ENTANGLEMENT ENTROPY IN A HOLOGRAPHIC MODEL FOR THE QCD CRITICAL POINT

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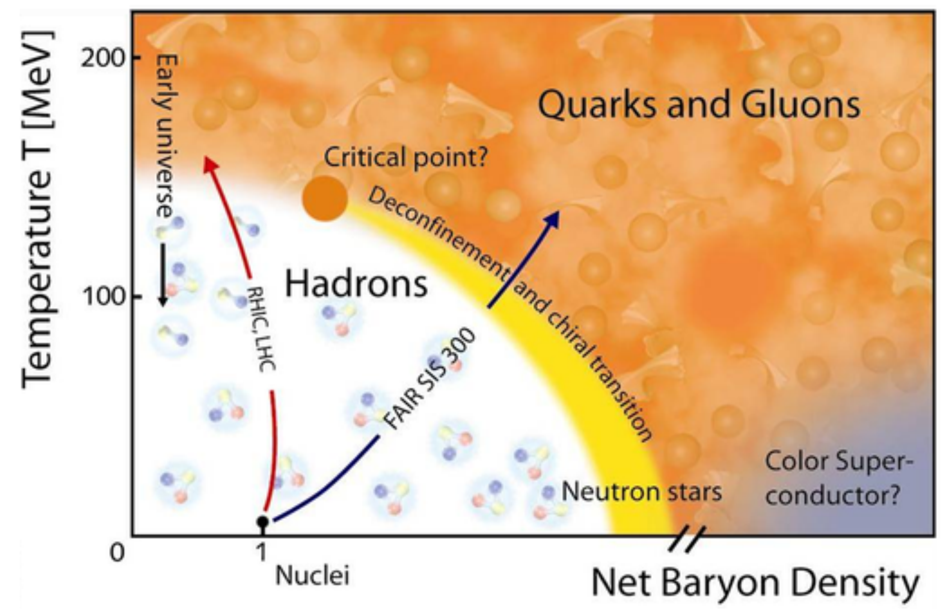
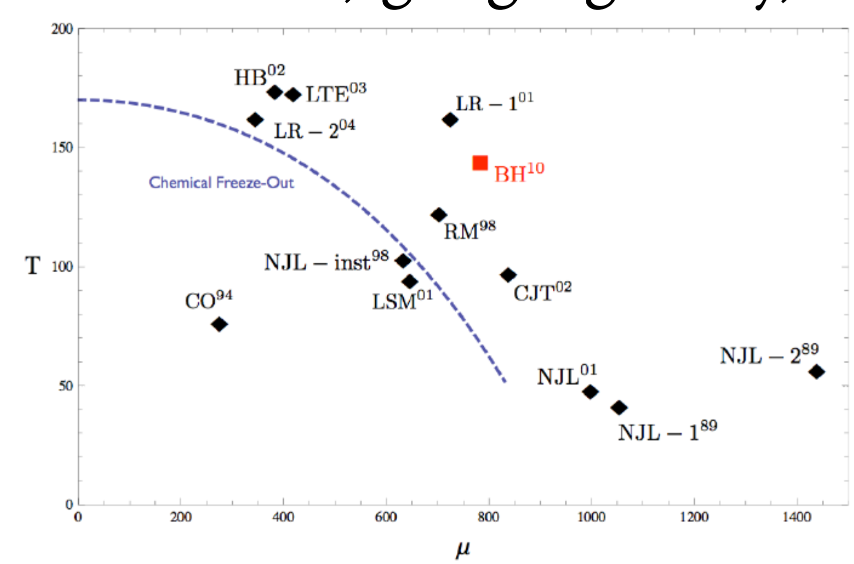
MOTIVATION

- QCD as theory of strong interactions is expected to exhibit various phases
- gas-liquid (GL) first-order phase transition (FOPT) in nuclear matter established



- for $\mu=0$ hadron-quark (HQ) deconfinement transition is crossover in 2+1 flavor lattice QCD at O(155 MeV)
- sign problem in lattice calculations for $\mu>0$
  $\Rightarrow$ imaginary μ, taylor expansions, reweighting, complex Langevin, …
- alternative: model building (NJL, linear sigma/quark meson, gauge/gravity, …)
- possible critical end point (CEP)
  fairly unconstrained,
  experimental attempts: LHC, RHIC, FAIR, …



A (VERY) SHORT PRIMER ON ADS/CFT

- realization of the holographic principle (t'Hooft, Susskind)
- correspondence between CFT in d dimensions and gravitational theory in d+1 dim.:

  $\mathcal{N}=4$ SYM w/ gauge group $SU(N)$ and g_{YM} is dynamically equivalent to IIB superstring theory w/ $l_s^2=\alpha'$ and g_s on $AdS_5\times S_5$ w/ curvature radius L and N units of $F_{(5)}$ flux on S_5.

- parameter mapping: $g_{YM}^2 = 2\pi g_s \;, \quad 2g_{YM}^2 N = L^4/\alpha'^2$
- map between two different theories; *duality* refers to opposite strong/weak coupling regimes
- operator-field duality allows one-to-one correspondence/dictionary between field theory operator $\mathcal{O}$ and gravity fields ϕ in same representation ($\mathrm{CFT_d}$ is defined on boundary of $\mathrm{AdS_{d+1}}$):

$$Z_{\mathcal{O}}[\phi_0]_{CFT} = \int \mathcal{D}\phi \; e^{-S+\int d^4x \mathcal{O}(x)\phi_0(x)} = Z_{\text{classical}}[\phi_0]_{AdS} = e^{-S_{\text{SUGRA}}[\phi[\phi_0]]}$$

  generating functional of CFT is identified w/ gravity partition function

HOLOGRAPHIC EMD MODEL

- 5D gravity (Einstein-Maxwell-dialton) model based on action

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left(R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) - \frac{f(\phi)}{4} F_{\mu\nu}^2 \right)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad A_\mu dx^\mu = \Phi dt$

- metric ansatz: $ds^2 = \mathrm{e}^{2A(r)} \left(-h(r) dt^2 + d\vec{x}^2 \right) + \frac{dr^2}{h(r)}$ $\Rightarrow$ asymptotically AdS_5 spacetime
  boundary at $r \to \infty$
  black hole horizon $h(r_H) \equiv 0$

- field eqns. are solved with $\phi_0 \equiv \phi(r_H)$ and $\Phi_1 \equiv \frac{\partial \Phi}{\partial r}\Big|_{r_H}$ as parameters

- Thermodynamic quantities follow from boundary asymptotics as:

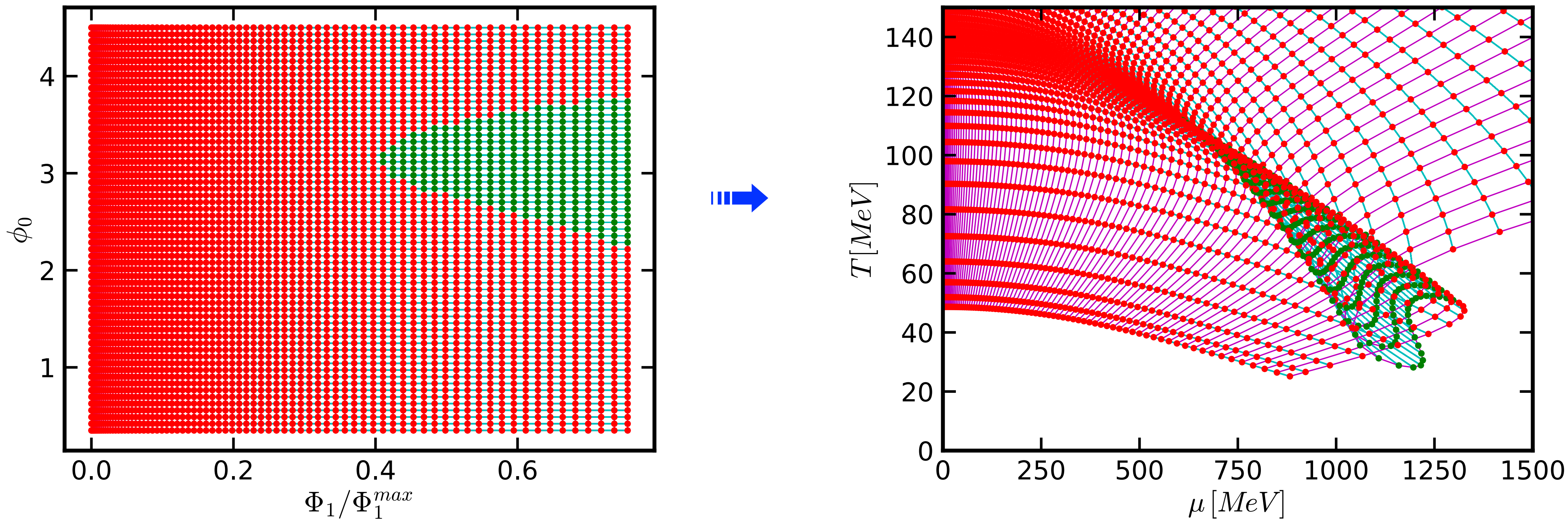
$$T = \lambda_T \frac{1}{4\pi \phi_A^{1/(4-\Delta)} \sqrt{h_0^\infty}}, \quad s = \lambda_s \frac{2\pi}{\phi_A^{3/(4-\Delta)}}$$

$$\mu = \lambda_\mu \frac{\Phi_0^\infty}{\phi_A^{1/(4-\Delta)}}, \quad n = \lambda_n \frac{Q_G}{2 f(0) \phi_A^{3/(4-\Delta)}}$$

- dimensional scaling factors $\lambda_T = \lambda_\mu := 1/L$ and $\lambda_s = \lambda_n := 1/\kappa_5^2$ restore physical units

- pressure from integration $dp(T, \mu) = s(T, \mu) dT + n(T, \mu) d\mu$

➤ $T-\mu$ plane is uncovered with properly chosen initial conditions ϕ_0, Φ_1



➤ Related literature:
model proposed in [DeWolfe, Gubser, Rosen, PRD 83 (2011)], dynamical critical phenomena analysed in [DeWolfe, Gubser, Rosen, PRD 84 (2011)] (fit to „old" lattice data);
updated to recent 2+1 flavor lattice QCD in [Noronha et al., PRL 115 (2015), JHEP 04 (2016), arXiv:1704.05558] (transport coefficients, quark energy loss etc.)
not yet addressed: possible CEP and related phase diagrams

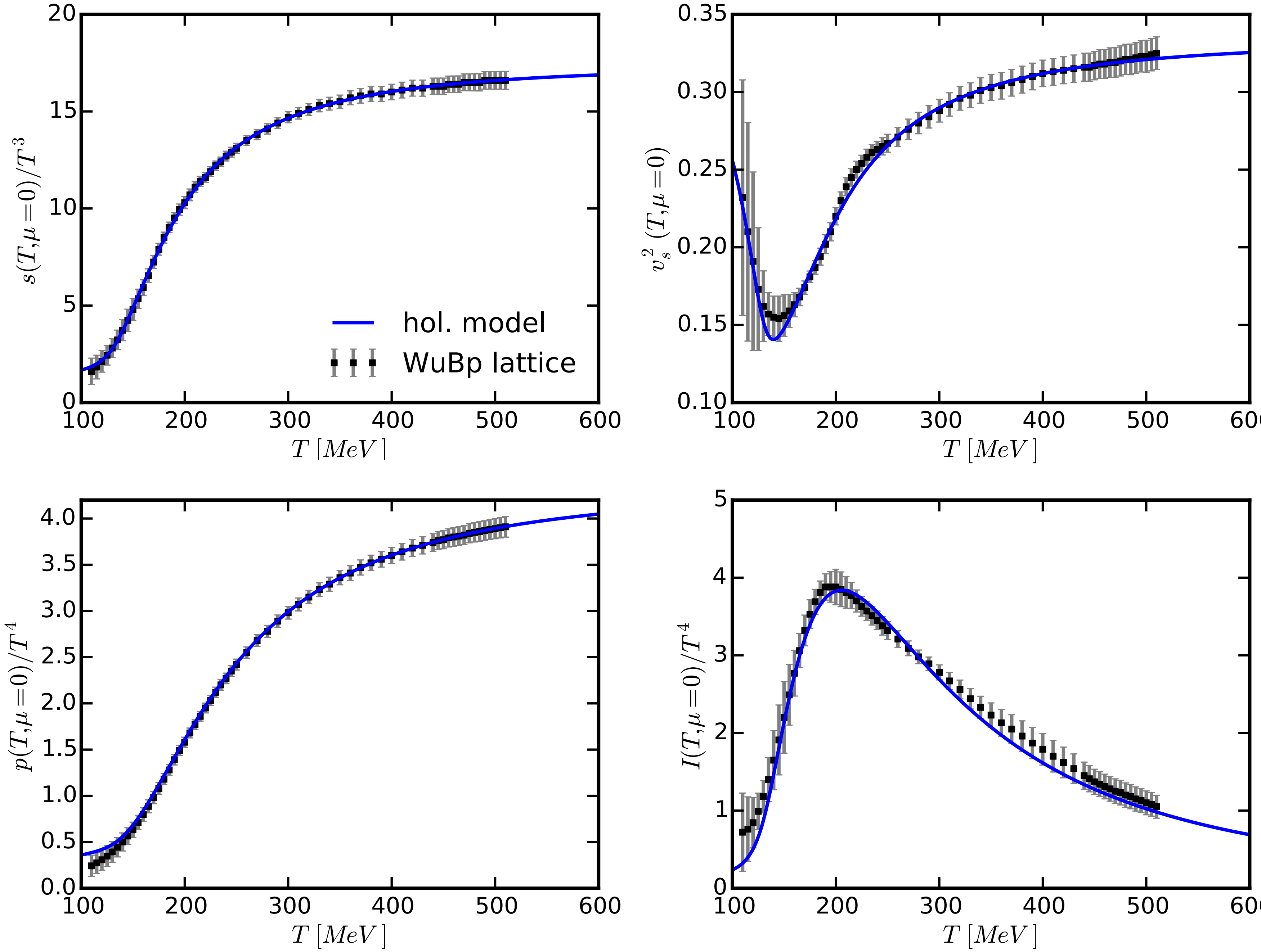
☛ our topic in [JK, Yaresko, Kämpfer (2017), arXiv:1702.06731]

ADJUSTMENT TO LATTICE QCD AT μ=0

- dilaton potential $V(\phi)$ and scaling factors are determined through fit to 2+1 flavor lattice QCD at μ=0 [Borsanyi et al., PLB 730 (2014); Bazavov et al., PRD 90 (2014)] to match EoS:

Equation of state of the updated holographic EMD model as functions of T for μ= 0:

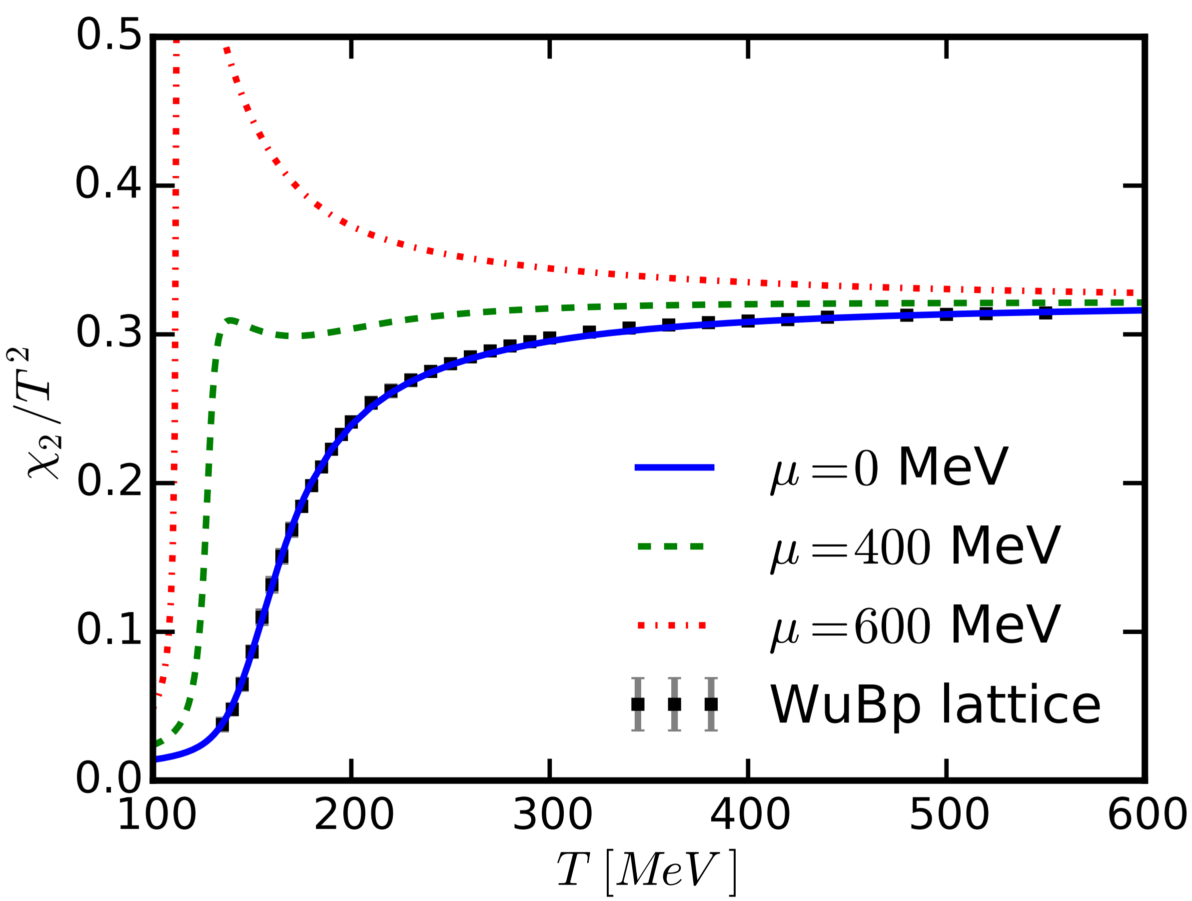
scaled entropy density (top left), speed of sound squared (top right), scaled pressure (bottom left) and scaled trace anomaly (bottom right).

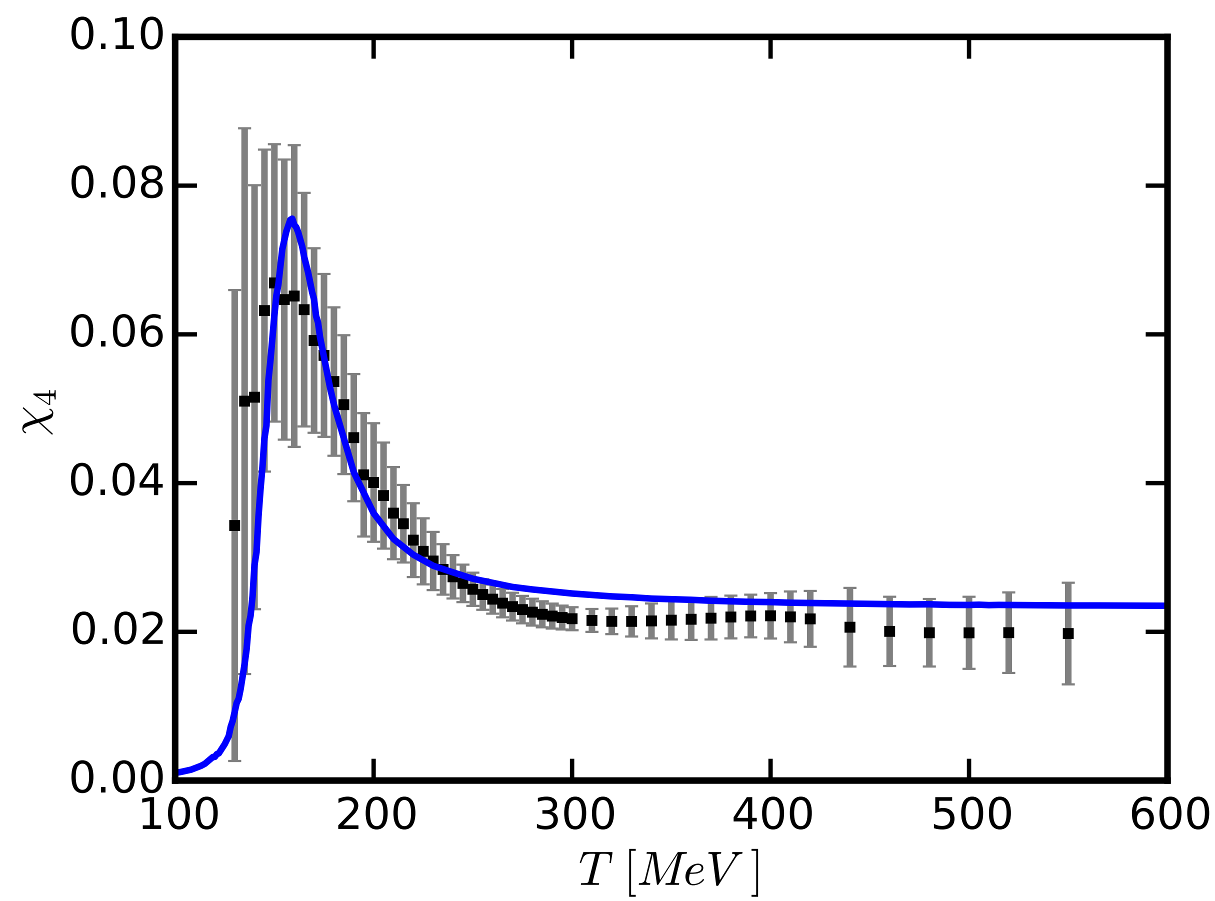


- susceptibilities are relevant fluctuation measures: $\chi_i(T,\mu) \equiv \frac{\partial^i p(T,\mu)}{\partial \mu^i}\Big|_T, \; i = 2,3,4,\ldots$
- $f(\phi)$ is chosen to match 2nd order quark number susceptibility χ_2 in [Bellwied et al., PRD 92 (2015)]:

$$\frac{\chi_2(T,0)}{T^2} = \frac{L}{16\pi^2 f(0)} \frac{s}{T^3} \frac{1}{\int_{r_H}^{\infty} dr \, \frac{e^{-2A}}{f(\phi)}}$$

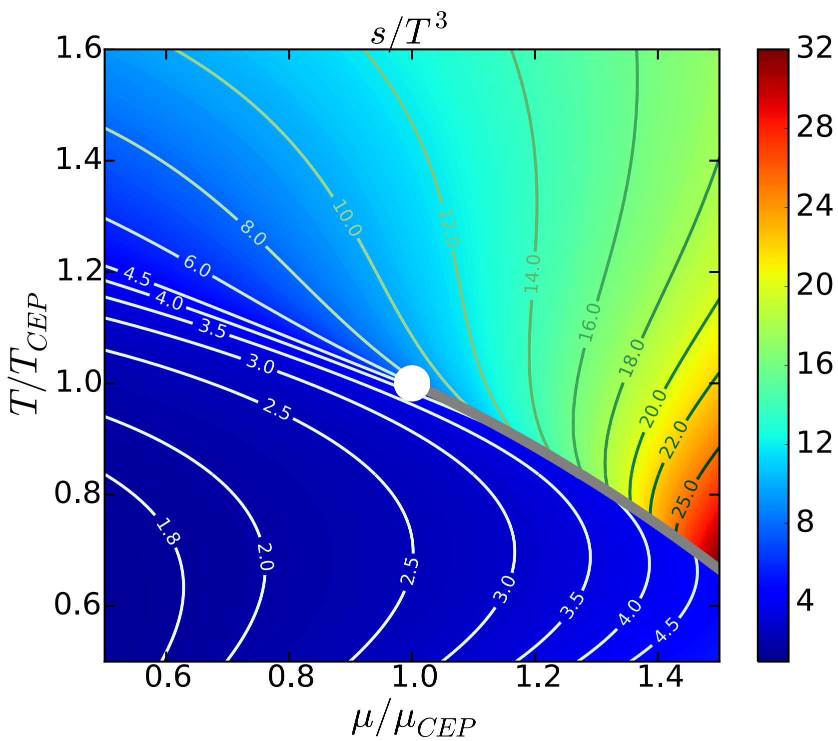
- holographic results:

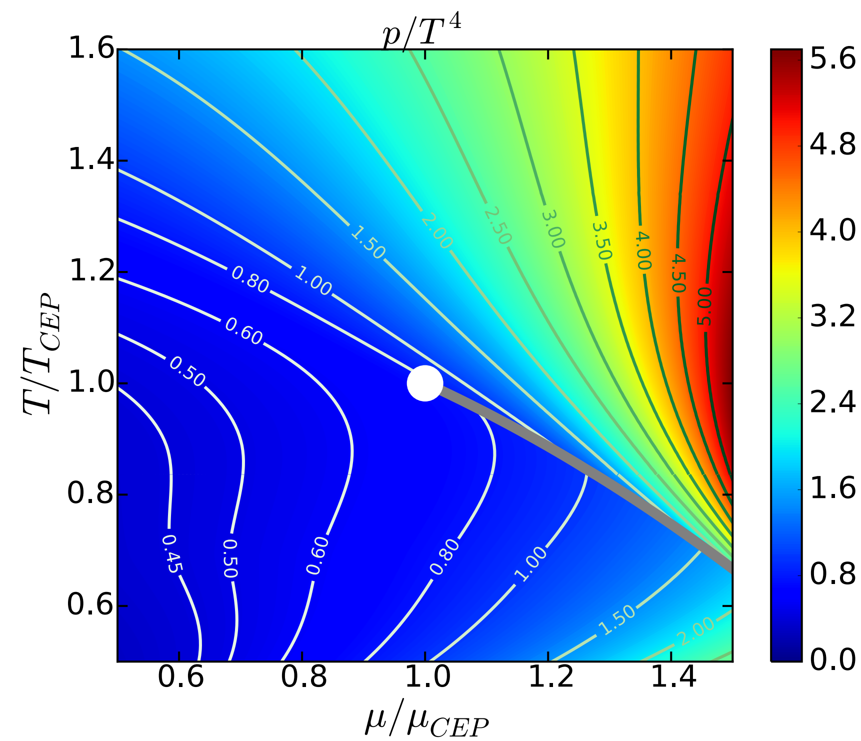




THERMODYNAMIC PHASE DIAGRAMS

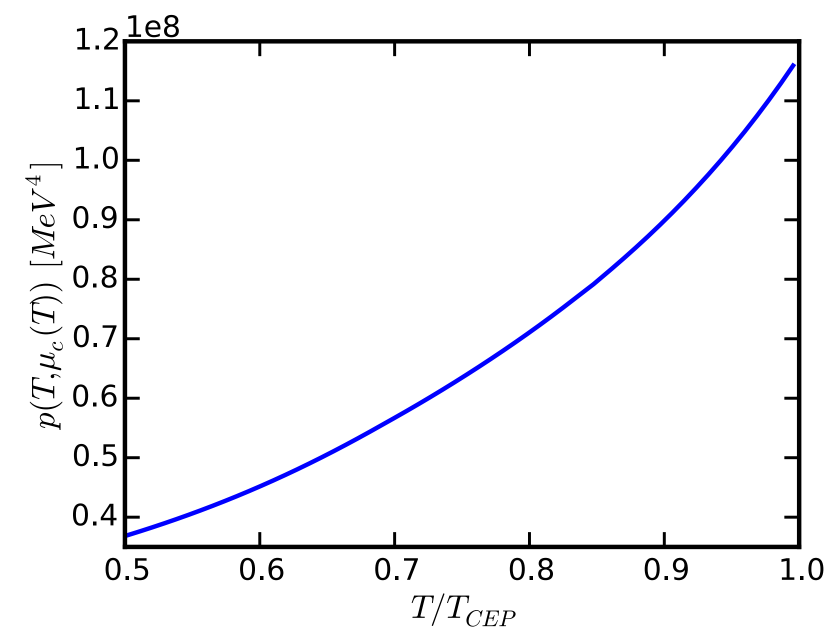
- EMD model exhibits CEP at $T_{CEP} = (112 \pm 5)\,\text{MeV}$ and $\mu_{CEP} = (612 \pm 50)\,\text{MeV}$

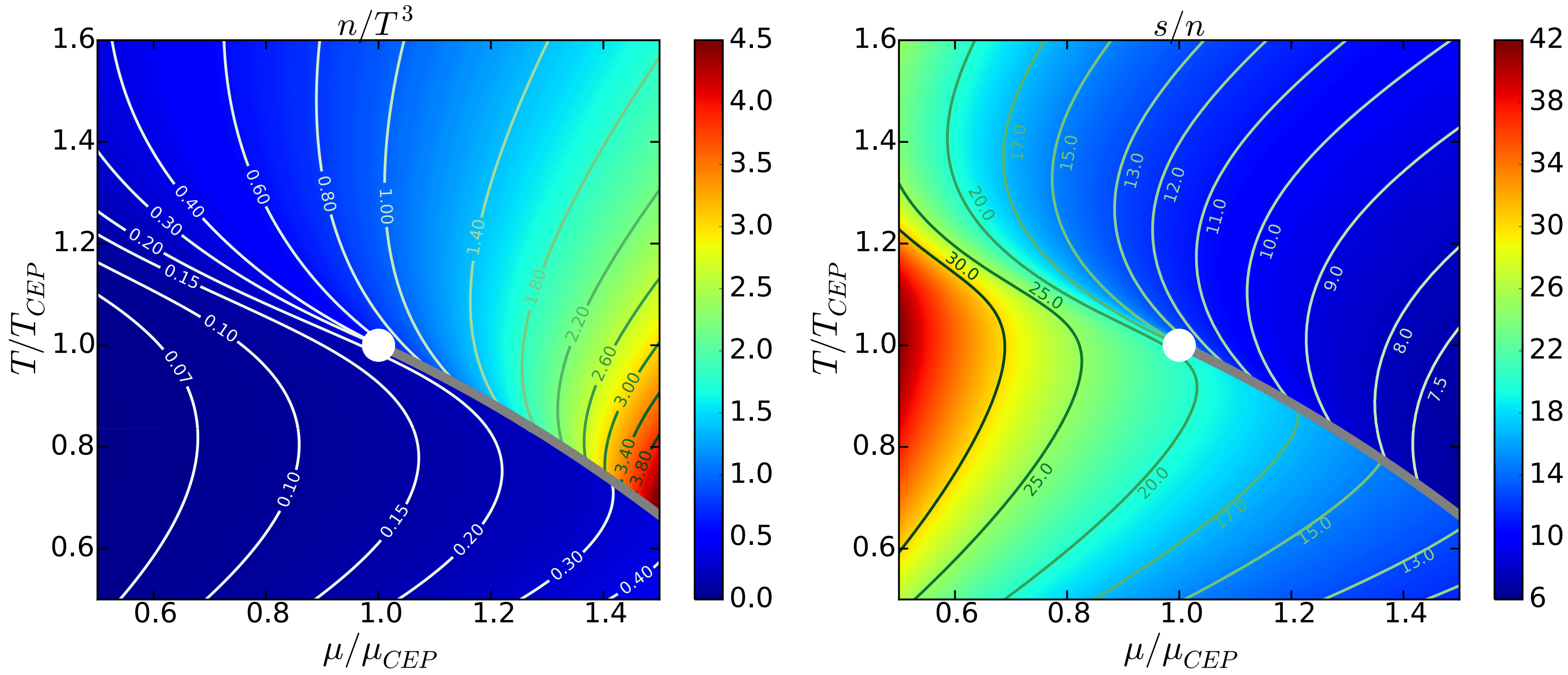




- FOPT curve shows up as kinky behavior of p/T^4 and jumpy behavior of s/T^3, n/T^3, s/n (stable phases)

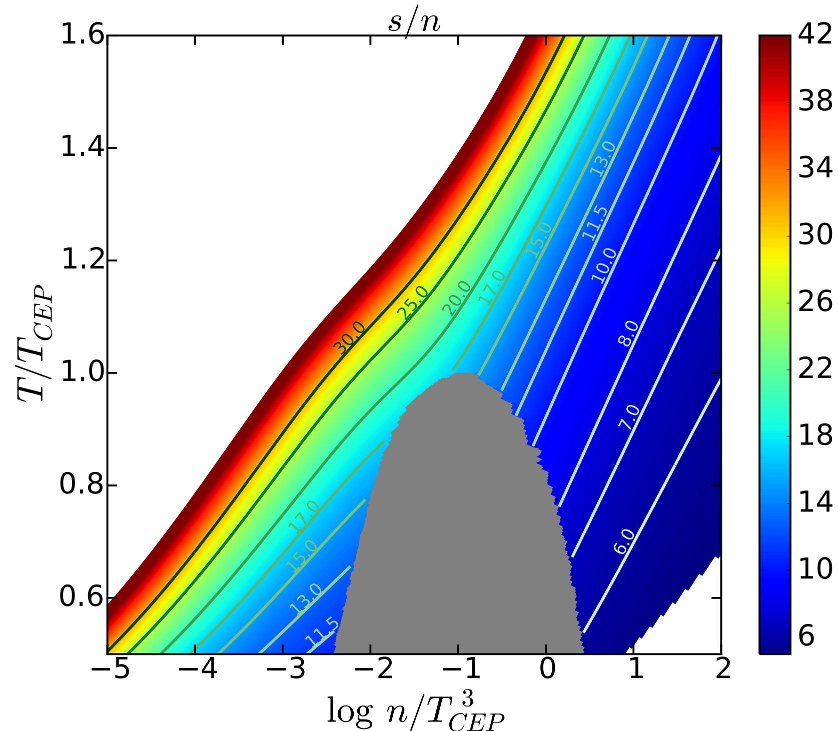
- GL transition:
  critical pressure increases in T-direction
  s/n jumps towards smaller values in T,μ-directions
  across FOPT





Contour plots of scaled baryon density and entropy-to-baryon ratio over the scaled T-μ plane for the updated holographic EMD model.

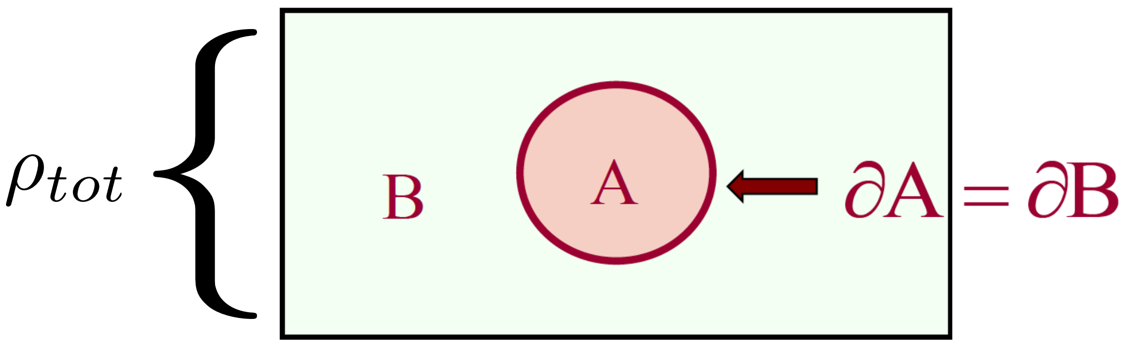
- isentropes enter coexistence region on deconfined/dense side and are leaving on confined/dilute side:



- CEP uncertainty estimated by parameter variations and different low-temperature asymptotics that take lattice uncertainties into account

HOLOGRAPHIC PROPOSAL FOR THE ENTANGLEMENT ENTROPY (HEE)

- Definition: pictures: [Takayanagi, Ahrenshoop Symposium (2012)]



$$H_{tot} = H_A \otimes H_B \quad .$$

Entanglement entropy = von Neumann entropy for reduced density matrix:

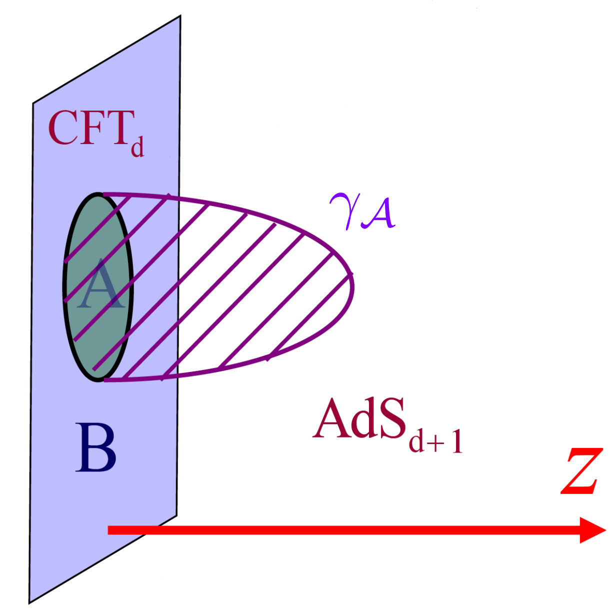
$$\rho_{\mathcal{A}} = Tr_{\mathcal{B}} \rho_{tot}$$

$$S_{\rm EE} := -Tr_{\mathcal{A}} \rho_{\mathcal{A}} \ln \rho_{\mathcal{A}}$$

- HEE for CFT_d is minimal surface in the bulk for a given boundary [Ryu, Takayanagi, PRL 96 (2006)]:

$$S_{\rm HEE} = \frac{\text{Area}(\gamma_{\mathcal{A}})}{4G_N^{(d+1)}}$$

$\gamma_{\mathcal{A}}$ ... *static minimal surface in* AdS_{d+1}
w/ boundary $\partial\gamma_{\mathcal{A}} = \partial\mathcal{A}$

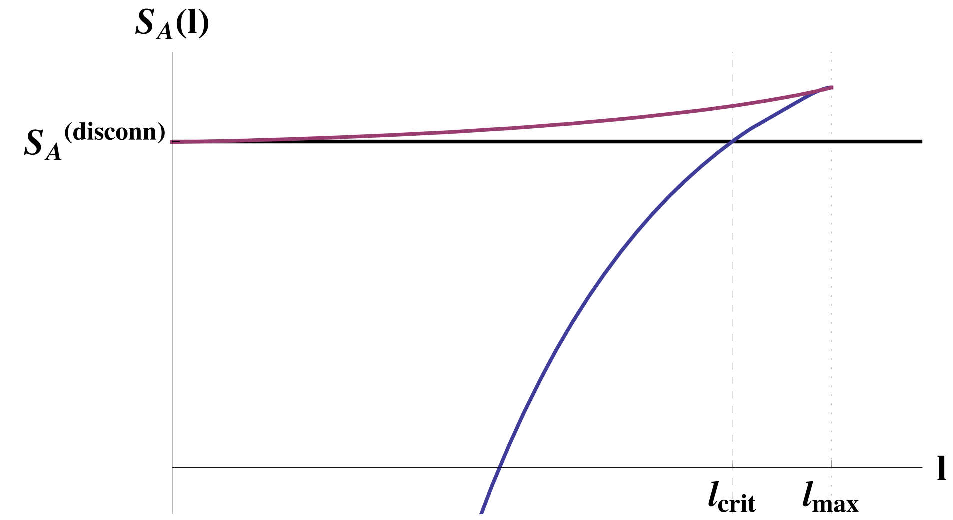


- lots of applications:
  - to study Van der Waals like phase transition in RN-AdS BHs & massive gravity
  - characterization of thermalization processes

  gravity/condensed matter correspondence: - holographic superconductors
  - metal-insulator transitions

- HEE can serve as probe of confinement in gravity duals of large N_c gauge theories [Klebanov et al., NPB 796 (2008)]:
  change between connected and disconnected surfaces in dependence of the length of the boundary area is interpreted as a signature of confinement

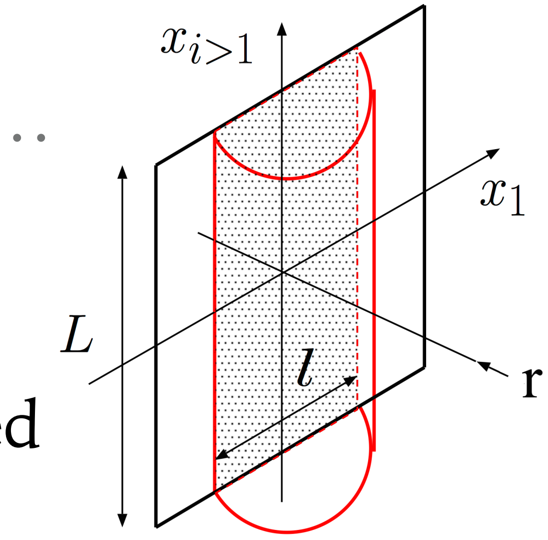
typical „butterfly" shape ⟶



- recent discussion in [Zhang, NPB 916 (2017)] for a fixed shape of entanglement region in dependence of temperature for a bottom-up model that mimics QCD properties at μ=0 [Gubser et al., PRD 78 (2008), PRL 101 (2008)]

  ☛ here: extension for μ>0 in holographic EMD model that mimics QCD phase diagram [JK, Kämpfer (2017), arXiv:1706.02647]

HEE IN THE EMD MODEL



- assume a fixed strip shape on boundary for entanglement region
  $\mathcal{A}: \quad x_1 \in [-l/2, l/2], \quad x_2, x_3 \in [-L/2, L/2] \quad$ w/ $\quad L \gg l$
  (translation invariance) $\longrightarrow$ minimal surface can be parameterised by $r = r(x_1)$

- induced metric on static minimal surface: $ds^2_{\gamma_\mathcal{A}} = \left(e^{2A} + \frac{r'^2}{h}\right) dx_1^2 + e^{2A}\left(dx_2^2 + dx_3^2\right)$

  HEE: $$S_\text{HEE} = \frac{1}{4}\int dx_1 dx_2 dx_3 \sqrt{\gamma} = \frac{V_2}{2}\int_0^{l/2} dx_1 e^{2A(r)} \sqrt{e^{2A(r)} + \frac{r'^2}{h(r)}}$$

  w/ $V_2 \equiv \int dx_2 dx_3$ and γ … determinant of induced metric

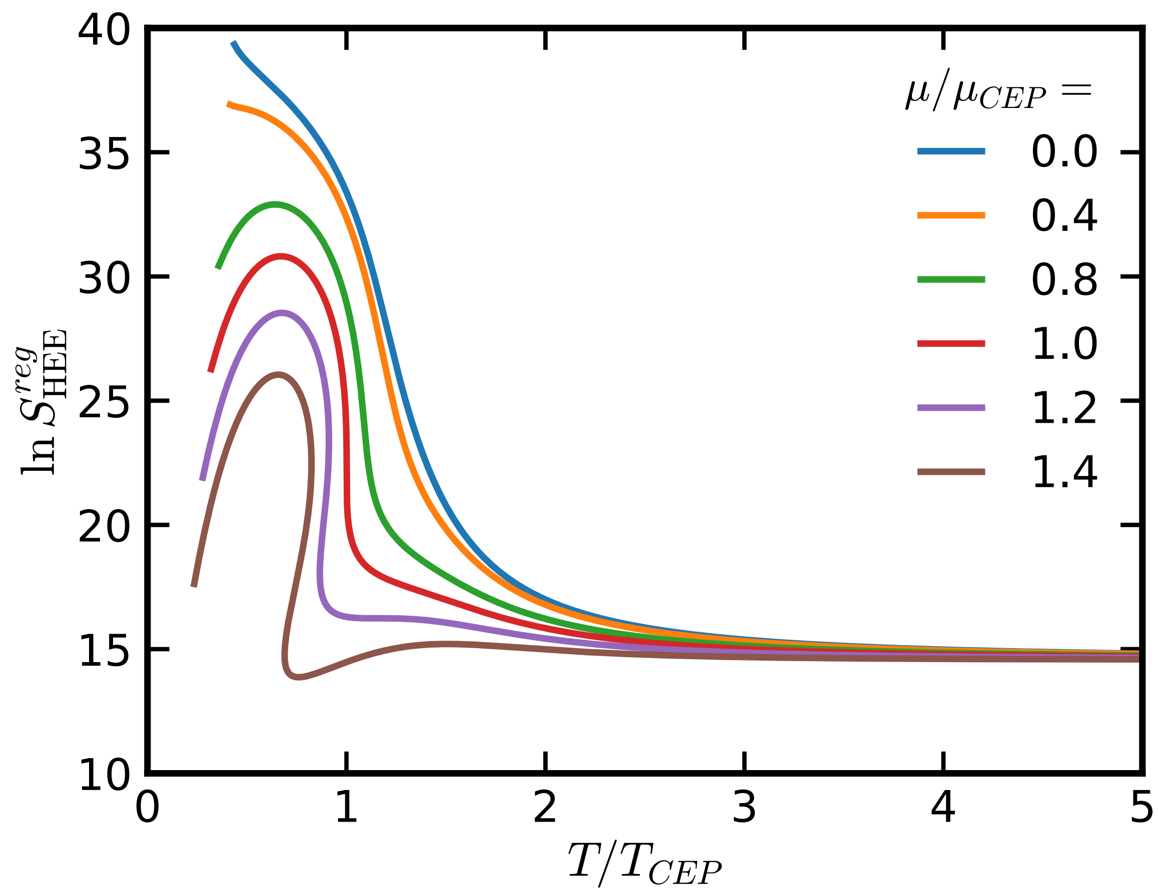
- Extremizing S_HEE similar to mechanics problem: one has conserved quantity and finds
  $r' = \sqrt{h(r)\left(e^{8A(r)-6A(r_*)} - e^{2A(r)}\right)}$ $\quad r_*$ … closest position of minimal surface to horizon

- boundary condition: $\frac{l}{2} = \int_{r_*}^\infty dr \frac{dx}{dr} = \int_{r_*}^\infty dr \left[h(r)\left(e^{8A(r)-6A(r_*)} - e^{2A(r)}\right)\right]^{-1/2}$
  $\Longrightarrow$ determines r_* for given l

- HEE, finally:
  $$S_\text{HEE} = \frac{V_2}{2}\int_{r_*}^\infty dr \frac{e^{6A(r)-3A(r_*)}}{e^{A(r)}\sqrt{h(r)\left(e^{6A(r)-6A(r_*)} - 1\right)}}$$ $\longrightarrow$ divergent

- regularized HEE density defined by cutoff:

$$S_{\mathrm{HEE}}^{reg} := \frac{1}{2}\int_{r_*}^{r_m} dr \frac{\mathrm{e}^{6A(r)-3A(r_*)}}{\mathrm{e}^{A(r)}\sqrt{h(r)\left(\mathrm{e}^{6A(r)-6A(r_*)}-1\right)}}$$



decreasing in crossover region,

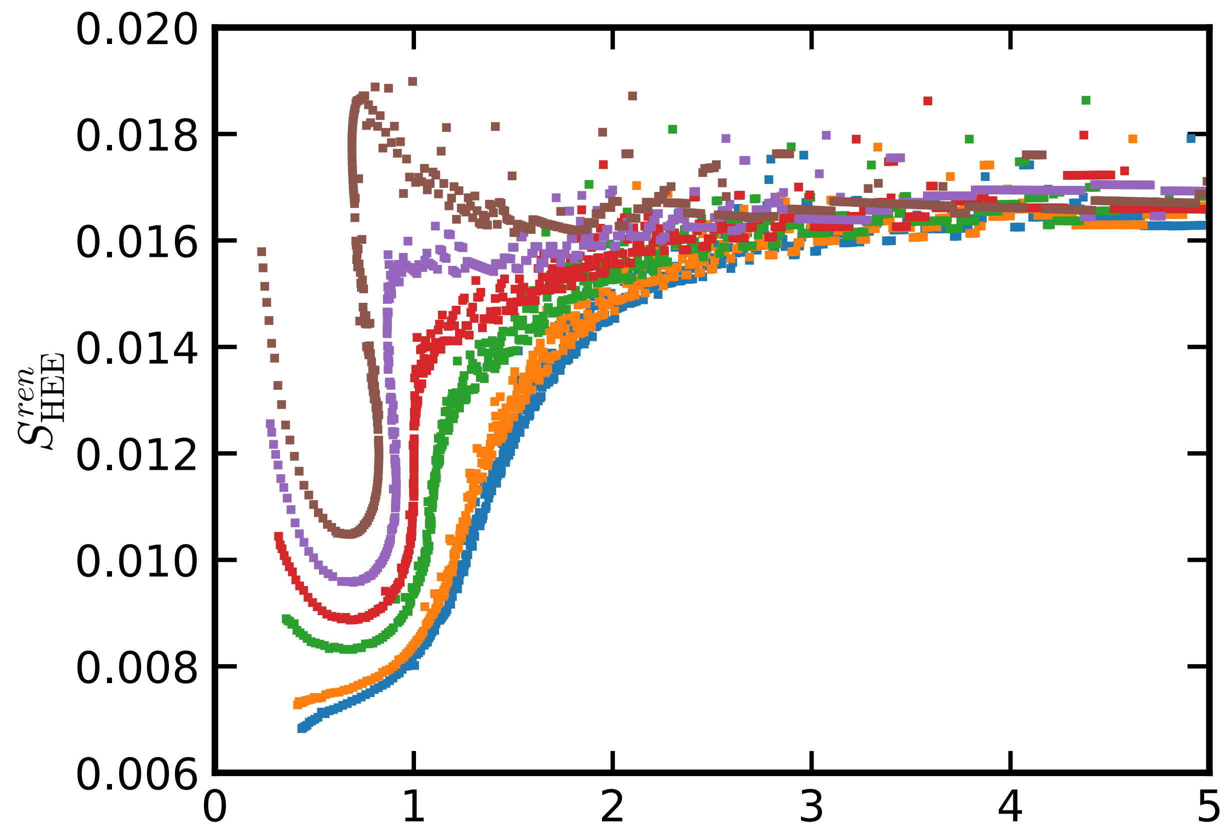
1st order phase transition signaled

by multivalued branch

- renormalized HEE density: $S_{\mathrm{HEE}}^{ren} := \frac{1}{2}\int_{r_*}^{r_m} dr \ln \frac{H(r)}{\tilde{H}(r)}$

$H(r)$ … integrand in S_{HEE}

$\tilde{H}(r)$ … $A(r_*) \equiv 0$ set in $H(r)$

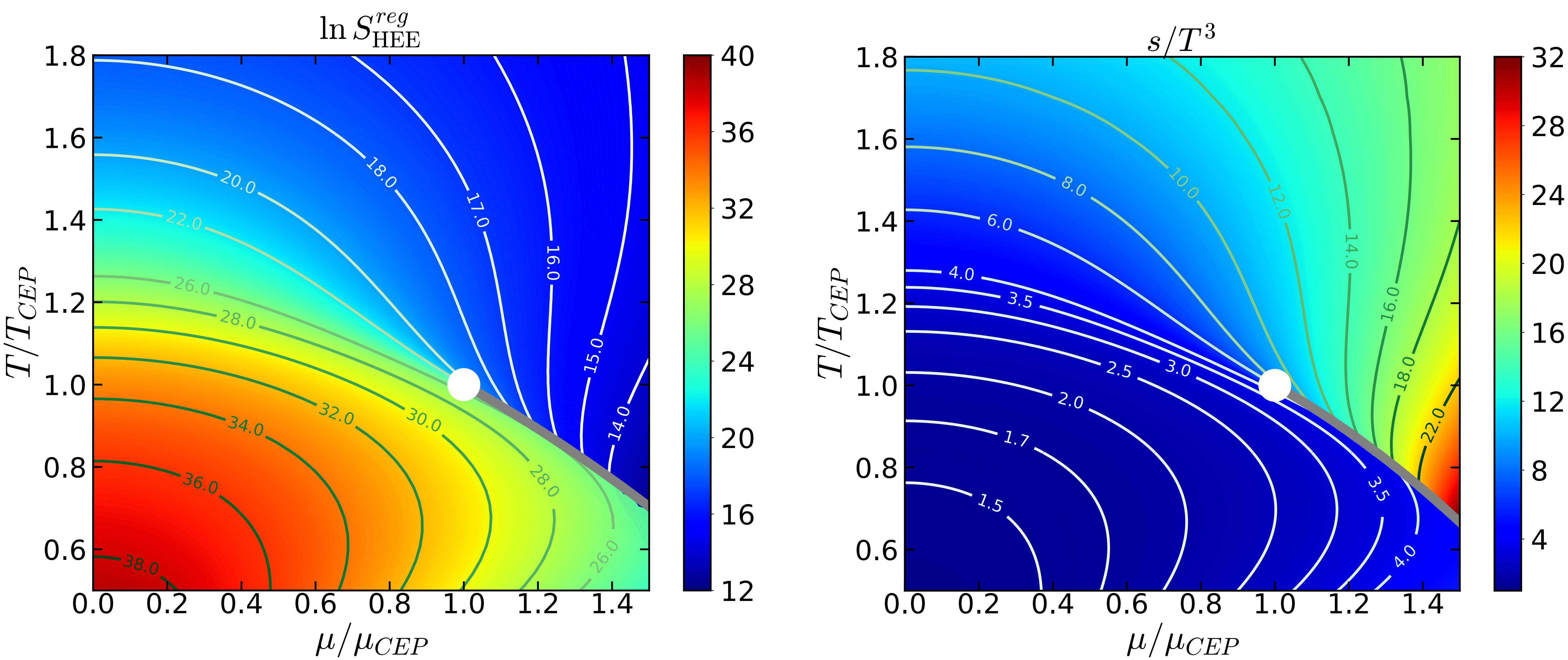


qualitative behavior similar to

thermodynamic entropy density,

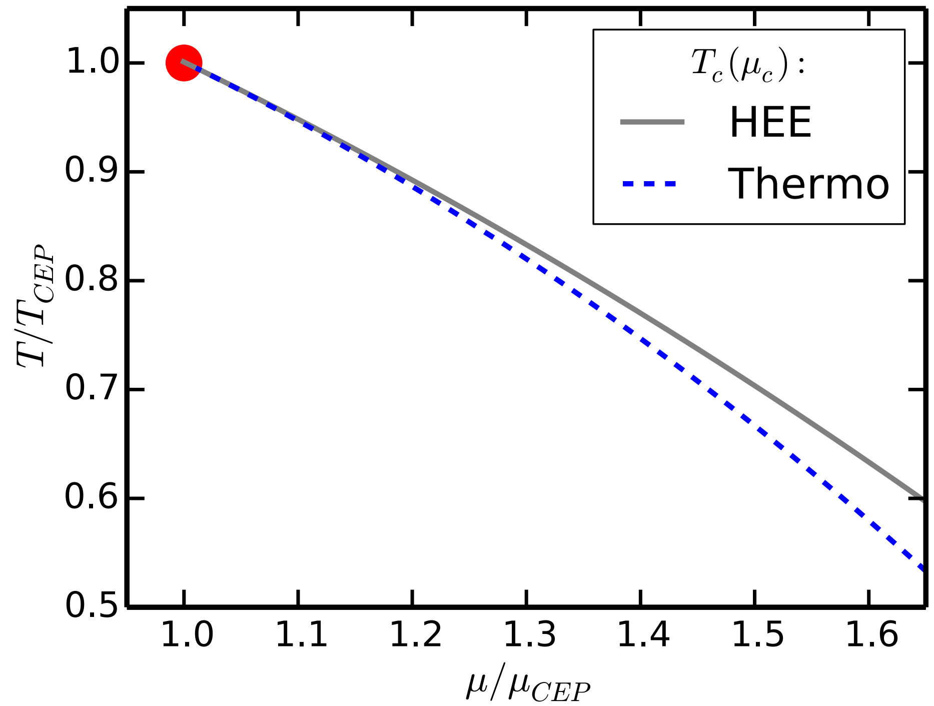
numerically difficult

➤ phase diagrams:



- pseudo-pressure defined as $dp_{\mathrm{HEE}} = \ln(S_{\mathrm{HEE}}^{reg})\mathrm{d}T$ for $\mu \equiv \mathrm{const}$ to determine T_c
- opposite qualitative behavior of HEE to scaled thermodynamic entropy
- HEE exhibits the same critical point
- remarkable similarity of „isentrope" pattern

➤ comparison of FOPT curves:



good agreement near CEP

➤ critical exponent for heat capacity at constant chemical potential:

$$C_\mu \equiv T \frac{\partial s}{\partial T}\Big|_\mu = -T \frac{\partial^2 f}{\partial T^2}\Big|_\mu \sim |T - T_{CEP}|^{-\alpha}, \qquad \mu \equiv \mu_{CEP}, \quad T < T_{CEP}$$

(α' similar for $T > T_{CEP}$)

for thermodynamic entropy $\longrightarrow$ $\alpha \approx 0.66, \quad \alpha' \approx 0.64$

for HEE w/ $\ln S_{\text{HEE}}^{reg}$ $\longrightarrow$ $\alpha \approx 0.65, \quad \alpha' \approx 0.66$

$\Bigg\} \approx \frac{2}{3}$

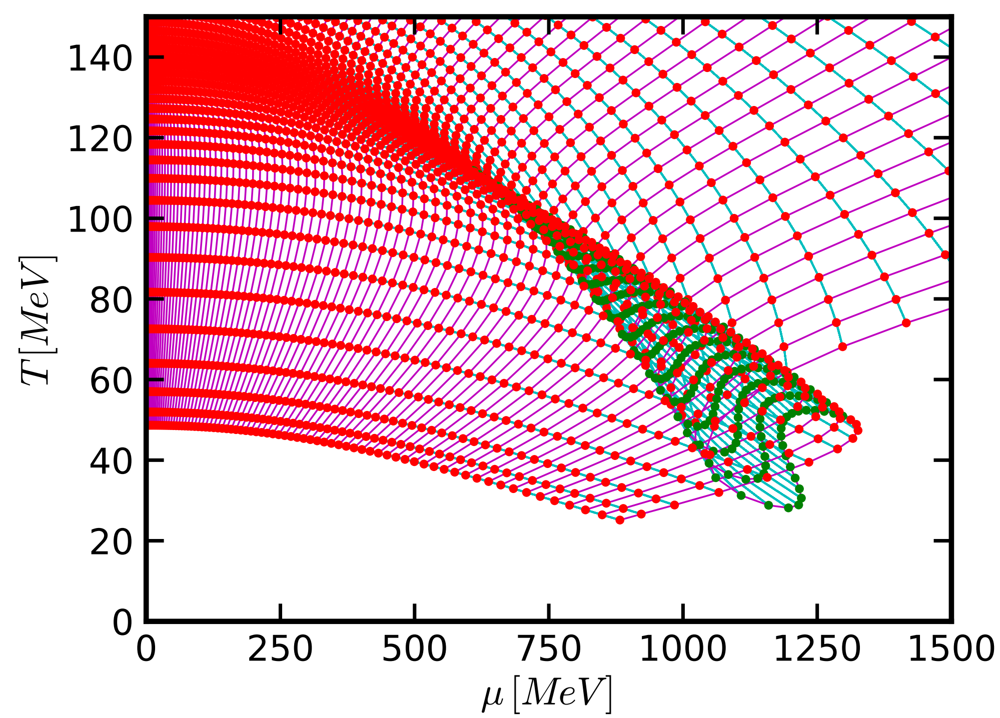
SUMMARY:

+ **Holographic model is adjusted to known lattice data at μ=0 and exhibits CEP at**
  $T_{CEP} = (112 \pm 5)\,\text{MeV}$ and $\mu_{CEP} = (612 \pm 50)\,\text{MeV}$
+ **Holographic QCD phase diagram has gas-liquid FOPT w/ in- and outgoing isentropes**
+ **HEE can characterize different phase structures in the T-μ plane**
+ **confinement/deconfinement transition of HEE at finite μ is described by FOPT curve starting at critical point in agreement w/ thermodynamic result**

BACKUP

- position of CEP can be estimated with determinant of susceptibility matrix:

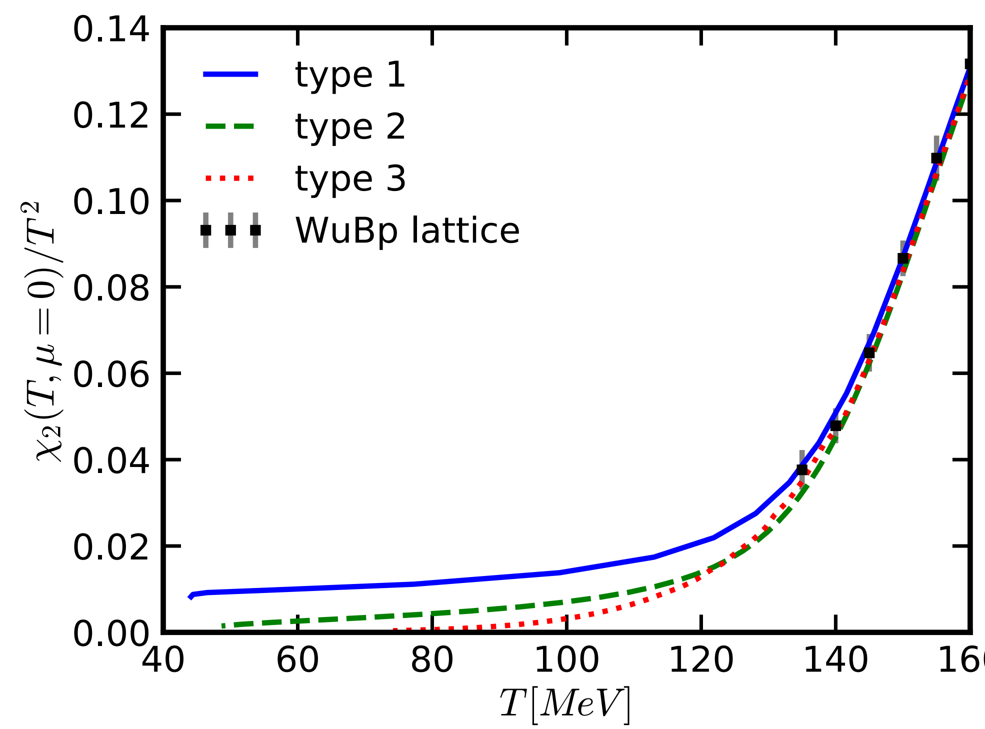
$$J \equiv \det S = \partial(s,\rho)/\partial(T,\mu)$$



red: J>0 (stable)

green: J<0 (unstable)

- accuracy of CEP estimated through different low-temperature asymptotics of χ_2 / EoS and parameter variations to take lattice uncertainties into account:



- Entanglement entropy in QFTs has UV divergences
  Area law:
  leading divergence of EE in (d+1) dim. QFT in its ground state is proportional to the area of the (d-1) dim. boundary ∂A :

$$S_{\rm EE} \sim \frac{\text{Area}(\partial A)}{a^{d-1}} + \text{subleading}$$

  (a… UV cutoff / lattice spacing) $\Longrightarrow$ initial interest in BH physics

- recent CEP estimation based on holographic model in [R. Critelli et al. (2017), arXiv:1706.00455] gives:

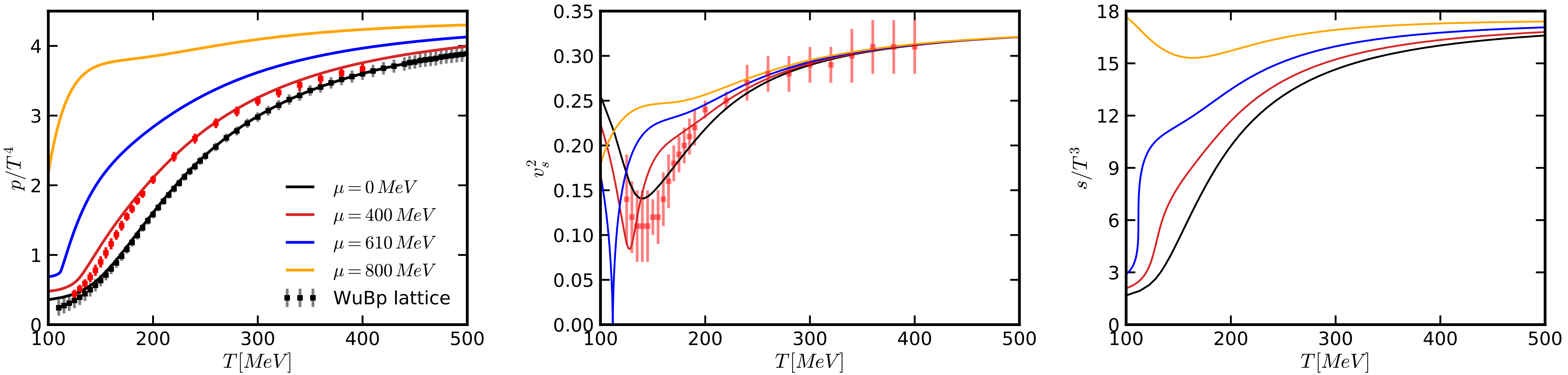
$$T_{CEP} = (89 \pm 11)\,\text{MeV} \text{ and } \mu_{CEP} = (723 \pm 36)\,\text{MeV}$$

  only marginally consistent with our result

$$T_{CEP} = (112 \pm 5)\,\text{MeV} \text{ and } \mu_{CEP} = (612 \pm 50)\,\text{MeV}$$

  $\Longrightarrow$ model is sensitive on input and adjustment;
  missing lattice data for low temperatures seems to hamper unique determination of CEP

- comparison to lattice QCD at μ>0:



lattice: [Borsanyi et al., JHEP 08 (2012)]

- phase diagram: [Günther et al., EPJ WoC (2017)]
(direct comparison might be not appropriate due to imposed conditions in lattice calculations)