Probing CP in differential distributions

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(DESY)

PRD 92 (2015) 076013, [1508.03054]
with Yuval Grossman, spin-0 multibody decays

JHEP 10 (2016) 005, [1608.03288]
spin-1/2 multibody decays

10 July 2017
LHC Physics Discussion

Multibody hadronic decays

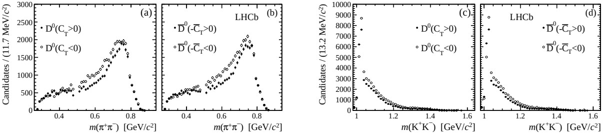
- Large statistics

$B_s^0 \to \phi\phi \to K^+K^-K^+K^-$	3950 ± 67 candidates	[1407.2222]
$D^0 \to K^+K^-\pi^+\pi^-$	$171\,300 \pm 600$	[1408.1299]
$\Lambda_b \to p\pi^-\pi^+\pi^-$	$6\,646 \pm 105$	[1609.05216]
$\Lambda_b \to \Lambda\phi \to p\pi^-K^+K^-$	89 ± 13	[1603.02870]
$\Lambda_b \to pK^-J/\psi \to pK^-\mu^+\mu^-$	$28\,834 \pm 204$	[1603.06961]

…

- Multidimensional phase space

e.g. 5d in $D^0 \to K^+K^-\pi^+\pi^-$:



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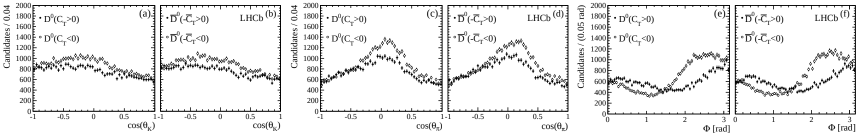
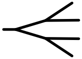
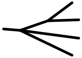
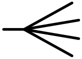


The paradox of richness and complexity

- Rich variety of interfering contributions

	Intermediate states in $D^0 \to K^+K^-\pi^+\pi^-$	Br $/10^{-4}$
	$(\phi\rho^0)_S, \quad \phi \to K^+K^-, \quad \rho^0 \to \pi^+\pi^-$	9.3 ± 1.2
	D	0.83 ± 0.23
	$(K^{*0}\overline{K}^{*0})_S, \quad K^{*0} \to K^\pm\pi^\mp$	1.48 ± 0.30
	$\phi(\pi^+\pi^-)_S, \quad \phi \to K^+K^-$	2.50 ± 0.33
	$(K^-\pi^+)_P(K^+\pi^-)_S$	2.6 ± 0.5
	$K_1^+K^-, \quad K_1^+ \to K^{*0}\pi^+$	1.8 ± 0.5
	$K_1^-K^+, \quad K_1^- \to \overline{K}^{*0}\pi^-$	0.22 ± 0.12
	$K_1^+K^-, \quad K_1^+ \to \rho^0K^+$	1.14 ± 0.26
	$K_1^-K^+, \quad K_1^- \to \rho^0K^-$	1.46 ± 0.25
	$K^*(1410)^+K^-, \; K^*(1410)^+ \to K^{*0}\pi^+$	1.02 ± 0.26
	$K^*(1410)^-K^+, \; K^*(1410)^- \to \overline{K}^{*0}\pi^-$	1.14 ± 0.25

[CLEO '12]

$\Longrightarrow$ Opportunities for CP violation searches
but also modelling challenges!

Probing CP in differential distributions

CP violation in differential distributions

- With or without strong phases
- With untagged samples / self-conjugate states

Multibody decays of mesons and baryons

- Spinless case: systematic modelling-independent analysis
- Spinful case: resolving ambiguities with modellisation

Other applications

Motion reversal $\hat{T}$ (often called *naive time reversal*)

$\hat{T}$ flips $\vec{p}$ and $\vec{s}$.

$\hat{T}$-oddity arises from $\epsilon_{\mu\nu\rho\sigma}\ p^\mu q^\nu r^\rho s^\sigma$ contractions ...

- ϵ from the Lagrangian: $i\tilde{F}^{\mu\nu} \equiv \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$
- ϵ from chiral fermions: $\gamma^5 \equiv \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$

... of four independent momenta or spin vectors.

$\rightarrow$ minimal multiplicity

e.g. spinless four-body decays

In the p restframe,

$$\epsilon_{\mu\nu\rho\sigma}\ p^\mu q^\nu r^\rho s^\sigma \quad \propto \quad \vec{q}\cdot(\vec{r}\times\vec{s})$$

is a scalar *triple product*.

Differential CP violation

Compare the CP-conjugate amplitudes (squared)

$$\mathcal{M}(\{\vec{p}_i, \sigma_i\}) \quad \text{and} \quad \bar{\mathcal{M}}(\{-\vec{p}_{\bar{\imath}}, -\sigma_{\bar{\imath}}\})\Big|_{\vec{p}_{\bar{\imath}}=\vec{p}_i, \sigma_{\bar{\imath}}=\sigma_i}$$

phase-space point by phase-space point.

Contributions of definite
- *strong* δ and *weak* φ phases
- $\hat{\mathrm{T}}$ transformation properties

$$\mathcal{M}(\{\vec{p}_i, \sigma_i\}) = \\ +a(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_a+\varphi_a)} \\ +b(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_b+\varphi_b)} \\ +c(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_c+[\varphi_c+\pi/2])} \\ +\cdots$$

$$\bar{\mathcal{M}}(\{-\vec{p}_i, -\sigma_i\}) = \\ +a(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_a-\varphi_a)} \\ +b(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_b-\varphi_b)} \\ +c(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_c-[\varphi_c+\pi/2])} \\ +\cdots$$

with
$$a(\{-\vec{p}_i, -\sigma_i\}) = +a(\{\vec{p}_i, \sigma_i\}) \quad \hat{\mathrm{T}}\text{-even}$$
$$b(\{-\vec{p}_i, -\sigma_i\}) = +b(\{\vec{p}_i, \sigma_i\}) \quad \hat{\mathrm{T}}\text{-even}$$
$$c(\{-\vec{p}_i, -\sigma_i\}) = -c(\{\vec{p}_i, \sigma_i\}) \quad \hat{\mathrm{T}}\text{-odd}$$
$$\cdots$$

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$\Longrightarrow$ The φ phases are defined to contain all 'CP-oddity'.

CP violation and strong phases

Distributions of definite CP and $\hat{\mathrm{T}}$ transformation properties

$$\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-}^{\text{even}}_{\text{odd}}}_{\mathrm{CP}\text{-}^{\text{even}}_{\text{odd}}} \equiv \frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2}\,\frac{\mathbb{I} \pm \mathrm{CP}}{2}\,\frac{d\Gamma}{d\Phi}$$

- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-even}}_{\mathrm{CP}\text{-even}} \propto \begin{array}{l} a\, a + b\, b + c\, c \\ +2\, a\, b\ \cos(\delta_a - \delta_b)\cos(\varphi_a - \varphi_b) \end{array}$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-odd}}_{\mathrm{CP}\text{-even}} \propto \begin{array}{l} 2\, a\, c\ \sin(\delta_a - \delta_c)\ \cos(\varphi_a - \varphi_c) \\ +2\, b\, c\ \sin(\delta_b - \delta_c)\ \cos(\varphi_b - \varphi_c) \end{array}$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-even}}_{\mathrm{CP}\text{-odd}} \propto -2\, a\, b\ \sin(\delta_a - \delta_b)\ \sin(\varphi_a - \varphi_b)$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-odd}}_{\mathrm{CP}\text{-odd}} \propto \begin{array}{l} 2\, a\, c\ \cos(\delta_a - \delta_c)\sin(\varphi_a - \varphi_c) \\ +2\, b\, c\ \cos(\delta_b - \delta_c)\sin(\varphi_b - \varphi_c) \end{array}$

$\Longrightarrow$ Four different sensitivities to strong and weak phases.

CP violation and strong phases

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- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-even}}_{\mathrm{CP}\text{-even}} \propto \; a\,a + b\,b + c\,c$ $\quad +2\,a\,b\,\cos(\delta_a - \delta_b)\cos(\varphi_a - \varphi_b)$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-odd}}_{\mathrm{CP}\text{-even}} \propto \; 2\,a\,c\,\sin(\delta_a - \delta_c)\,\cos(\varphi_a - \varphi_c)$ $\quad +2\,b\,c\,\sin(\delta_{\ldots} - \delta_{\ldots})\,\cos(\varphi_{\ldots} - \varphi_{\ldots})$

$\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-even}}_{\mathrm{CP}\text{-odd}} \propto$ '$\sin\delta\sin\varphi$'	Sensitivity to small differences of CP-odd phases between decay amplitudes of different CP-even phases.

$\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-odd}}_{\mathrm{CP}\text{-odd}} \propto$ '$\cos\delta\sin\varphi$'	Sensitivity to small differences of CP-odd phases between decay amplitudes of identical —or vanishing— CP-even phases.

$\Longrightarrow$ Four different sensitivities to strong and weak phases.

Untagged samples / self-conjugate states

- Tagging CP-conjugate processes may cost efficiency, and is N/A with self-conjugate initial and final states.
- An untagged sample is: [as in 1503.05362]

e.g. $$\begin{cases} B_s^0 \to & K^+(+\vec{p}_1) \quad \pi^-(+\vec{p}_2) \quad K^-(+\vec{p}_3) \quad \pi^+(+\vec{p}_4) \\ \bar{B}_s^0 \to & K^-(-\vec{p}_1) \quad \pi^+(-\vec{p}_2) \quad K^+(-\vec{p}_3) \quad \pi^-(-\vec{p}_4) \end{cases}$$

$$\frac{\mathbb{I} + \mathrm{CP}}{2} \, \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}}}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}E}^*}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}E^*}}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

It has two CP-odd distributions, $\hat{\mathrm{T}}$-odd or E^*-odd:

$$\frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2} \frac{\mathbb{I} \mp \mathrm{E}^*}{2} \left(\frac{\mathbb{I} + \mathrm{CP\hat{T}E^*}}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi} \right) = \frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2} \frac{\mathbb{I} \mp \mathrm{E}^*}{2} \frac{\mathbb{I} - \mathrm{CP}}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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CP violation in differential distributions
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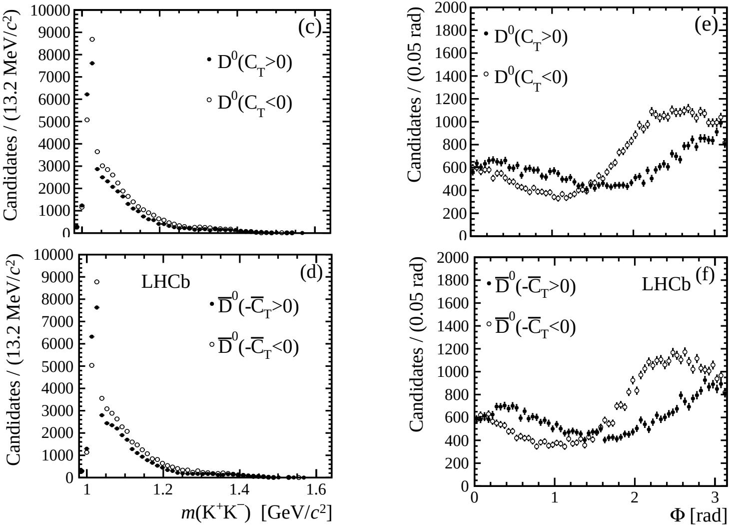
Multibody decays of mesons and baryons
- Spinless case: systematic modelling-independent analysis
- Spinful case: resolving ambiguities with modellisation

Other applications

Spinless case: $\hat{T}$-folding of the phase space

$$\frac{d\Gamma}{d\Phi}(D^0 \to K^+K^-\pi^+\pi^-)$$

[1408.1299]



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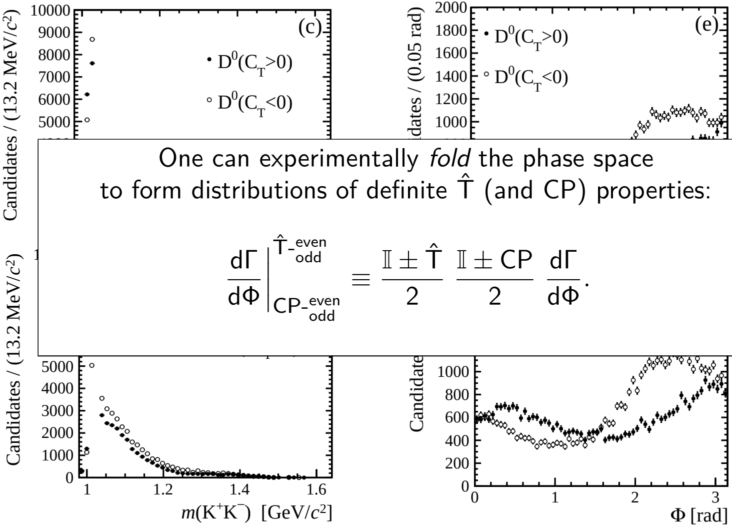
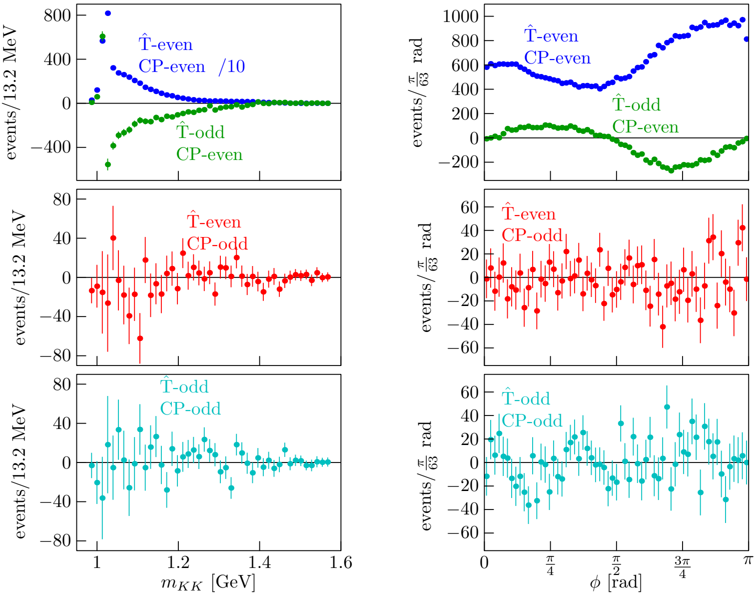


One can experimentally *fold* the phase space to form distributions of definite $\hat{T}$ (and CP) properties:

$$\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-}^{\text{even}}_{\text{odd}}}_{CP\text{-}^{\text{even}}_{\text{odd}}} \equiv \frac{\mathbb{I} \pm \hat{T}}{2}\,\frac{\mathbb{I} \pm CP}{2}\,\frac{d\Gamma}{d\Phi}.$$

Spinless case: T̂-folding of the phase space

$$\frac{d\Gamma}{d\Phi}(D^0 \to K^+K^-\pi^+\pi^-)$$



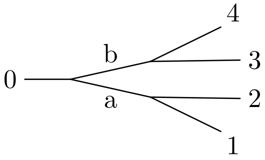
Spinless case: systematic modelling-independent analysis

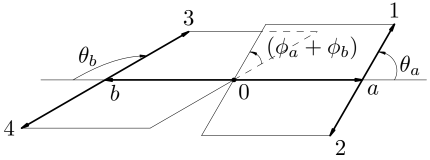
Define CP-odd asymmetries (or moments) systematically
exploiting the full angular distributions

$$\mathcal{A}^{kl}_{no} \equiv \int d\Omega \left(\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} - \frac{1}{\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\Omega} \right) \text{sign}\left\{ f_k(\cos\theta_a)\, f_l(\cos\theta_b)\, \sin\left(n\phi_a + n\phi_b + o\frac{\pi}{2} \right) \right\}$$

with $o = 0$: $\hat{\text{T}}$-odd
$o = 1$: $\hat{\text{T}}$-even

Given a phase-space parametrisation
a set of angles and invariant masses (biasing the analysis sensitivity)





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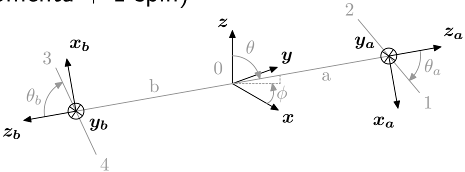
Other applications

The richness of polarization

$\hat{\text{T}}$-oddity with lower multiplicity

in the three-body decay of a polarized particle
($\epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu r^\rho s^\sigma$ with 3 momenta + 1 spin)

e.g. $\Lambda_b \to \Lambda^* \gamma \to pK\gamma$
$\Lambda_b \to N^* K \to p\pi K$



More angles

One polarization component breaks two rotation symmetries.

New $\hat{\text{T}}$-odd variables

- one polarization component
  - P_z: P-even–$\hat{\text{T}}$-odd ($\perp$ production plane)
  - $P_{x,y}$: P-odd–T-even
- angular variables: $\cos\theta$, $\cos\phi_a$, $\cos\phi_b$

The ambiguity of polarization

From the decay perspective:

- specific values cannot be selected
- $\hat{T}$-folding along P_z cannot be performed.
- Distributions of definite $\hat{T}$ properties cannot be defined.
- Linking amplitudes and kinematics through additional modelling:

  from helicity formalism [Jacob, Wick 59']

  with resonance+spin structure specified

Spinful case: resolving ambiguities

e.g. for $\frac{1}{2} \to \frac{1}{2}\,1 \to (\frac{1}{2}\,0)\,(0\,0)$ like $\Lambda_b \to N^*\rho \to p\pi\pi\pi$, $\Lambda_b \to \Lambda\,\phi \to p\pi KK$

$\frac{d\Gamma}{d\Phi} \propto$

$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$					$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$					$\sin^2\theta_b$	
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$	α_a			$\cos\theta_a$	$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$	α_a			$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} - \mathrm{Re}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$		P_z	$\cos\theta$		$\cos^2\theta_b$	
$-3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$		P_z	$\cos\theta$		$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} - \mathrm{Re}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\cos\phi_b$
$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$	α_a	P_z	$\cos\theta$	$\cos\theta_a$	$\cos^2\theta_b$	
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$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} + \mathrm{Re}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} + \mathrm{Re}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\cos\phi_b$
-6	$\mathrm{Re}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\cos\phi_a$
$+3$	$\mathrm{Re}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\cos(\phi_a+2\phi_b)$
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} + \mathrm{Im}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\sin\phi_b$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} - \mathrm{Im}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} - \mathrm{Im}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\sin\phi_b$
-6	$\mathrm{Im}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\sin\phi_a$
$+3$	$\mathrm{Im}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\sin(\phi_a+2\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} + \mathrm{Im}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$

Spinful case: resolving ambiguities

e.g. for $\frac{1}{2} \to \frac{1}{2}\, 1 \to (\frac{1}{2}\, 0)\, (0\, 0)$ like $\Lambda_b \to N^* \rho \to p\pi\pi\pi$, $\Lambda_b \to \Lambda\, \phi \to p\pi KK$

$\frac{d\Gamma}{d\Phi} \propto$

$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$					$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$					$\sin^2\theta_b$	
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$	α_a			$\cos\theta_a$	$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$	α_a			$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} - \mathrm{Re}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$		P_z	$\cos\theta$		$\cos^2\theta_b$	
$-3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$		P_z	$\cos\theta$		$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} - \mathrm{Re}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\cos\phi_b$
$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$	α_a	P_z	$\cos\theta$	$\cos\theta_a$	$\cos^2\theta_b$	
$-3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$	α_a	P_z	$\cos\theta$	$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} + \mathrm{Re}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} + \mathrm{Re}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\cos\phi_b$
-6	$\mathrm{Re}\{A_+^* A_-\}$						
$+3$	$\mathrm{Re}\{B_+^* B_-\}$						
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} + \mathrm{Im}\{A_-^* B_-\}$						
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} - \mathrm{Im}\{A_-^* B_+\}$						
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} - \mathrm{Im}\{A_-^* B_-\}$						
-6	$\mathrm{Im}\{A_+^* A_-\}$						
$+3$	$\mathrm{Im}\{B_+^* B_-\}$						
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} + \mathrm{Im}\{A_-^* B_+\}$						

$$
\begin{aligned}
P_{x,y} &= 0 \\
A_\pm &\equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}1}(\pm\tfrac{1}{2}, 0) \\
B_\pm &\equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}1}(\pm\tfrac{1}{2}, \pm 1) \\
1 &\equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(+\tfrac{1}{2}, 0) + \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(-\tfrac{1}{2}, 0) \\
\alpha_a &\equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(+\tfrac{1}{2}, 0) - \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(-\tfrac{1}{2}, 0) \\
1 &\equiv \mathcal{M}_{1\to 00}(0, 0)
\end{aligned}
$$

Spinful case: resolving ambiguities

e.g. for $\frac{1}{2} \to \frac{1}{2}\, 1 \to (\frac{1}{2}\, 0)\, (0\, 0)$ like $\Lambda_b \to N^* \rho \to p\pi\pi\pi$
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$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$					$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$					$\sin^2\theta$	
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$	α_a					
$+3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$	α_a			$\cos\theta_a$	$\sin\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} - \mathrm{Re}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3$			P_z	$\cos\theta$		$\cos^2\theta_b$	
$-3/2$			P_z	$\cos\theta$		$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} - \mathrm{Re}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\cos\phi_b$
$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$	α_a	P_z				
$-3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$	α_a	P_z				
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} + \mathrm{Re}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} + \mathrm{Re}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\cos\phi_b$
-6	$\mathrm{Re}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\cos\phi_a$
$+3$	$\mathrm{Re}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\cos(\phi_a+2\phi_b)$
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} + \mathrm{Im}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\sin\phi_b$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} - \mathrm{Im}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$
$-3/\sqrt{2}$	Im	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\sin\phi_b$
-6	$\mathrm{Im}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\sin\phi_a$
$+3$	$\mathrm{Im}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\sin(\phi_a+2\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} + \mathrm{Im}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$

$\hat{T}$-even angular distrib.

→ 'sin δ sin φ'

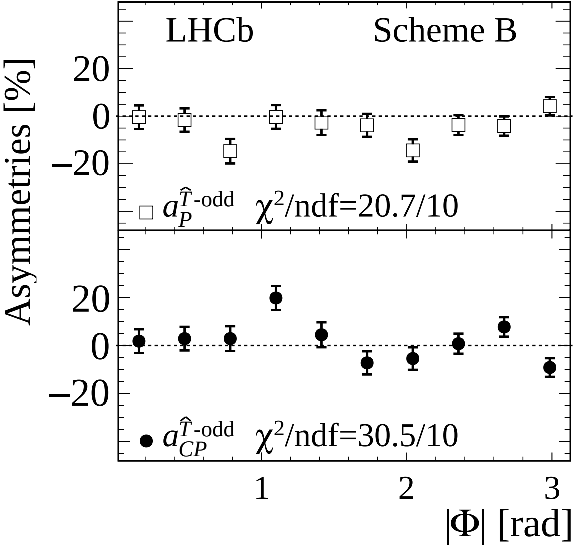
$\hat{T}$-odd angular distrib.

→ 'cos δ sin φ'

A first hint for CPV?

In $\Lambda_b \to \pi^-\pi^+\pi^- p$

[LHCb '16]
[News and views, Nature Phys.]



Probing CP in differential distributions

CP violation in differential distributions

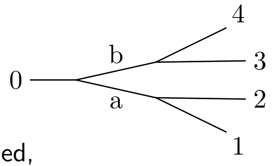
- With or without strong phases
- With untagged samples / self-conjugate states

Multibody decays of mesons and baryons

- Spinless case: systematic modelling-independent analysis
- Spinful case: resolving ambiguities with modellisation

Other applications

$H \to 4\ell$ and $e^+e^- \to HZ$



Modelling is non longer an issue.

Distributions of interested can be identified, e.g., in the SM EFT

[Beneke, Boito, Wang '14]

$$\begin{aligned}\mathcal{J}(q^2,\theta_1,\theta_2,\phi) = \; & J_1(1+\cos^2\theta_1\cos^2\theta_2+\cos^2\theta_1+\cos^2\theta_2)\\ & + J_2\sin^2\theta_1\sin^2\theta_2 + J_3\cos\theta_1\cos\theta_2\\ & + (J_4\sin\theta_1\sin\theta_2 + J_5\sin 2\theta_1\sin 2\theta_2)\sin\phi\\ & + (J_6\sin\theta_1\sin\theta_2 + J_7\sin 2\theta_1\sin 2\theta_2)\cos\phi\\ & + J_8\sin^2\theta_1\,\sin^2\theta_2\,\sin 2\phi + J_9\sin^2\theta_1\,\sin^2\theta_2\,\cos 2\phi.\end{aligned}$$

$$J_1 = 2\,r\,s\,\left(g_A^2+g_V^2\right)\left(|H_{1,V}|^2+|H_{1,A}|^2\right),$$
$$J_2 = \kappa\,\left(g_A^2+g_V^2\right)[\kappa\,(|H_{1,V}|^2+|H_{1,A}|^2) + \lambda\,\mathrm{Re}\,(H_{1,V}H_{2,V}^* + H_{1,A}$$
$$J_3 = 32\,r\,s\,g_A\,g_V\,\mathrm{Re}\left(H_{1,V}\,H_{1,A}^*\right),$$
$$J_4 = 4\kappa\,\sqrt{r\,s\,\lambda}\,g_A\,g_V\,\mathrm{Re}\left(H_{1,V}H_{3,A}^* + H_{1,A}H_{3,V}^*\right),$$
$$J_5 = \frac{1}{2}\kappa\,\sqrt{r\,s\,\lambda}\,\left(g_A^2+g_V^2\right)\mathrm{Re}\left(H_{1,V}H_{3,V}^* + H_{1,A}\,H_{3,A}^*\right),$$
$$J_6 = 4\sqrt{r\,s}\,g_A\,g_V\,[4\kappa\,\mathrm{Re}\left(H_{1,V}H_{1,A}^*\right) + \lambda\,\mathrm{Re}\,(H_{1,V}H_{2,A}^* + H_{1,A}H_{2,V}^*$$
$$J_7 = \frac{1}{2}\sqrt{r\,s}\,\left(g_A^2+g_V^2\right)[2\kappa\,\left(|H_{1,V}|^2+|H_{1,A}|^2\right) + \lambda\,\mathrm{Re}\,(H_{1,V}H_{2,V}^* +$$
$$J_8 = 2\,r\,s\,\sqrt{\lambda}\,\left(g_A^2+g_V^2\right)\mathrm{Re}\left(H_{1,V}H_{3,V}^* + H_{1,A}H_{3,A}^*\right),$$
$$J_9 = 2\,r\,s\,\left(g_A^2+g_V^2\right)\left(|H_{1,V}|^2+|H_{1,A}|^2\right).$$

$$\mathcal{A}_\phi^{(1)} = \frac{1}{d\Gamma/dq^2}\int_0^{2\pi} d\phi\;\mathrm{sgn}(\sin\phi)\,\frac{d^2\Gamma}{dq^2 d\phi} = \frac{9\,\pi}{32}\,\frac{J_4}{4J_1+J_2},$$
$$\mathcal{A}_\phi^{(2)} = \frac{1}{d\Gamma/dq^2}\int_0^{2\pi} d\phi\;\mathrm{sgn}(\sin(2\phi))\,\frac{d^2\Gamma}{dq^2 d\phi} = \frac{2}{\pi}\,\frac{J_8}{4J_1+J_2},$$

or, model-independently.

$(pp)^{|m|} \to X^J \to (\gamma\gamma)^S$

[Panico, Vecchi, Wulzer '16]

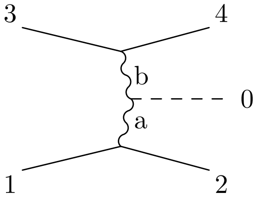
- List angular distributions for $J = 0, 2, 3, \ldots$ without Lagrangian.
- Allow the determination of spin provide *some* CP information.

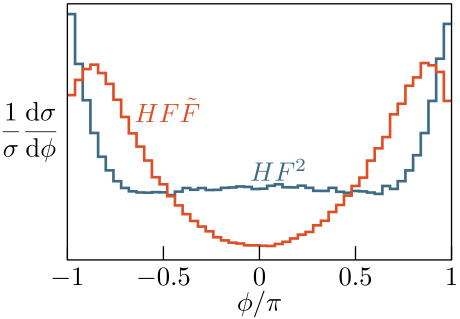
$\mathbf{J=0}$	$\mathcal{D}^{(0)}_{0,0} = 1$
$\mathbf{J=2}$	$\mathcal{D}^{(2)}_{\lvert m\rvert,S} = \begin{bmatrix} \frac{5}{4}(3c^2-1)^2 & \frac{15}{8}s^4 \\ \frac{15}{2}s^2c^2 & \frac{5}{4}s^2(1+c^2) \\ \frac{15}{8}s^4 & \frac{5}{16}(1+6c^2+c^4) \end{bmatrix}$
$\mathbf{J=3}$	$\mathcal{D}^{(3)}_{\lvert m\rvert,S} = \begin{bmatrix} \frac{7}{4}c^2(3-5c^2)^2 & \frac{105}{8}s^4c^2 \\ \frac{21}{16}s^2(5c^2-1)^2 & \frac{35}{32}s^2(1-2c^2+9c^4) \\ \frac{105}{8}s^4c^2 & \frac{7}{16}(4-15c^2+10c^4+9c^6) \end{bmatrix}$

VBF (pseudo-)scalar production

The phase space and pdf have a non-trivial angular dependence

$$d\sigma(\Phi) = \text{pdf}(\Phi)\ \ \text{ps}(\Phi)\ \ |\mathcal{M}|^2(\Phi)$$

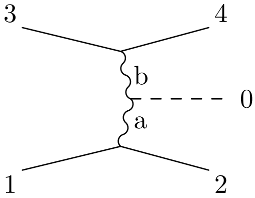


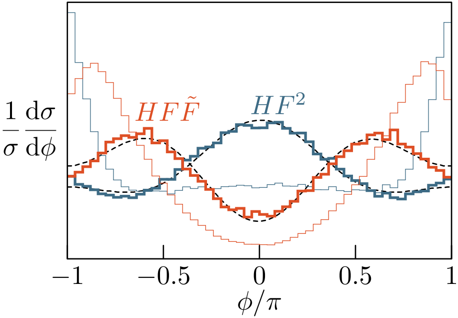


VBF (pseudo-)scalar production

The phase space and pdf have a non-trivial angular dependence

$$\left[d\sigma(\Phi) = \mathrm{pdf}(\Phi)\ \ \mathrm{ps}(\Phi)\ \ |\mathcal{M}|^2(\Phi)\ \right] \Big/ \mathrm{ps}(\Phi)$$





A 5D reweighing *almost* yields the intrinsic amplitude dependence

... on which one can perform a model-independent angular analysis.

Probing CP in differential distributions

Differential distributions are rich of opportunities
to search for CP violation.

$\hat{\mathrm{T}}$-odd distributions allow to probe CP violation:

- in the absence of CP-even *strong* phases
- in untagged samples / with self-conjugate states

In spinless decays, systematic search procedures
are free of modelling limitations.

Beyond this simplest case, the Jacob–Wick helicity formalism
yields valuable model-independence.