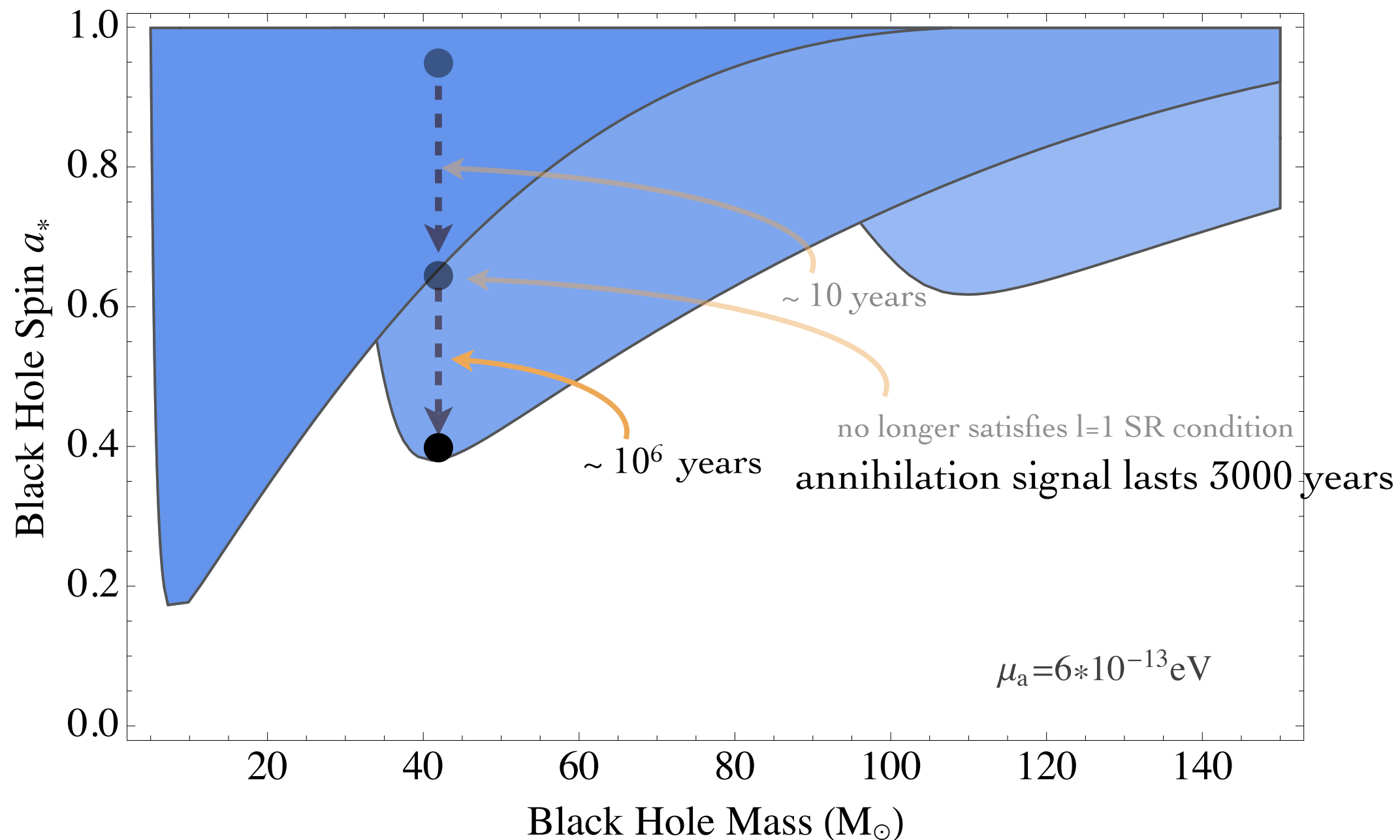
Once BH angular velocity matches that of the level, growth stops



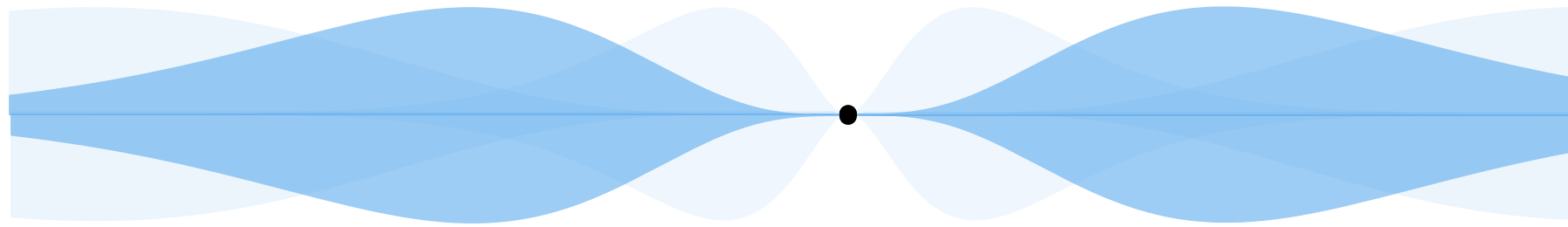
Superradiance: a stellar black hole history



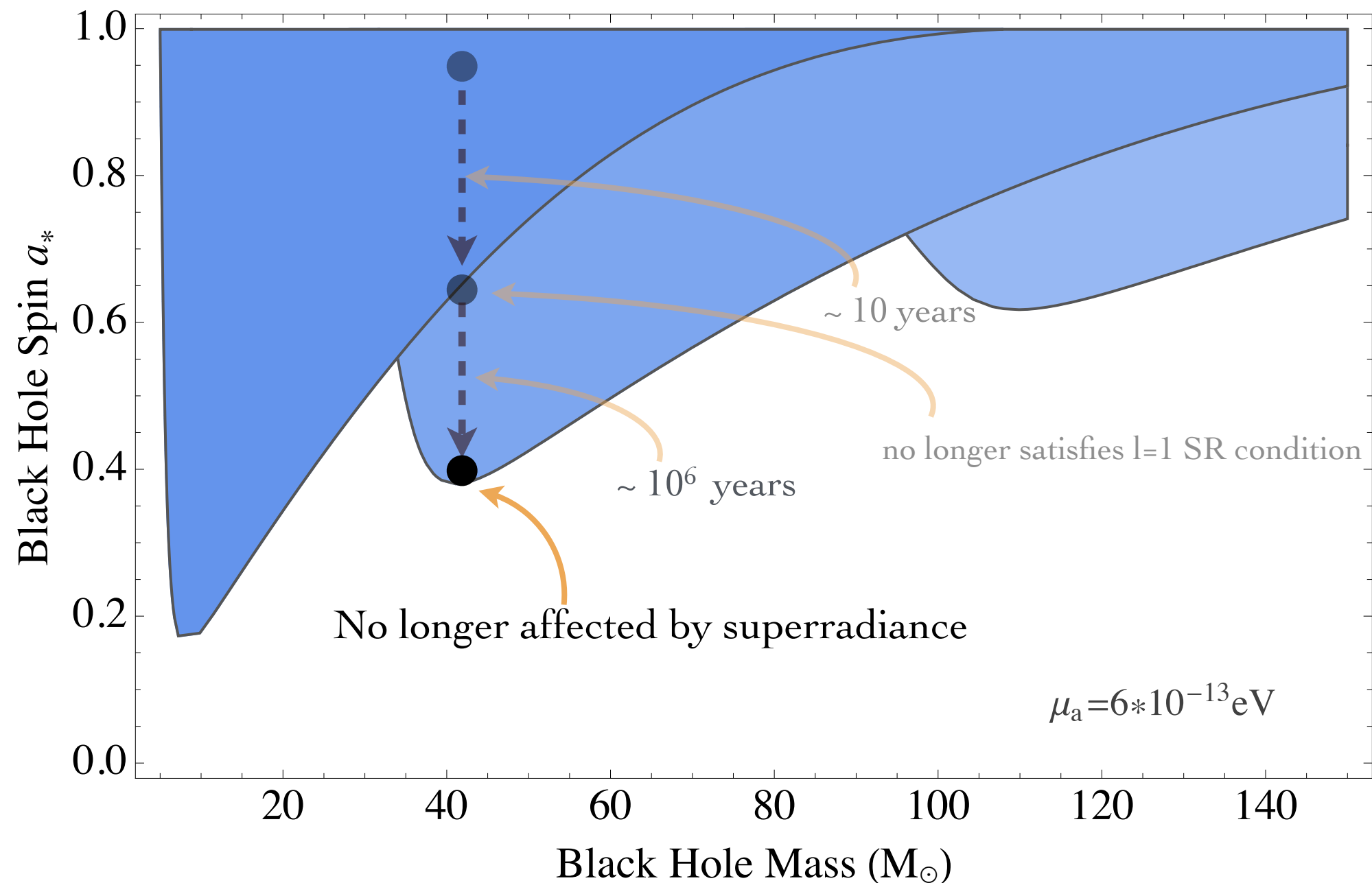
BH spins down and *next* level formed;
annihilations to GWs deplete first level



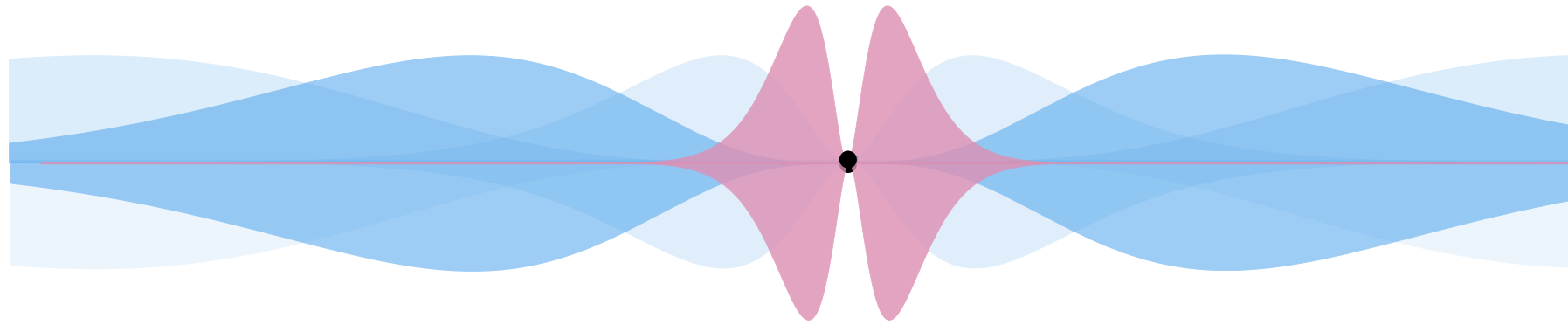
Superradiance: a stellar black hole history



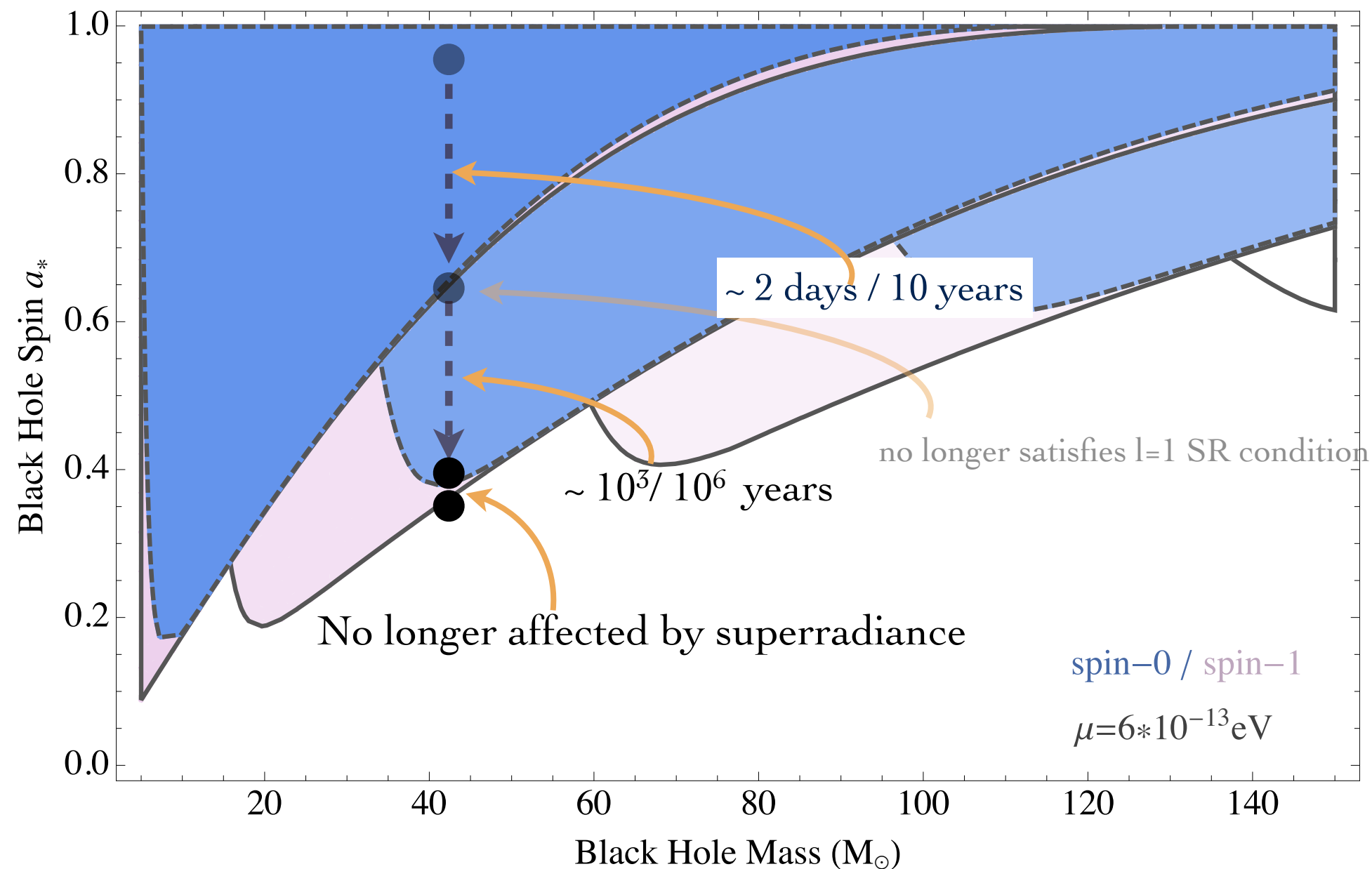
The following level has a superradiance rate exceeding age of BH



Superradiance: a stellar black hole history



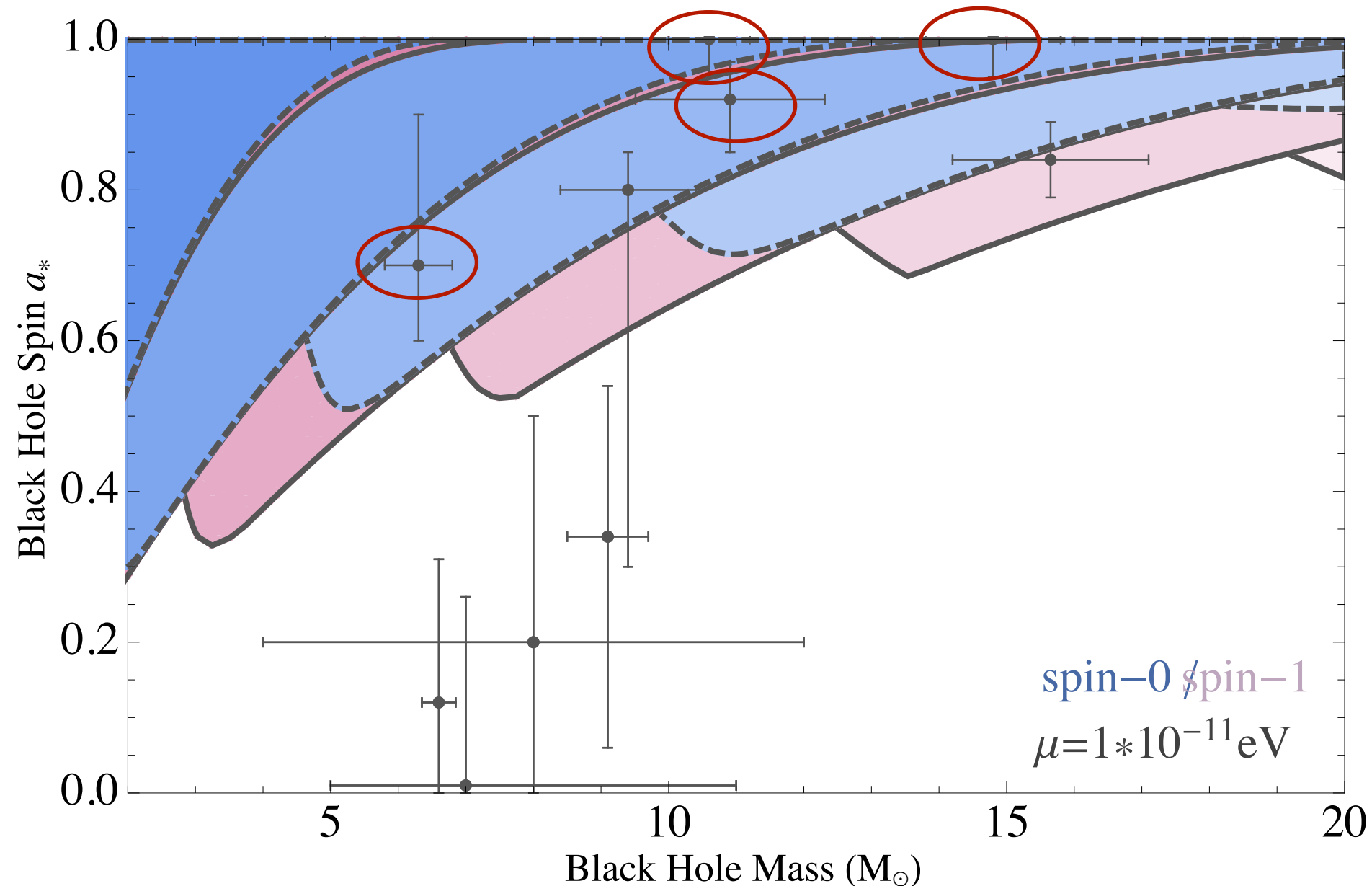
Spin 1 particles: faster superradiance rate for the same mass particle



Vectors can have higher total angular momentum for a given orbital angular momentum

Black Hole Spins

Black hole spin and mass measurements from X-ray binaries:
several black holes disfavor this scalar/vector mass

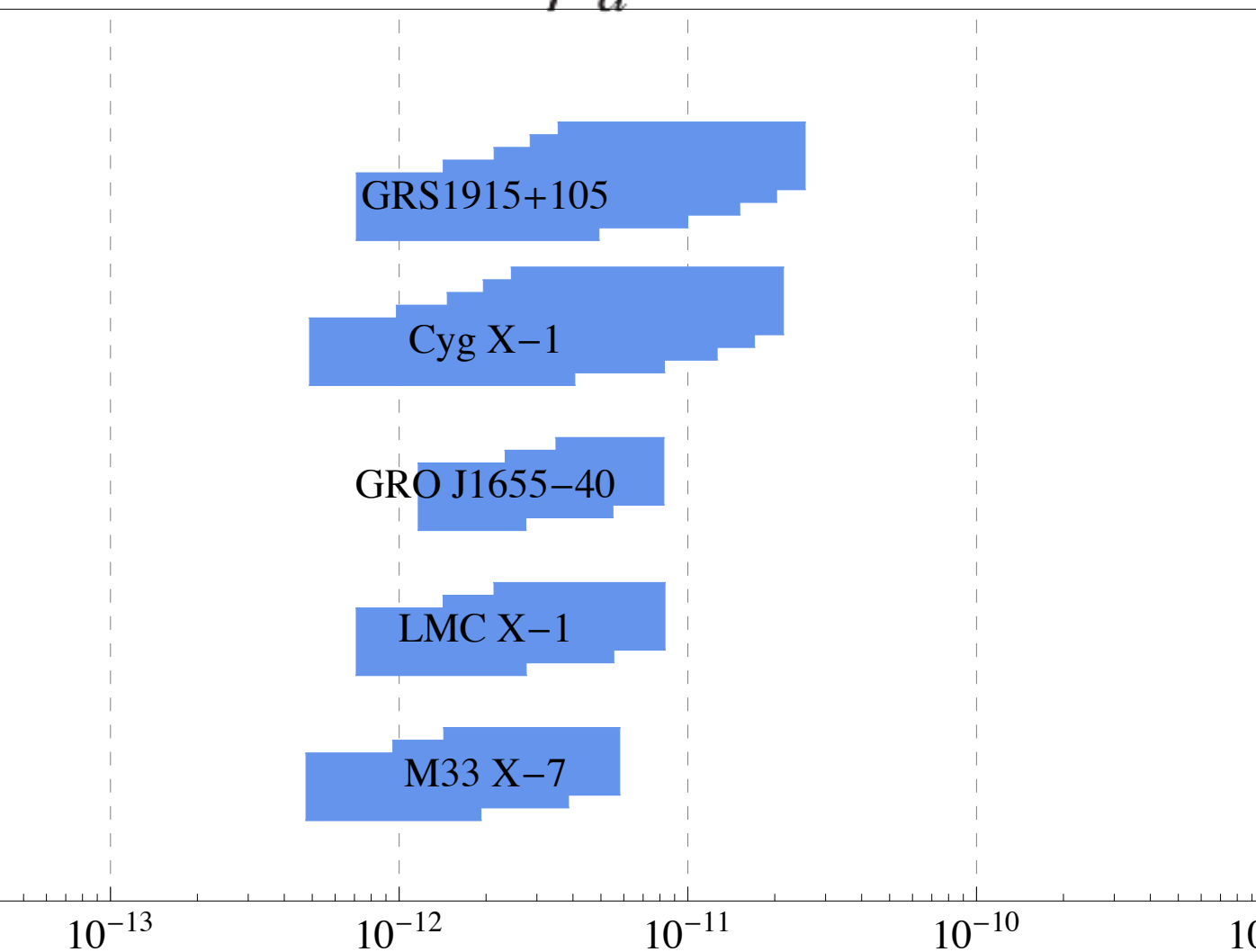


Black Hole Spins

Five stellar black holes and four SMBHs combine to disfavor the range:

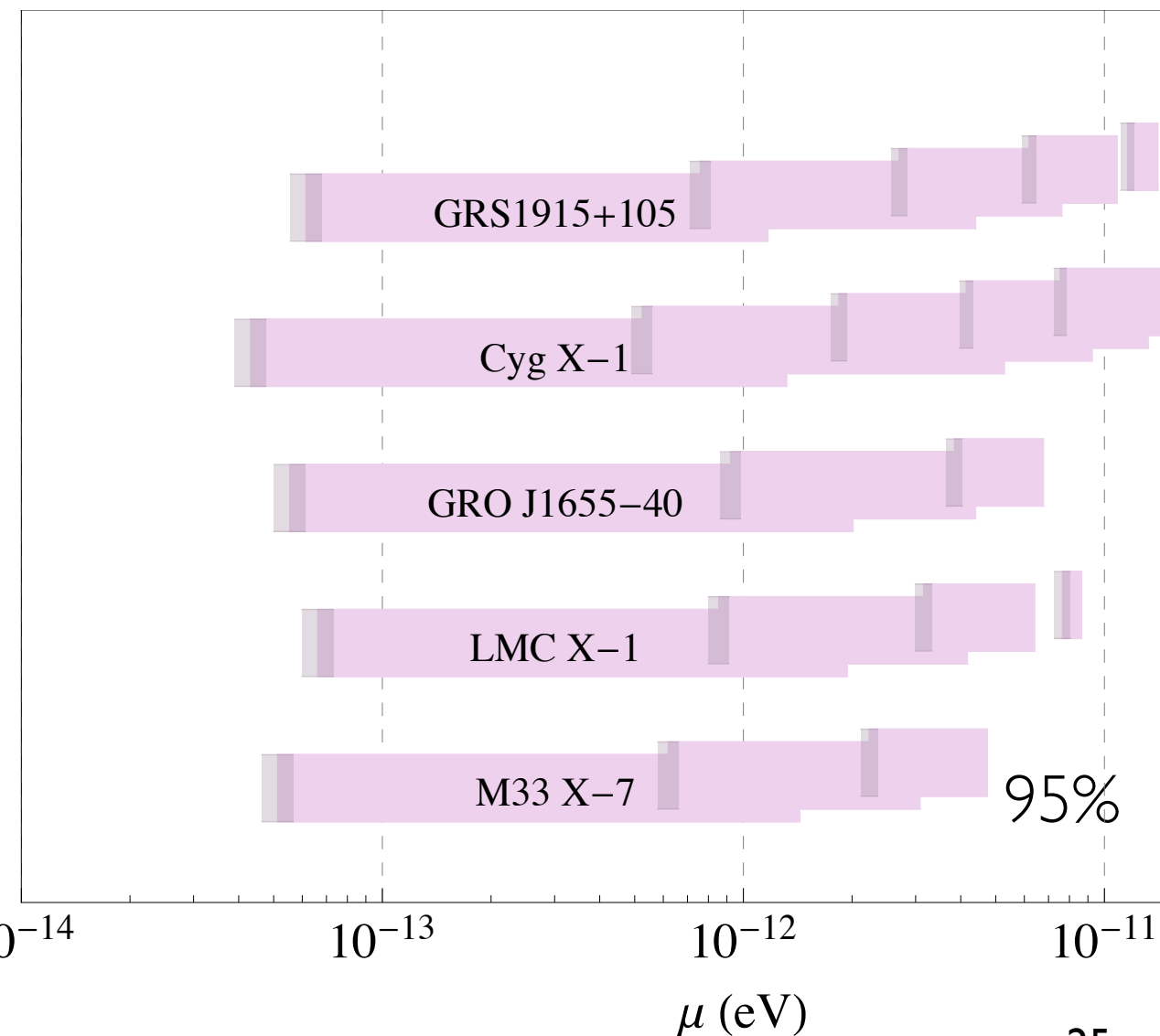
scalar

$$2 \times 10^{-11} > \mu_a > 6 \times 10^{-13} \text{ eV}$$



vector

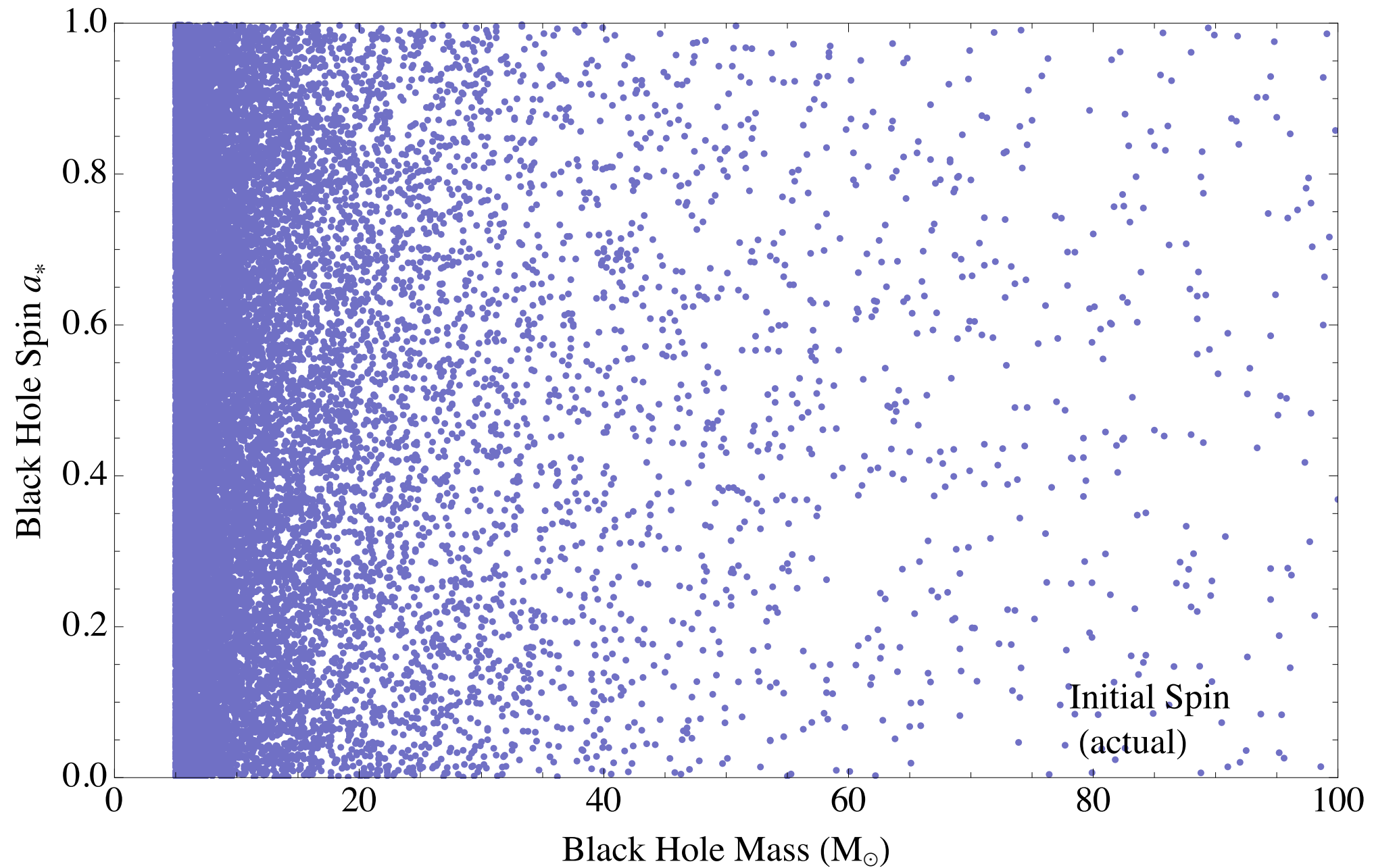
$$2 \times 10^{-11} \gtrsim \mu_V \gtrsim 5 \times 10^{-14} \text{ eV}$$



$$3 \times 10^{17} < f_a < 1 \times 10^{19} \text{ GeV}$$

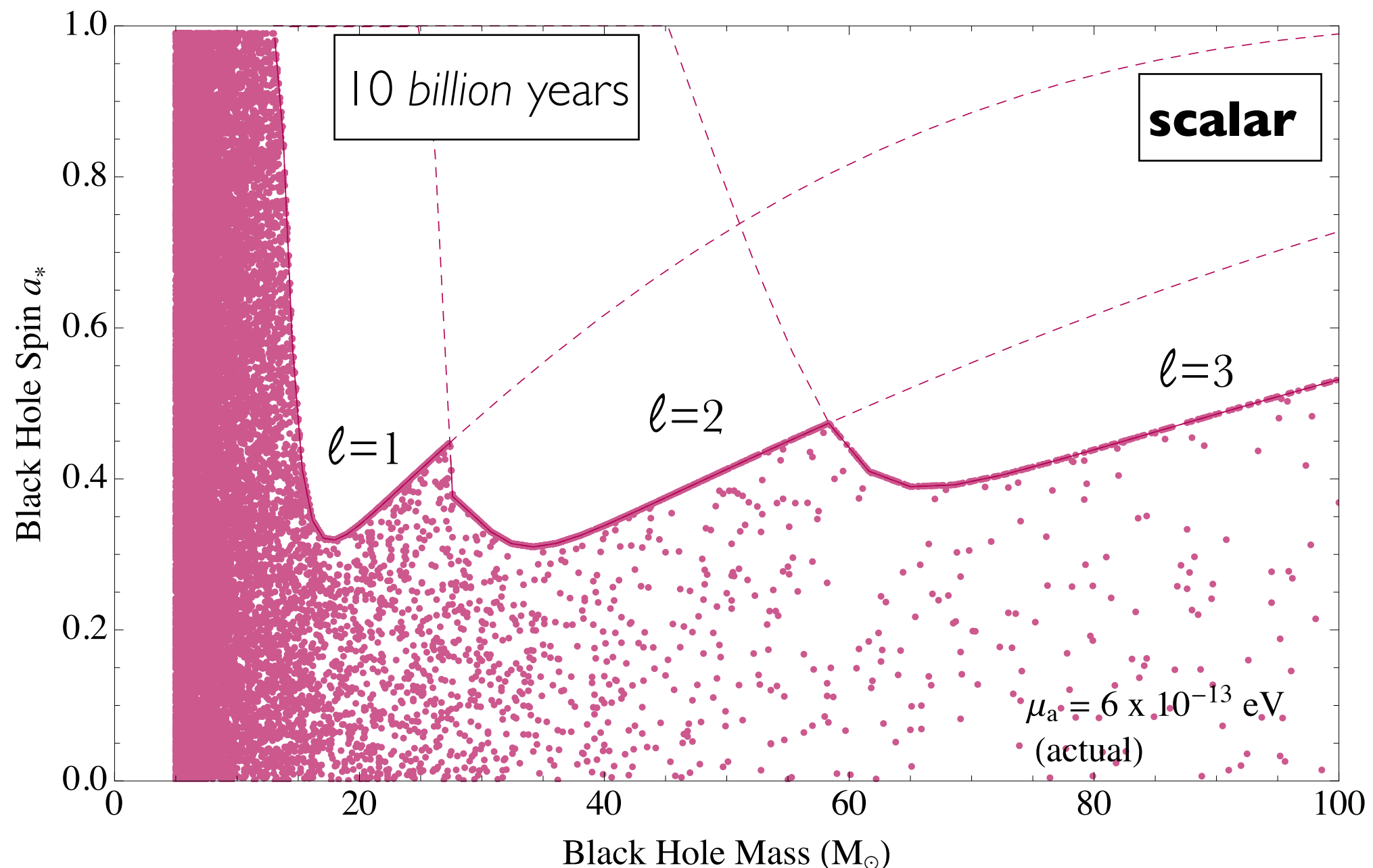
Black Hole Spins at LIGO

9-240 BBHs/Gpc³/yr. — 1000s of BHs merging in
low-redshift universe —



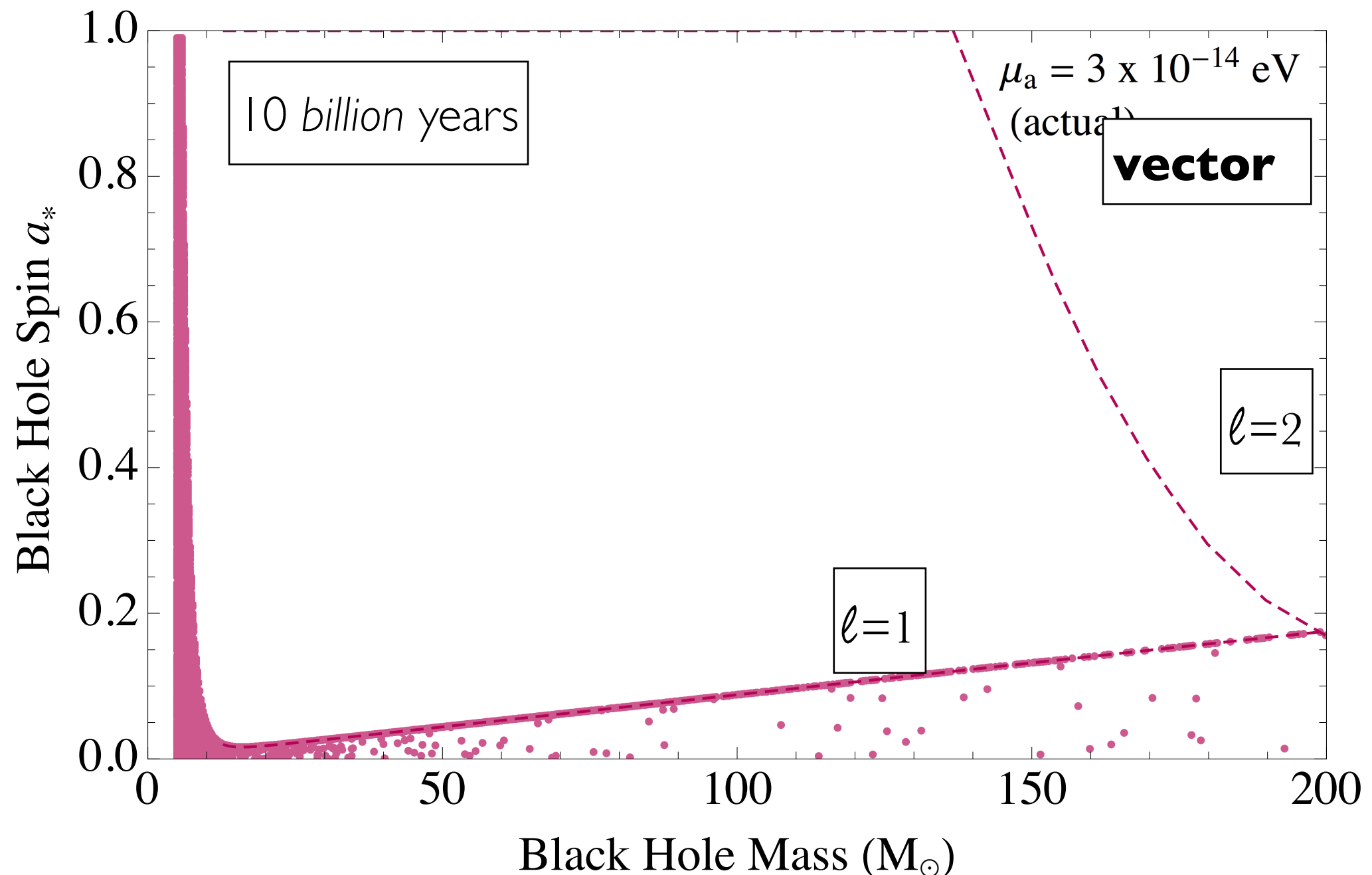
Black Hole Spins at LIGO

If light axion exists, many initial BHs would have low spin due to superradiance, limited by age and radius of binary system



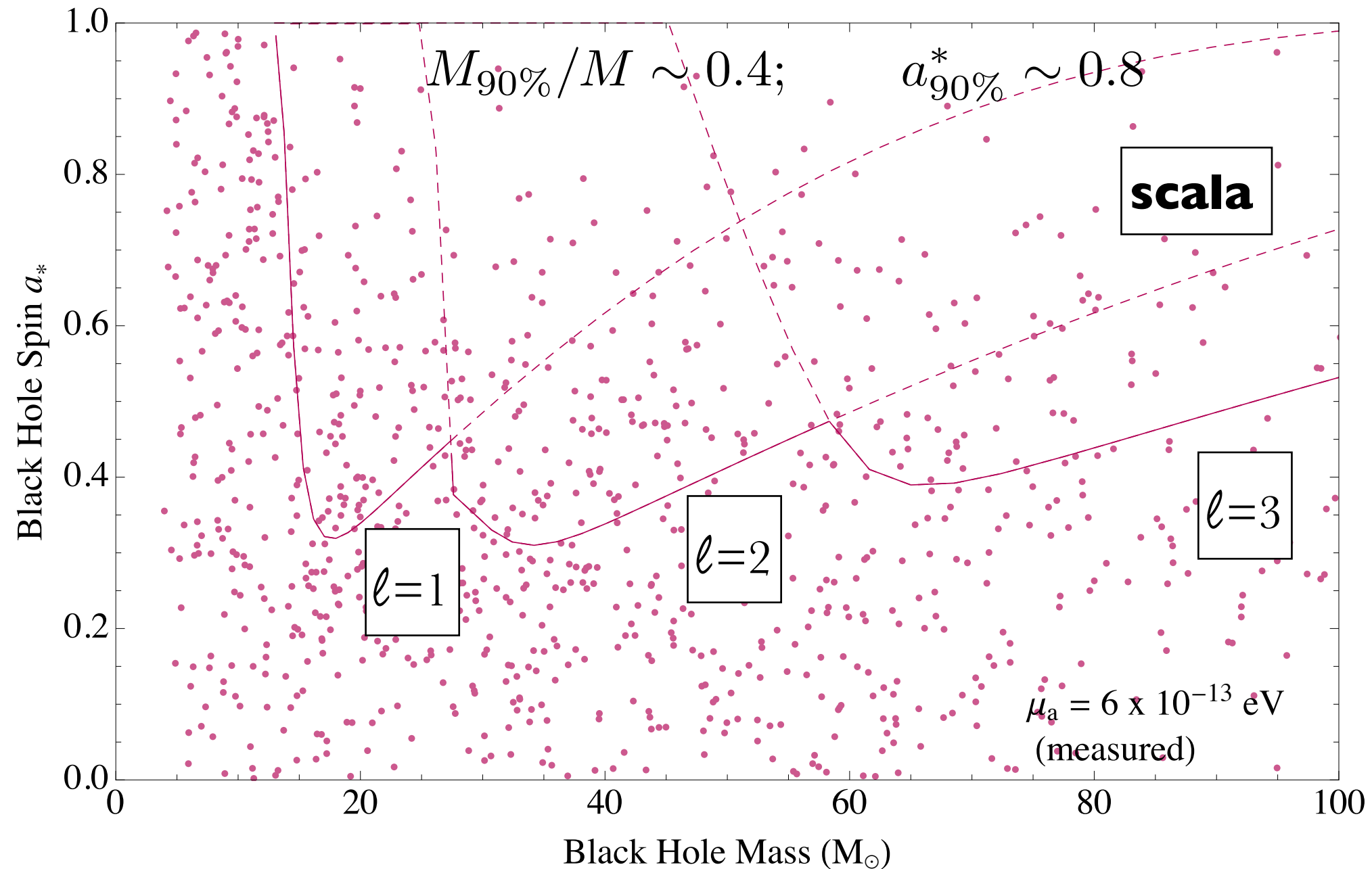
Black Hole Spins at LIGO

For light vector, the spindown is even more dramatic, limited by age
binary system; first level not affected by mixing



Black Hole Spins

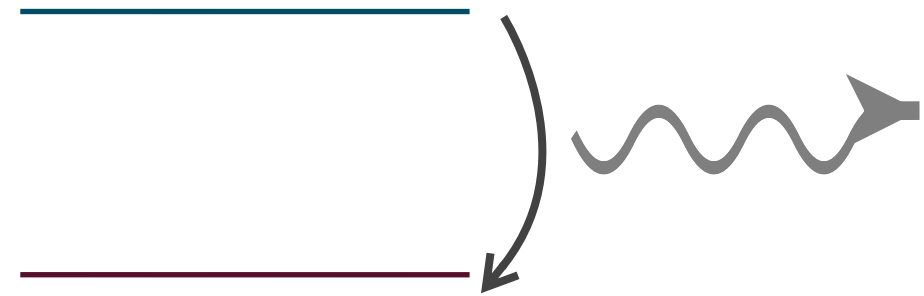
LIGO may measure hundreds of BHs in spin-mass plane



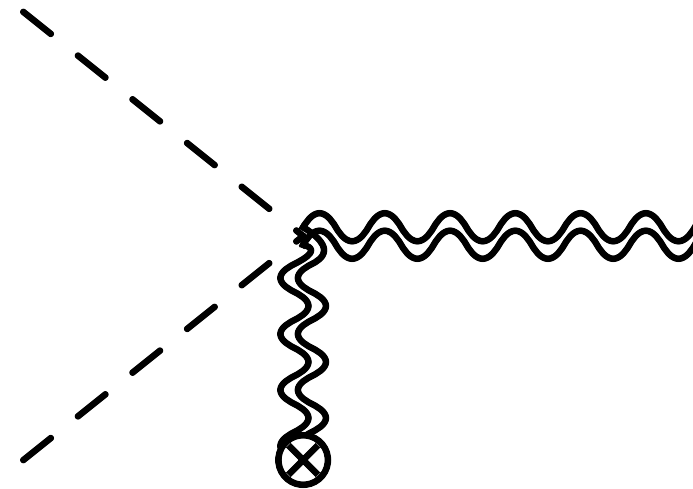
Can find statistical evidence for superradiance-like features with 50-200 merger measurements

Gravitational Wave Signals

- Transitions between levels

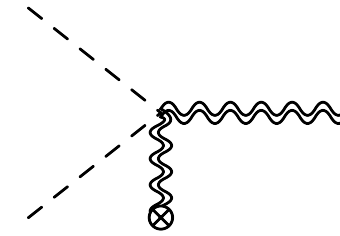


- Annihilations to gravitons

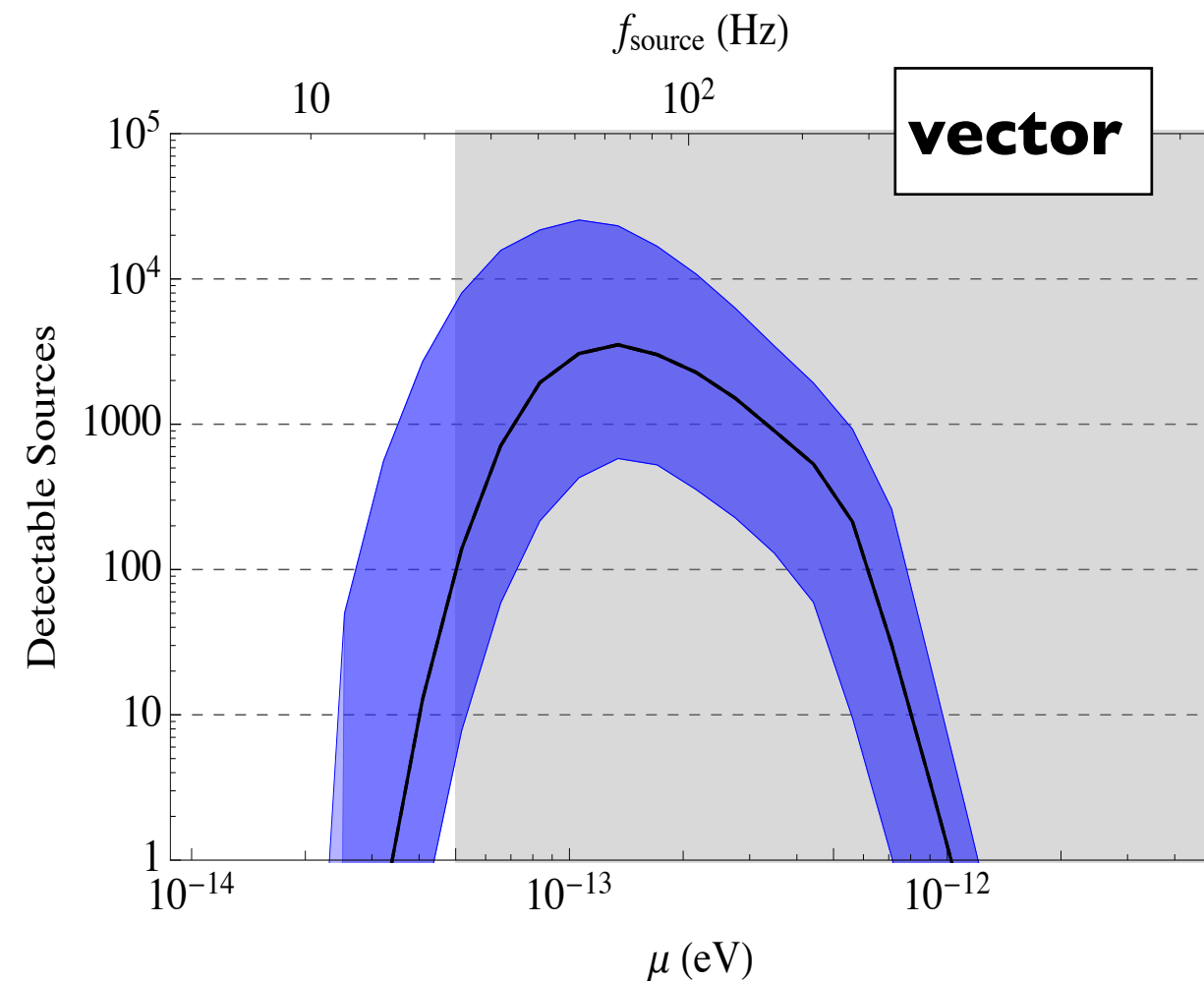
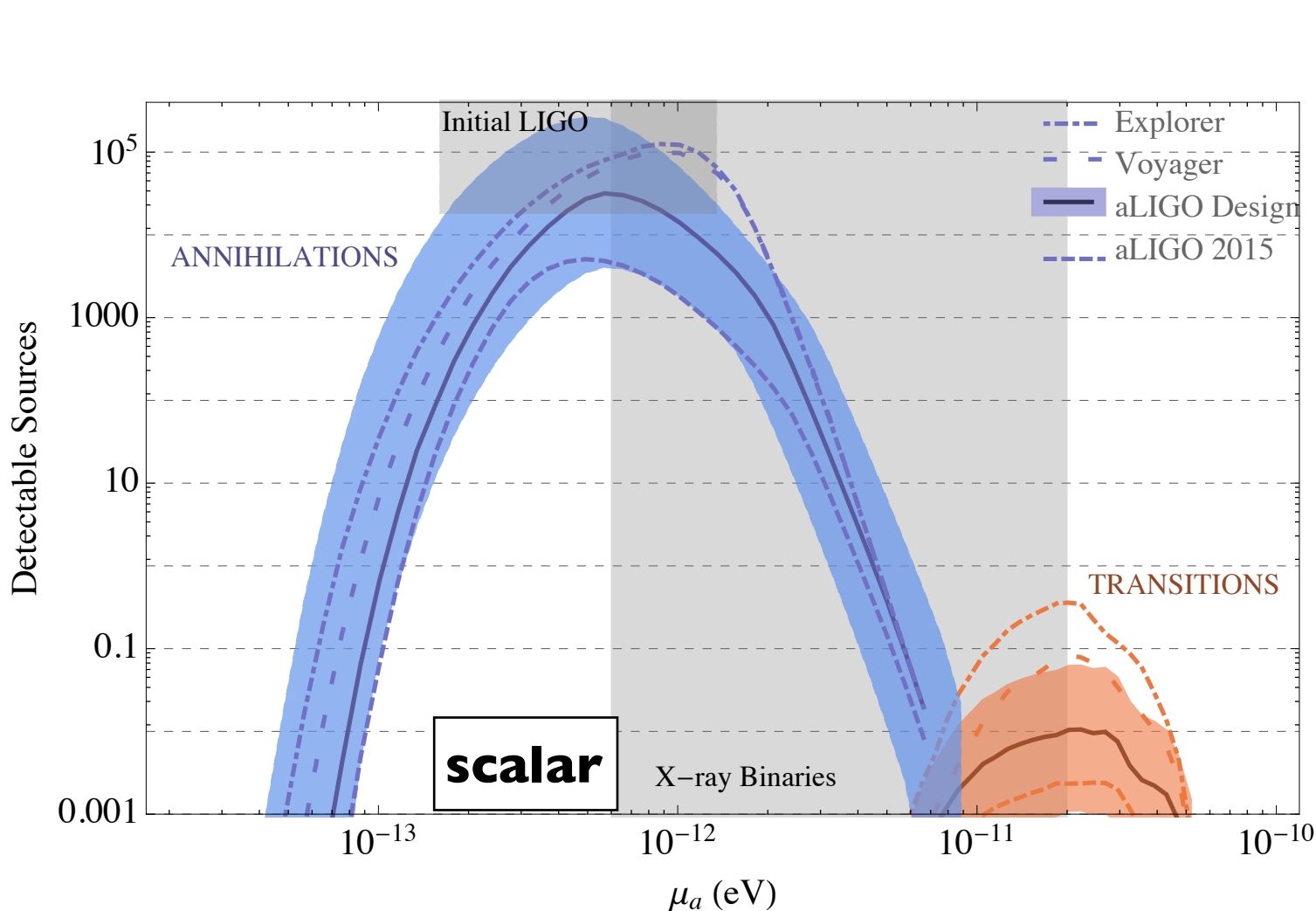


- Signals coherent, monochromatic, last hours to millions of years

Annihilations



- Event rates up to 10,000 — can be observed and studied in detail
- Uncertainty dominated by BH mass distribution at higher masses

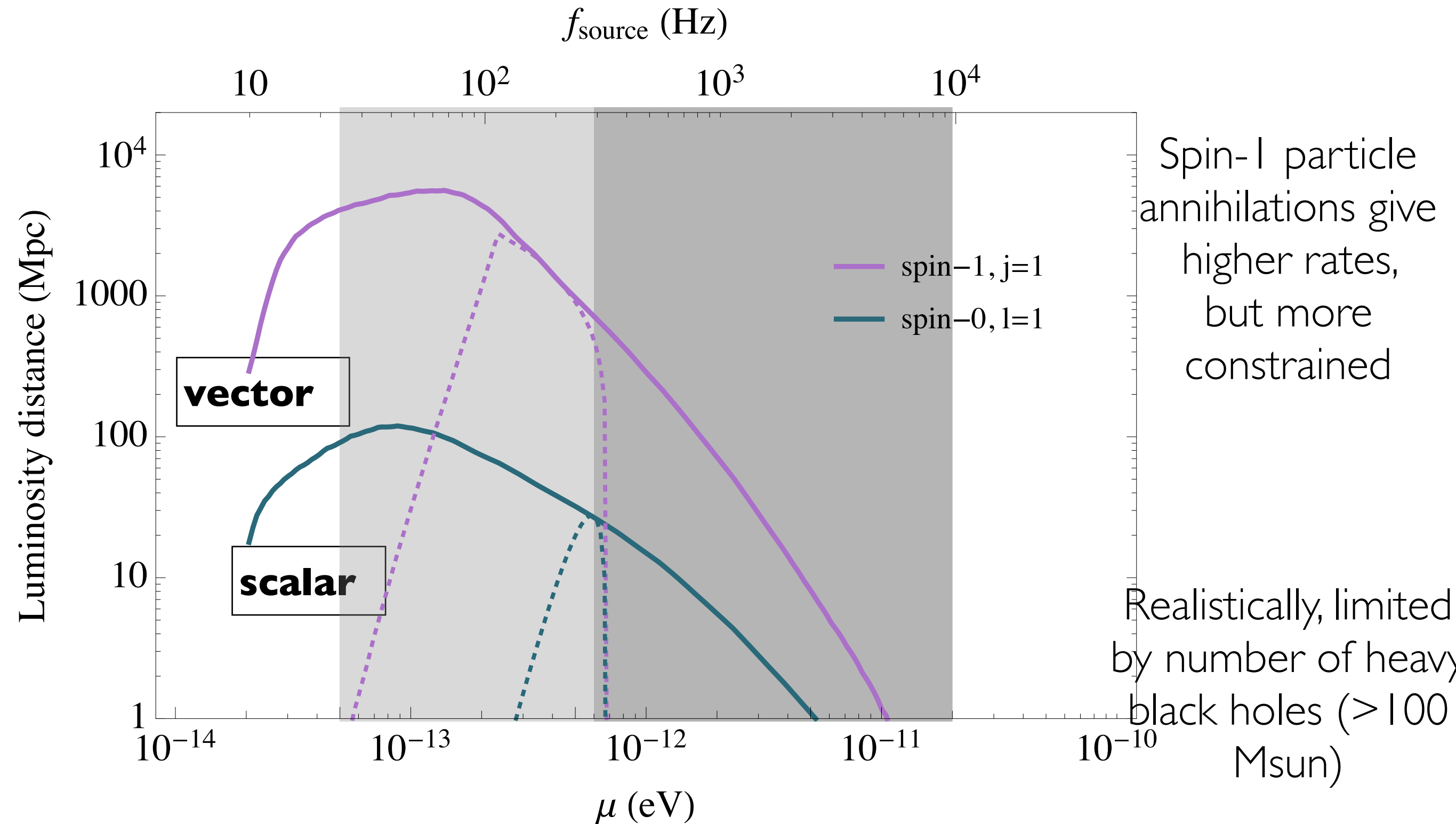
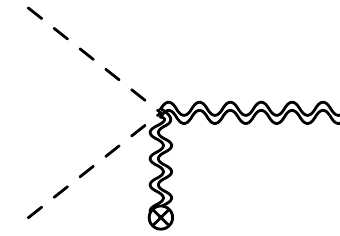


Long, weak signals visible from galactic center, limited by LIGO noise floor

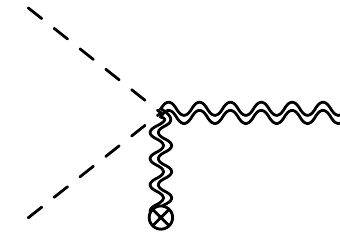
Signals coherent and monochromatic: Fits into searches for **long, continuous, monochromatic** gravitational waves (“mountains” on neutron stars)

Cross-check spin limits

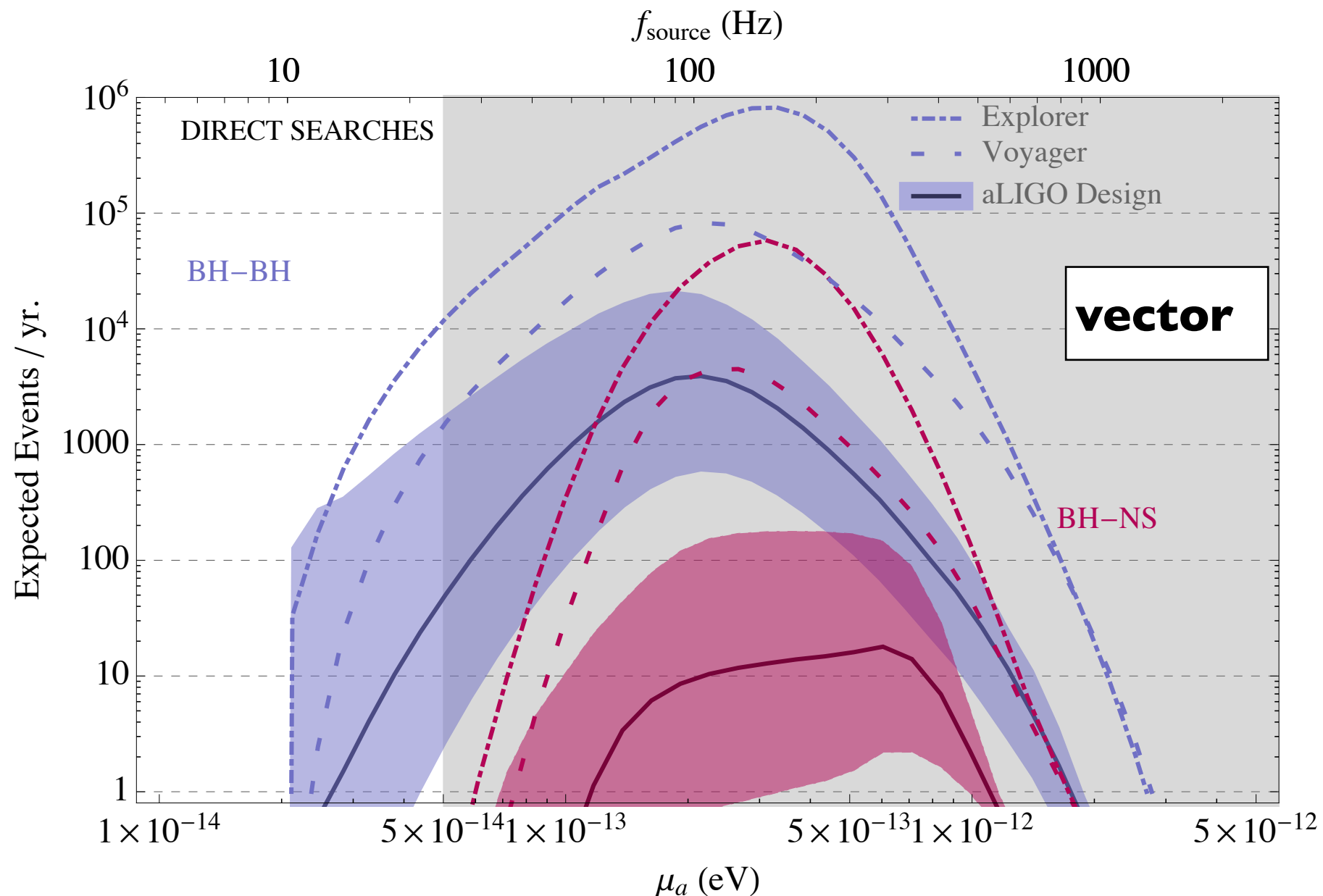
Annihilations



Annihilations



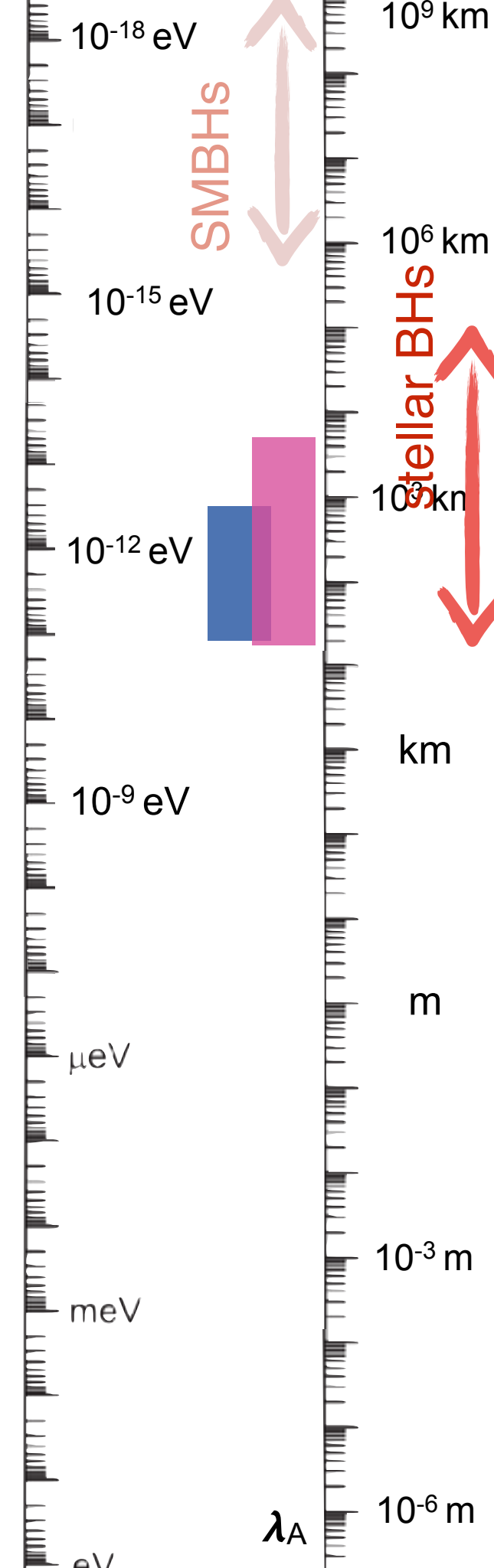
- Mergers at LIGO: a black hole is born!
- Follow up with continuous wave search to see if superradiance creates a cloud of axions around the new BH
- Targeted searches especially promising at future GW observatories



Conclusions

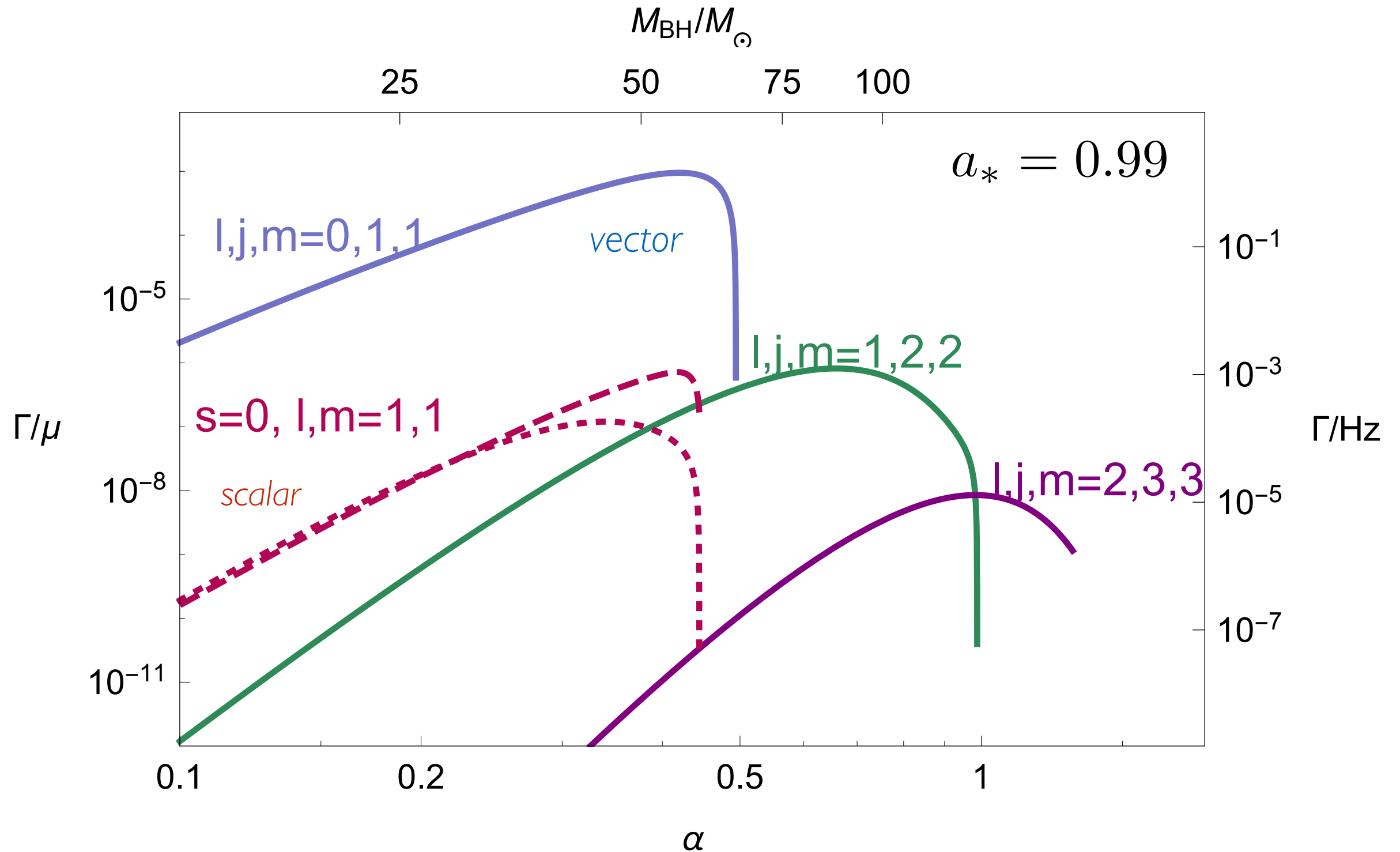
- Ultra light axions, scalars, vectors, ..., can be constrained or discovered by measurements of astrophysical black holes
- Independent of background density and coupling
- BH spin measurements exclude previously open parameter space
- Advanced LIGO may measure thousands of BH spins and provide evidence of a new light particle
- Continuous GW signals may be observable from annihilations of scalars or vectors
- May observe growth of gravitational atom after a merger in real time

Thank you!



Extra

Analytic superradiance rates



- For massless boson, scattering probability depends on $\mathbf{j}$, wavefunction value depends on $\mathbf{l}$

$$\Gamma_{sr}^{nlm} \sim \mu \alpha^{2l+2j+4} (m\Omega_{BH} - \mu) r_+ C_{njl m} \lesssim 10^{-3} \mu$$

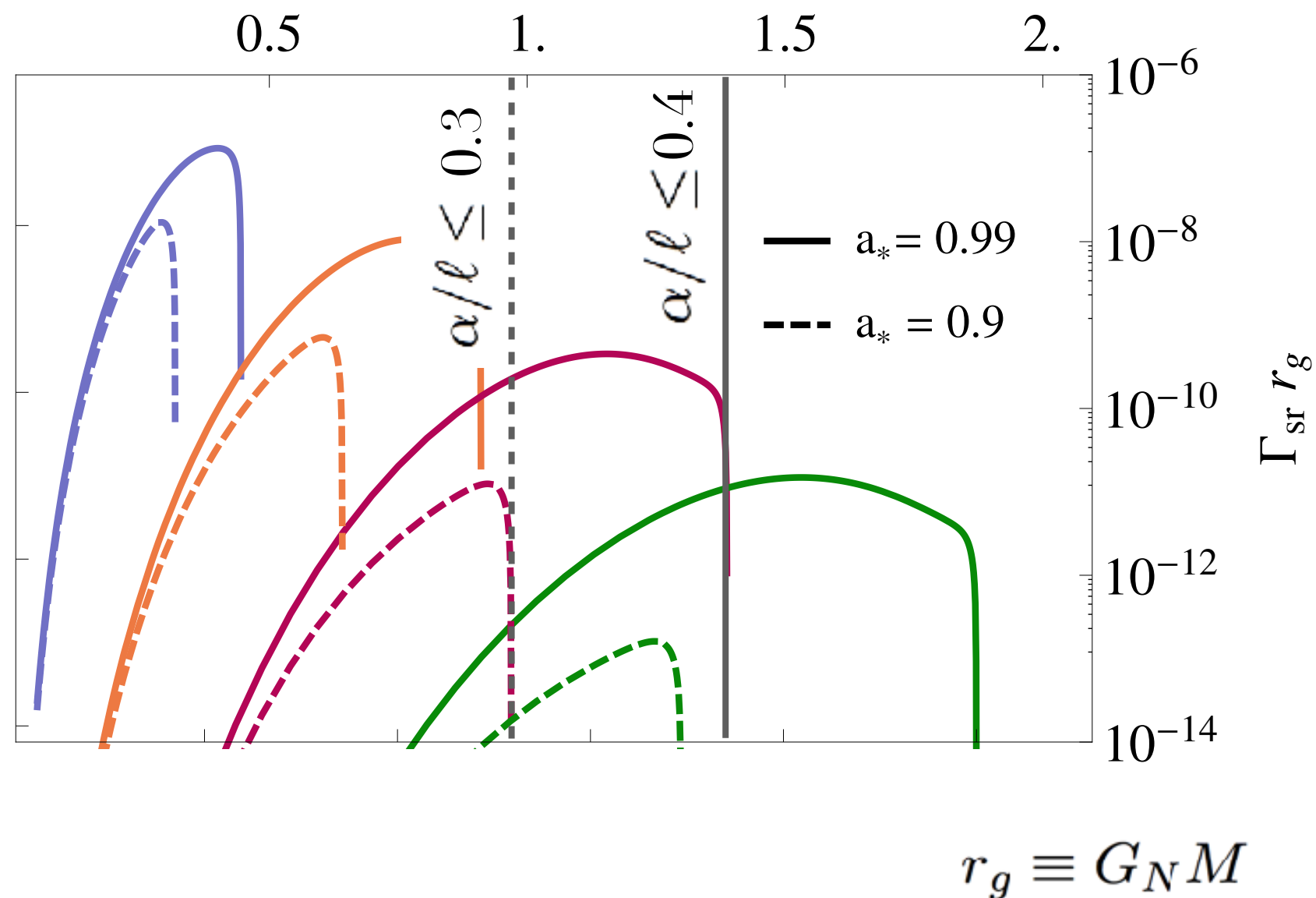
Superradiance

SR boundary function of α and spin a_* : $\alpha/\ell \leq 1/2$

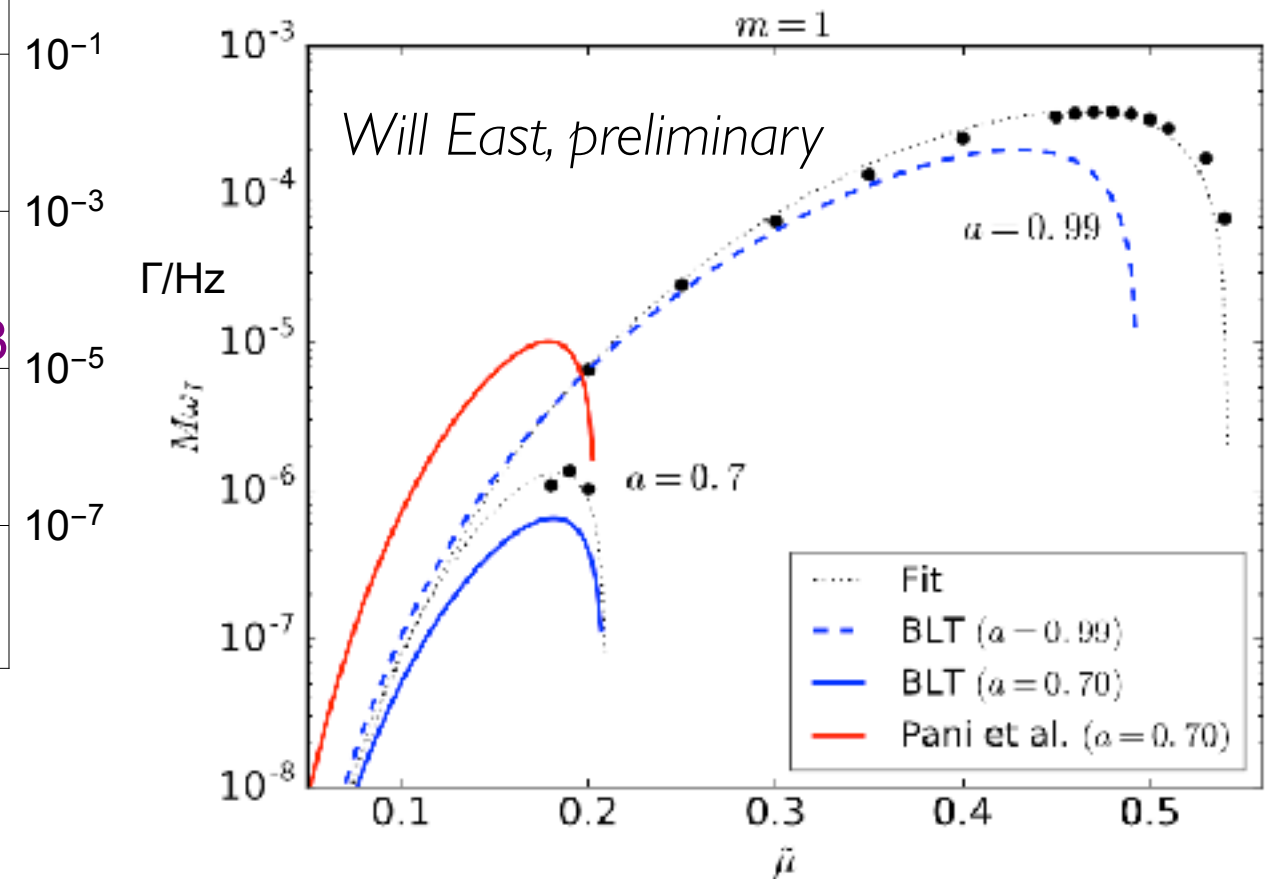
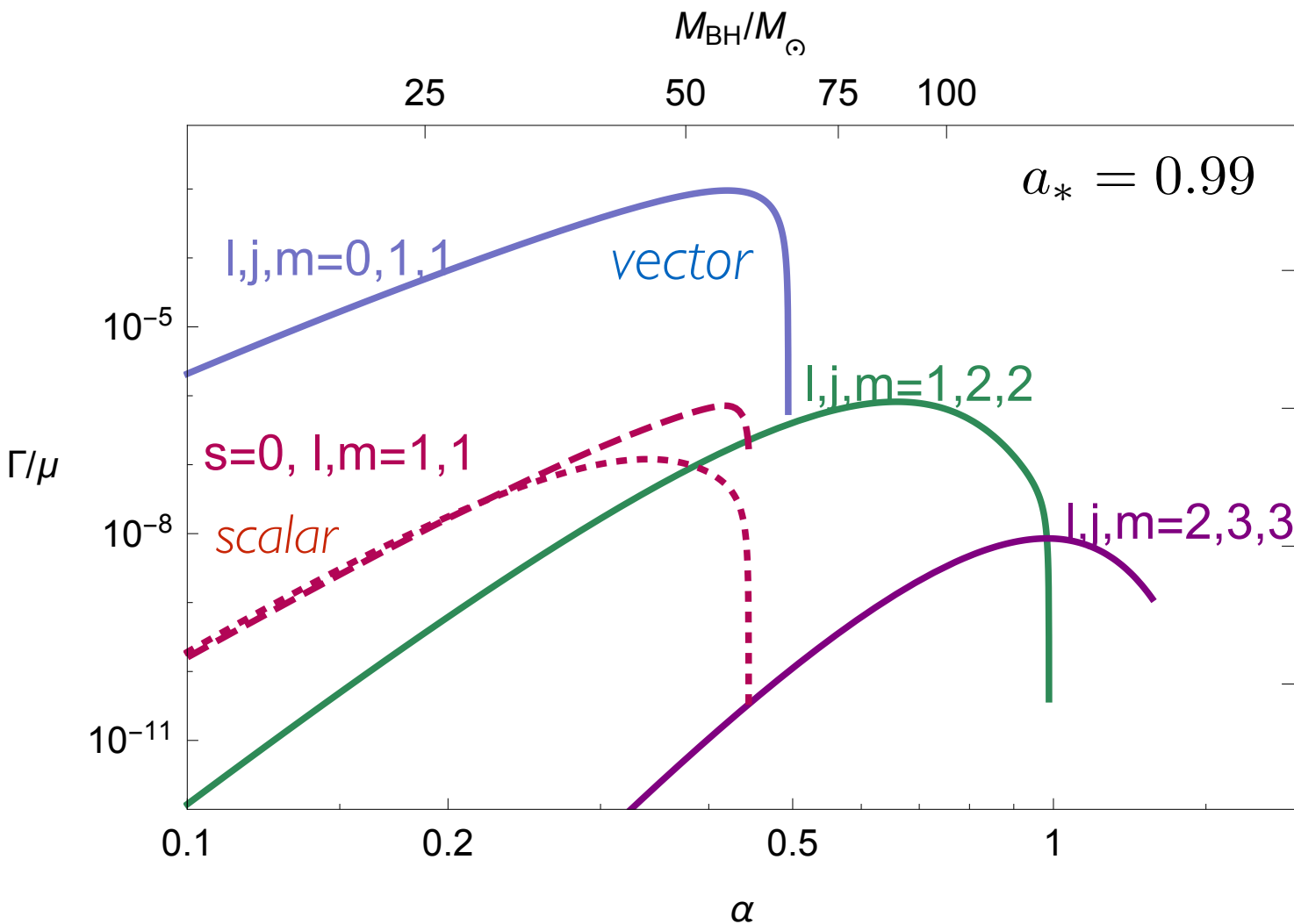
$$\frac{\omega}{m} < \omega^+, \quad \omega^+ \equiv \frac{1}{2} \left(\frac{a_*}{1 + \sqrt{1 - a_*^2}} \right) r_g^{-1}.$$

$$\alpha = G_N M_{\text{BH}} \mu_a$$

- Strong dependence on ℓ
- Steep function of coupling α
- Depends on BH spin a_*
- One superradiance time lasts between 100 s and 100 years



Vector superradiance rates



- Analytic expression is accurate to leading order in α , for given a_*
- Not a small a_* approximation

Analytic superradiance rates

- For massless boson, absorption probability is

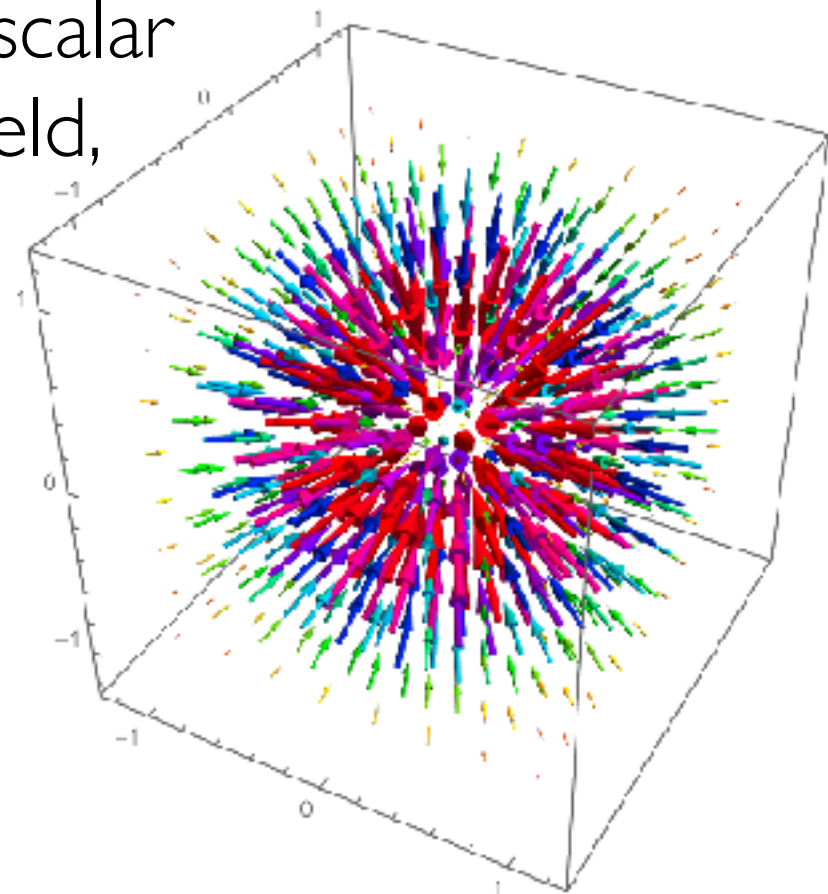
$$\mathbb{P}_{\text{abs}} = \left(\frac{(j-s)!(j+s)!}{(2j)!(2j+1)!!} \right)^2 \prod_{n=1}^j \left[1 + \left(\frac{\omega - m\Omega}{n\kappa} \right)^2 \right] 2 \left(\frac{\omega - m\Omega}{n\kappa} \right) \left(\frac{A\kappa}{2\pi} \omega \right)^{2j+1}$$

- Split light vector into transverse (“massless vector”) and longitudinal (“massless scalar”) modes. Massless scalar component corresponds to A_0 part of vector field,

$$D_\mu A^\mu = 0 \Rightarrow \partial_i A_i \simeq \partial_0 A_0$$

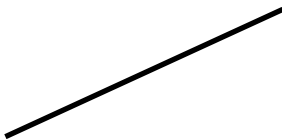
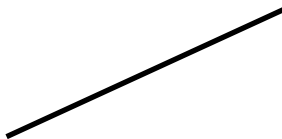
- e.g. monopole mode: $A_0 \sim \frac{1}{\mu a_0} |A_i|_{a_0} Y^{00}$

Decay “through scalar mode” with $\Gamma \simeq \frac{3}{2} \alpha^7 \mu$



$$j = 0, l = 1$$

Analytic superradiance rates

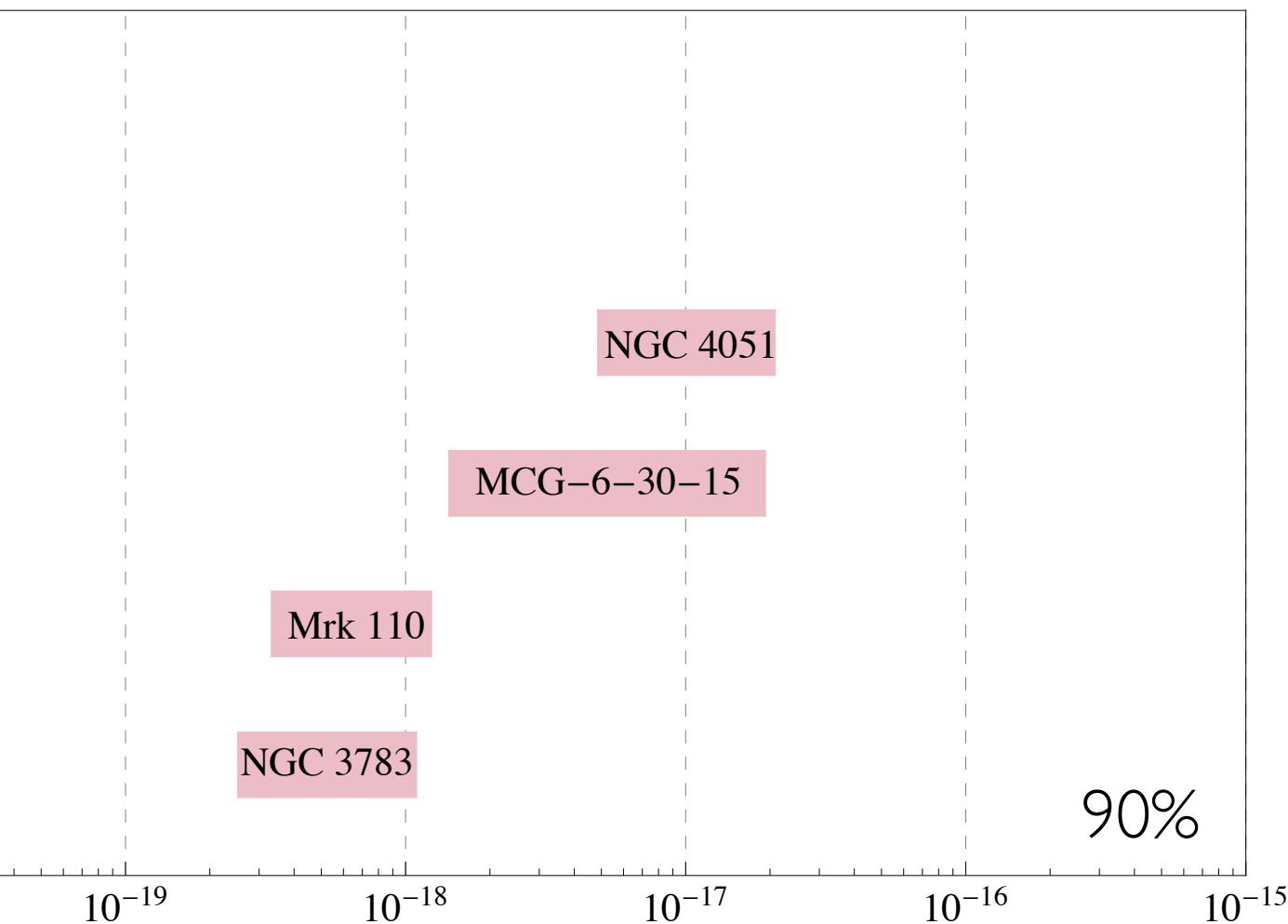
	vecto			scala
	$l = j - 1$	$l = j$	$l = j + 1$	l
Leading growth via:	vector	vector	scalar (+vector)	scalar
$j = 0$			$-\frac{3}{2}\alpha^7\mu + \dots$	$-8\alpha^5\mu$
$j = 1$	$4a_*\alpha^6\mu + \dots$	$\frac{1}{6}a_*\alpha^8\mu + \dots$	$\sim a_*\alpha^{10}\mu + \dots$	$\frac{1}{24}a_*\alpha^8\mu + \dots$
$j = 2$	$\sim a_*\alpha^{10}\mu + \dots$	$\sim a_*\alpha^{12}\mu + \dots$	$\sim a_*\alpha^{14}\mu + \dots$	$\sim a_*\alpha^{12}\mu + \dots$

Vector rates are faster for the same total angular momentum,
and more robust to perturbations

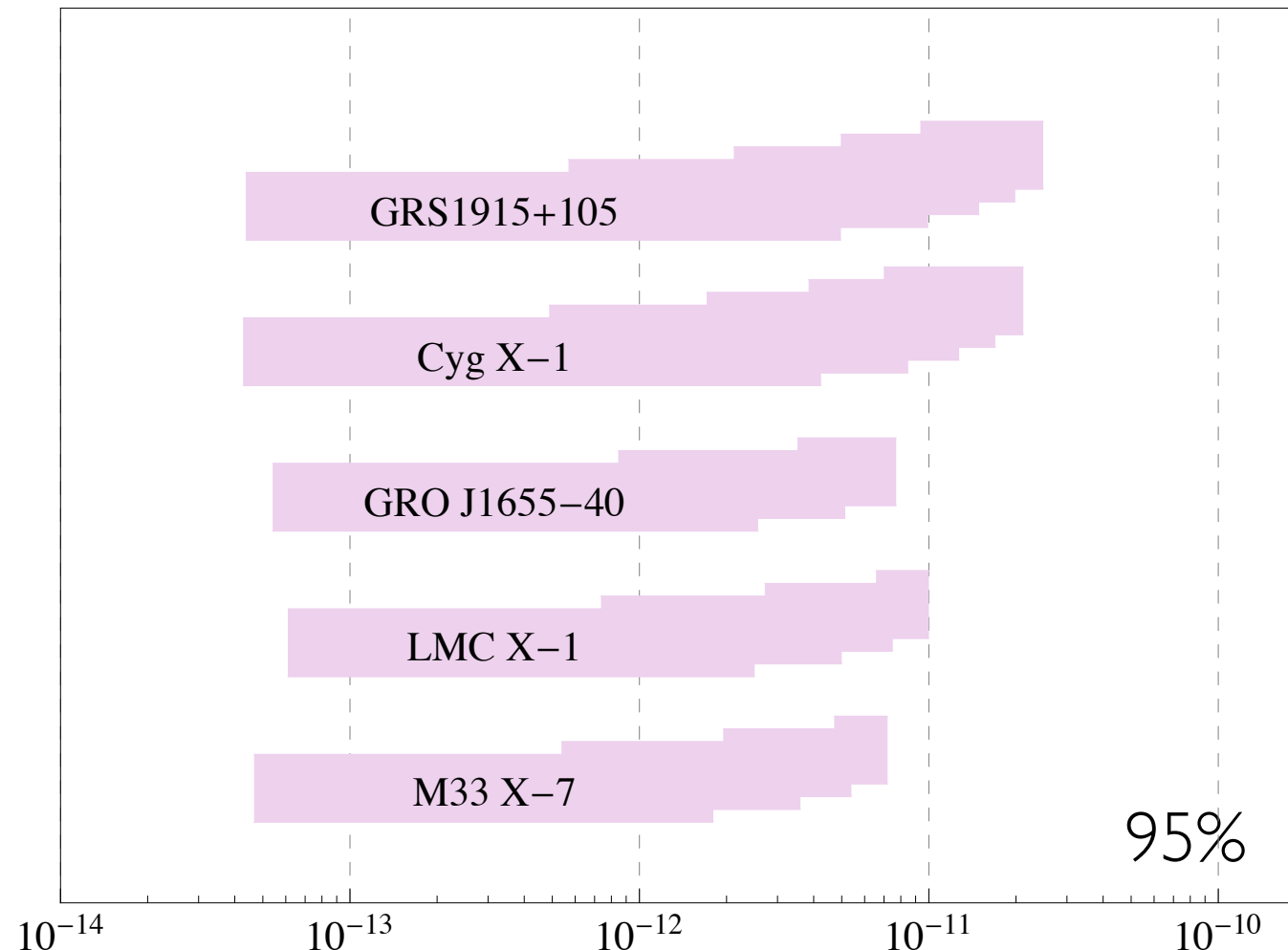
Black Hole Spins

Five stellar black holes and four SMBHs combine to disfavor the range:

$$2.5 \times 10^{-19} < \mu_V < 2.1 \times 10^{-17} \text{ eV}$$

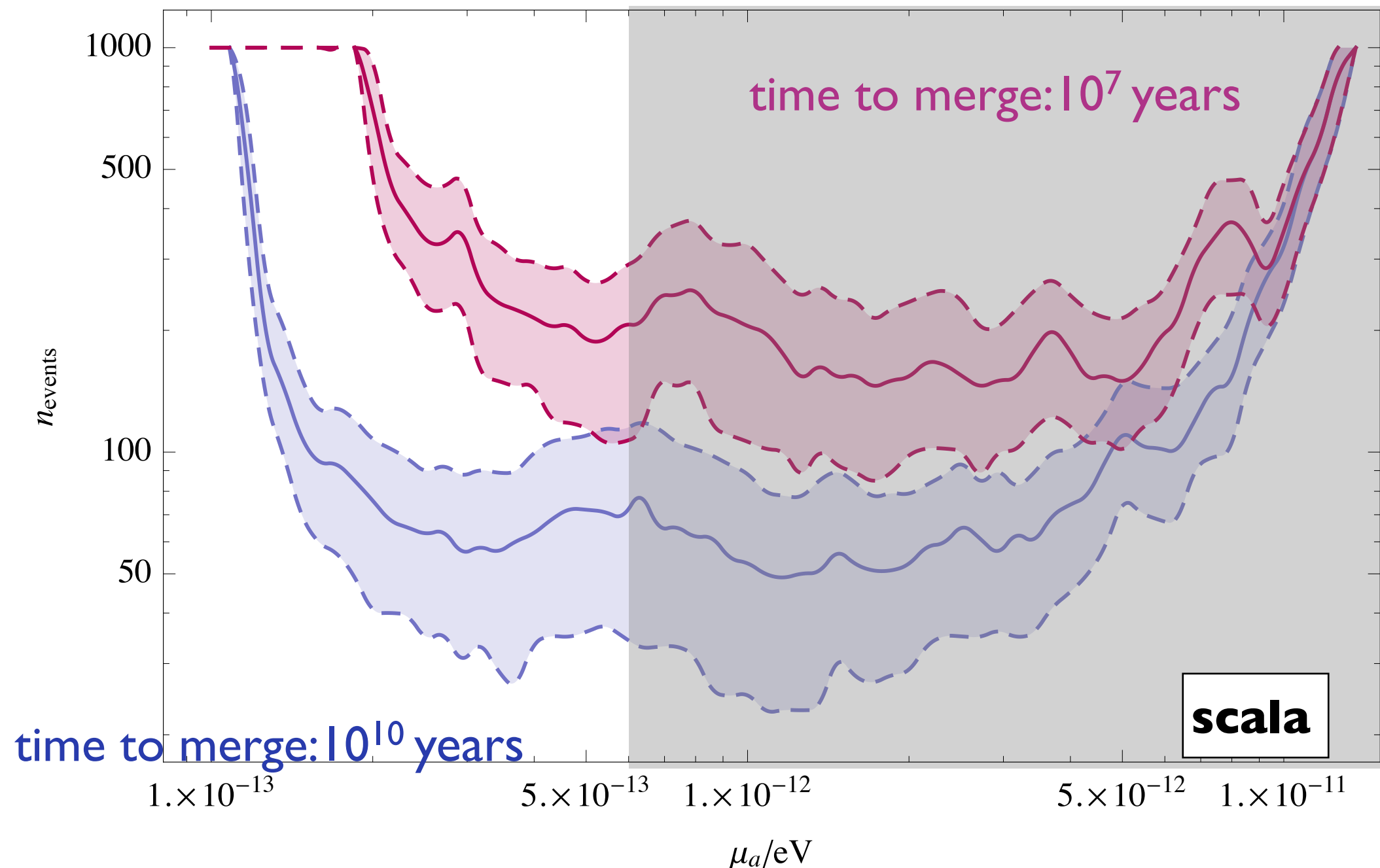


$$2 \times 10^{-11} \gtrsim \mu_V \gtrsim 5 \times 10^{-14} \text{ eV}$$



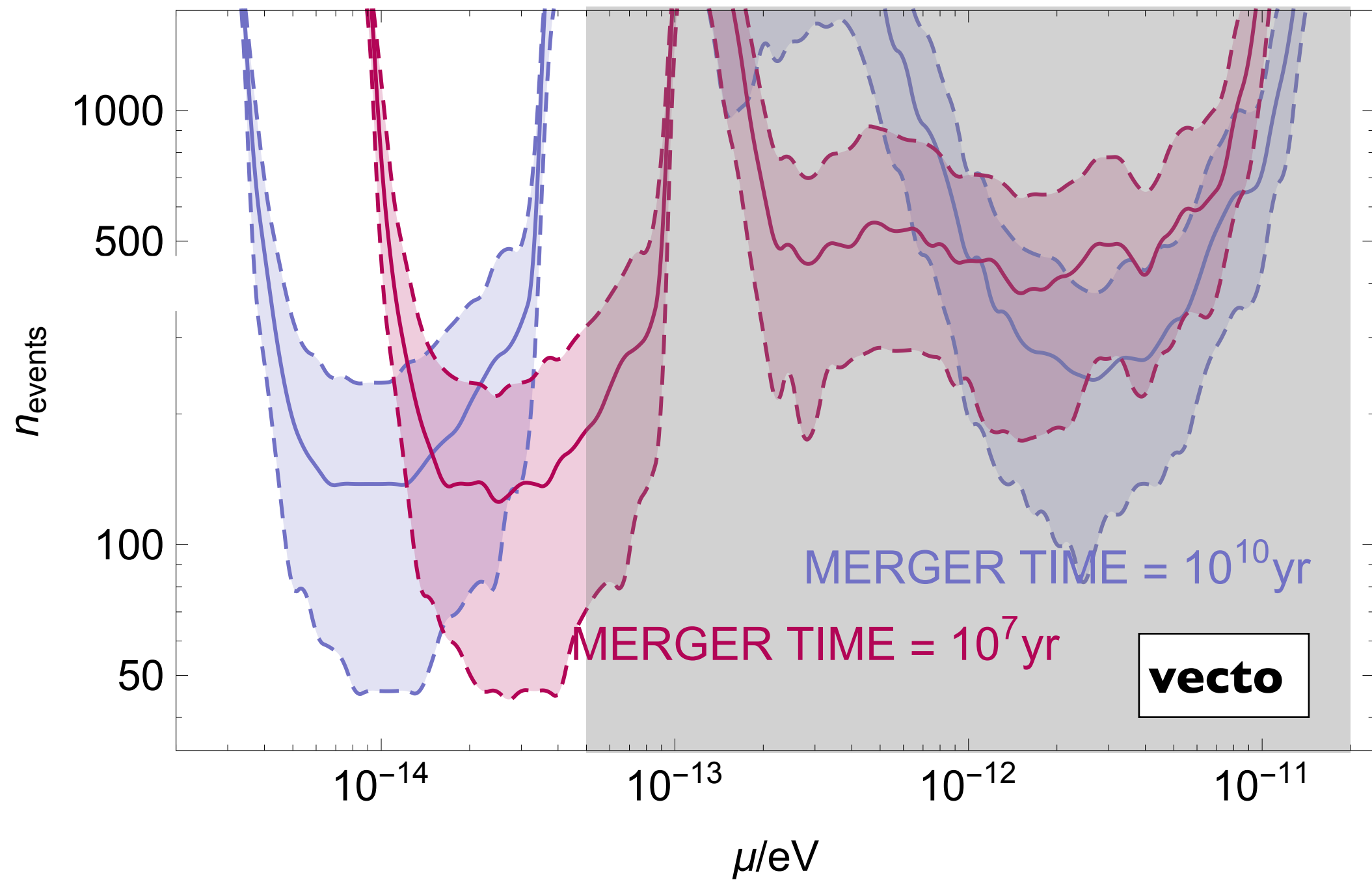
Black Hole Spins

Can find statistical evidence for deficit of high spins in a range of BH masses with 50-200 measurements:



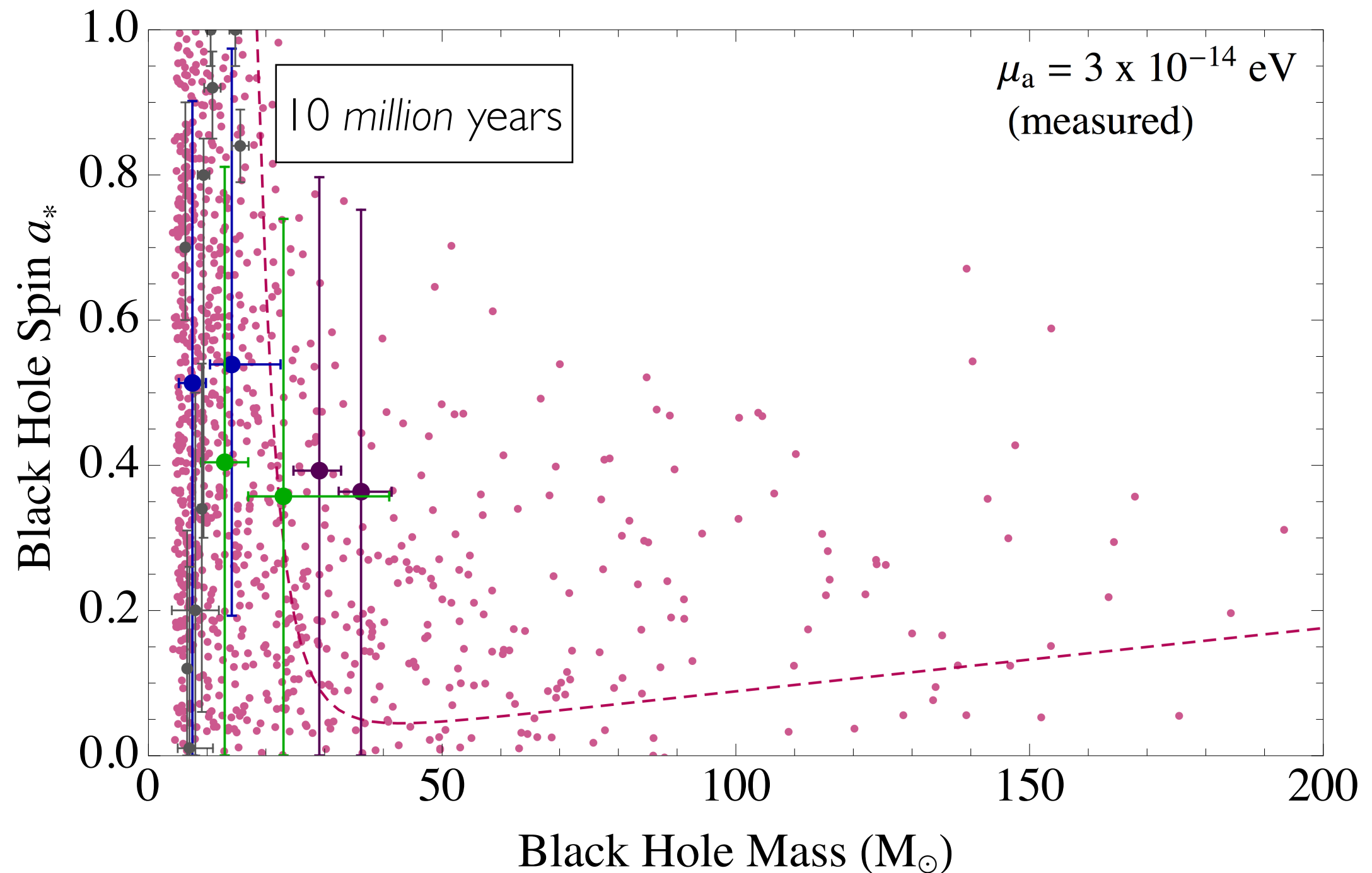
Black Hole Spins

Can find statistical evidence for deficit of high spins in a range of BH masses with 50-200 measurements:



Black Hole Spins

May see spin-down of black holes at LIGO outside of excluded region

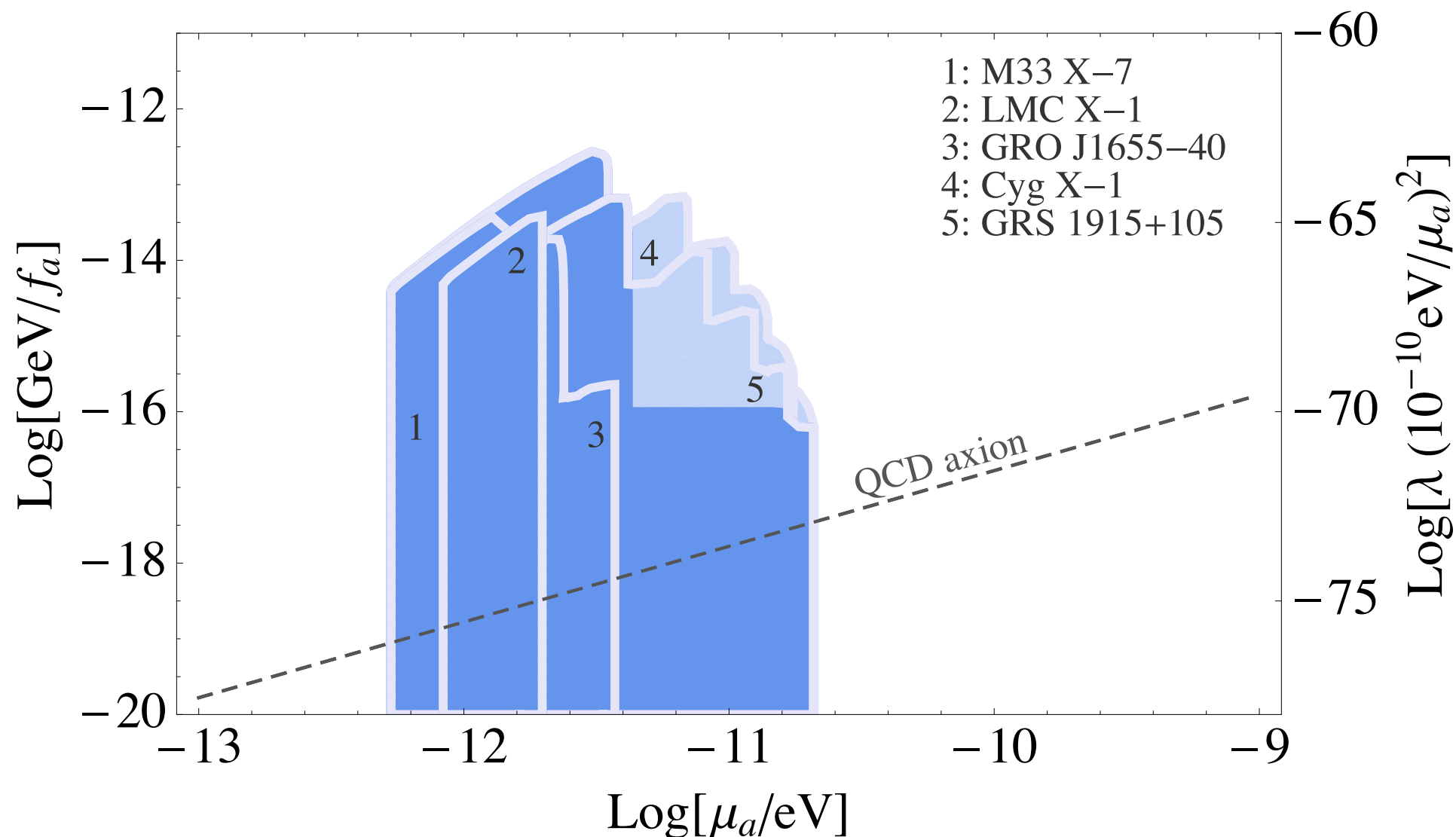


Black Hole Spins, coupling

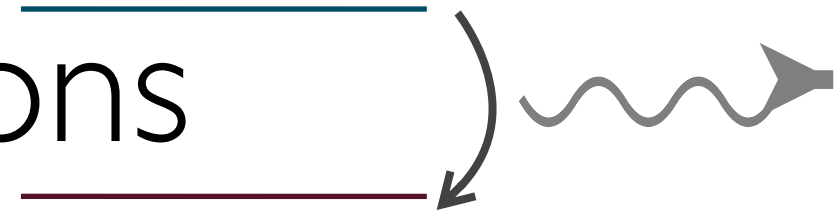
Five currently measured black holes combine to set limit:

$$2 \times 10^{-11} > \mu_a > 6 \times 10^{-13} \text{ eV}$$

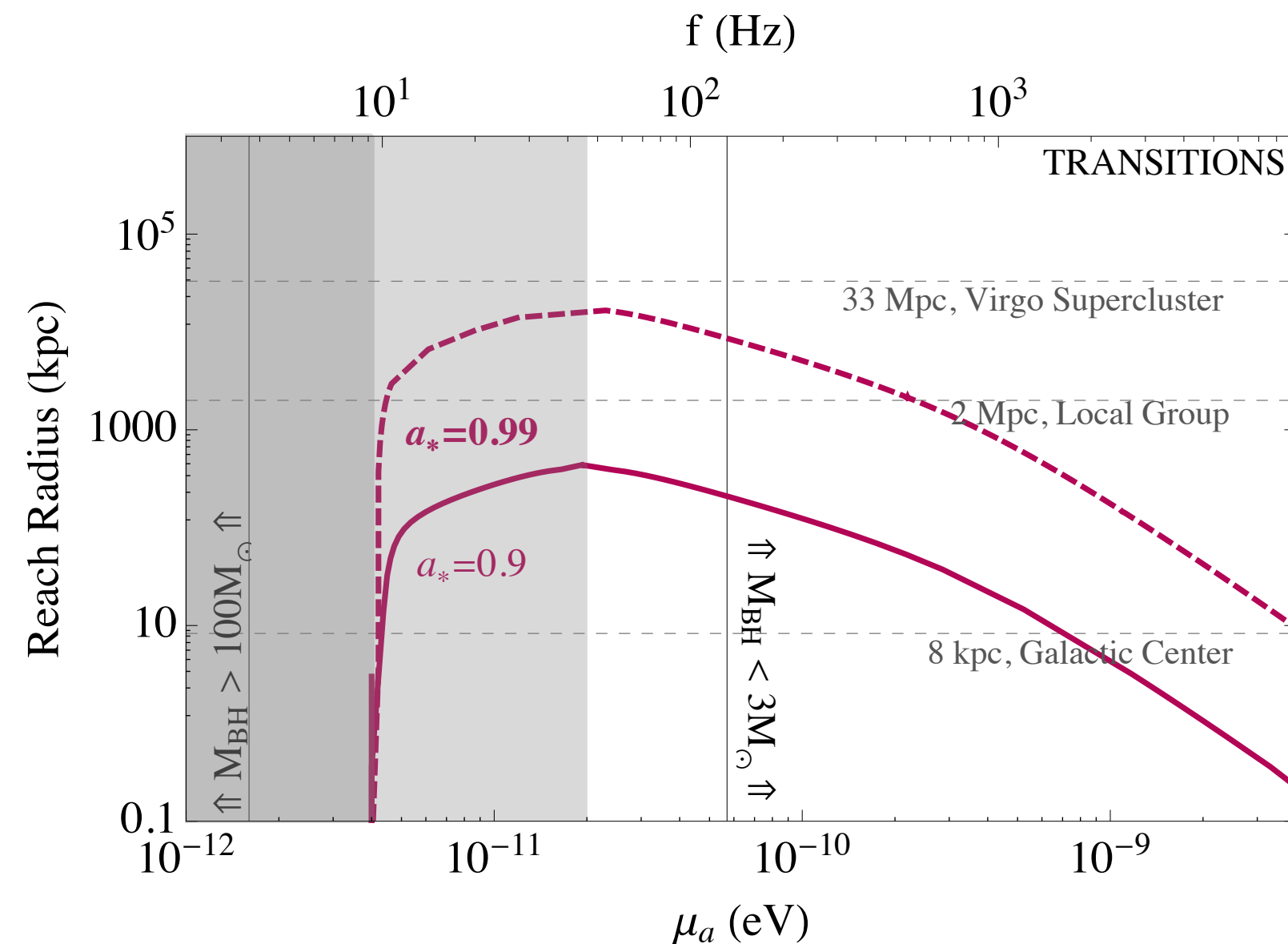
$$3 \times 10^{17} < f_a < 1 \times 10^{19} \text{ GeV}$$



Transitions



Optimal reach of advanced LIGO



Using
monochromatic
search for rotating
neutron stars

Heavier BHs give
bigger signals

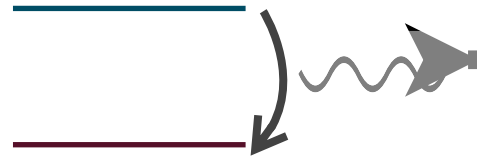
Cut off at low
masses by LIGO
sensitivity

Signals visible from
galactic center
typically last 10-100
yrs

BH with high spin, optimal mass, and is currently superradiating

Gravitational Wave Signals

Transitions

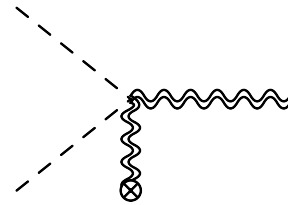


$$\frac{dN_e}{dt} = \Gamma_e^{\text{sr}} N_e - \Gamma_t N_e N_g$$

$$\frac{dN_g}{dt} = \Gamma_g^{\text{sr}} N_g + \Gamma_t N_g N_e$$

$$\frac{df}{dt} \simeq 10^{-11} \frac{\text{Hz}}{\text{s}} \left(\frac{f}{90 \text{ Hz}} \right) \left(\frac{M}{10 M_\odot} \right) \left(\frac{10^{17} \text{ GeV}}{f_a} \right)^2 \left(\frac{5 \text{ yr}}{T} \right)^2$$

Annihilations

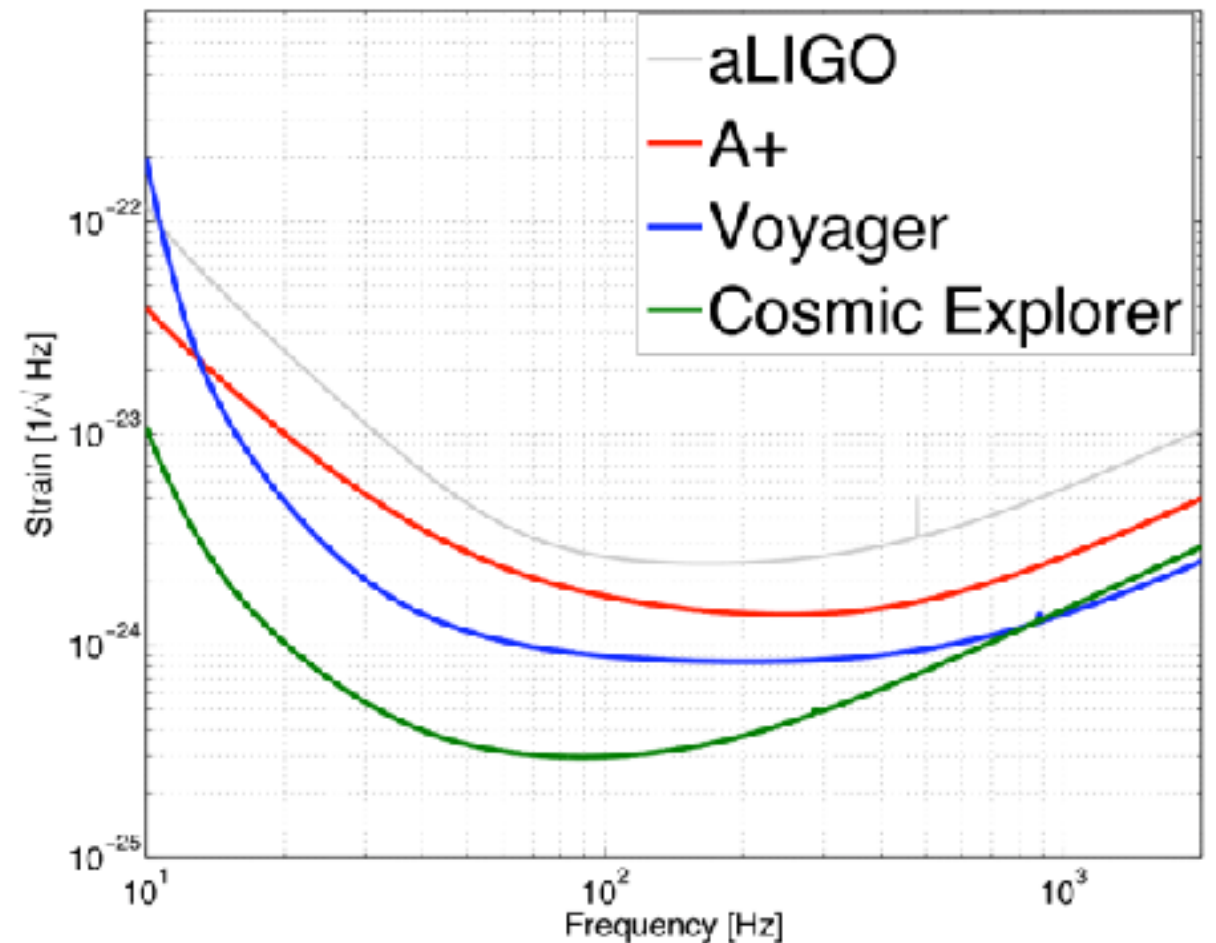
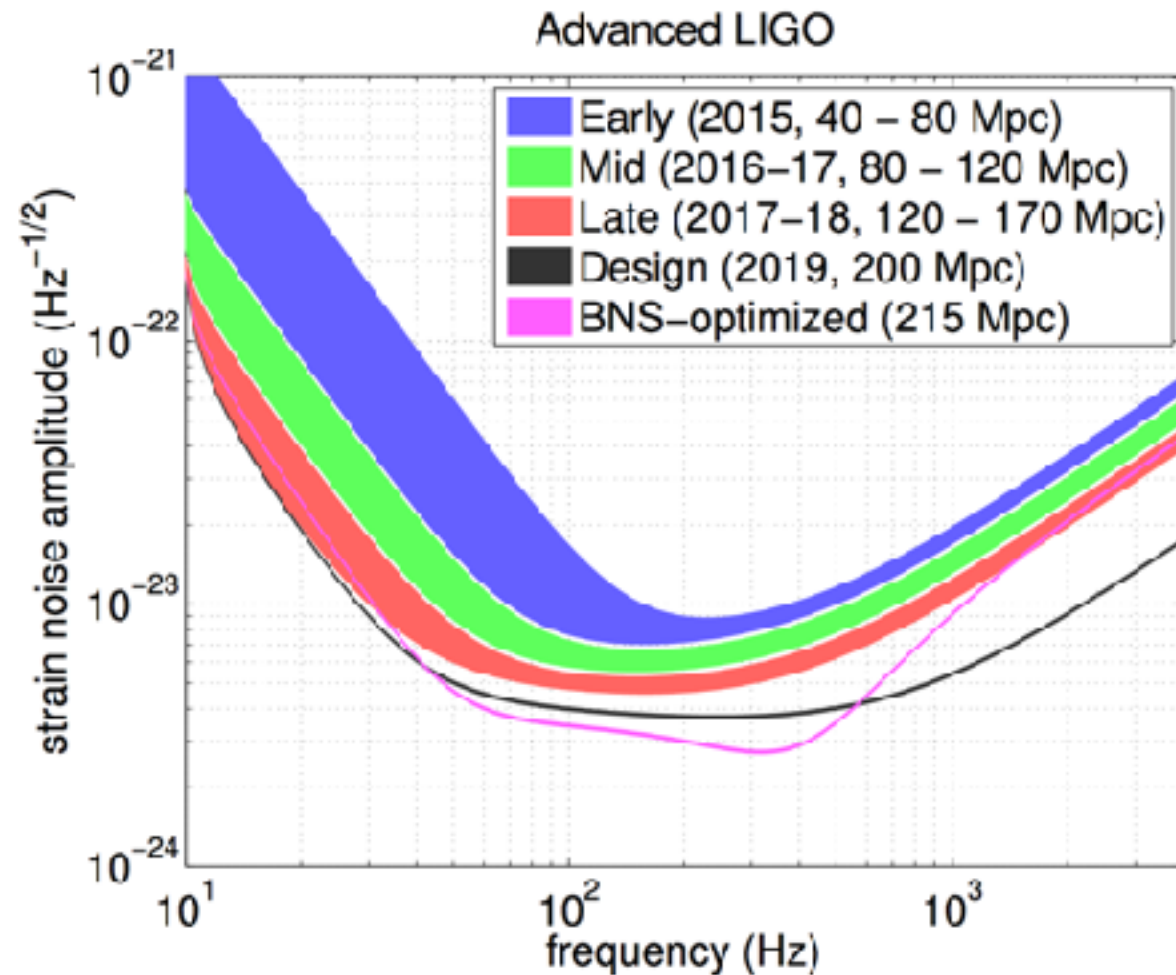


$$\frac{dN}{dt} = \Gamma_{\text{sr}} N - \Gamma_a N^2$$

$$\frac{df}{dt} \simeq 10^{-12} \frac{\text{Hz}}{\text{s}} \left(\frac{f}{\text{kHz}} \right) \left(\frac{M_{\text{Pl}}}{f_a} \right)^2 \left(\frac{10^3 \text{ yr}}{T} \right)$$

Gravitational Wave Signals

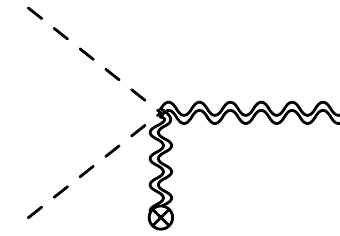
Advanced LIGO sensitivity



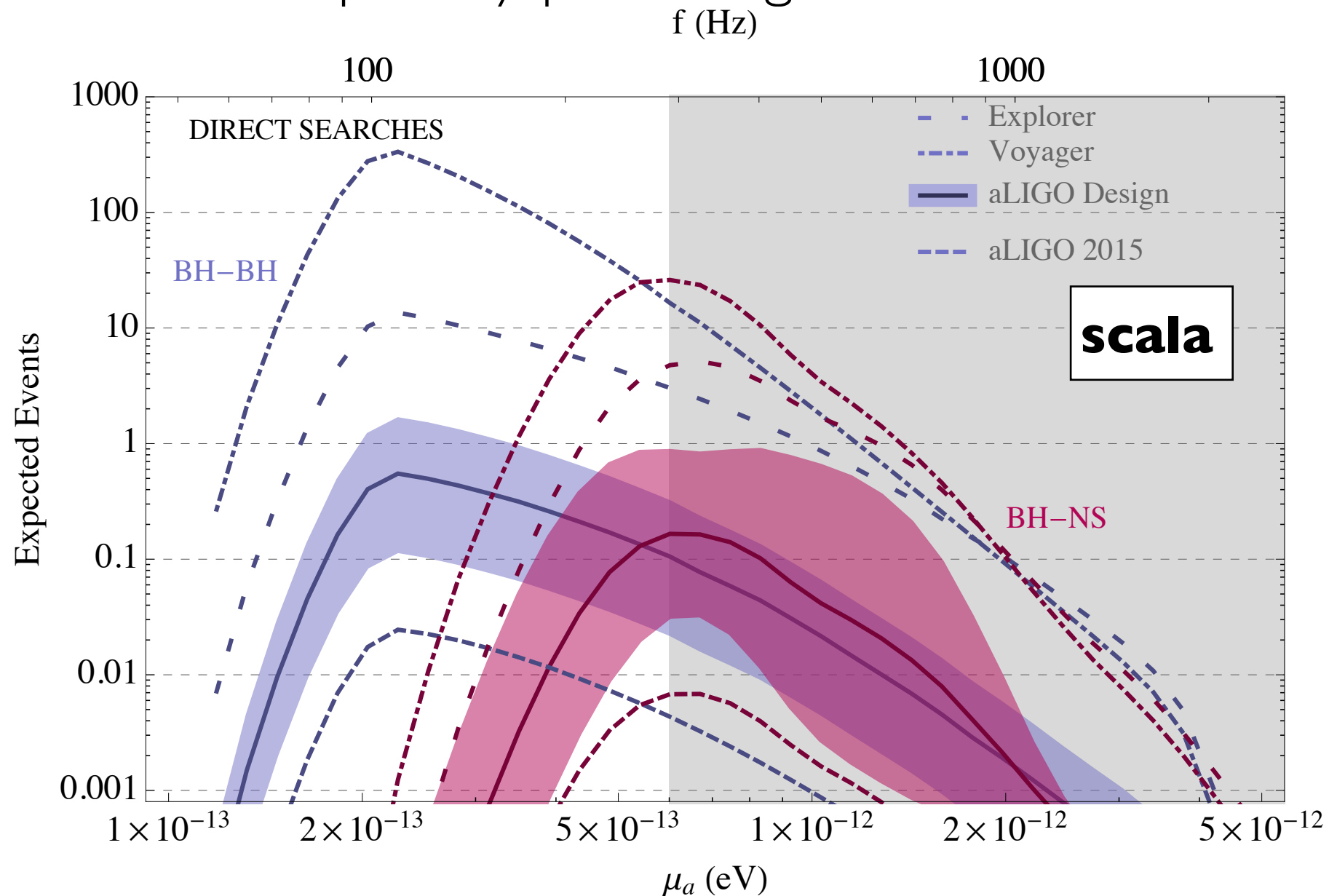
- Fits into searches for **long, continuous, monochromatic** gravitational waves
- Currently looking for “mountains” on neutron stars



Annihilations

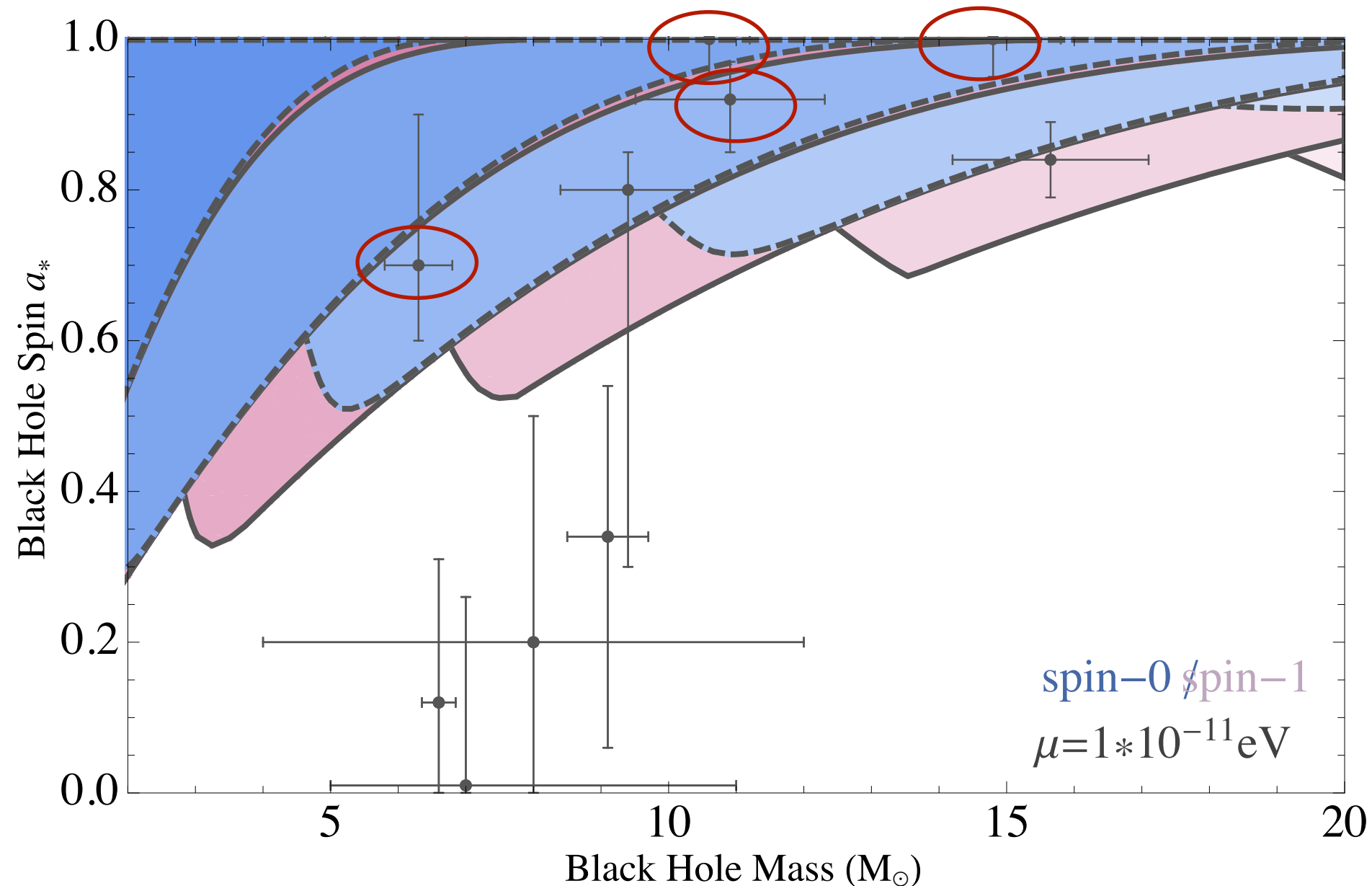


- Mergers at LIGO: a black hole is born!
- Follow up with continuous wave search to see if superradiance creates a cloud of axions around the new BH
- Targeted searches especially promising at future GW observatories



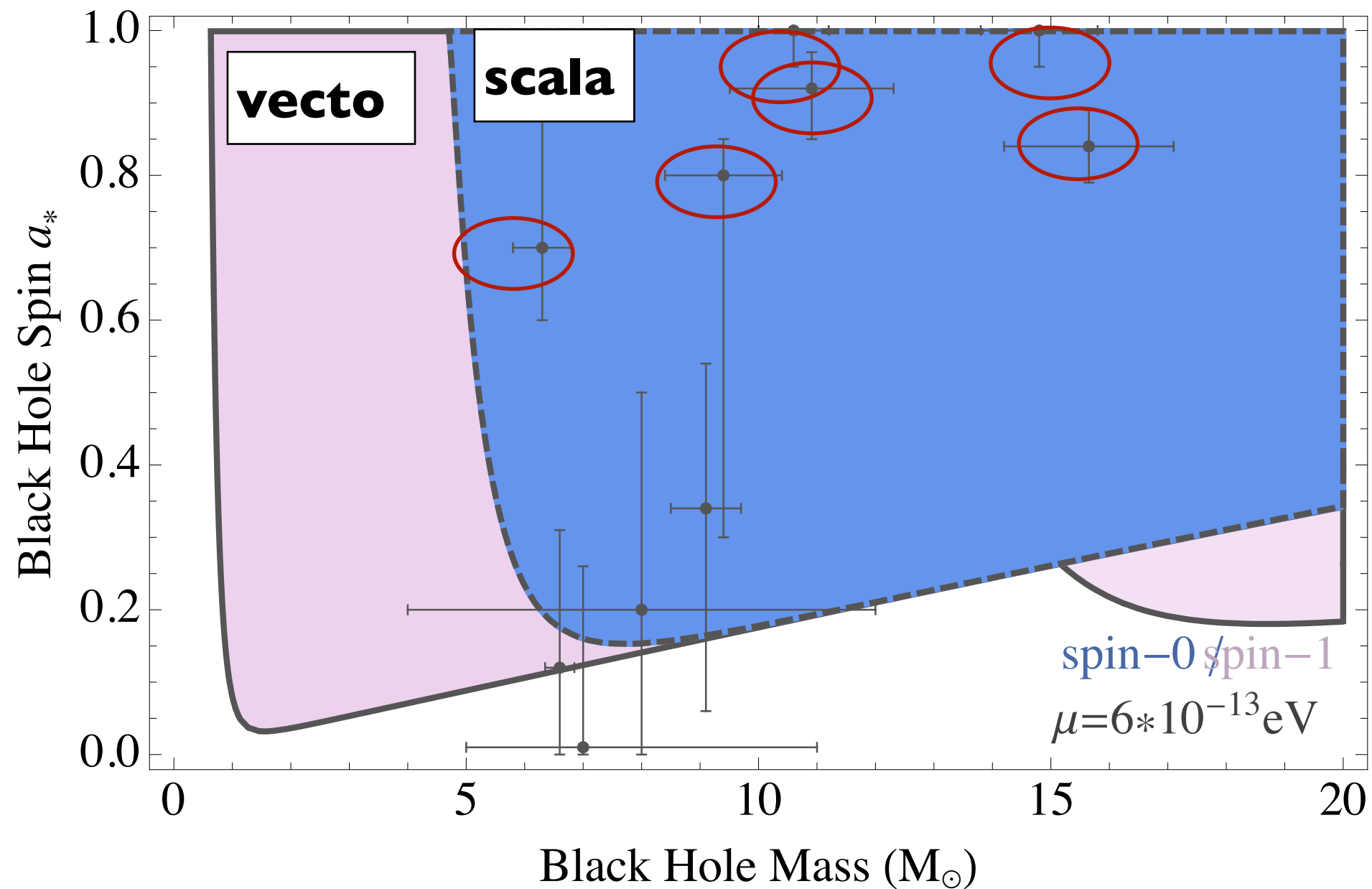
Black Hole Spins

Black hole spin and mass measurements from X-ray binaries:
several black holes disfavor this scalar/vector mass



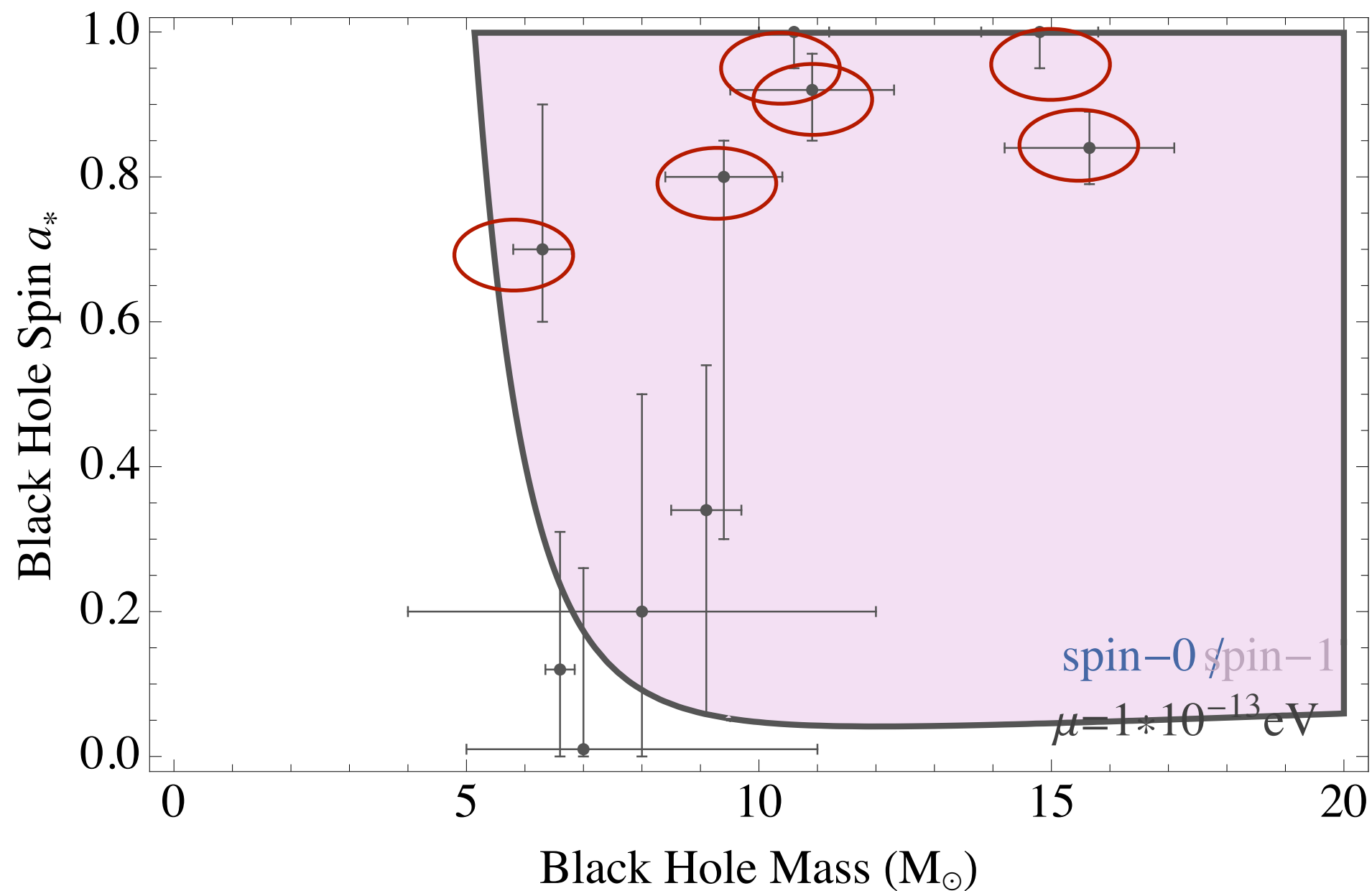
Black Hole Spins

More constrained at lighter masses

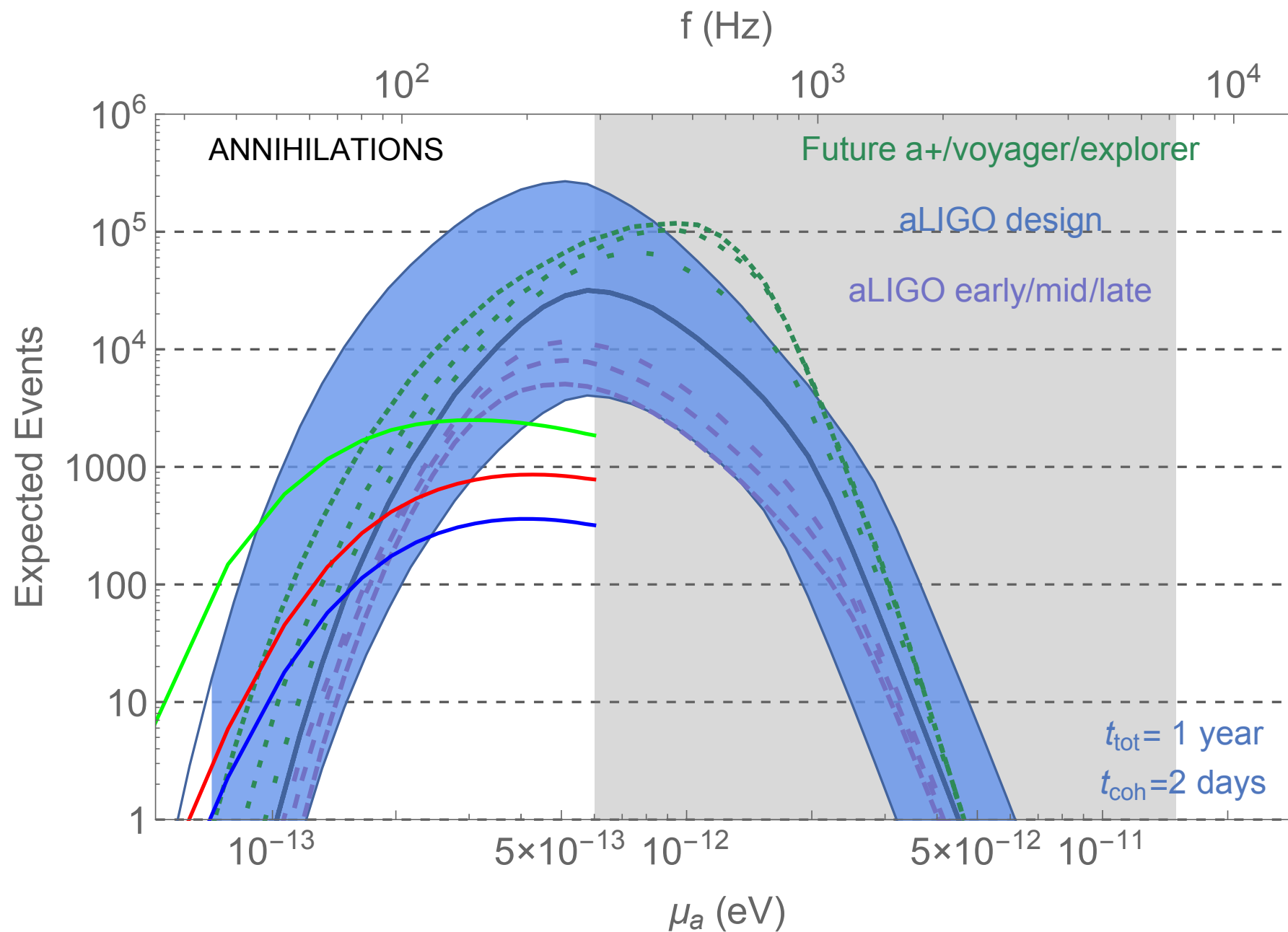


Black Hole Spins

At even lighter masses, constraint is relaxed; SR time is too long



Gravitational Wave Signals stochastic bkgd



Black Hole Environment

Perturbations from non-axisymmetric matter can lead to level mixing, disrupting SR.

$$\left| \frac{\Gamma_{\text{dump}}^{n'\ell'm'}}{\Gamma_{\text{sr}}^{n\ell m}} \right|^{1/2} \left| \frac{\langle \psi_{\text{dump}}^{n'\ell'm'} | \delta V(\vec{r}) | \psi_{\text{sr}}^{n\ell m} \rangle}{\Delta E} \right| < 1,$$

Black holes themselves are perfectly axisymmetric

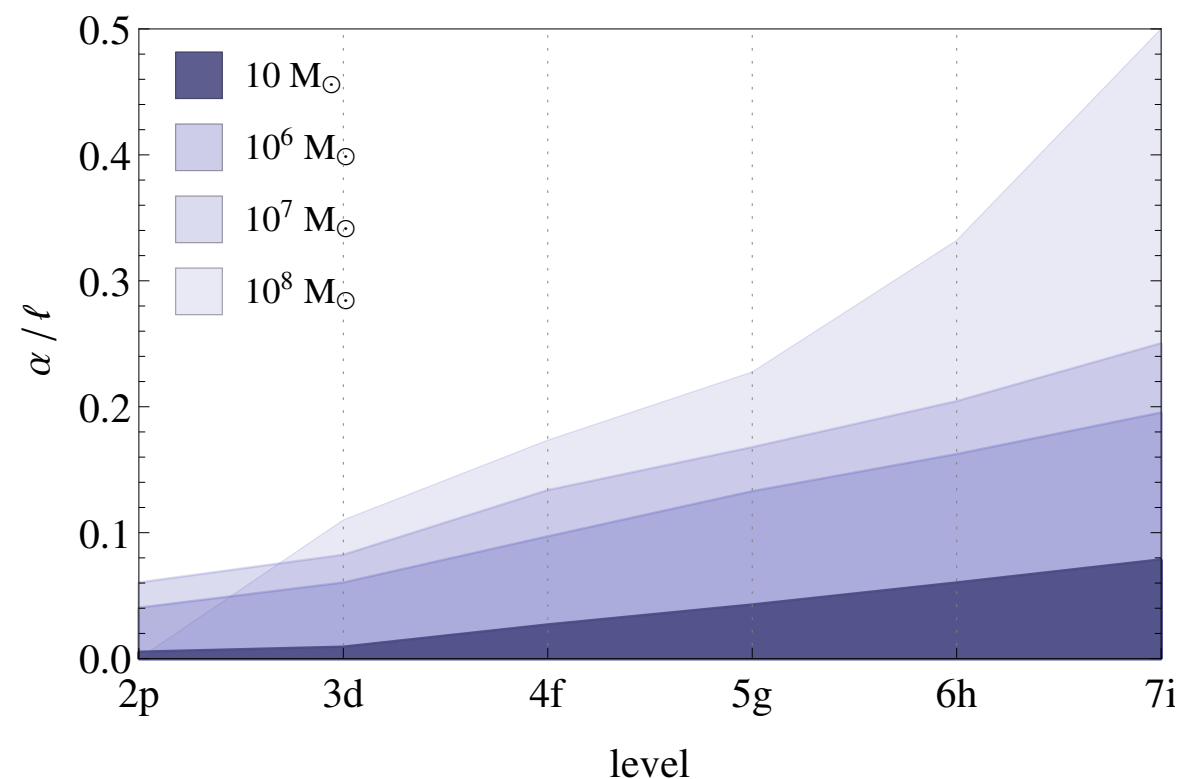
Environments of BHs are very clean

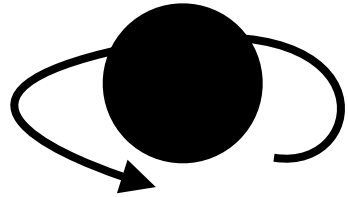
If present, companion star/accretion disk can slightly constrain small coupling parameter space

Companion star

$$\left(\frac{\alpha}{\ell} \right) > (0.05) \left(\frac{M_*}{M} \right)^{1/8} \left(\frac{M}{10M_{\odot}} \right)^{1/6} \left(\frac{\text{day}}{T} \right)^{1/6}$$

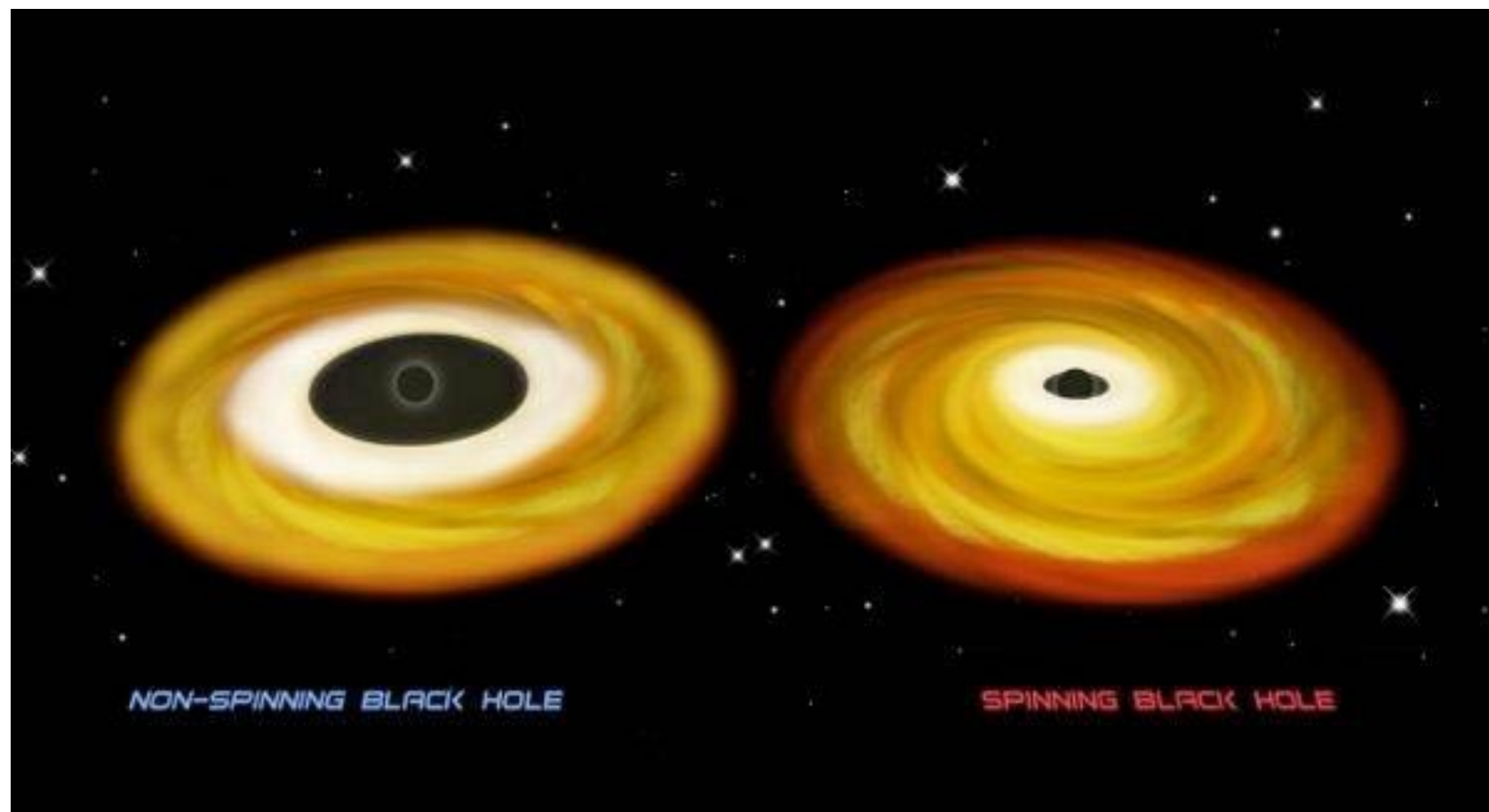
Accretion disk



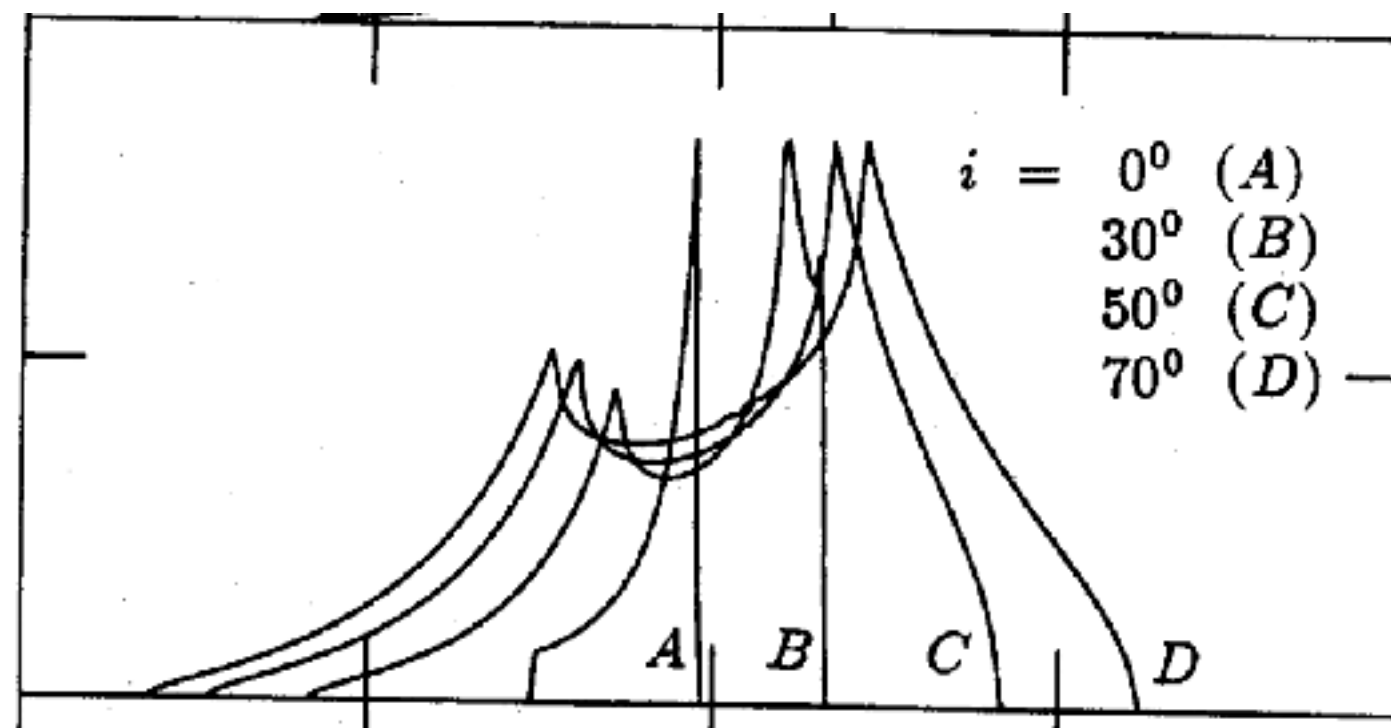
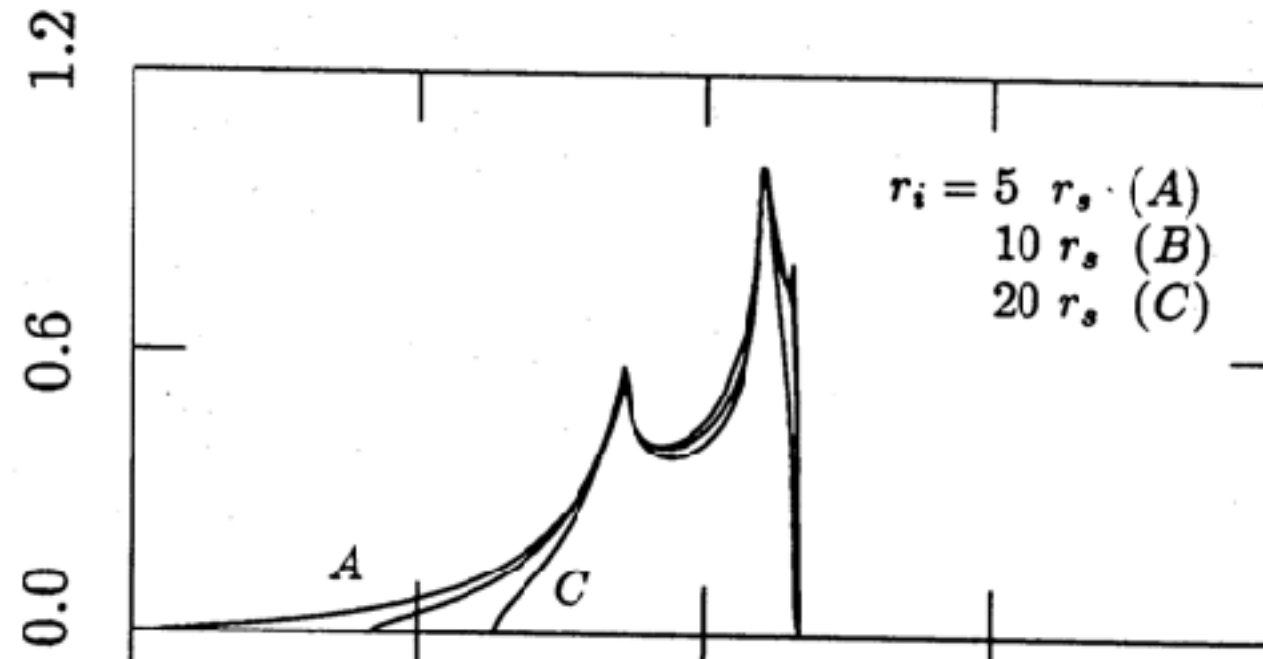


Black Hole Spins

- Two leading methods: continuum fitting and X-ray reflection
- Based on finding the innermost stable orbit of the accretion disk
- Uncertainty dominated by observational errors; smaller at extremal spins



Xray line BH spin measurement



Continuum measurement

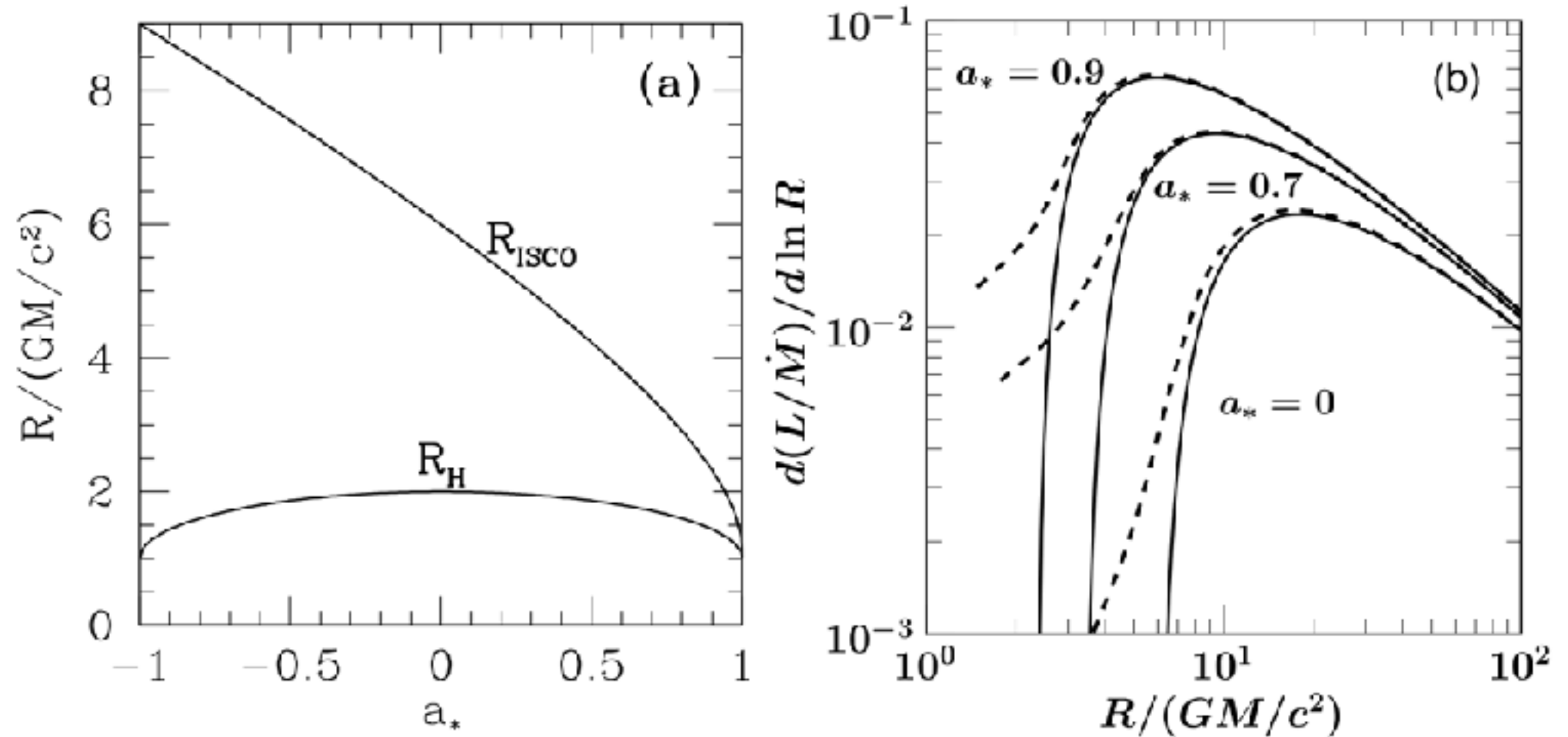


Fig. 3 (a) Radius of the ISCO R_{ISCO} and of the horizon R_{H} in units of GM/c^2 plotted as a function of the black hole spin parameter a_* . Negative values of a_* correspond to retrograde orbits. Note that R_{ISCO} decreases monotonically from $9GM/c^2$ for a retrograde orbit around a maximally spinning black hole, to $6GM/c^2$ for a non-spinning black hole, to GM/c^2 for a prograde orbit around a maximally spinning black hole. (b) Profiles of $d(L/\dot{M})/d\ln R$, the differential disk luminosity per logarithmic radius interval normalized by the mass accretion rate, versus radius $R/(GM/c^2)$ for three values of a_* . Solid lines are the predictions of the NT model. The dashed curves from Zhu et al. (2012), which show minor departures from the NT model, are discussed in Section 5.2.