Damping of gravitational waves by matter

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Can gravitational waves be damped by matter?

TT gauge (lapse, shift are trivial)

$$g_{00} = -1,\ g_{i0} = 0,\ g_{ij}(x,t) = a^2(t)\left[\delta_{ij} + h_{ij}(x,t)\right]$$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2}h_{ij} = 16\pi G_N \pi_{ij}$$

Can gravitational waves be damped by matter?

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$$T_{ij} = \bar{p}g_{ij} + a^2\pi_{ij}$$

anisotropic stress

Induced metric on the hypersurfaces orthogonal to the fluid flow:

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$\pi^{\mu\nu} = -2\eta\left(\tfrac{1}{2}\left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu\right] - \tfrac{1}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right)$$

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= 0 for a perfect fluid

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"shear viscosity"

$$\pi^{\mu\nu} = -2\eta\left(\tfrac{1}{2}\left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu\right] - \tfrac{1}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right)$$

What is shear viscosity?

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In presence of a velocity gradient, e.g. $\frac{\partial u^z}{\partial x} \neq 0$

Momentum transfer in x direction b/c microscopic motion of fluid particles…
Fast moving layers transfer momentum to slow moving layers.

(friction normal to fluid flow -> resistance against shear)

inviscous fluid needs zero mean free path -> ideal fluid

Imperfect fluid in hydrodynamical limit

$$\pi_{ij} = -\eta \dot{h}_{ij}$$ (Hawking 1966)

hydro limit: $\frac{\lambda_{\rm mf}}{\lambda_{\rm obs}} \ll 1$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2} h_{ij} = -16\pi G_N \eta \, \dot{h}_{ij}$$

dissipation rate $\sim G_N \eta$

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Use to bound η from observations, learn about DM self interactions?

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But what if there are too few collisions for hydrodynamic limit to be valid?

Outline and summary

$$\pi^{\mu\nu} = -2\eta\left(\tfrac{1}{2}\left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu\right] - \tfrac{1}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right)$$

A more detailed analysis that interpolates between kinetic and hydro regimes:

- Interactions produce η, but also serve to erase anisotropies
- Two distinct regimes – collisional damping, and Landau damping
- Independent of details of the collisions, efficient only when $\omega \sim H$

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A more detailed analysis that interpolates between kinetic and hydro regimes:

- Damping for astrophysical sources (even in dense environments) inefficient
- Primordial GW's can be damped (Misner 1967, Weinberg 2004) – we interpret the former in terms of Landau damping
- (Generalize Weinberg's calculation to include collisions, derive η from microphysics, outline future directions – ultra light DM?)

GW's at low redshift

(e.g. to begin with, neglect expansion and work on a Minkowski background)

$$\pi^{\mu\nu} = ?$$

Collisional Boltzmann equation:

$$\left(\frac{\partial}{\partial t} + \vec{\nabla}_p \epsilon \cdot \vec{\nabla}_r - \vec{\nabla}_r \epsilon \cdot \vec{\nabla}_p \right) f = \mathcal{C}$$

Liouville operator

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Liouville operator

$$\frac{\partial f}{\partial t} - \{\epsilon, f\} = \mathcal{C}$$

GW's at low redshift

(e.g. neglect expansion and work on a Minkowski background)

$$T^j_{i,M} = \int_p p_i v^j f = g^{jk} \int_p \frac{p_i p_k}{\epsilon} f$$

$$\epsilon^2 = g^{ij} p_i p_j + m^2 \qquad\qquad \delta\epsilon = \frac{1}{2} h^{ij} \frac{p_i p_j}{\epsilon_0} = -\frac{1}{2} h_{ij} \frac{p_i p_j}{\epsilon_0}$$

$$\pi_{ij} = \delta T_{ij} - \frac{1}{3} g_{ij} T = \int_p \frac{p_i p_j}{\epsilon_0} \left[\delta f - \delta\epsilon \frac{\partial f_0}{\partial \epsilon} \right]$$

distribution function in presence of wave (not necessary to assume thermal!)

$$f \to f_h = \frac{1}{e^{\beta(\epsilon-\mu)} \mp 1}$$

GW's at low redshift

(e.g. neglect expansion and work on a Minkowski background)

$$f_h = f_0 + \delta\epsilon \frac{\partial f}{\partial \epsilon}$$

Collision time approximation (spin 2):

$$\mathcal{C} \approx \frac{\Delta f}{\Delta t} = -\frac{f - f_h}{\tau} = -\frac{1}{\tau}\left(\delta f - \delta\epsilon \frac{\partial f}{\partial \epsilon}\right)$$

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau} + \vec{v} \cdot \vec{\nabla}_{\vec{r}}\right) \delta f = \left(\vec{v} \cdot \nabla_r \delta\epsilon + \frac{1}{\tau}\delta\epsilon\right) \frac{\partial f_0}{\partial \epsilon}$$

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(e.g. neglect expansion and work on a Minkowski background)

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau} + \vec{v} \cdot \vec{\nabla}_{\vec{r}}\right) \delta f = \left(\vec{v} \cdot \nabla_r \delta\epsilon + \frac{1}{\tau}\delta\epsilon\right) \frac{\partial f_0}{\partial \epsilon}$$

$$\delta f - \frac{\partial f_0}{\partial \epsilon}\delta\epsilon = -\left(\frac{\omega}{\omega - \vec{q}\cdot\vec{v} + i/\tau}\right)\frac{\partial f_0}{\partial \epsilon}\delta\epsilon$$

$$\pi_{ij}(q,\omega) = \int_p \frac{p_i p_j}{\epsilon_0}\delta\epsilon \left(\frac{\omega}{\vec{q}\cdot\vec{v} - \omega - i/\tau}\right)\frac{\partial f_0}{\partial \epsilon}$$

GW's at low redshift

(e.g. neglect expansion and work on a Minkowski background)

Damping governed by the imaginary part of the response:

$$\Im\left(\frac{\pi_{ij}}{h_{ij}}\right) = -\omega\int_p \left(\frac{p_i p_j}{\epsilon}\right)^2 \frac{1/\tau}{(\omega - \vec{q}\cdot\vec{v})^2 + 1/\tau^2}\frac{\partial f_0}{\partial \epsilon}$$

$$\begin{aligned}\pi_{ij} &= i\tau\omega\int_p \frac{p_i p_j}{\epsilon_0}\delta\epsilon\,\frac{\partial f_0}{\partial \epsilon} = -\frac{i\tau\omega}{15}\int_p \frac{p^4}{\epsilon_0^2}\frac{\partial f_0}{\partial \epsilon}h_{ij} \\ &= -\eta\dot{h}_{ij}.\end{aligned}$$

$$\eta = -\int_p \left(\frac{p_i p_j}{\epsilon_0}\right)^2 \frac{\partial f_0}{\partial \epsilon_0}\tau$$

$$\tau \to 0, \infty \to 0$$

GW's at low redshift

(e.g. neglect expansion and work on a Minkowski background)

Maximal collisional damping when $\omega\tau \sim 1$

$$\mathrm{Max}(|\Im\omega|) \lesssim -\frac{8\pi G}{\omega}\int_p \frac{p^4}{15\epsilon_0^2}\frac{\partial f_0}{\partial \epsilon} \leq \frac{8\pi G P}{\omega}$$

collisional damping only efficient for waves of frequencies

$$\omega \sim H$$

GW's at low redshift

(e.g. neglect expansion and work on a Minkowski background)

In a dense environment, collisional damping limited by

$$|\Im\omega| < \gamma_{\rm max} \qquad \gamma_{\rm max} \sim \frac{1}{M_{\rm pl}^2}\frac{P}{\omega} = \frac{w}{M_{\rm pl}^2}\frac{\rho}{\omega}$$

along line of sight, need:

$$\int_0^R dr\, \gamma_{\rm max} \sim 1$$

GW's at low redshift

(e.g. neglect expansion and work on a Minkowski background)

Consider a GW propagating from the core of a DM halo:

$$\int_0^R dr\,\gamma_{\max} \sim w\frac{10^{-27}}{(R_c/\mathrm{kpc})^2}\left(\frac{M}{M_\odot}\right)\frac{1}{(\nu/\mathrm{Hz})}$$

$$M \sim 10^{12} M_\odot,\ R_c \sim 100\ \mathrm{kpc}$$

$$\int_0^R dr\,\gamma_{\max} \sim w\frac{10^{-19}}{(\nu/\mathrm{Hz})}$$

… feeble for all astrophysical sources

GW's at low redshift

(e.g. neglect expansion and work on a Minkowski background)

Collisionless limit:

$$\Im\left(\frac{\pi_{ij}}{h_{ij}}\right) = \pi\omega \int_p \left(\frac{p_i p_j}{\epsilon_0}\right)^2 \delta(\omega - \vec{q}\cdot\vec{v})\frac{\partial f_0}{\partial \epsilon} \to 0$$

Including cosmological expansion unveils another limit – Landau damping

GW's at cosmological redshifts

$$\pi_{ij} = -4\bar{\rho} \int_{u_0}^{u} du' e^{-\ell(u,u')} K\left(q(u-u')\right) h'_{ij}(u')$$

$$K(s) = j_2(s)/s^2$$

$$\ell(u,u') = \int_{u'}^{u} \frac{du''}{\tau_c(u'')}$$

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} + q^2 h_{ij} = 16\pi G a^2 \pi_{ij}$$

$$h_{ij}(\vec{r},u) = \int \frac{d^3q}{(2\pi)^3} \frac{e^{-iqu}}{a(u)} F(q)$$

GW's at cosmological redshifts

$$\frac{\partial E}{\partial u} = \frac{\bar{\rho}}{4} \int \frac{d^3 q}{(2\pi)^3} |h'_{ij}(q,u)|^2 \int_{-1}^{1} \frac{d\zeta}{2} \frac{\mathcal{H}(1-\zeta^2)^2}{\mathcal{H}^2 + q^2(1-\zeta)^2}$$

$$\zeta := \cos\theta$$

Expansion induces a spread in frequencies $\sim \mathcal{H}$ around q…

$$\frac{\partial E}{\partial u} \sim \frac{4\mathcal{H}^3}{(a\bar{q})^2} E_{gw}$$

characteristic absorption time $\sim \omega^2 / H^3$; negligible for astrophysical, but not primordial sources

GW's at cosmological redshifts

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Ultra light DM processing of stochastic backgrounds during matter dom?