Introduction to Bimetric Theory

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DESY THEORY WORKSHOP Hamburg, 27.09.17



Navigation



Motivation



Massless & Massive Spin-2 Fields



The Ghost-Free Theory



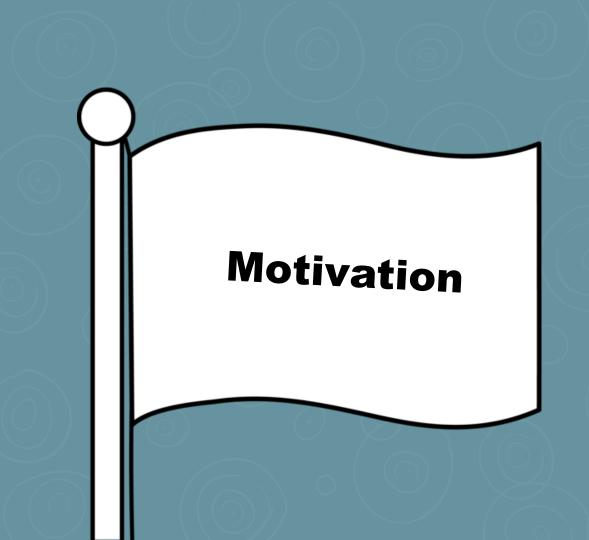
Physics of Massive Spin-2 Fields



Cosmology



Summary



Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks $\,\psi^a$

Spin 1: gluons, photon, W- & Z-boson A_{μ}

Spin 2: graviton $g_{\mu\nu}$

Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

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+ Supersymmetry

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new models are usually built using more copies of these particles

less understood...

Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson ϕ

Spin 1/2: leptons, quarks $\,\psi^a$

Spin 1: gluons, photon, W- & Z-boson A_{μ}

Spin 2: graviton $g_{\mu\nu}$ MASSLESS!

massless & massive



How do we make a spin-2 field massive?



Massless & Massive **Spin-2 Fields**

Massless Theory

General Relativity

= classical nonlinear field theory for metric tensor $g_{\mu\nu}$



Einstein-Hilbert action:
$$S_{
m EH}[g] = M_{
m P}^2 \int {
m d}^4 x \sqrt{g} \, \left(R(g) - 2 \Lambda
ight)$$



Einstein's equations:
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0$$



describes the two degrees of freedom of a self-interacting, massless spin-2 particle

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describes the two degrees of freedom of a self-interacting, massless spin-2 particle



General Relativity

=

unique description of self - interacting massless spin-2 field



... should not contain derivatives nor loose indices.

Examples:

scalar (spin 0)

$$-\partial_{\mu}\phi\partial^{\mu}\phi - m^2\phi^2$$

vector (spin 1)

$$-F^{\mu\nu}F_{\mu\nu}-m^2A^{\mu}A_{\mu}$$

... should not contain derivatives nor loose indices.

Examples: scalar (spin 0) vector (spin 1) $-g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi - m^{2}\phi^{2} \qquad -g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}F_{\mu\nu} - m^{2}g^{\mu\nu}A_{\mu}A_{\nu}$

For the spin-2 tensor contracting indices of the metric gives: $g^{\mu\nu}g_{\mu\nu}=4$ This is not a mass term.

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$$-g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi - m^2\phi^2$$

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For the spin-2 tensor contracting indices of the metric gives: $g^{\mu\nu}g_{\mu\nu}=4$ This is not a mass term.

Simplest way out: Introduce second "metric" to contract indices:

$$g^{\mu\nu}f_{\mu\nu} = \text{Tr}(g^{-1}f)$$
 $f^{\mu\nu}g_{\mu\nu} = \text{Tr}(f^{-1}g)$

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$$>$$
 Massive gravity action: $S_{
m MG}[g] = S_{
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m d}^4 x \; V(g,f)$

kinetic term mass term

... should

indices.

Examples: scalar (sr

$$-g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}$$

What determines $f_{\mu\nu}$? Shouldn't it be dynamical ?

For the spin-2 tensor cont

This is not a mass term.

Simplest way out: Introduce sec

$$g^{\mu\nu}f_{\mu\nu} = \operatorname{Tr}\left(g^{-}\right)$$



Massive gravity action:

$$S_{\mathrm{MG}}[g] = S_{\mathrm{EH}}[g] - \int \mathrm{d}^4 x \ V(g, f)$$

kinetic term

mass term

Bimetric Theory

Nonlinear action for two interacting tensors:

$$S_{b}[g, f] = m_{g}^{2} \int d^{4}x \sqrt{g} \left(R(g) - 2\Lambda \right)$$

$$+ m_{f}^{2} \int d^{4}x \sqrt{f} \left(R(f) - 2\tilde{\Lambda} \right) - \int d^{4}x V(g, f)$$



both metrics are dynamical and treated on equal footing



should describe massive & massless spin-2 field (5+2 d.o.f.)

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should describe massive & massless spin-2 field (5+2 d.o.f.)

This looks good, but there is a major problem...





Ghosts

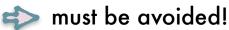
Ghost = field with negative kinetic energy

$$\mathcal{L} = (\partial_t \phi)^2 \cdots$$
 healthy

$$\mathcal{L} = -(\partial_t \phi)^2 \cdots$$
 ghost



consequences: classical instability, negative probabilities at quantum level





explicit check for ghosts by computing the Hamiltonian



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explicit check for ghosts by computing the Hamiltonian

Modifications of General Relativity tend to be haunted by ghosts. Modifying gravity is EXTREMELY difficult!



Ghosts

Ghost = field with negative kinetic energy

$$\mathcal{L} = (\partial_t \phi)^2 \cdots$$
 healthy

$$\mathcal{L} = -(\partial_t \phi)^2 \cdots$$
 ghost



consequences: classical instability, negative probabilities at quantum level must be avoided!



explicit check for ghosts by computing the Hamiltonian



massive spin-2: six instead of five propagating degrees of freedom



need extra constraint to remove the ghost

Avoiding the Ghost?

$$S_{b}[g, f] = m_{g}^{2} \int d^{4}x \sqrt{g} \left(R(g) - 2\Lambda \right)$$

$$+ m_{f}^{2} \int d^{4}x \sqrt{f} \left(R(f) - 2\tilde{\Lambda} \right) - \int d^{4}x V(g, f)$$

For generic interaction potentials the theory suffers from a ghost. Is there a particular potential which avoids the ghost?

Fierz & Pauli (1939):

Linear mass term avoiding the ghost ()



Avoiding the Ghost?

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Boulware & Deser (1972):

Beyond linear order it is impossible! (💌



No consistent nonlinear massive gravity / bimetric theory?





- free Bimetric Theory

de Rham, Gabadadze, Tolley (2010); Hassan, Rosen, ASM, von Strauss (2011/12)

$$S_{b}[g, f] = m_{g}^{2} \int d^{4}x \sqrt{g} R(g)$$

$$+ m_{f}^{2} \int d^{4}x \sqrt{f} R(f) - \int d^{4}x V(g, f)$$

$$V(g,f) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1}f} \right) = m^4 \sqrt{f} \sum_{n=0}^{4} \beta_{4-n} e_n \left(\sqrt{f^{-1}g} \right)$$



lacktriangledown arbitrary spin-2 mass scale m





igotimes square-root matrix S defined through $S^2=g^{-1}f$



- free Bimetric Theory

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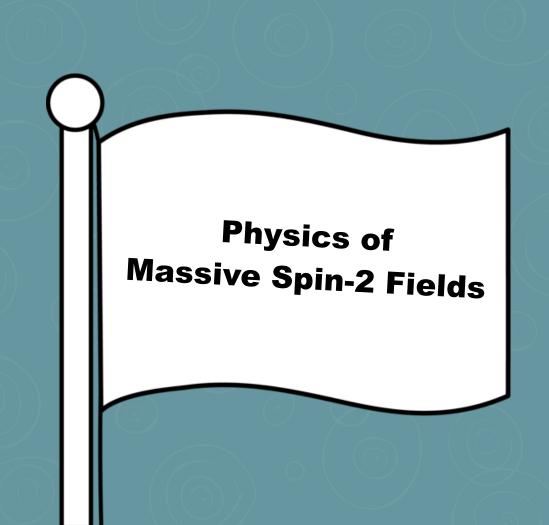
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elementary symmetric polynomials:

$$e_1(S) = \text{Tr}[S]$$
 $e_2(S) = \frac{1}{2} \left((\text{Tr}[S])^2 - \text{Tr}[S^2] \right)$
 $e_3(S) = \frac{1}{6} \left((\text{Tr}[S])^3 - 3 \text{Tr}[S^2] \text{Tr}[S] + 2 \text{Tr}[S^3] \right)$



Mass spectrum

Perturbations around proportional backgrounds:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad f_{\mu\nu} = \bar{g}_{\mu\nu} + \delta f_{\mu\nu}$$

Can be diagonalised into mass eigenstates ($\alpha=m_f/m_g$):

$$\delta G_{\mu\nu} = \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}$$
 massless (2 d.o.f.)
 $\delta M_{\mu\nu} = \delta f_{\mu\nu} - \delta g_{\mu\nu}$ massive (5 d.o.f.)

Linearised equations: $\bar{\mathcal{E}}_{\mu\nu}^{\ \rho\sigma}\delta G_{\rho\sigma} = 0$ $\bar{\mathcal{E}}_{\mu\nu}^{\ \rho\sigma}\delta M_{\rho\sigma} + \frac{m_{\mathrm{FP}}^2}{2}\left(\delta M_{\mu\nu} - \bar{g}_{\mu\nu}\delta M\right) = 0$

with mass $m_{\mathrm{FP}} = m_{\mathrm{FP}}(lpha, eta_n)$ and kinetic operator $\bar{\mathcal{E}} \sim
abla
abla + \Lambda$



Ghost-free bimetric theory

unique description of massless + massive spin-2 field





What is the physical metric?

How does matter couple to the tensor fields?



$$S_{b}[g, f] = m_{g}^{2} \int d^{4}x \sqrt{g} R(g)$$

$$+ m_{f}^{2} \int d^{4}x \sqrt{f} R(f) - \int d^{4}x V(g, f)$$

$$+ \int d^{4}x \sqrt{g} \mathcal{L}_{matter}(g, \phi)$$

Absence of ghosts: only one metric can couple to matter!



 $\Rightarrow g_{\mu\nu}$ is gravitational metric

$$S_{b}[g, f] = m_{g}^{2} \int d^{4}x \sqrt{g} R(g)$$

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(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu}$$
 $(\alpha \equiv m_f/m_g)$ massless massive

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The gravitational metric is not massless but a superposition of mass eigenstates. Max, Platscher, Smirnov (2017): analysis of gravitational wave oscillations

$$S_{b}[g, f] = m_g^2 \int d^4x \sqrt{g} R(g)$$

$$+ m_f^2 \int d^4x$$

(linearised) gravitational

 $\delta g_{\mu\nu}$

mo

See Moritz Platscher's talk Wednesday, 17:47 Seminar room 4b

 $m_g)$

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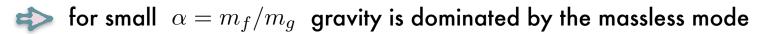
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(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - lpha^2 \delta M_{\mu\nu} \qquad (lpha \equiv m_f/m_g)$$
 massless massive





Mass Eigenstates

Baccetti, Martin-Moruno, Visser (2012); Hassan, ASM, von Strauss (2012/14); Akrami, Hassan, Koennig, ASM, Solomon (2015)

$$S_{b}[g, f] = m_{g}^{2} \int d^{4}x \sqrt{g} R(g)$$

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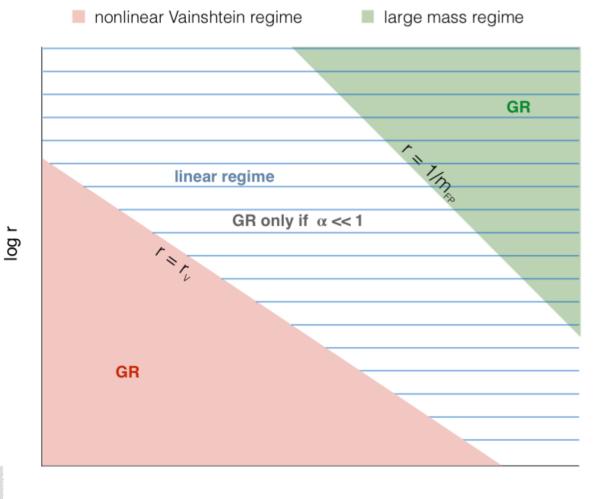
$$\alpha = m_f/m_q \rightarrow 0$$

is the General Relativity limit of bimetric theory

Recovery of GR

$$r_{
m V} = \left(rac{r_{
m S}}{m_{
m FP}^2}
ight)^{1/3}$$

$$V_{
m Yuk} \sim rac{e^{-m_{
m FP} r}}{r}$$



Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

 $log \; m_{\text{FP}}$



Ghost-free bimetric theory

=

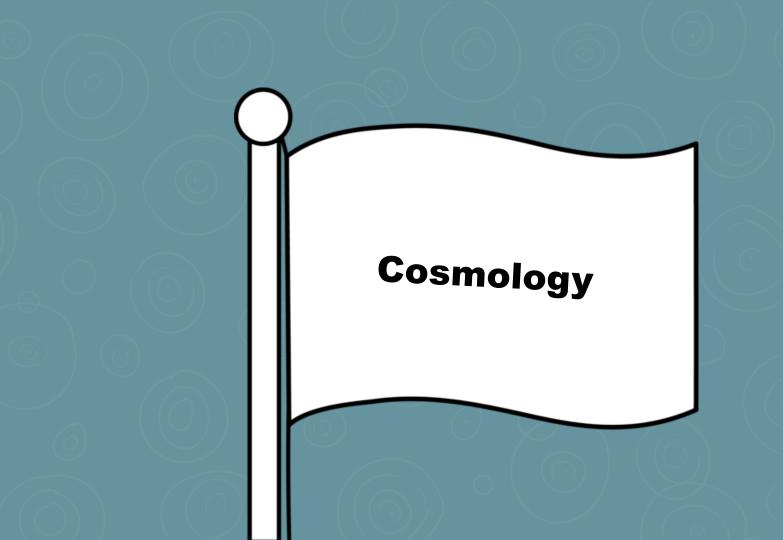
General Relativity + additional tensor field





What are the consequences for cosmology?















Spin-2 Dark Matter

Recall the (linearised) gravitational metric:

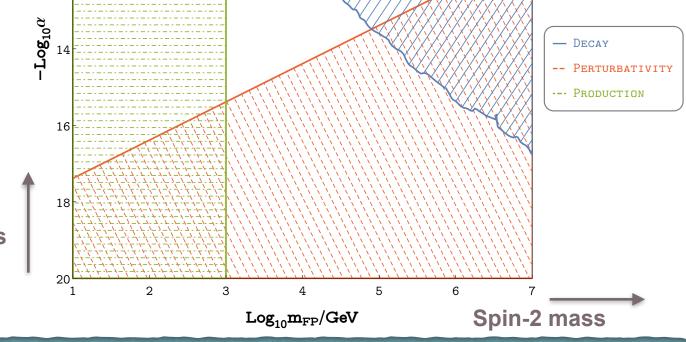
$$\delta g_{\mu
u} \propto \delta G_{\mu
u} - lpha^2 \delta M_{\mu
u}$$
 massless massive

and the General Relativity limit of bimetric theory: $\alpha = m_f/m_g o 0$

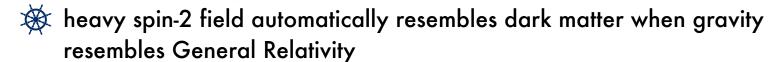
- gravity is weak because the physical Planck mass is large
- massive spin-2 field decouples from matter, interacts only with gravity

Constraints — DECAY --- PRODUCTION

ratio of **Planck masses**



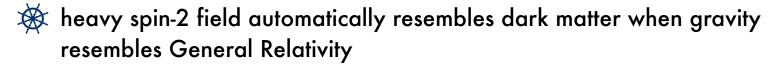
Features



interactions with baryonic matter are suppressed by the Planck mass

spin-2 mass and interaction scale are on the order of a few TeV

Features



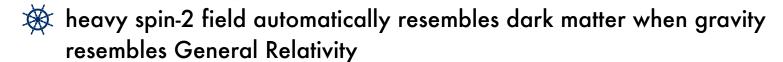
interactions with baryonic matter are suppressed by the Planck mass

spin-2 mass and interaction scale are on the order of a few TeV

Chu & Garcia-Cely (2017): may be lowered to MeV by taking into account self-interactions of massive spin-2

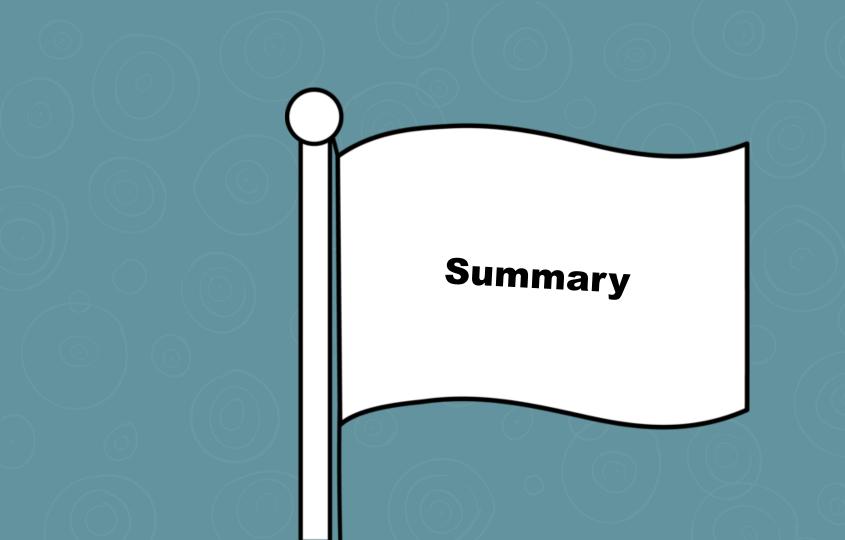
Gonzalez, ASM, von Strauss (2017): interesting new effects for more than one massive spin-2 field

Features



- interactions with baryonic matter are suppressed by the Planck mass
- spin-2 mass and interaction scale are on the order of a few TeV

- no need for extra fields, artificial symmetries or fine tuning
- bimetric theory could explain dark matter in the context of gravity
- massive spin-2 field is a natural addition to the Standard Models



Bimetric Theory ...

review: ASM, Mikael von Strauss; 1512.00021



is one of the few known consistent modifications of General Relativity



can be interpreted as gravity in the presence of an extra spin-2 field



contains an interesting dark matter candidate whose coupling to baryonic matter is suppressed by the Planck scale



Larger theoretical framework: String Theory?



Can we detect/observe the massive spin-2?



Back-up slides

Proportional solutions

Ansatz:

$$\bar{f}_{\mu\nu}=c^2\bar{g}_{\mu\nu}$$
 with $c=$ const.

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_g(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_f(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$

 \nearrow consistency condition: $\Lambda_g(\alpha, \beta_n, c) = \Lambda_f(\alpha, \beta_n, c)$ determines c



Dark Energy

Volkov; von Strauss, ASM, Enander, Mörtsell, Hassan; Comelli, Crisostomi, Nesti, Pilo (2011)



Cosmological solutions in analogy to GR



Homogeneous & isotropic ansatz for both metrics



Use bimetric equations to eliminate components of $f_{\mu
u}$



Obtain modified Friedmann equation for scale factor of physical metric:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} + F\left[\rho(t)\right]$$

GR:
$$F[\rho(t)] = \frac{\rho(t)}{3M_P^2}$$



Use full bimetric equations to compute linearised perturbations

Comparison to Data

Akrami, Koivisto, Mota, Sandstad (2013); Könnig, Patil, Amendola (2014); Akrami, Hassan, Könnig, ASM, Solomon (2015)



set vacuum energy to zero and look for self-accelerating solutions



impose conditions for viable cosmology (background & perturbations)



fit to data

self-accelerating solutions exist, meet the viability conditions Outcome: and fit the data as well as ACDM

Perturbations are well-behaved in the GR limit, i.e. for small $\alpha=m_f/m_q$

Structure of Vertices

(bimetric action expanded in mass eigenstates)

Quadratic (Fierz-Pauli)

δG^2	$\delta G \delta M$	δM^2
$1,\Lambda$	0	$1,\Lambda,m_{ ext{FP}}^2$

$$\delta G_{\mu\nu} = \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}$$
 massless $\delta M_{\mu\nu} = \delta f_{\mu\nu} - \delta g_{\mu\nu}$ massive

$$S_{(2)} = \frac{1}{2} \int d^4x \left[\delta G_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} \delta G_{\rho\sigma} + \delta M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} \delta M_{\rho\sigma} - \frac{m_{\rm FP}^2}{2} (\delta M^{\mu\nu} \delta M_{\mu\nu} - \delta M^2) - \frac{1}{m_{\rm Pl}} \left(\delta G^{\mu\nu} - \alpha \, \delta M^{\mu\nu} \right) T_{\mu\nu} \right]$$

Structure of Vertices

(bimetric action expanded in mass eigenstates)

Quadratic (Fierz-Pauli)

δG^2	$\delta G \delta M$	δM^2
$1,\Lambda$	0	$1,\Lambda,m_{ m FP}^2$

Cubic

δG^3	$\delta G^2 \delta M$	$\delta G \delta M^2$	δM^3
$1,\Lambda$	0	$1,\Lambda,m_{ ext{FP}}^2$	$\begin{array}{c} \alpha , \alpha \Lambda , \alpha m_{\mathrm{FP}}^2 \\ \frac{1}{\alpha} , \frac{1}{\alpha} \Lambda , \frac{1}{\alpha} m_{\mathrm{FP}}^2 \end{array}$



self-interactions of massless spin-2 sum up to General Relativity



no vertices giving rise to decay of massive into massless spin-2



massive spin-2 particle gravitates like baryonic matter



self-interactions of massive spin-2 are enhanced in the GR limit

