Primordial GWs and PBHs from axion inflation

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Outline

Inflation: the basic picture

Inflation is an early phase of nearly exponential expansion.

The metric of $g_{\mu\nu}$ of an homogeneous and isotropic Universe ($k = 0$) is:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$$

Einstein Equations ($\Lambda = 0, \kappa^2 \equiv 8\pi G_N$) read:

$$3\left(\frac{\dot{a}}{a}\right)^2 \equiv 3H^2 = \rho\kappa^2, \qquad -2\dot{H} = (p+\rho)\kappa^2$$

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Homogeneous scalar field ϕ in a homogeneous and isotropic universe:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{\dot{\phi}^2}{2} - V(\phi)\right)$$

In this case the pressure and energy density are:

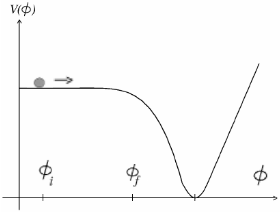
$$p = \frac{\dot{\phi}^2}{2} - V \qquad \rho = \frac{\dot{\phi}^2}{2} + V$$

Slow-roll inflation

Slow-roll inflation:
$$\left|\dot{\phi}^2/2\right| \ll |V(\phi)|.$$

The evolution is fixed by:

$$\begin{aligned} 3H^2 &= \rho\kappa^2 \simeq V\kappa^2 \\ -2\dot{H} &= (p+\rho)\kappa^2 = \dot{\phi}^2\kappa^2 \\ 3H\dot{\phi} &\simeq -V_{,\phi} \end{aligned}$$



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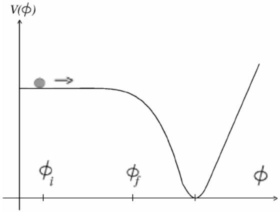

To discuss this problem it is useful to introduce the slow-roll parameters:

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2\kappa^2}\left(\frac{V_{,\phi}}{V}\right)^2 \equiv \epsilon_V$$

$$\epsilon_2 \equiv \frac{\mathrm{d}\ln(\epsilon_1)}{\mathrm{d}\ln a} \simeq -\frac{2}{\kappa^2}\frac{V_{,\phi\phi}}{V} + \frac{2}{\kappa^2}\left(\frac{V_{,\phi}}{V}\right)^2 \equiv -2\eta_V + 4\epsilon_V$$

and the number of e-foldings (from the end of inflation):

$$N(t) \equiv -\int_{a_f}^{a} \mathrm{d}\ln\hat{a} = -\int_{t_f}^{t} H(\hat{t})\mathrm{d}\hat{t} \simeq \int_{\phi_f}^{\phi} \kappa^2 \frac{V(\hat{\phi})}{V_{,\phi}(\hat{\phi})}\mathrm{d}\hat{\phi}$$

Model classification

Inflationary models can be classified using:

Mukhanov 2013, Roest 2014, Garcia-Bellido and Roest 2014, Binetruy, Kiritsis, Mabillard, Pieroni and Rosset 2015

$$\epsilon_1 \simeq \frac{\beta}{(1+N)^p}$$

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$$\epsilon_1 \simeq \frac{\beta}{(1+N)^p}$$

- (p=1) $\longrightarrow$ Chaotic models:

$$V(\phi) = V_0\ \phi^q$$

- (p=2) $\longrightarrow$ Starobinsky-like models:

$$V(\phi) \simeq V_0 \left(1 - \exp\{-\gamma\phi\}\right)^2$$

- (p=3) $\longrightarrow$ Hilltop models:

$$V(\phi) \simeq V_0 \left[1 - \left(\frac{\phi}{v}\right)^4\right]^2$$

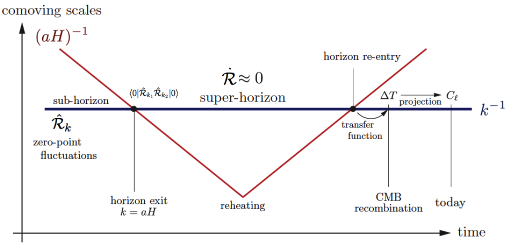
- (p=4) $\longrightarrow$ Hilltop models:

$$V(\phi) \simeq V_0 \left[1 - \left(\frac{\phi}{v}\right)^3\right]^2$$

Power spectra and CMB Observables

Perturbations around the homogeneous background:

$$\Phi(t,\vec{x}) = \phi(t) + \delta\phi(t,\vec{x}) \qquad \mathbf{g}_{\mu\nu}(t,\vec{x}) = g_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\vec{x})$$



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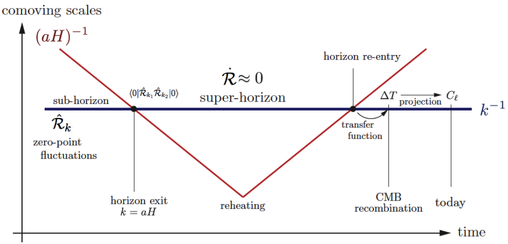


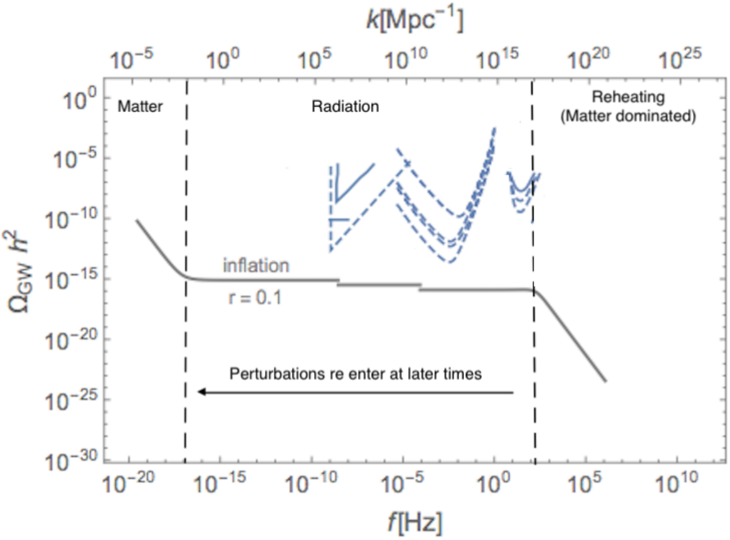
Scalar and tensor power spectra:

$$\Delta_s^2(k,\tau)\Big|_{\tau=k^{-1}} = \frac{1}{8\pi^2}\frac{H^2\kappa^2}{\epsilon_V} \qquad \Delta_t^2(k,\tau)\Big|_{\tau=k^{-1}} = 2\left(\frac{\kappa H}{\pi}\right)^2$$

Tensor-to-scalar ratio (r) and the scalar spectral index (n_s):

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2}\Big|_{k=aH} \simeq 16\epsilon_V \qquad n_s \equiv 1 + \frac{\mathrm{d}\ln\Delta_s^2(k)}{\mathrm{d}\ln k}\Big|_{k=aH} \simeq 1 + 2\eta_V - 6\epsilon_V$$

Detection of Primordial GWs



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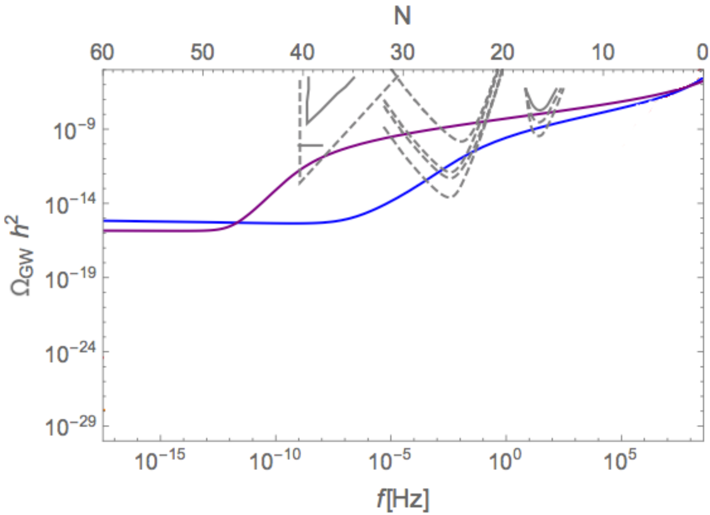

Primordial Black Hole formation

If large fluctuations are generated during inflation (*i.e.* for large Δ_s^2), when they re-enter into the horizon they may lead to the formation of PBHs.

PBHs are viable candidates for CDM!

García-Bellido, Linde and Wands 1996

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Given $P_N(\zeta)$ probability distribution of fluctuations (in terms of N) we have:

$$\beta(N) = \int_{\zeta_c}^{\infty} \frac{M(\zeta, N)}{M_H(N)} P_N(\zeta) d\zeta = \int_{\zeta_c}^{\infty} \gamma \, P_N(\zeta) d\zeta$$

fraction of the energy density which collapses into PBHs at any given t_N.

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The mass of the horizon at the time of horizon re-entry t_N is:

$$M_H(N) \simeq \gamma \, \frac{4\pi M_P^2}{H_{\text{inf}}} e^{jN} \simeq 55 \, \gamma \left(\frac{10^{-6} \, M_P}{H_{\text{inf}}} \right) e^{jN} \, g$$

We can express the fraction of the energy that collapeses in terms of M!

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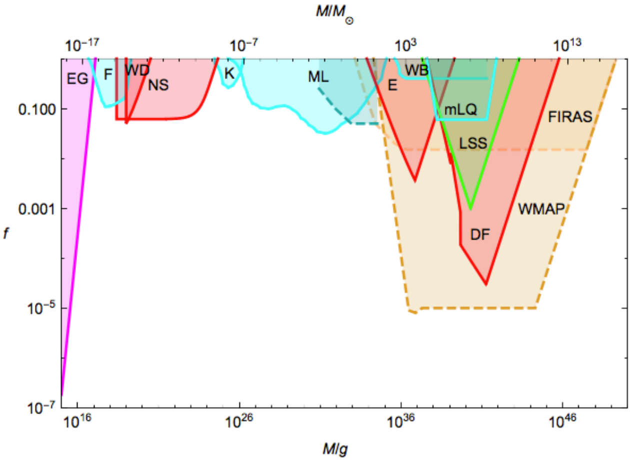
The fraction of DM composed by PBHs (with mass M) today is:

$$f(M) = \frac{M n_{\rm PBH}(t_0)}{\Omega_{CDM}\, \rho_c} \simeq 4.1\times 10^8\, \gamma^{1/2} \left(\frac{g_*(t_N)}{106.75} \right)^{-1/4} \left(\frac{h}{0.68} \right)^{-2} \left(\frac{M}{M_\odot} \right)^{-1/2} \beta(M)$$

A summary of constraints on non-evaporated PBHs

PBHs with $M \lesssim 10^{15} g$ evaporate through Hawking radiation



B. Carr, F. Kuhnel and M. Sandstad 2016

Outline

Axion inflation

Inflaton non-minimally coupled to some Abelian gauge fields:

$$\mathcal{L} = \frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4\Lambda}\phi\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Turner, Widrow '88,
Garretson, Field, Caroll '92,
Anber, Sorbo '06./'10/'12,
Barnaby, Namba, Peloso '11,
Barnaby, Pajer, Peloso '12 ,
.....

The equations of motion for the fields are:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial\phi} = \frac{\alpha}{\Lambda}\langle\vec{E}\cdot\vec{B}\rangle$$

$$\frac{\mathrm{d}^2\,\vec{A}^a(\tau,\vec{k})}{\mathrm{d}\tau^2} - \vec{\nabla}^2\vec{A}^a = \frac{\alpha}{\Lambda}\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\vec{\nabla}\times\vec{A}^a$$

$$\mathrm{d}t \equiv a\,\mathrm{d}\tau$$

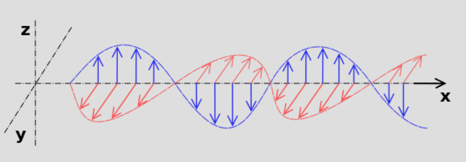
$$N \equiv -\int H\,\mathrm{d}t$$

Friedmann equation reads:

$$3H^2\kappa^{-2} = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\langle\vec{E}^2 + \vec{B}^2\rangle$$

Gauge field amplification

Assuming
$\vec{k}$ parallel to $\hat{x}$



$$\vec{e}_\pm \equiv (\hat{y} \pm i\hat{z})/\sqrt{2}$$
$$\vec{A}^a \equiv \vec{e}_\pm A^a_\pm$$

The equations of motion for the gauge fields (in Fourier transform) read:

$$\frac{\mathrm{d}^2 \; A^a_\pm(\tau,\vec{k})}{\mathrm{d}\tau^2} + \left[k^2 \pm 2k\frac{\xi}{\tau}\right] A^a_\pm(\tau,\vec{k}) = \; 0$$

$$\boxed{\xi \equiv \frac{\alpha}{2\Lambda}\left|\frac{\dot{\phi}}{H}\right| \propto \sqrt{\epsilon_1}}$$

If ξ is nearly constant one mode (A^a_+) is exponentially growing with ξ.

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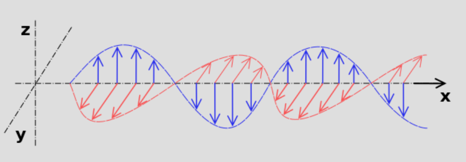

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Substituting $\langle \vec{E} \cdot \vec{B} \rangle$ into the equation of motion for ϕ we get:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = \frac{\alpha}{\Lambda}\langle \vec{E} \cdot \vec{B} \rangle \simeq \frac{\alpha}{\Lambda} 2.4 \cdot 10^{-4} \mathcal{N} \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

Friction term that dominates the last part of the evolution.

Gauge field amplification

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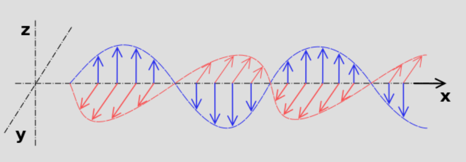

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Friction term that dominates the last part of the evolution.

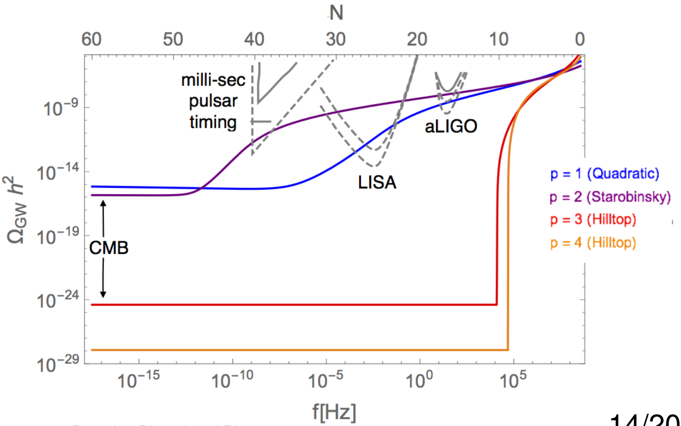
Modified dynamics also affects the scalar and tensor power spectra!

Modified tensor spectrum

GW spectrum $\longrightarrow$ $$\Delta_t^2(k) = \frac{1}{12}\left(\frac{\kappa H}{\pi}\right)^2\left(1 + 4.3\cdot 10^{-7}\frac{\kappa^2 H^2}{\xi^6}e^{4\pi\xi}\right)$$

N-frequency relation $\longrightarrow$ $$N = N_{\rm CMB} + \ln\frac{k_{\rm CMB}}{0.002\ {\rm Mpc}^{-1}} - 44.9 - \ln\frac{f}{10^2\ {\rm Hz}}$$

- Spectra asymptote to an universal value at small scales
- Low scale models ($p = 3, 4$) have a stronger increase
- Some models produce GW in the observable range of direct GW detectors



Domcke, Pieroni and Binetruy 2016

Modified scalar spectrum

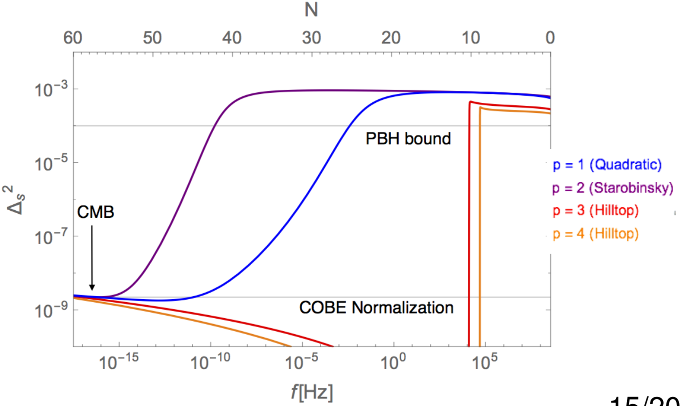
Scalar spectrum $\longrightarrow$

$$\Delta_s^2(k) = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 + \left(\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3bH\dot{\phi}}\right)^2$$

where:

$$b \equiv 1 - 2\pi\xi\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3\Lambda H\dot{\phi}}$$

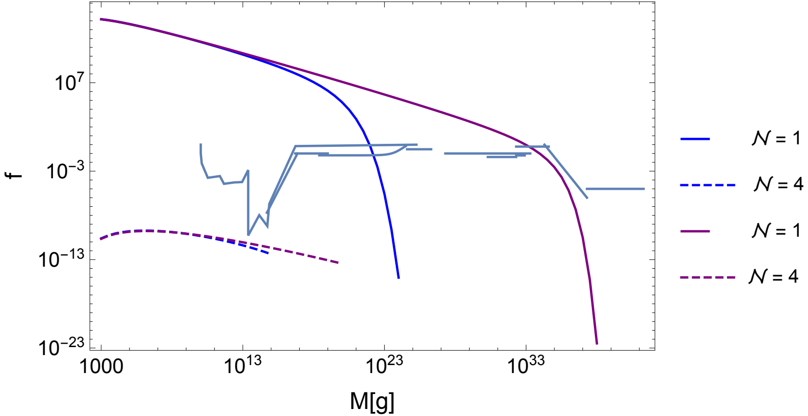
- COBE normalization fixes V_0
- Nearly universal behavior at large scales
- $\Delta_s^2(k) \simeq \frac{1}{\mathcal{N}(2\pi\xi)^2}$ at small scales (Linde, Mooij, Pajer 2013)
- Strong increase at small scales $\to$ PBHs



Domcke, Pieroni and Binetruy 2016

Comparison with PBH bounds

The PBHs can account for part (or all) of the DM in our Universe!
Predictions must be consistent with all the bounds:



A generalized case

Assuming the inflaton to be non-minimally coupled with gravity:

$$\mathcal{L} = \Omega(\phi)\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

where $\Omega(\phi) = 1 + \varsigma g(\phi)$ and $V(\phi) = V_0 g^2(\phi)$.

Kallosh and Linde 2010, Kallosh, Linde and Roest 2014, . . .

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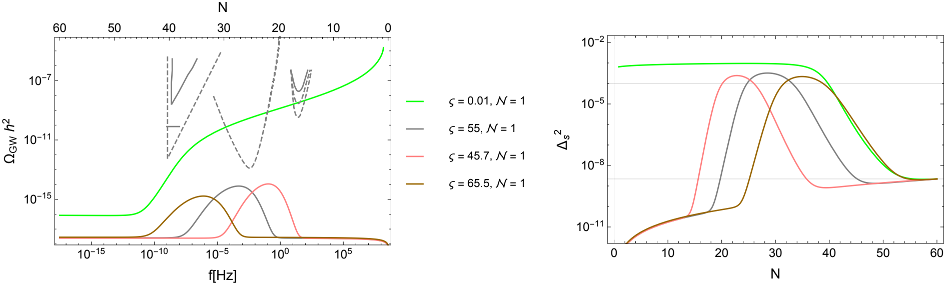
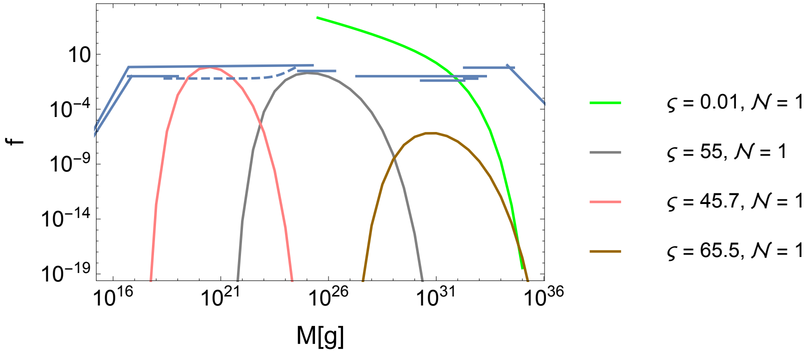
For $g(\phi) = 1 - 1/\phi$ we get:

Domcke, Muia, Pieroni and Witkowski 2017



Comparison with PBH bounds

The corresponding PBH distributions are:



Leading to :

$$f_{\rm tot}^{\varsigma=45.7} = 98.6\%, \quad f_{\rm tot}^{\varsigma=55} = 39.4\%, \quad f_{\rm tot}^{\varsigma=65.5} = 1.2 \cdot 10^{-4}\,\%\,.$$

Domcke, Muia, Pieroni and Witkowski 2017

Conclusions and future perspectives

Conclusions:

- Possibility of generating of an observable GW background.
- If observed informations on the microphysics of inflation.
- Possibility of generating a distribution of PBHs.

Future perspectives:

- Non-abelian gauge fields
- Reheating
- Embedding in high energy theories
- New models

Last Slide

The End

Thank you

Model classification

The system is specified by four parameters: α/Λ, β, p, V_0.

No gauge fields $\longrightarrow$ $n_s \simeq 1 - \frac{\mathcal{O}(1)}{N}$ $\quad r \propto \epsilon_V \simeq \frac{\mathcal{O}(1)}{(1+N)^p}$

The gauge fields introduce an additional friction term.

- The CMB observables are effectively ‘shifted’ at a ‘later’ point N_*:

$$N_* < N_{CMB} \simeq 60 \qquad \longrightarrow \qquad n_s \simeq 1 - \frac{\mathcal{O}(1)}{N_*} \qquad r \propto \epsilon_V \simeq \frac{\mathcal{O}(1)}{(1+N_*)^p}$$

  We get reduced n_s and increased r with respect to the standard case.
- As $\xi \propto \sqrt{\epsilon_1} \simeq \sqrt{\epsilon_V}$, the effects on models with big p will be stronger.

Domcke, Pieroni and Binétruy 2016

CMB constraints

The presence of the gauge fields modifies the scalar and tensor power spectra but CMB constraints should not be violated.

- **COBE Normalization**: Sets the value of the scalar power spectrum at the CMB scales:

$$\Delta_s^2\Big|_{N_{CMB}} = (2.21 \pm 0.07) \cdot 10^{-9}$$

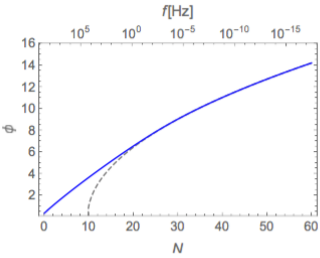
- **Planck constraints on n_s and r**:

$$n_s = 0.9645 \pm 0.0049 \qquad\qquad r < 0.10$$

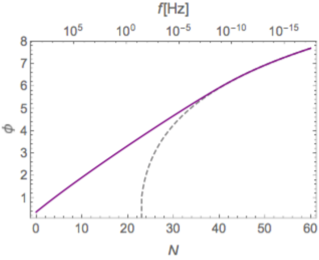
- **Non gaussianities**: The gauge fields induce a non-gaussian component in the power spectrum. To be consistent we need:

$$\xi_{CMB} = \frac{\alpha}{2\Lambda}\left|\frac{\dot{\phi}}{H}\right|_{N=N_{CMB}} \lesssim 2.5$$

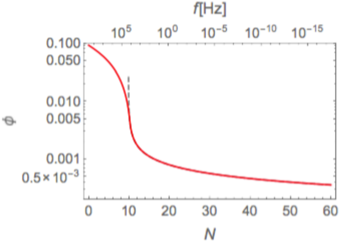
Modified scalar evolution

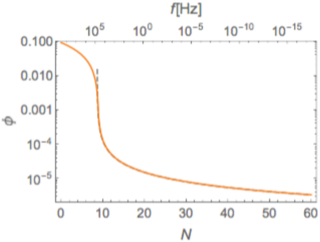


(a) *Quadratic model with* $\alpha/\Lambda = 35$ *and* $V_0 = 1.418 \cdot 10^{-11}$.

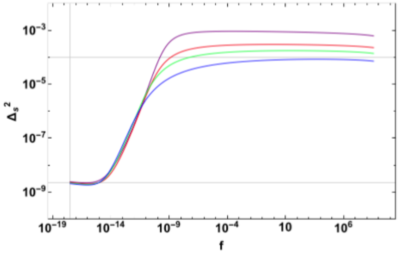


(b) *Starobinsky model with* $\alpha/\Lambda = 75$, $\gamma = 0.3$, $V_0 = 1.17 \cdot 10^{-9}$.

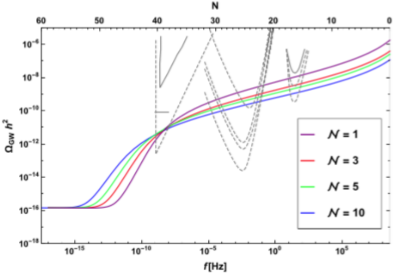




Several gauge fields



(a) *Scalar power spectra.*

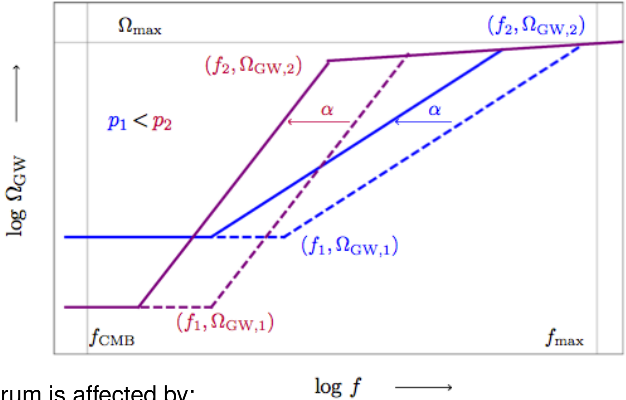


(b) *Tensor power spectra.*

General features of the GW spectrum

Notice that:

- Gauge fields take over at f_1
- Gauge fields' friction dominates after f_2
- Ω_{GW}^{CMB} is fixed by COBE and r.
- Ω_{GW}^{Max} is fixed by $\epsilon_1 \leq 1$.



The shape of the spectrum is affected by:

- p : the slope and the vacuum amplitude
- β : vacuum amplitude
- α/Λ : shifts the spectrum horizontally

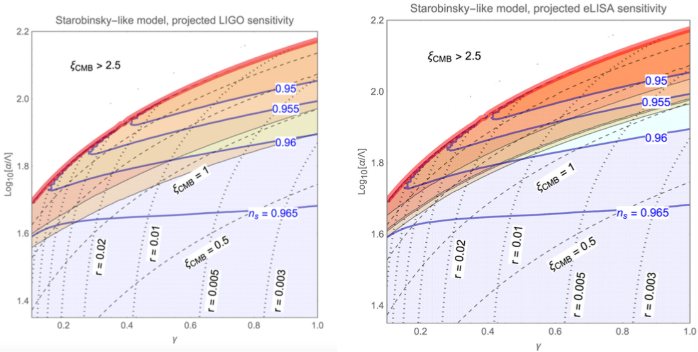
$$\epsilon_1 \simeq r \simeq \beta/(1+N)^p$$
$$\mathcal{L}_{int} = \frac{\alpha}{4\Lambda}\phi F_{\mu\nu}\tilde{F}_{\mu\nu}$$

Starobinsky-like model parameter space

Choosing:

- $p = 2$
- $\beta = 1/(2\gamma)^2$

leads to: $V(\phi) \simeq V_0\,(1 - \exp\{-\gamma\phi\})^2$



(a) *LIGO plot.* (b) *eLISA plot.*

The complementarity between different measures can be used to restrict the parameter space!