Early Kinetic Decoupling:

a case when the standard thermal dark matter relic-density calculation fails

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Fundamental physics in the cosmos:
The early, the large and the dark Universe

DESY Hamburg, Germany

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Outline

1. The Standard WIMP freeze-out formalism

2. Refined treatment

   Coupled 0th and 2nd moment of Boltzmann Equations

   Full phase-space Boltzmann Equation

3. Applied to Singlet Scalar DM

   — Order of magnitude impact on relic abundance Ω_{DM}

Standard formalism

Boltzmann Equation*

Liouville operator = Collision terms

$$E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f_\chi = C[f_\chi]\,.$$

- ➡ Provides the time evolution Eq. for the Phase Space density $f_\chi(t,p)$

- ➡ For CP invariant 2 ↔ 2 annihilation and scattering processes

$C[f_\chi]$:

$$C_{\rm ann} = E\int\frac{d^3\tilde{p}}{(2\pi)^3}\, v\sigma_{\bar{\chi}\chi\to\bar{f}f}\left[f_{\chi,\rm eq}(E)f_{\chi,\rm eq}(\tilde{E}) - f_\chi(E)f_\chi(\tilde{E})\right],$$

$$C_{\rm el} \simeq \frac{E}{2}\gamma(T)\left[TE\partial_p^2 + \left(p + 2T\frac{E}{p} + T\frac{p}{E}\right)\partial_p + 3\right]f_\chi$$

Semi-relativistic: small momentum transfer approx.

[Binder et al. JCAP (2016)]

DM Particle Model Dependent

- ➡ Solving this **(stiff) partial integero differential equation** would provide

$$n_\chi(t) = \int\frac{\mathrm{d}^3p}{(2\pi)^3}\, f_\chi(t,\mathbf{p})$$

* [From Kinetic theory: dilute gas, molecular chaos, instantaneous int. … + homogeneous FRW background]

The Standard Assumption

➡ Integrate the BE over momentum $\mathbf{p}$ ($C_{\rm el} \to 0$), i.e. study BE 0th moment

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \int \frac{d^3p}{(2\pi)^3 E} C_{\rm ann}[f_\chi] \,,$$

➡ **Assume kinetic equilibrium** during chemical decoupling, i.e.

$$f_\chi = \frac{n_\chi}{n_{\chi,\rm eq}} f_{\chi,\rm eq} \;\propto\; e^{-E/T}$$

$$E = \sqrt{m_\chi^2 + \mathbf{p}^2}$$

➡ With this assumption, the RHS can be evaluated to give:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle \sigma v \rangle_{\rm eq} \left(n_{\chi,\rm eq}^2 - n_\chi^2 \right) \,,$$

DM annihilation x-section

where $\langle \sigma v \rangle_{\rm eq} \equiv \frac{1}{n_{\chi,\rm eq}^2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3\tilde{p}}{(2\pi)^3} \sigma_{\bar{\chi}\chi \to \bar{f}f} v f_{\chi,\rm eq}(\mathbf{p}) f_{\chi,\rm eq}(\tilde{\mathbf{p}})$

This 6D integral can further more be reduced to a 1D integral.

Gondolo & Gelmini, Nucl. Phys. (1991)

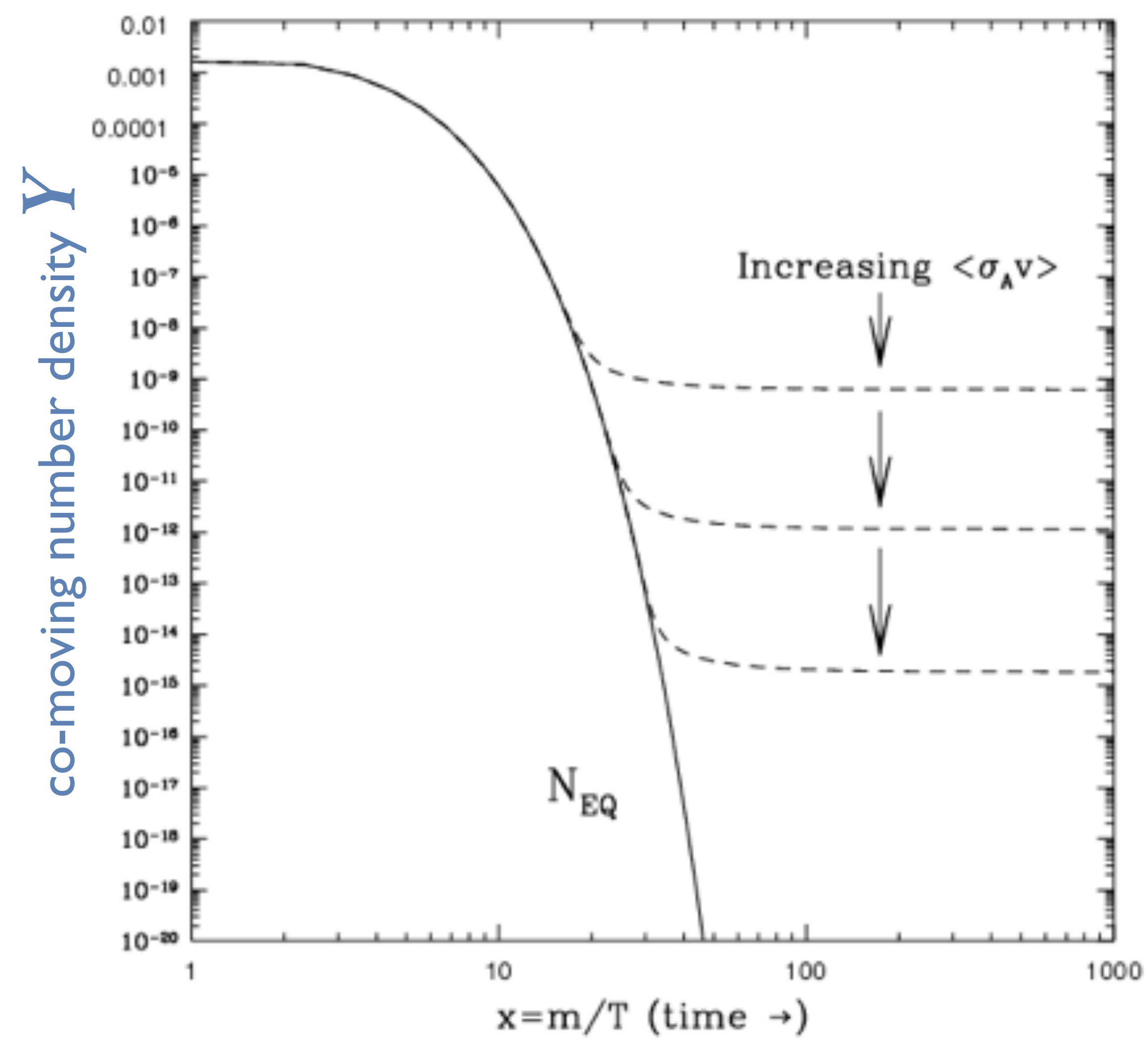
The Standard Treatment

➡ Rewriting 0th moment of BE into dimensionless variables $x \equiv m_\chi/T$, $Y \equiv n_\chi/s$ gives

$$\frac{Y'}{Y} = \frac{sY}{x\tilde{H}} \langle \sigma v \rangle_{\rm eq} \left[\frac{Y_{\rm eq}^2}{Y^2} - 1 \right]$$

➡ **Numerical codes** guarantee to solve this equation to sub-percent lever for sophisticated DM models (e.g. **DarkSuSy**, **micrOMEGAs** …)

$$\Omega_\chi h^2 = 2.8 \cdot 10^{10} \frac{m_\chi}{100 \text{ GeV}} \cdot Y(x \to \infty)$$



➡ **What if** the DM is **not** in **kinetic equilibrium** during chemical decoupling?

⟶ Would results change?

⟶ How to compute $\Omega_{\rm DM}$?

⟶ **A refined formalism required**

Notes on kinetic decoupling ($\sigma_{\bar{\chi}\chi\to\bar{f}f} = 0$)

➡ Look at the **2nd** moment of $f_\chi(p,t)$ — *aka* DM "temperature" :

$$T_\chi = \frac{1}{n_\chi}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{3E} f_\chi(\mathbf{p}) \qquad \text{or} \qquad y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$$

➡ **Assume thermal** distribution **but a separate** DM **"temperature"** $T_\chi \neq T$

$$f_\chi \propto e^{-E/T} \to f_\chi \propto e^{-E/T_\chi}$$

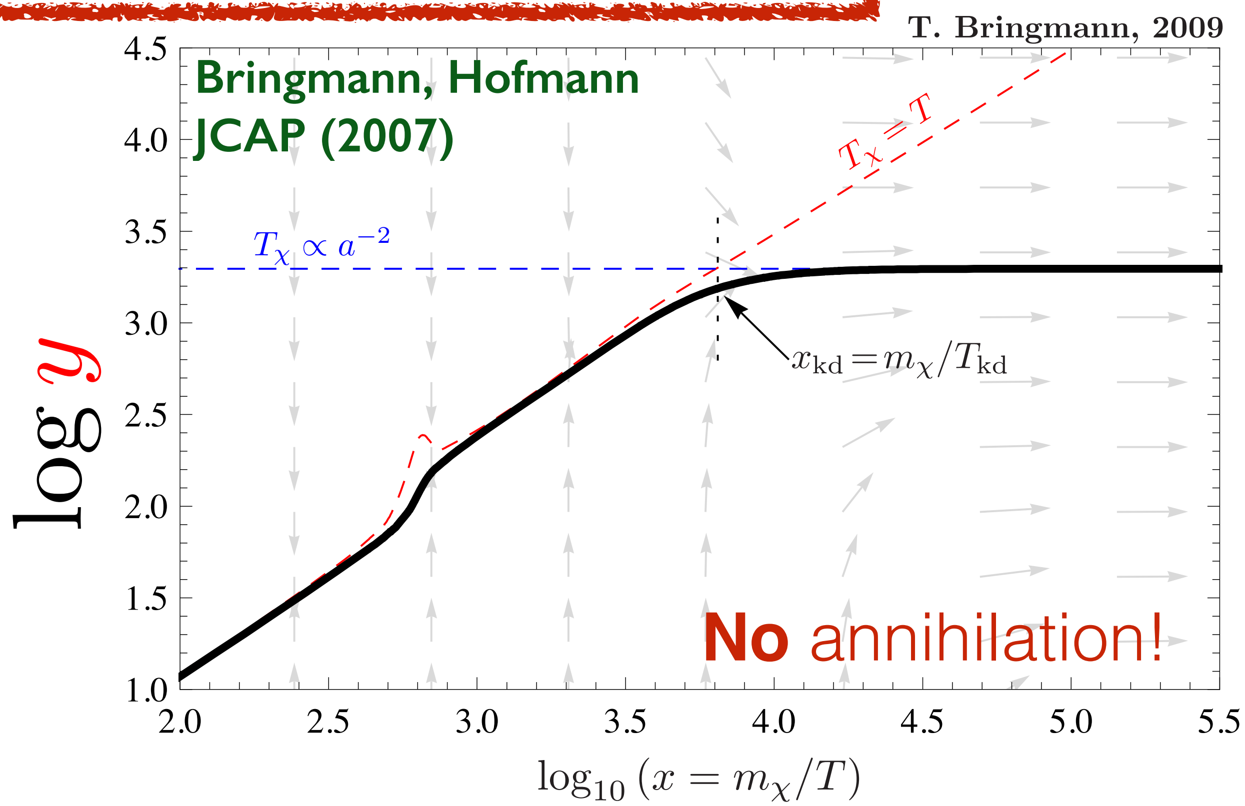
➡ Then the **2nd** moment (non-rel.) BE gives

$$\dot{T}_\chi + 2HT_\chi = \gamma(T)\,(T - T_\chi)\,.$$

where

$$\gamma(T) = \int d\omega\, \frac{8k^4 f_q^\pm (1\mp f_q^\pm)}{3\pi^2 m_\chi T} \int d\Omega\,(1-\cos\theta)\frac{d\sigma_{\chi q\to\chi q}}{d\Omega}$$

Scattering x-section of DM with thermal bkg. particles



A refined treatment

1. 0^{th} and 2^{nd} moments of the BE

➡ With $\sigma_{\bar{\chi}\chi\to\bar{f}f} \neq 0$ and $\sigma_{\bar{\chi}q\to\bar{\chi}q} \neq 0$, CD and KD are coupled:

$$\frac{Y'}{Y} = \frac{sY}{x\tilde{H}}\left[\frac{Y_{\rm eq}^2}{Y^2}\langle\sigma v\rangle_{\rm eq} - \langle\sigma v\rangle_{\rm neq}\right]$$

production and annihilation thermal averages are with different $f_{\chi,\rm eq}$ and f_χ ,

$$\frac{y'}{y} = \frac{\gamma(T)}{x\tilde{H}}\left[\frac{y_{\rm eq}}{y} - 1\right] + \frac{Y'}{Y}\left[\frac{\langle\sigma v\rangle_{2,\rm neq}}{\langle\sigma v\rangle_{\rm neq}} - 1\right] + \frac{H}{x\tilde{H}}\frac{\langle p^4/E^3\rangle_{\rm neq}}{3T_\chi}\,.$$

$y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$

Elastic scatterings

For brevity, $Y_{\rm eq}^2/Y^2$ terms are left out

Relativistic term

$\langle p^4/E^3\rangle \equiv \frac{1}{n_\chi}\int\frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E^3}f_\chi(\mathbf{p})$

with $$\langle\sigma v\rangle_{2,\rm neq} \equiv \frac{1}{T_\chi n_\chi^2}\int\frac{d^3p\, d^3\tilde{p}}{(2\pi)^6}\frac{p^2}{3E}\sigma v_{\bar{\chi}\chi\to\bar{f}f}f_\chi(\mathbf{p})f_\chi(\tilde{\mathbf{p}})$$

6D integral reduces to 2D integral

$\langle\ldots\rangle_{\rm neq}$ are with $f_\chi \neq f_{\chi,\rm eq}$ and in order to close the equations we assume equilibrium momentum distribution — with separate DM "temperature"

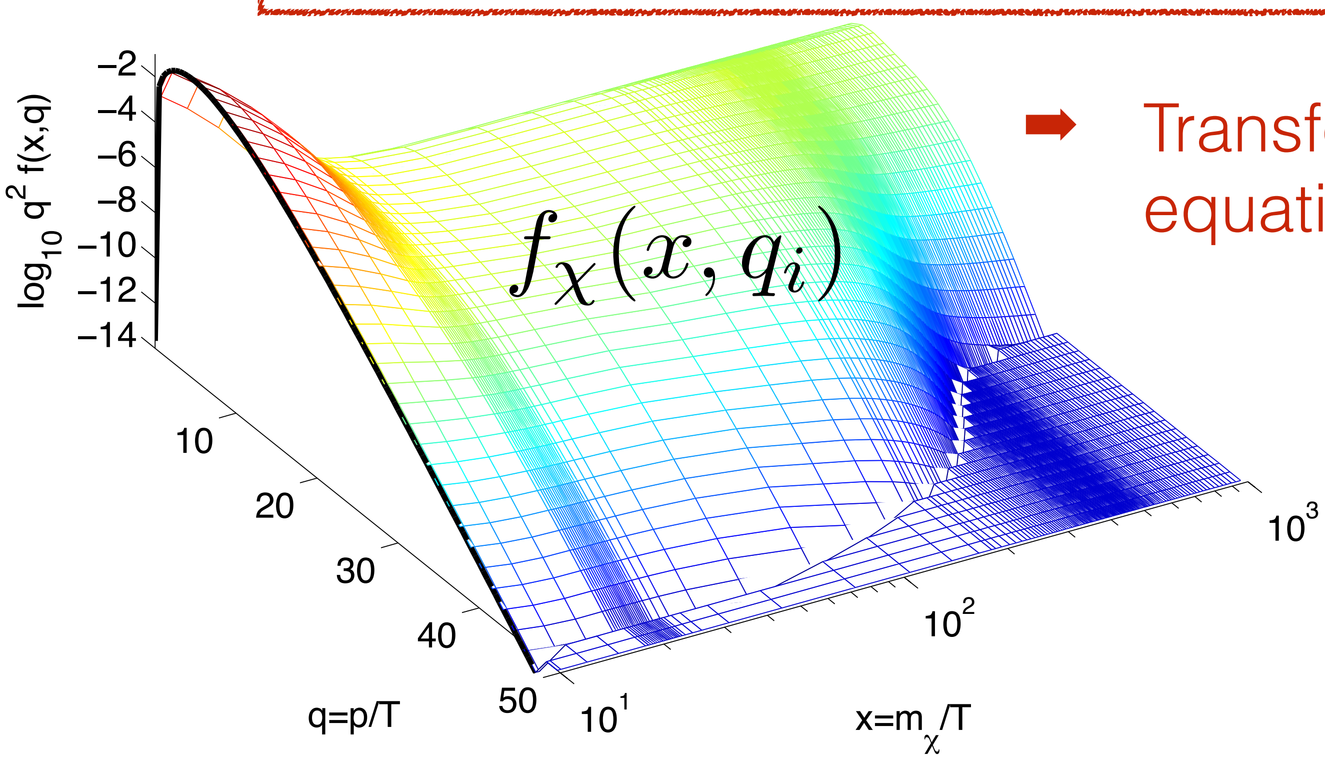
$$\langle\ldots\rangle_{\rm neq} \to \langle\ldots\rangle_{T_\chi = y\, s^{2/3}/m_\chi}$$

2. Solving the full phase-space BE

➡ Rewrite BE in $x(t,p) \equiv m_\chi/T$ and $q(t,p) \equiv p/T$

➡ Discretize $q \to q_i$ and impose boundary conditions at $q_{\min}$ and $q_{\max}$

$$\frac{d}{dx} f_i = \frac{m_\chi^3}{\tilde{H} x^4} \frac{1}{2\pi^2} \sum_{j=1}^{N-1} \frac{\Delta q_j}{2} \left[\sum_{k=j}^{j+1} q_k^2 \, \langle v_{\mathrm{M\o l}} \sigma_{\bar{\chi}\chi \to \bar{f} f} \rangle_{i,k}^{\theta} \left(f_i^{\mathrm{eq}} f_k^{\mathrm{eq}} - f_i f_k \right) \right]$$
$$+ \frac{\gamma(x)}{2\tilde{H}x} \left[x_{q,i} \partial_q^2 f_i + \left(q_i + \frac{2 x_{q,i}}{q_i} + \frac{q_i}{x_{q,i}} \right) \partial_q f_i + 3 f_i \right]$$
$$+ \tilde{g} \frac{q_i}{x} \partial_q f_i$$

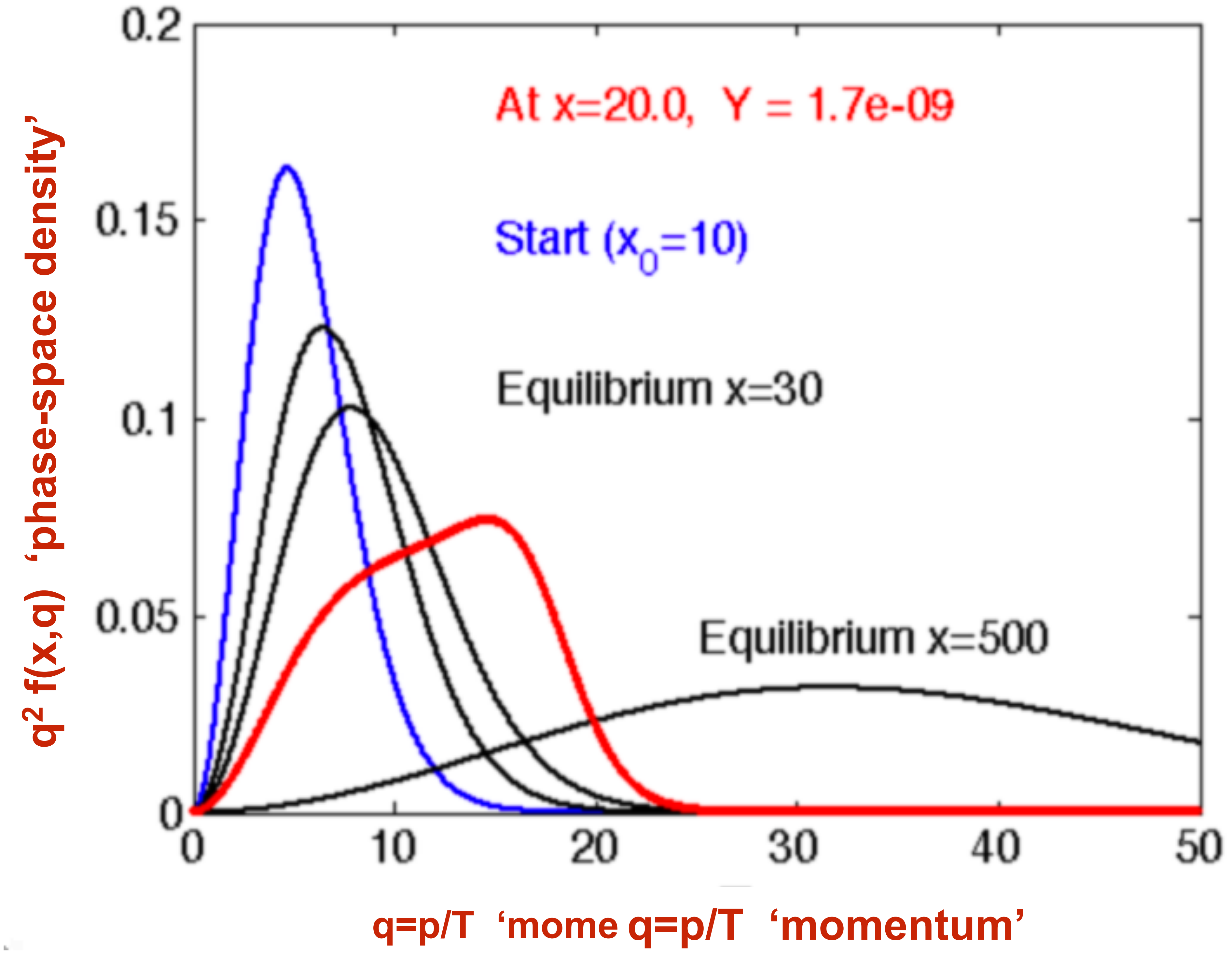


➡ Transformed the partial integero differential equation into ***N* coupled ODEs !**

$$f_i \equiv f_\chi(x, q_i) \to f_\chi(x, q)$$

DM's full phase-space evolution

A non-trivial evolution of f(x,q) for a realistic DM model



Singlet Scalar: m_S=62.5 GeV, $\Omega_{DM}h^2$ = 0.1188

An example:
The Singlet Scalar model

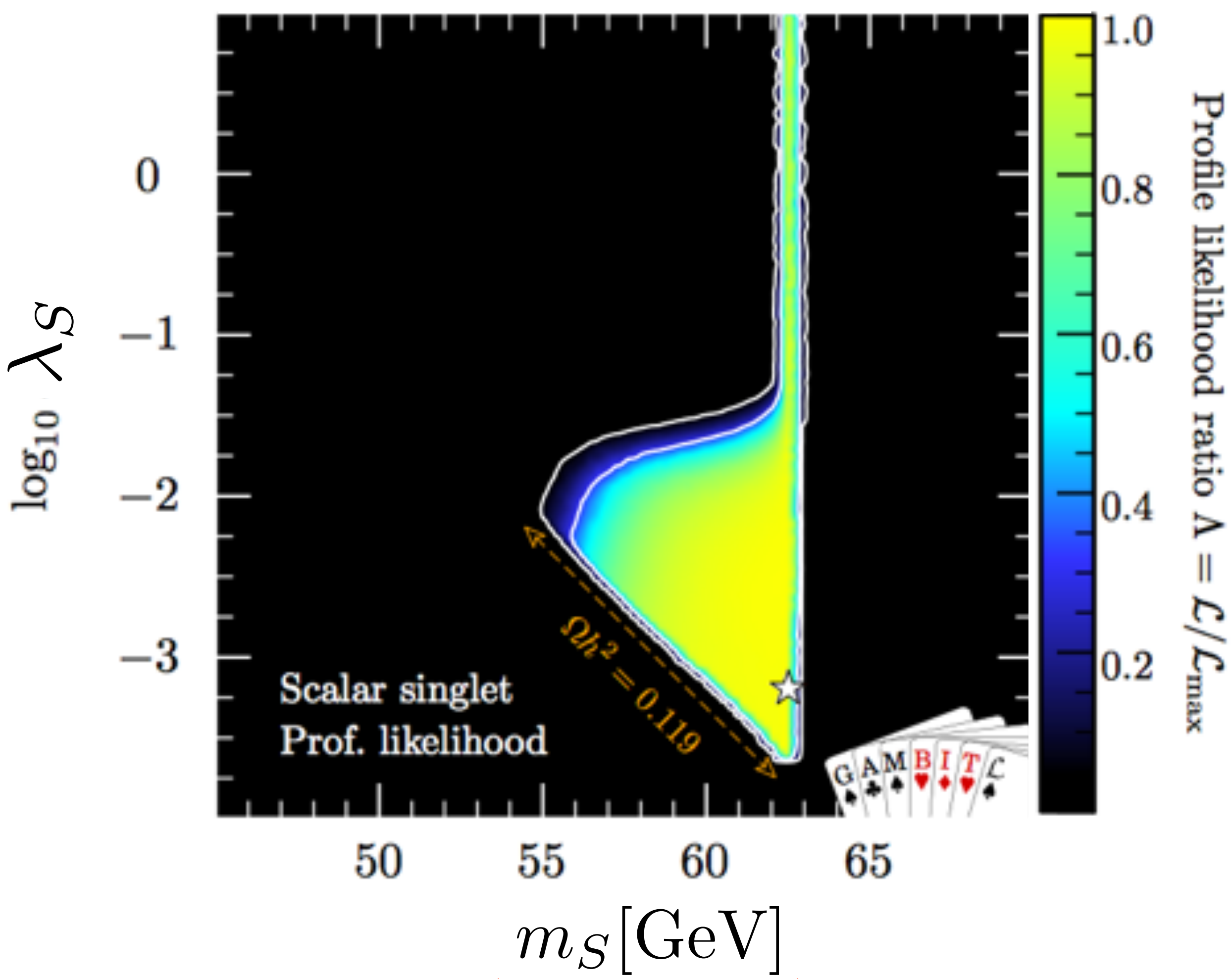
Singlet Scalar Model

➡ Add to the SM one Singlet Scalar field S that interacts with the Higgs:

$$\mathcal{L}_{\rm SS} = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_S S^2 H^\dagger H$$

$(m_h = 125.1\mathrm{GeV})$

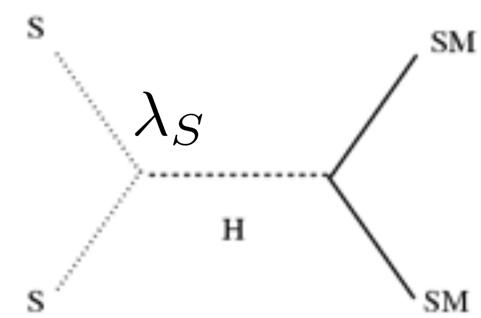
GAMBIT collaboration, 1705.07931



$$m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$

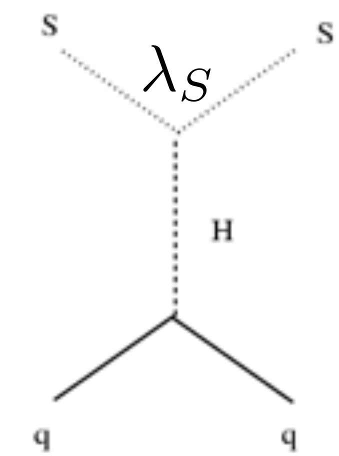
Annihilation processes:

resonant



Elastic scattering processes:

non-resonant

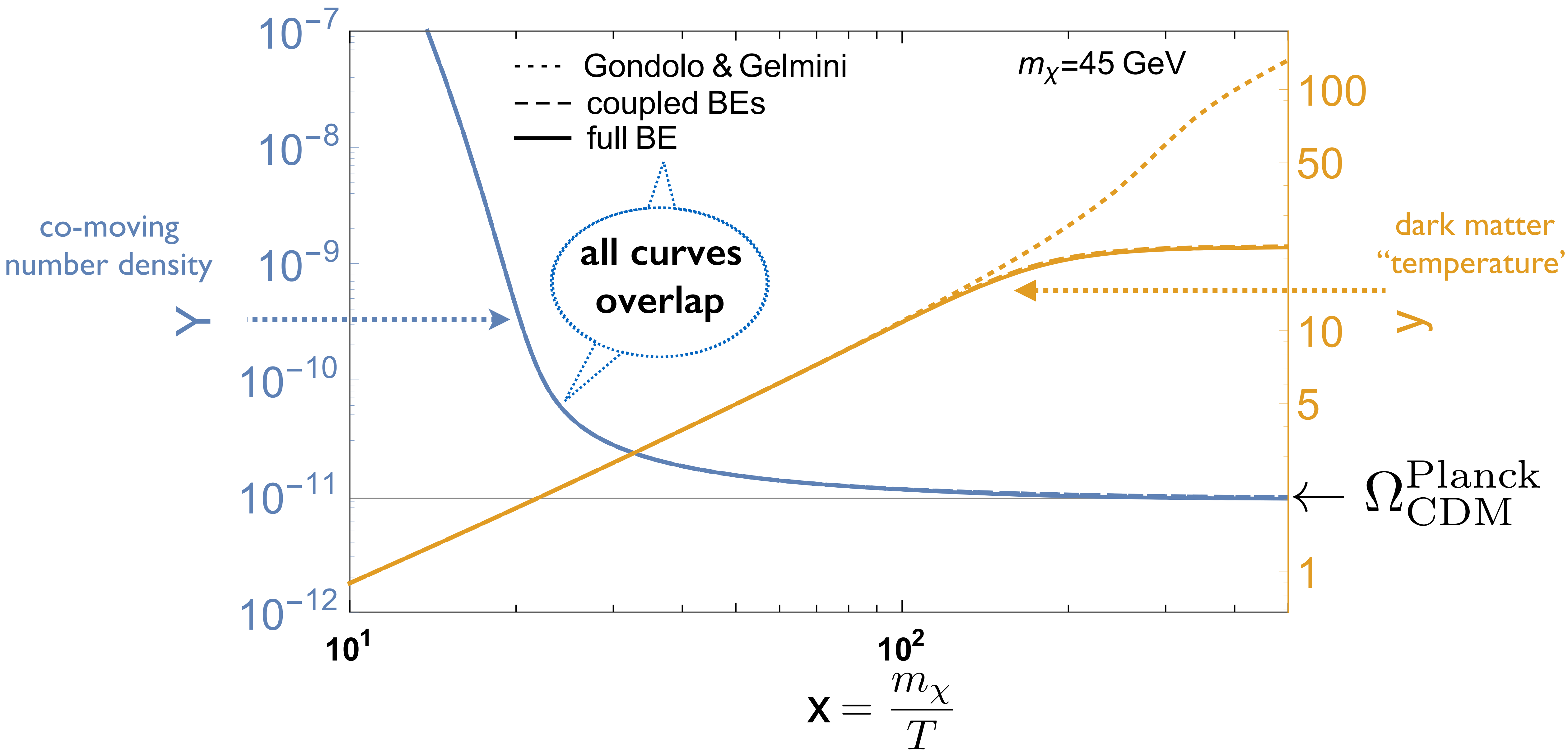


➡ **a DM scenario with** large annihilation and weak scattering x-sections

↳ **Early Kinetic Decoupling**

m_{DM} = 45 GeV, far below resonance

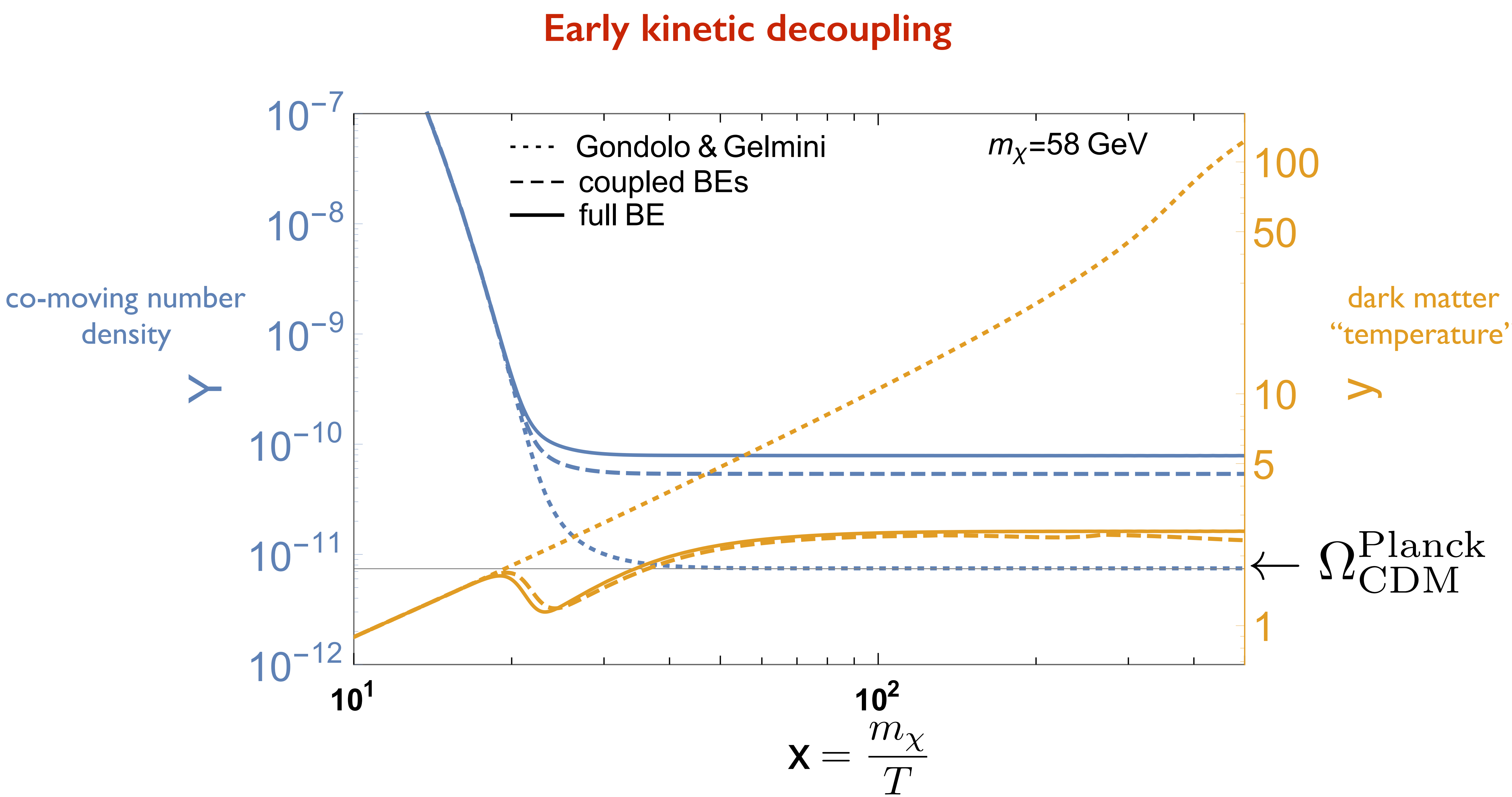
Late kinetic decoupling



Standard formalism (= Gondolo & Gelmini) works excellent

DM leaves chemical freeze-out ($x \approx 25$) before kinetic decoupling ($x \approx 150$)

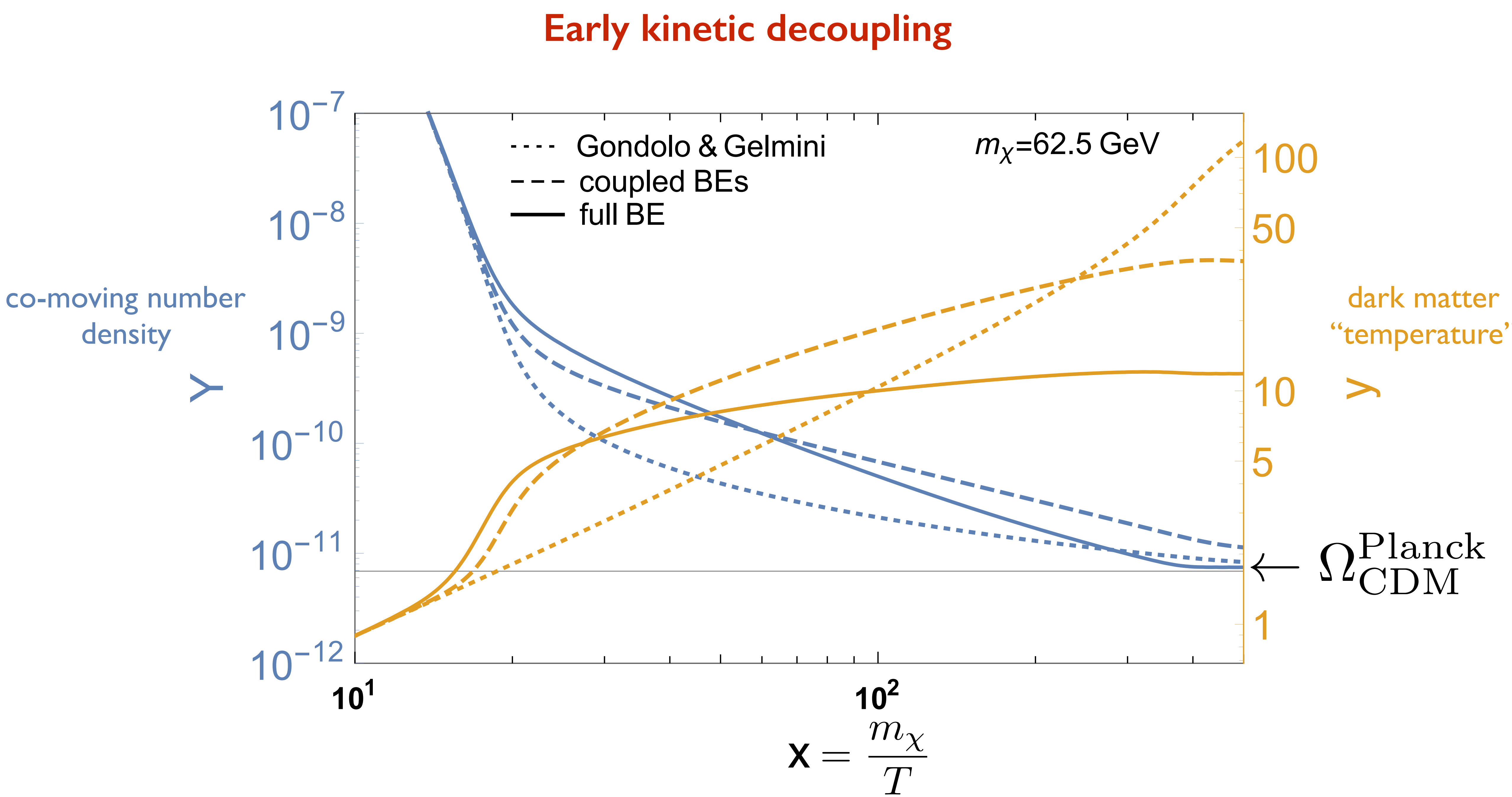
m_{DM} = 58 GeV — closer to the resonance



Resonant Annihilation most effective for high momenta

DM goes through a "cooling" phase and annihilation quickly loose efficiency

m_{DM} = 62.5 GeV — just on resonance:



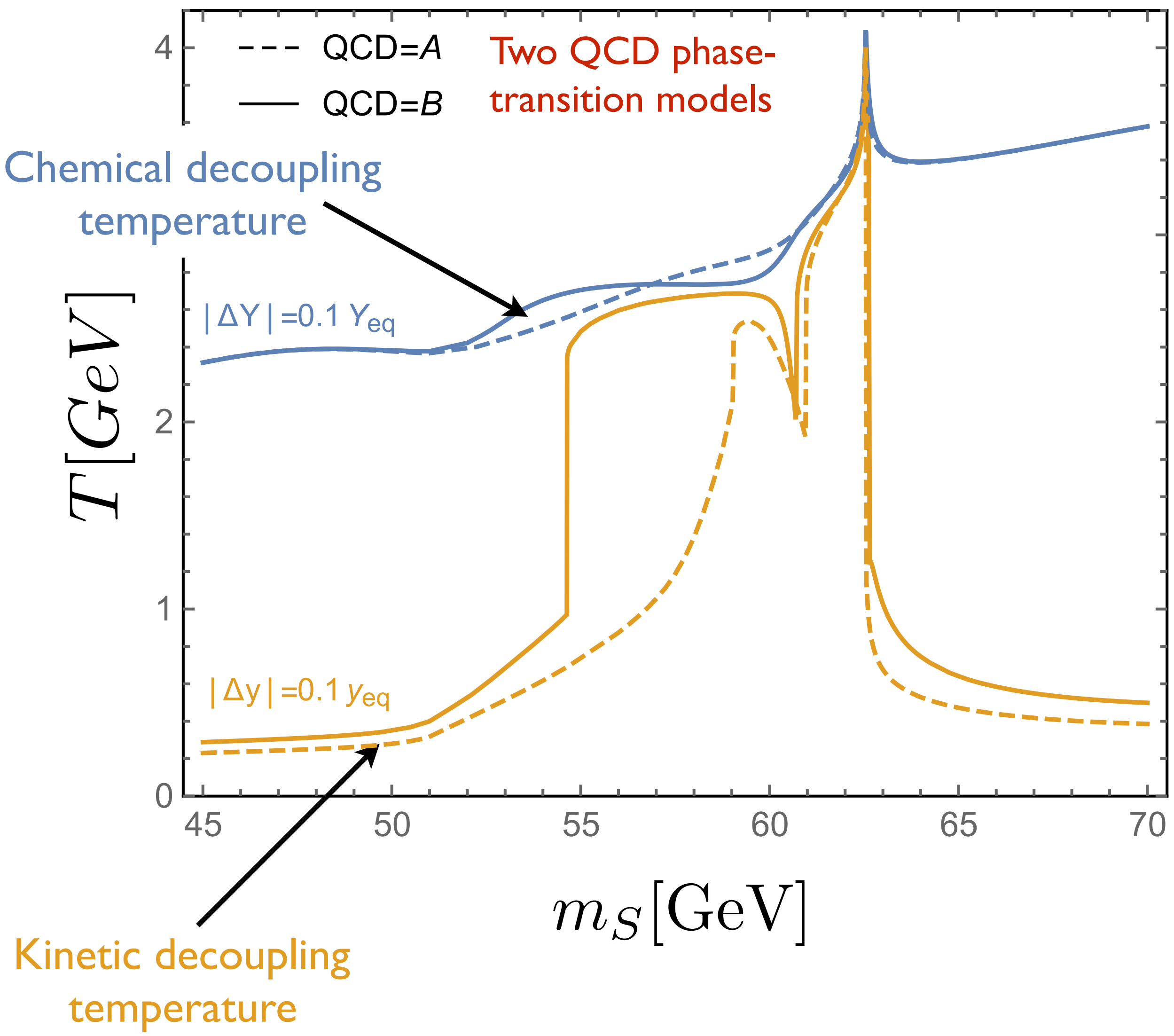
Resonant annihilation most effective for low momenta

DM goes through a"heating" phase and a prolonged annihilation phase

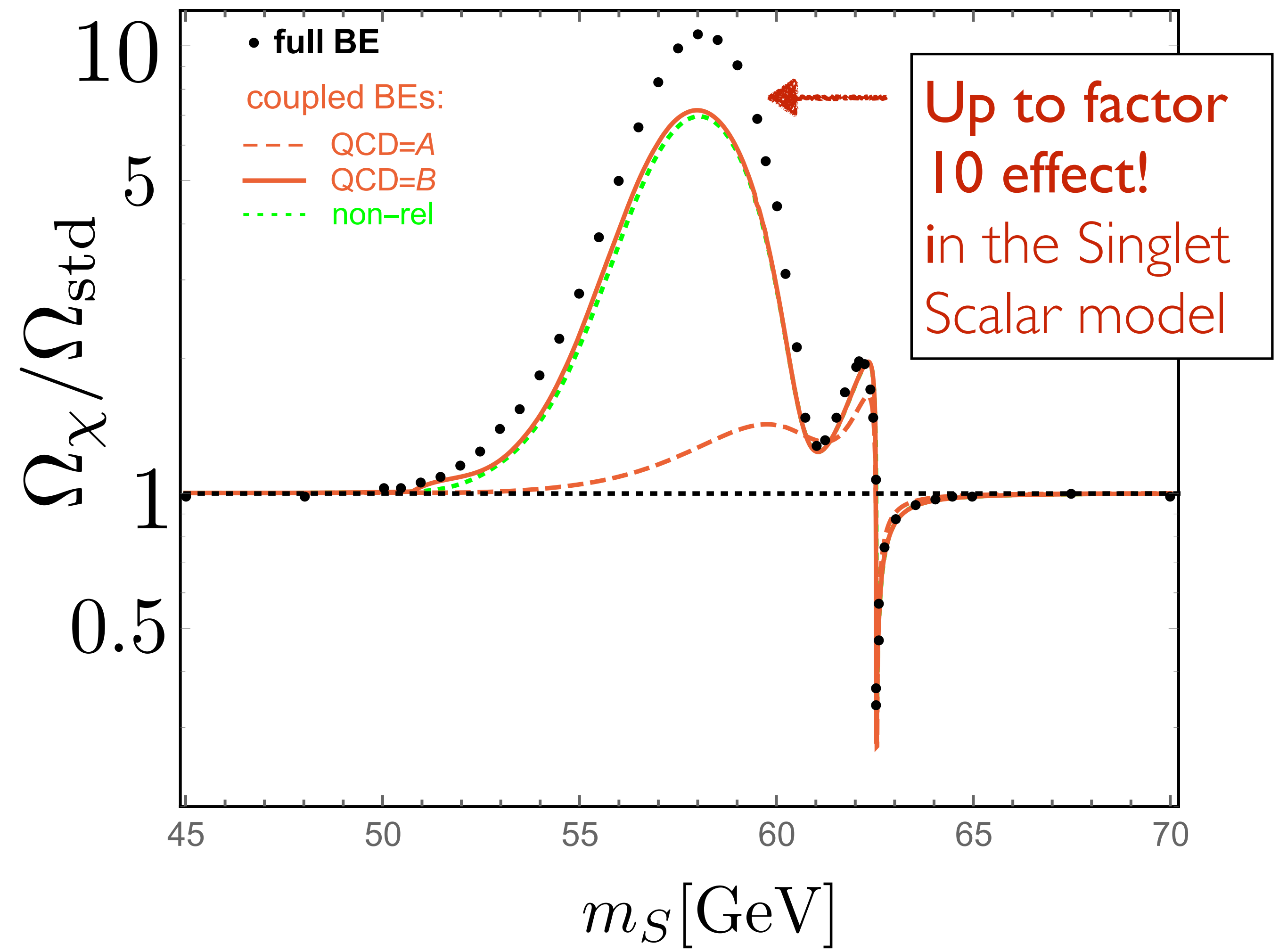
The coupled BEs and the full phase-space BE calc. do not capture the same physics here!

Global effect on relic density

Kinetic and chemical decoupling *temp.*:

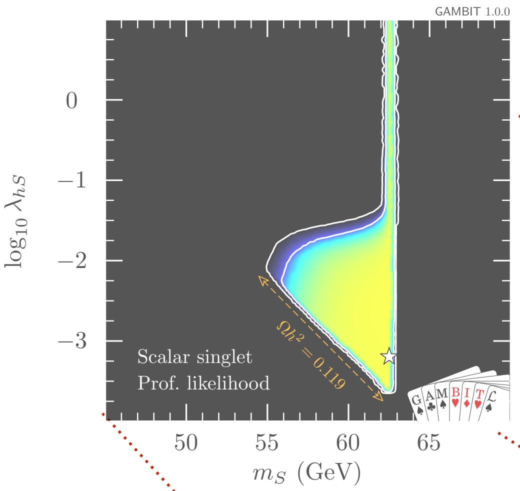


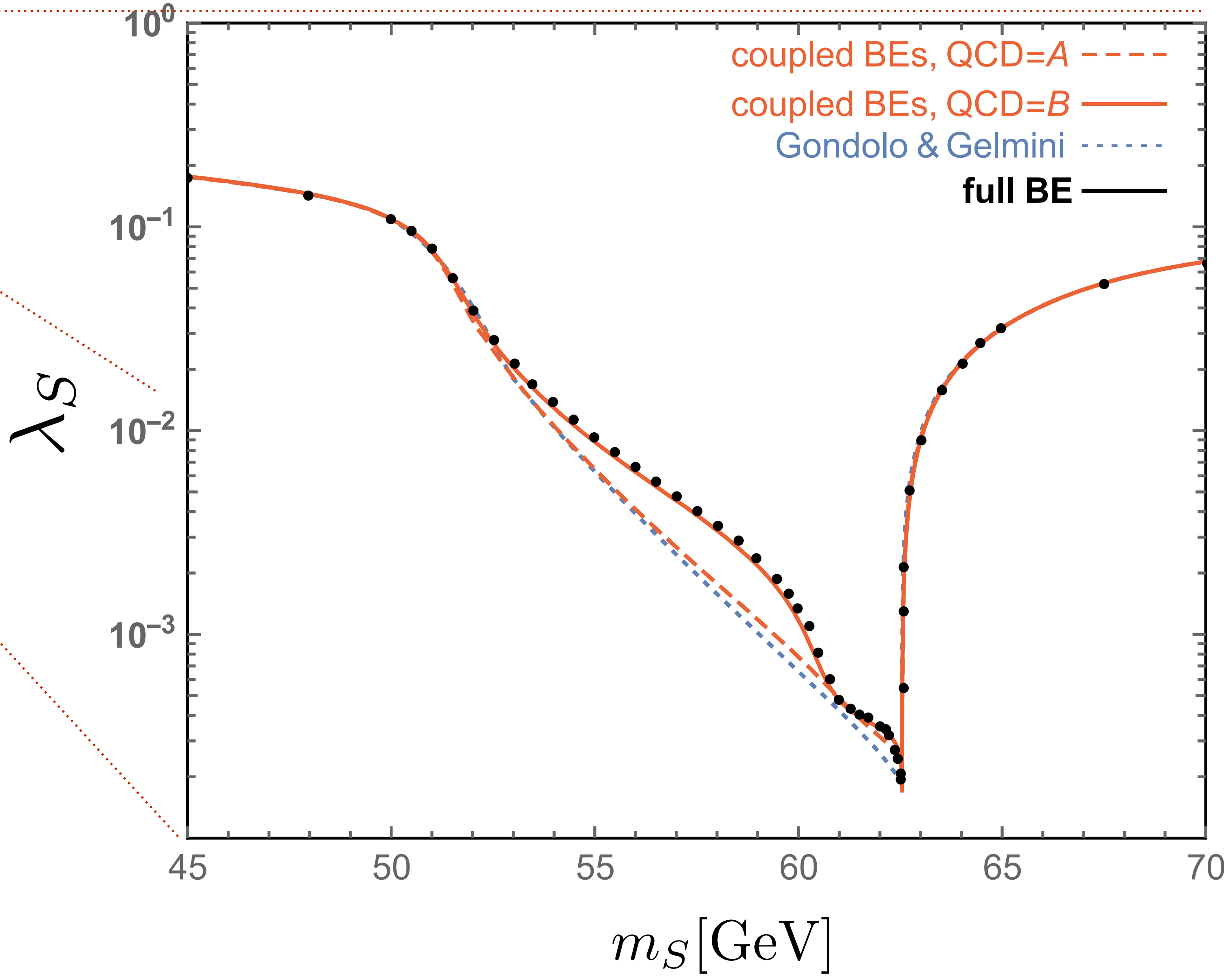
Change in relic density $\frac{\Omega_\chi}{\Omega_{\rm std}}$:



→ **Early kinetic decoupling in the epoch of chemical freeze-out significantly effects the relic abundance**

Impact on model parameters





Significant modification of the correct relic density contour in the Scalar Singlet DM model

→ larger coupling needed

Summary

Summary

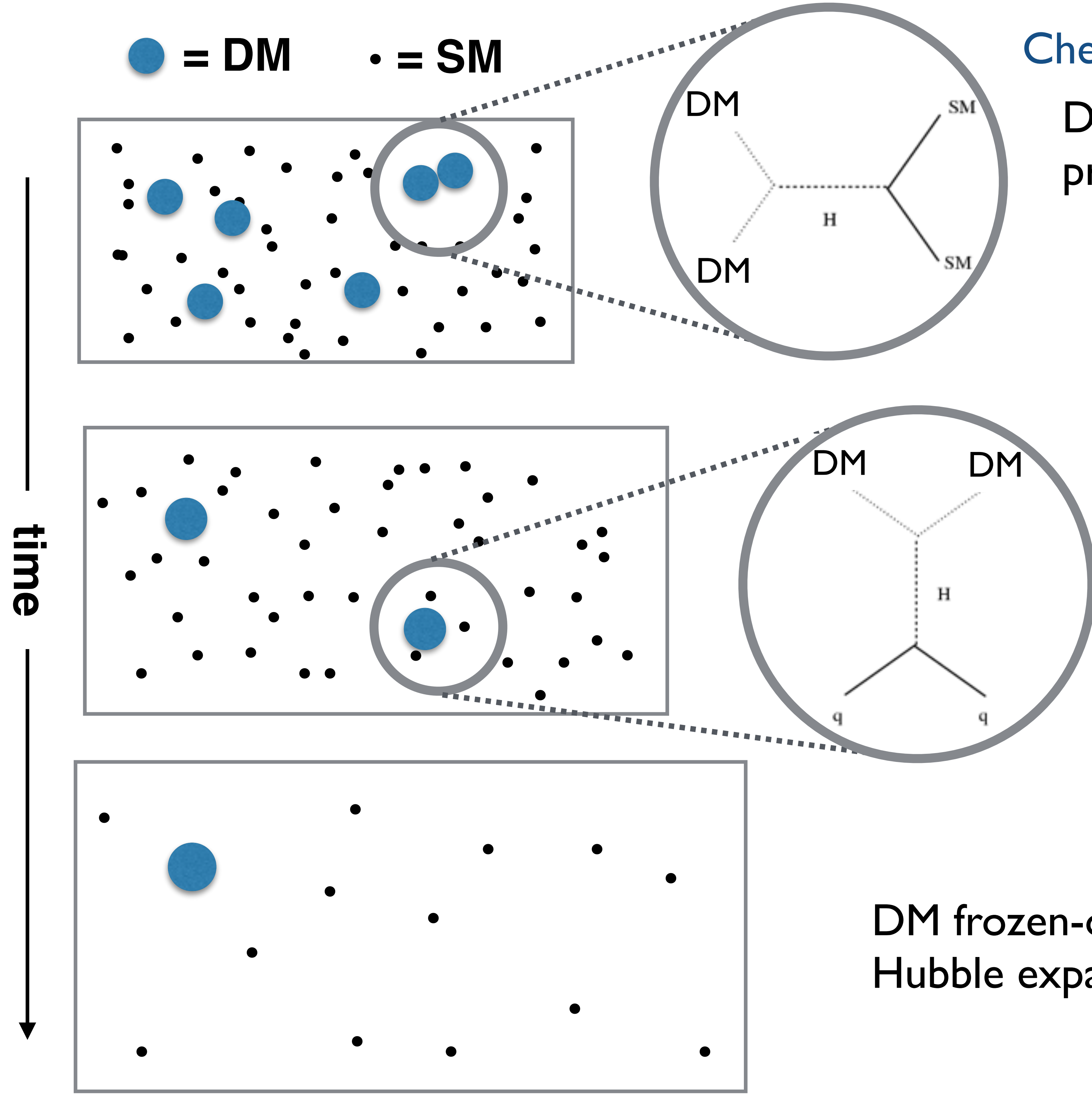
- ➡ **Kinetic equilibrium** is a necessary assumption in standard relic density calculation formalism
- ➡ A **coupled system of 0th and 2nd moments of the Boltzmann equation** allows to accurately treat kinetic decoupling and its impact on relic density
- ➡ A **full phase-space Boltzmann equation code** was setup — for non-trivial momentum distributions during freeze-out

A pitfall to lookout for …

… kinetic decoupling can happen early

Backups

Thermal freeze-outs



Chemical decoupling (CD)

DM Annihilation processes stop

Typical order

Kinetic decoupling (KD)

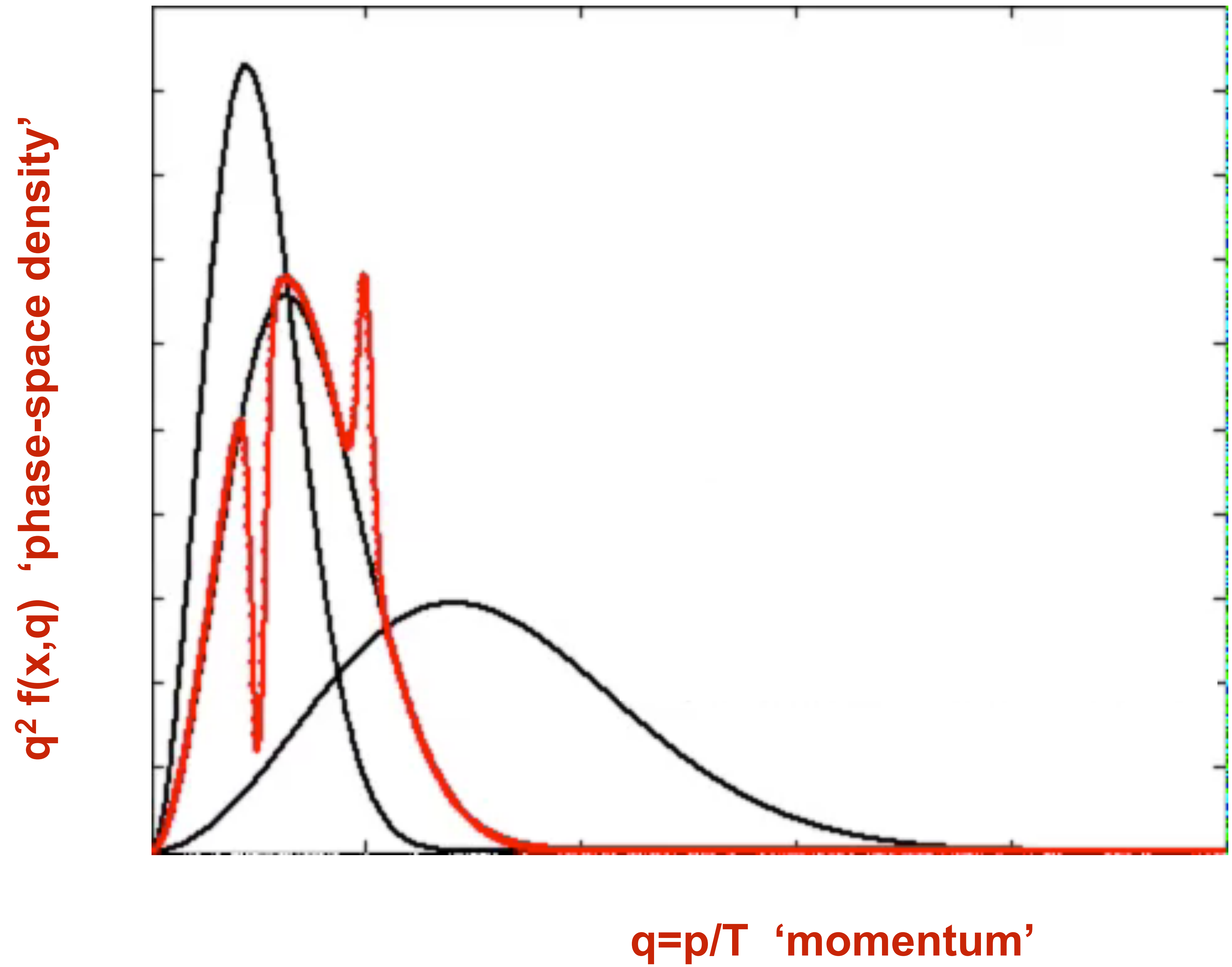
DM Elastic scattering processes with SM stop

What if KD before CD ?

DM frozen-out and density evolves with Hubble expansion (until structure formations)

Full phase-space evolution

A first test: local disturbances in f(q)



Notations

$$v_{\text{Møl}} \equiv (E\tilde{E})^{-1}[(p\cdot\tilde{p})^2 - m_\chi^4]^{1/2}$$

$$v_{\text{lab}} \equiv [s(s-4m_\chi^2)]^{1/2}/(s-2m_\chi^2)$$

$$\tilde{H} \equiv H/\left[1+\tilde{g}(x)\right]$$

$$\tilde{g} \equiv \frac{1}{3}\frac{T}{g_{\text{eff}}^s}\frac{dg_{\text{eff}}^s}{dT}$$

$$\langle\sigma v\rangle = \int_1^\infty d\tilde{s}\,\sigma_{\bar{\chi}\chi\to\bar{f}f}v_{\text{lab}}\frac{2m_\chi\sqrt{\tilde{s}-1}(2\tilde{s}-1)K_1\left(\frac{2\sqrt{\tilde{s}}m_\chi}{T}\right)}{T{K_2}^2(m_\chi/T)}.$$

$$\tilde{s} \equiv s/(4m_\chi^2)$$

$$\langle\sigma v\rangle_2 \equiv \frac{g_\chi^2}{Tn_{\chi,\text{eq}}^2}\int\frac{d^3p\,d^3\tilde{p}}{(2\pi)^6}\frac{p^2}{3E}\sigma v_{\bar{\chi}\chi\to\bar{f}f}f_{\chi,\text{eq}}(\mathbf{p})f_{\chi,\text{eq}}(\tilde{\mathbf{p}})$$

$$= \int_1^\infty d\tilde{s}\,\sigma_{\bar{\chi}\chi\to\bar{f}f}v_{\text{lab}}\frac{4\tilde{s}(2\tilde{s}-1)x^3}{3{K_2}^2(x)}$$

$$\int_1^\infty d\epsilon_+ e^{-2\sqrt{\tilde{s}}x\epsilon_+}\left[\epsilon_+\sqrt{(\tilde{s}-1)(\epsilon_+^2-1)}\right.$$

$$\left.+\frac{1}{2\sqrt{\tilde{s}}}\log\left(\frac{\sqrt{\tilde{s}}\epsilon_+ - \sqrt{(\tilde{s}-1)(\epsilon_+^2-1)}}{\sqrt{\tilde{s}}\epsilon_+ + \sqrt{(\tilde{s}-1)(\epsilon_+^2-1)}}\right)\right]$$

$$f_\chi = A(T)f_{\chi,\text{eq}} = \frac{n_\chi}{n_{\chi,\text{eq}}}f_{\chi,\text{eq}}$$

$$s = (2\pi^2/45)g_{\text{eff}}^s T^3$$

Singlet Scalar — cross sections

$$\sigma v_{\rm CMS} = \frac{2\lambda_S^2 v_0^2}{\sqrt{s}} \left|D_h(s)\right|^2 \Gamma_{h\to{\rm SM}}(\sqrt{s})$$

$$\left|D_h(s)\right|^2 = \frac{1}{(s-m_h^2)^2 + m_h^2\Gamma_h^2}$$

$$\gamma(T) = \frac{1}{48\pi^3 g_\chi m_\chi^3} \int d\omega\, g^{\pm} \partial_\omega \left(k^4 \left\langle |\mathcal{M}|^2 \right\rangle_t\right),$$

with

$$\left\langle |\mathcal{M}|^2 \right\rangle_t \equiv \frac{1}{8k^4} \int_{-4k^2}^{0} dt(-t) \left|\mathcal{M}\right|^2 = 16\pi m_\chi^2\, \sigma_T.$$

$$\left|\mathcal{M}_{Sf\to Sf}\right|^2 = \frac{N_f \lambda_S^2 m_f^2}{2} \frac{4m_f^2 - t}{(t-m_h^2)^2}$$

A: all quarks are free and present in the plasma down to temperatures of $T_c = 154\,\mathrm{MeV}$ (largest scattering scenario, as adopted in [20])

B: only light quarks (u, d, s) contribute to the scattering, and only for temperatures above $4T_c \sim 600\,\mathrm{MeV}$, below which hadronization effects start to become sizeable [63] (smallest scattering scenario, as adopted in [12]).

$$\left\langle |\mathcal{M}|^2 \right\rangle_t = \sum_f \frac{N_f \lambda_S^2 m_f^2}{8k^4} \Big[\frac{2k^2 - 2m_f^2 + m_h^2}{1 + m_h^2/(4k^2)} - \left(m_h^2 - 2m_f^2\right) \log\left(1 + 4k^2/m_h^2\right) \Big]$$

Based on

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Arxiv:1706.07433