



UNIVERSITY OF JYVÄSKYLÄ

Electroweak Vacuum Stability During and After Inflation

Mindaugas Karčiauskas

Enqvist, MK, Lebedev, Rusak, Zatta (2016) [arXiv:1608.08848](#)
Ema, MK, Lebedev, Zatta (2017) [arXiv:1703.04681](#)

LINUX



Higgs Mass and the Standard Model Consistency



1. SM is consistent up to m_{Pl} : $111 \text{ GeV} < M_h < 175 \text{ GeV}$

Higgs mass: $M_h = 125.09 \pm 0.24 \text{ GeV}$ Aad et al. (2015)

2. No other states discovered

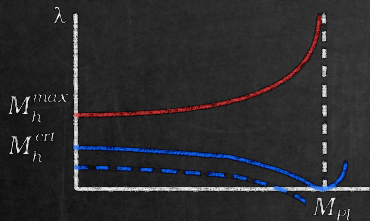
- Ev. Vacuum metastability
 - $\lambda = 0$ at $\sim 10^{10} \text{ GeV}$
 - $\mu_{\text{max}} \sim 10^{10} \text{ GeV}$
- Lifetime $\gg$ age of the universe

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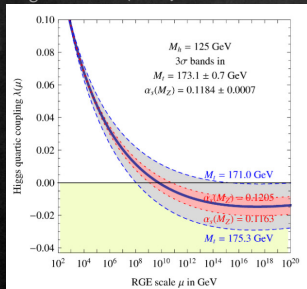
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Degrassi et al. (2012)



- EW vacuum metastability

- $\lambda = 0$ at $\sim 10^{10} \text{ GeV}$

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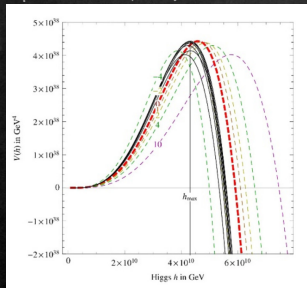
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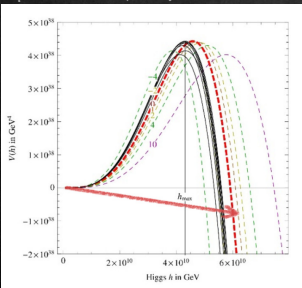
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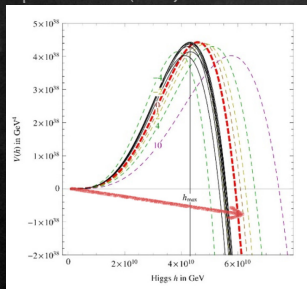
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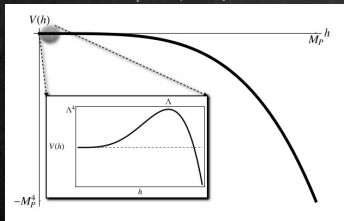
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• Questions:

1. Why isn't Higgs in an energetically favourable state?
2. How EW vacuum survived inflation?

Production of Infrared Modes



- Langevin equation

$$\frac{dh}{dN} \simeq -\frac{\lambda h^3}{3H^2} + \frac{H}{2\pi} \xi(N)$$

- High energy scale of inflation

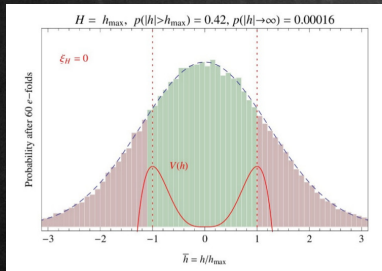
$$H < 10^{12} \text{ GeV} \quad \Leftrightarrow \quad V_{\text{inf}}^{1/4} < 10^{13} \text{ GeV}$$

- If GW are detected ($r > 10^{-4}$)

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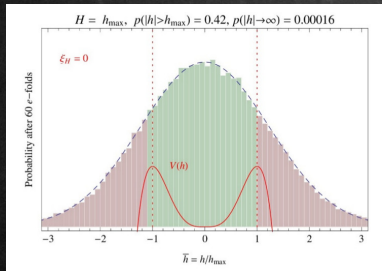
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Stabilising EW Vacuum



- Non-SM interactions
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 - Reheating $\Rightarrow$ Higgs-inflaton interaction

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$$\mathcal{L} \supset \xi h^2 R$$

See the next talk by Marco.



Stabilising EW Vacuum



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Stabilising EW Vacuum



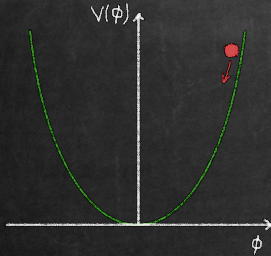
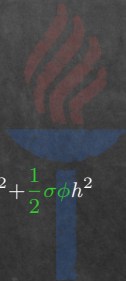
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Suppresses quantum fluctuations during inflation

Enhances parametric and tachyonic resonances after inflation

Higgs-Inflaton Coupling



- Higgs-inflaton coupling

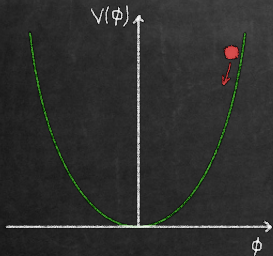
$$V(\phi, h) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda(h)h^4 + \frac{1}{4}\lambda_{h\phi}\phi^2h^2 + \frac{1}{2}\sigma\phi h^2$$

- Constraints

$$10^{-10} < \lambda_{h\phi} < 10^{-6}$$

- Perturbations

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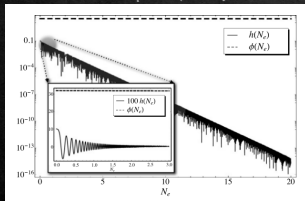


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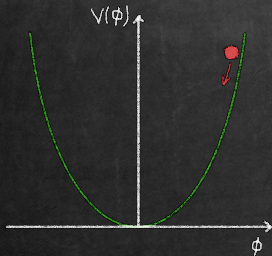
Lebedev & Westphal (2013)



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- Initial inflaton value

$$\phi \sim 10 m_{\text{Pl}}$$

- Constraints

Heavy Higgs

$$10^{-10} <$$

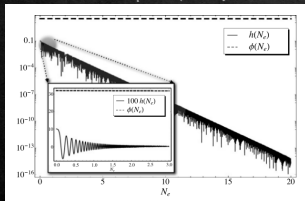
$$\lambda_{h\phi}$$

Small rad. corr.

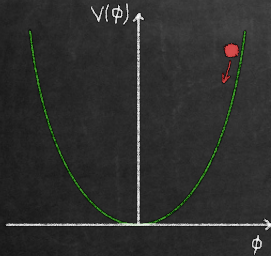
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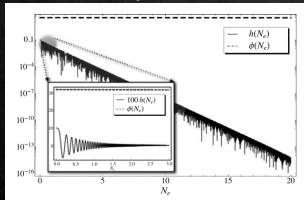
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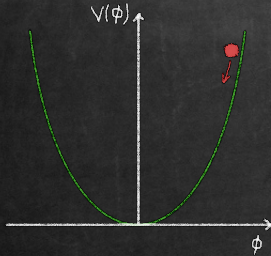
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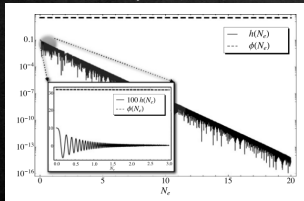
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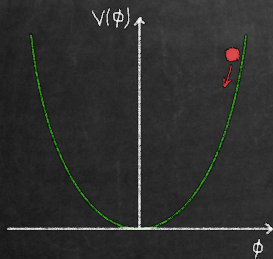
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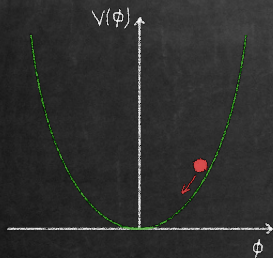
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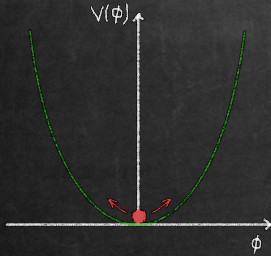
Parametric Resonance



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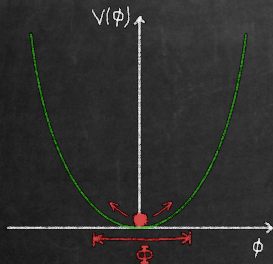
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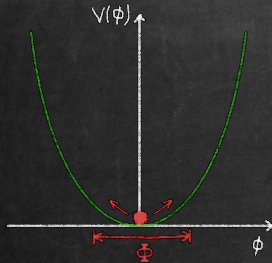
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$$\Phi(t) \propto t^{-1}$$



Parametric Resonance

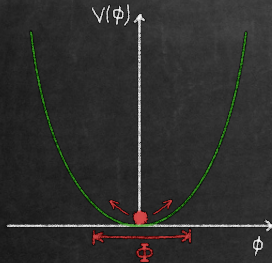
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$$V(\phi, \chi) \supset \frac{g}{2} \phi^2 \chi^2$$



Parametric Resonance



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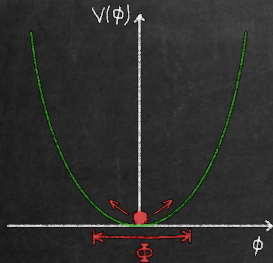
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$$\ddot{\chi}_k + \left[\frac{k^2}{a^2} + g\Phi^2(t) \cos^2(mt) \right] \chi_k = 0$$



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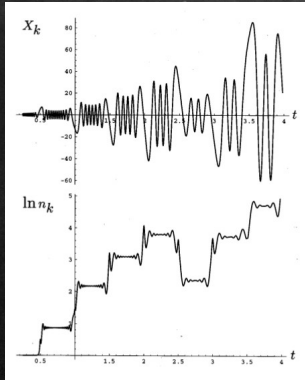
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- Parametric resonance $\left| \frac{\omega'}{\omega^2} \right| \gg 1$



Parametric Resonance

Kofman et al. (1997)



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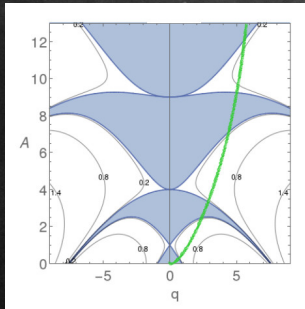
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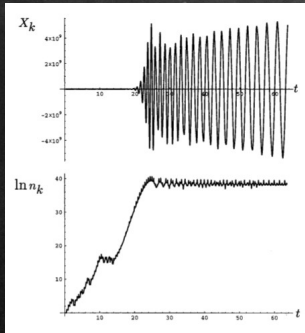
- Parametric resonance $\left| \frac{\omega'}{\omega^2} \right| \gg 1$

$$A(t) \equiv \frac{k^2}{a^2} + q(t)$$

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EW Vacuum Stability: Parametric Resonance



- Mirror symmetry

$$\phi \leftrightarrow -\phi \quad \Rightarrow \quad V \supset \frac{1}{4} \lambda_{h\phi} \phi^2 h^2$$

- Higgs EoM: $\ddot{h}_k + \omega_k^2 h_k \simeq 0$

$$\omega_k^2 = \left(\frac{k}{a}\right)^2 + \frac{1}{2} \lambda_{h\phi} \Phi(t)^2 \cos^2(mt) \quad \simeq \frac{\lambda(h)}{a^{3/2}} \langle h^2 \rangle$$

- Numerical results

$$\lambda = 0.01 \cdot \text{sign} \left(h_{\text{crt}}^{\text{SM}} - \sqrt{\langle h^2 \rangle} \right) \quad \Rightarrow \quad \lambda = -0.01$$

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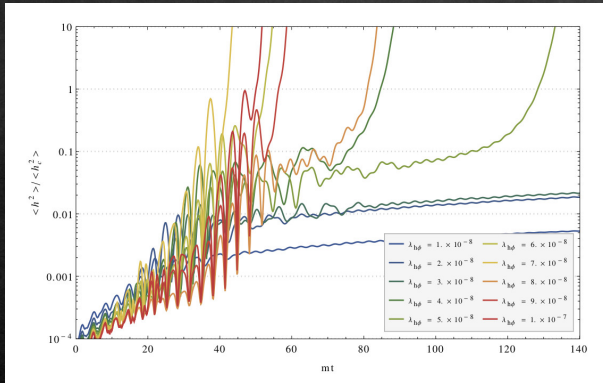
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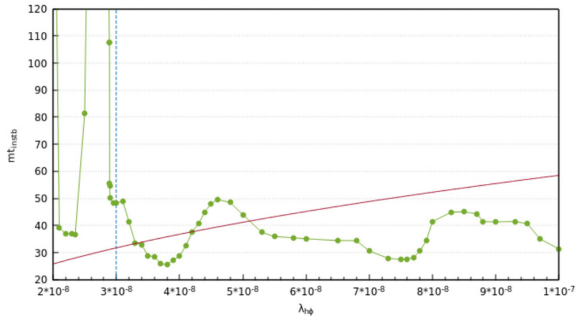
Hartree approximation: $h^4 \rightarrow \langle h^2 \rangle h^2$



EW Vacuum Stability: Parametric Resonance



Lattice:



EW Vacuum Stability: Combined Resonance

$$V(\phi, h) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda(h)h^4 + \frac{1}{4}\lambda_{h\phi}\phi^2h^2 + \frac{1}{2}\sigma\phi h^2$$

$$\omega_k^2 = \frac{1}{2}m^2 \left[\left(\frac{k}{ma} \right)^2 + p(t) \cos(mt) + 2q(t) \cos^2(mt) + 3 \frac{\lambda_h(h)}{a^{3/2}} \frac{\langle h^2 \rangle}{m^2} \right]$$

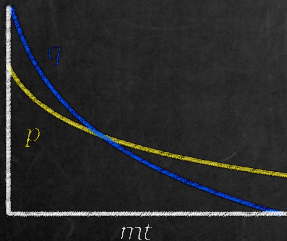


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- Amplitudes

$$q \equiv \frac{\lambda_{h\phi} \Phi^2}{2m^2} \quad p \equiv 2 \frac{\sigma_{h\phi} \Phi}{m^2}$$

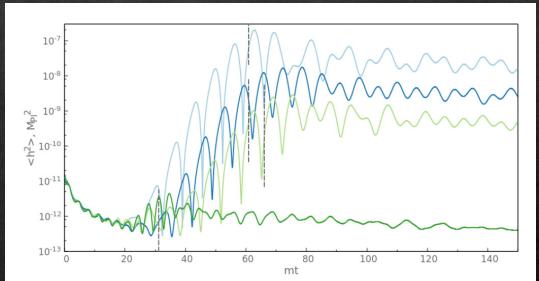
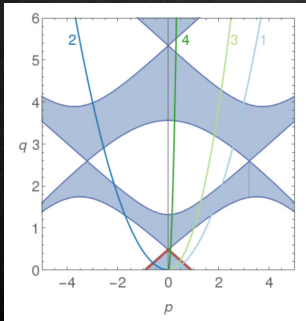
- Trilinear interaction grows

$$\frac{p}{q} \propto t$$

EW Vacuum Stability: Combined Resonance

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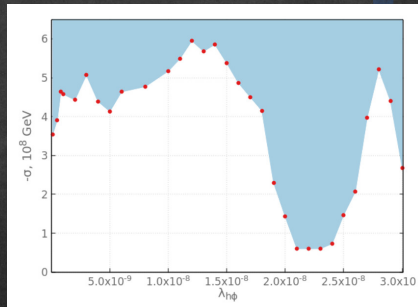
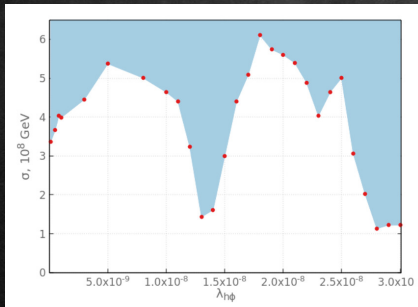
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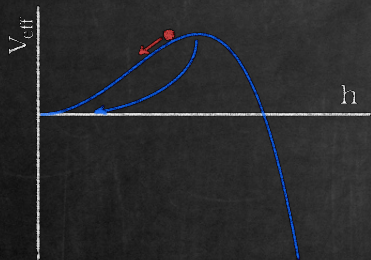


Lattice easy:

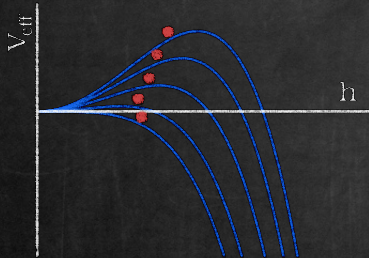


$$|\sigma| < 10^8 \text{ GeV}$$

Post-Resonance: A Possible Scenario



Post-Resonance: A Possible Scenario



- Ema et al. (2016):
 - Destabilised for allowed $\lambda_{h\phi}$
 - Stabilised by thermal bath (?)

Conclusions



- SM is a consistent theory up to m_{Pl}
- But:
 1. Why EW vacuum?
 2. Stability during inflation?
- Reheating $\Rightarrow$ Higgs-Inflaton coupling

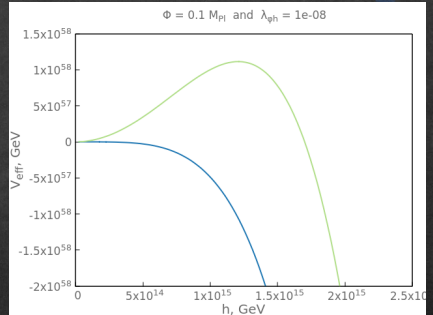
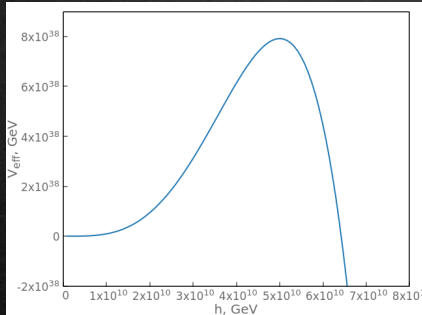
$$V \supset \frac{1}{4} \lambda_{h\phi} \phi^2 h^2 + \frac{1}{4} \sigma \phi h^3$$

- EW vacuum is stable during preheating if

$$\begin{aligned} 10^{-10} < \lambda_{h\phi} < 3 \times 10^{-8} \\ |\sigma| < 10^8 \text{ GeV} \end{aligned}$$

- After preheating EW vacuum is stabilised by thermal corrections (?)

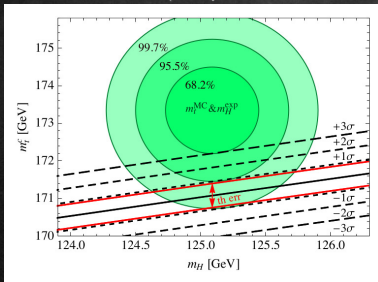
Inflaton Stabilised Potential



Metastability Bound



Iacobellis & Masina (2016)

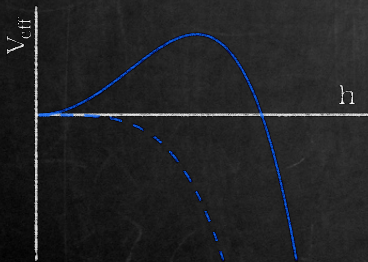


EW Vacuum Stability: Parametric Resonance



$$\omega_k^2 = m^2 \left[\left(\frac{k}{am} \right)^2 + q(t) \cos^2(mt) + 3 \frac{\lambda_h(h)}{a^{3/2}} \frac{\langle h^2 \rangle}{m^2} \right]$$

$$h \equiv \sqrt{\langle h^2 \rangle} \quad q(t) \equiv \frac{\lambda_{h\phi} \Phi(t)^2}{2m^2}$$

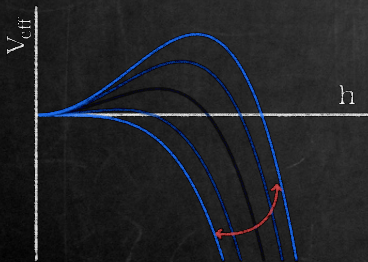


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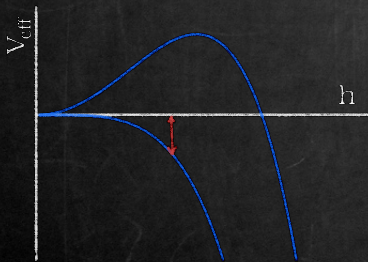


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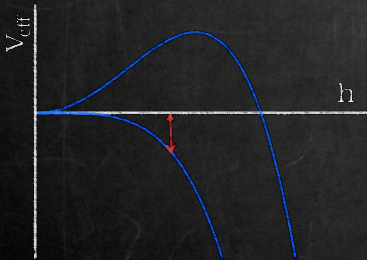
- Tachyonic region $h \propto e^{m_h^{\text{eff}} \Delta t}$

$$(m_h^{\text{eff}})^2 = 3 |\lambda_h| \langle h^2 \rangle$$

$$\Delta t = \sqrt{\frac{6 |\lambda_h| \langle h^2 \rangle}{\lambda_{h\phi} \Phi^2 m^2}}$$

- Instability scale

$$m_h^{\text{eff}} \Delta t < 1 \Rightarrow \lambda_{h\phi} < 3 \times 10^{-8}$$



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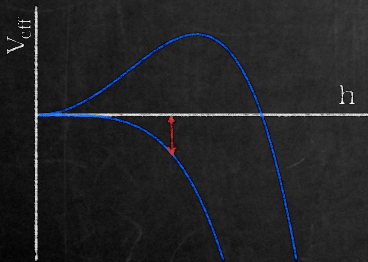
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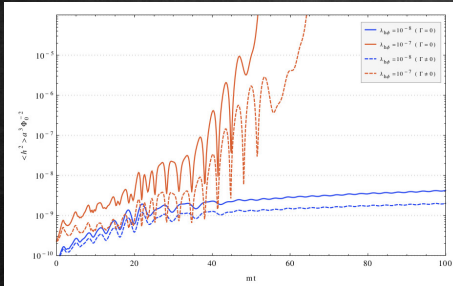
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Perturbative Decay



- Perturbative decay

$$\Gamma(h \rightarrow t\bar{t}) = \frac{3y_t^2 m_h^{\text{eff}}}{16\pi}$$

where

$$y_t(m_h^{\text{eff}}) \sim \frac{1}{2} \text{ and } m_h^{\text{eff}} \simeq \sqrt{\frac{1}{2}\lambda_{h\phi}|\phi|}$$

- The reduction

$$\frac{\langle h^2 \rangle}{\langle h_{\Gamma}^2 \rangle} \sim \text{few}$$

A Model Example



$$-\Delta\mathcal{L} = \frac{1}{2}\lambda_\nu\phi\nu_R\nu_R + y_\nu\bar{l}_L \cdot H^*\nu_R + \frac{1}{2}M\nu_R\nu_R + \text{h.c.}$$

- We take

$$\lambda_{h\phi}(m_{\text{Pl}}) = 0 \quad \text{and} \quad \sigma(m_{\text{Pl}}) = 0$$

- At leading order

$$\lambda_{h\phi} \simeq \frac{|\lambda_\nu y_\nu|^2}{2\pi^2} \ln \frac{m_{\text{Pl}}}{\mu}$$

$$\sigma \simeq -\frac{M|y_\nu|^2 \text{Re}\lambda_\nu}{2\pi^2} \ln \frac{m_{\text{Pl}}}{\mu}$$

- The bound

$$\lambda_{h\phi} < 3 \times 10^{-8} \quad \Rightarrow \quad y_\nu < 0.2$$

$$|\sigma| < 10^8 \text{ GeV} \quad M < 4 \times 10^{12} \text{ GeV}$$