Are tiny gauge couplings out of the Swampland?

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Introduction

QFT's are complicated. Consistency requirements not always transparent:

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Obstruction arises only after coupling to A_μ: Not there if $g = 0$.

- Not every theory can be coupled to a gauge field.
- But we have nice criteria (anomaly cancellation) on theories that can.

Introduction

The same idea works for gravity: We call it the [Vafa '05]

Swampland

Not every EFT is consistent with quantum gravity/ string theory

But which ones? More complicated story:

- Gravitational/mixed anomalies
- No (continuous) global symmetries in QG [Banks-Dixon '88]
- The Swampland conjectures [Ooguri-Vafa '06]
  - Can constrain inflationary models
- The Weak Gravity Conjecture (WGC) [Arkani-Hamed-Nicolis-Motl-Vafa ' 06]
  - Constrains inflation [...]
  - Constrains relaxion, clockwork...even the SM itself! [...]

What is the support for these Swampland criteria?

- A lot of black hole heuristics
- A few concrete calculations (WGC in pert. ST or AdS_3)

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- ...but mostly, a lot of stringy examples!

In this talk I will present a firm bound [MM '17, arXiv: 1708.02249] on (the large N behaviour of) gauge couplings in AdS.

- Motivation: Study WGC heuristics in AdS
- Ultimate goal: Holographic **proof** of the WGC?

WGC heuristics

Brief summary of WGC heuristics in flat space:

1. Consider (near) extremal RN black holes: $M = gQM_p$
2. Take $g \to 0$ limit. Too many remnants!
3. Postulate WGC particle w. $m < gqM_P$. Now extremal black holes are unstable.

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We get a **different** Swampland constraint: Gauge couplings cannot be tiny.

Holography 101

- Basic idea

$$Z_{CFT} = Z_{AdS}, \quad \langle TT \rangle = C_T \sim N^2 \leftrightarrow \ell^{d-3}/G_N$$

- As $C_T \to \infty$ only a small number of light fields with $s \leq 2$ remain. Then $Z_{AdS} \approx Z_{EFT}$. This defines a **holographic** CFT.
- In such a CFT, Z_{EFT} is the generating function for a $1/G_N$ expansion of correlators:

$$\langle \phi_1 \phi_2 \ldots \rangle = \sum_{n=0}^{\infty} (G_N)^n \langle \phi_1 \phi_2 \ldots \rangle_n$$

  This is the 't Hooft $1/N$ expansion of the dual theory.
- We also have a bulk gauge field: $A \leftrightarrow J_{CFT}, \quad g \leftrightarrow C_J$.

AdS Black holes

Consider the Einstein-Maxwell system in $d+1$ dimensions

$$\int d^{d+1}x \left(\frac{R}{2\kappa_{d+1}^2} + \frac{d(d-1)}{\ell^2} \right) - \frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu}.$$

and the corresponding RN-AdS solutions

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_{d-1}, \quad V(r) \equiv 1 - \frac{m}{r^{d-2}} + \frac{q^2}{r^{2d-4}} + \frac{r^2}{\ell^2}.$$

with

$$M = \frac{(d-1)\omega_{d-1}}{16\pi G} m, \quad Q = \sqrt{(d-2)(d-1)} \frac{\omega_{d-1}}{\sqrt{8\pi G} g} q.$$

The Euclidean version of this solution is a saddle contributing to several interesting partition functions.

The game:

$$Z_{CFT} = \sum_{\text{gravity solutions}} e^{-S_E}.$$

In particular,

$$Z_Q(\beta) = \sum_{\text{charge}=Q} e^{-\beta E} = Z_{BH,Q}(\beta) + \ldots, \quad Z_{BH,Q}(\beta) = \exp(-S_E).$$

Even if the WGC is satisfied, we have $Z_Q(\beta) \geq Z_{BH,Q}(\beta)$. For $Q = 0$, can do better:

$$Z_0(\beta) \geq Z_{AdS} + Z_{BH,0}(\beta) \equiv Z_{\text{leading}}$$

Hawking-Page

Another nice partition function: The canonical one!

$$Z(\beta) = Z_{AdS} + Z_{\text{Schwarzschild-AdS}} + \ldots = Z_{\text{leading}} + \ldots$$

Z_{leading} is the dominant contribution: For $\beta \gg 1$, Z_{AdS} dominates. For $\beta \ll 1$, $Z_{\text{Schwarzschild-AdS}} = Z_{BH,0}$ dominates.

$$Z(\beta) = \sum_{\text{states}} e^{-\beta E} = \sum_{Q} \left(\sum_{\text{charge} = Q} e^{-\beta E} \right)$$
$$= \sum_{Q} Z_Q(\beta) = Z_{\text{leading}}(\beta) + \mathcal{Z}(\beta), \quad \mathcal{Z}(\beta) \equiv \sum_{Q \neq 0} Z_Q(\beta).$$

Consistency demands that $\mathcal{Z}(\beta)$ is **subleading**!

$$\log Z_{\text{leading}}(\beta) \approx \begin{cases} 0 & \text{if } \beta > \beta_{HP} \\ F(x_+^0) & \text{if } \beta < \beta_{HP} \end{cases} + \dots$$

$$\log \mathcal{Z}(\beta) \approx \begin{cases} F\left(\frac{1}{\Lambda \ell}\right) + \log\left(\frac{\kappa_{d+1}}{g\ell}\right) & \text{if } \beta > \beta_{HP} \\ \\ F\left(x_+\right) + \log\left(\frac{\kappa_{d+1}}{g\ell}\right) & \text{if } \beta < \beta_{HP} \end{cases} + \dots.$$

The logarithm is just a sum over black holes with almost degenerate charge. If $g \sim \exp(-N^2) \sim \exp(-1/G_N)$, it is **not** subleading.

Bounds on the gauge coupling

- For $\beta \gg \beta_{HP}$, we get

$$\Lambda^{d-2} < \frac{2\pi\omega_{d-1}\ell}{\log\left(\frac{\kappa_{d+1}}{g\ell}\right)\kappa_{d+1}^2}.$$

AdS version of cutoff found by [Saraswat '16] in flat space from Bekenstein's bound.

- For $\beta \ll \beta_{HP}$,

$$F(x_+^0) - F(x_+) \approx -F'(x_+^0)\delta x_+ > \log\left(\frac{\ell^{\frac{d-3}{2}}}{g}\right) \sim \log C_J$$

The lhs is subleading in N, which means the rhs is as well:
~~$g \sim \exp(-N^2) \sim \exp(-1/G_N)$~~

Comments

- $g \sim \exp(-1/G_N)$ = nonperturbative gauging, invisible in EFT
- Would have meant “no global symmetries” is not a Swampland constraint!
- Statement nonperturbative in $1/N$: Out of reach of the $1/N$ conformal bootstrap.

Summary

- Gaugings are visible in the EFT.
- Logarithmic relation between EFT cutoff and g.
- WGC black hole heuristics gives something different in AdS!

Outlook:

- Stronger constraints for non-abelian groups?
- What about scalars? Relevant for **inflation**
- Want to understand the WGC in a holographic context.

Danke Schön!