QCD at the LHC

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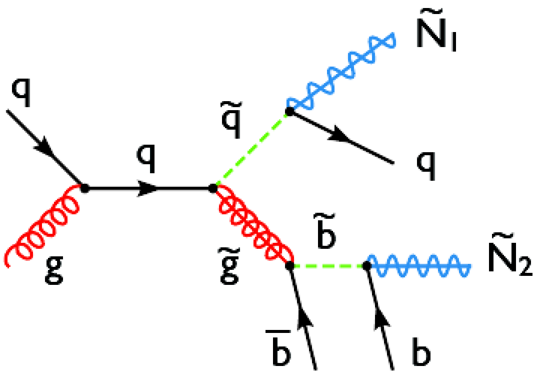
Lepton Photon 2009, 17-22 August 2009, Hamburg

Present Status of QCD

- ✓ Thanks to LEP, HERA and the TEVATRON
  QCD now firmly established theory of strong interactions
- ✓ We have gained a lot of confidence in comparing theoretical predictions with experimental data
- ✓ No major areas of discrepancies
- ?? But LHC brings new frontiers in energy and luminosity
- ?? typical SM process is accompanied by multiple radiation to form multi-jet events
- ?? most BSM signals involve pair-production and subsequent chain decays

SUSY:



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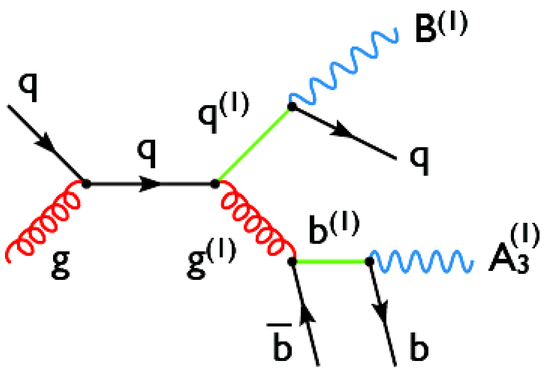
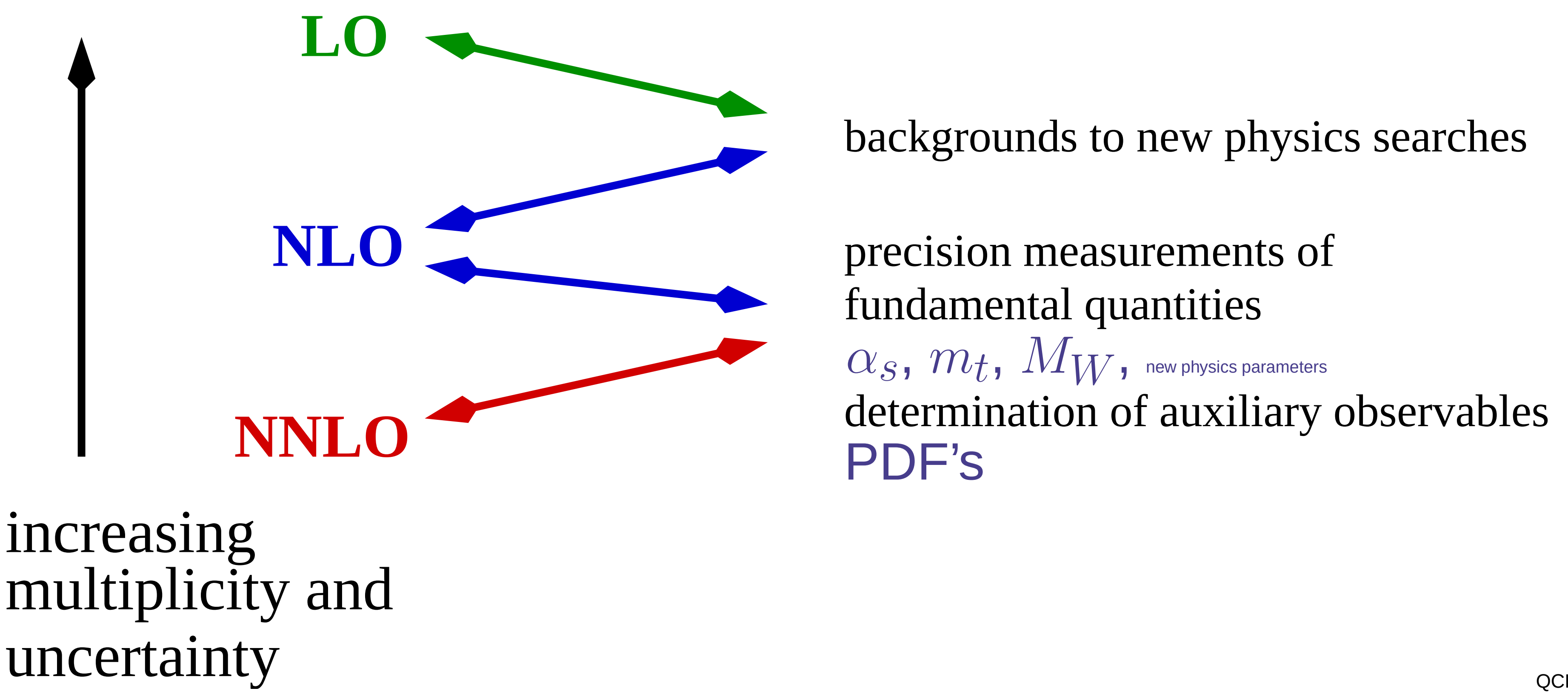


Matching onto Physics Goals

Twin Goals:

1. Identification and study of New Physics
2. Precision measurements (e.g. α_s, PDF’s) leading to improved theoretical predictions



What is covered in this talk

1. Overview
2. NLO multiparticle production
3. Jets
4. NNLO

State of the Art - at a glance

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3	NNNLO	NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- LO Automated and under control, even for multiparticle final states
- NLO Well understood for $2 \to 1$ and $2 \to 2$ in SM and beyond
- NLO Many new $2 \to 3$ calculations from Les Houches wish list since 2007
- NLO Very first $2 \to 4$ LHC cross section in 2008 $q\bar{q} \to t\bar{t}b\bar{b}$
- NLO Important developments in automation, $W + 3$ jets (2009)
- NNLO Inclusive and exclusive Drell-Yan and Higgs cross sections
- NNLO $e^+e^- \to 3$ jets, but still waiting for $pp \to \text{jets},\ W + \text{jet},\ t\bar{t}, VV$
- NNNLO F_2, F_3 and form-factors

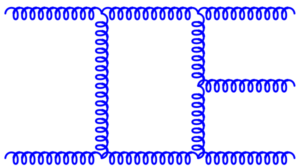
2. NLO multiparticle production

Limitations of LO

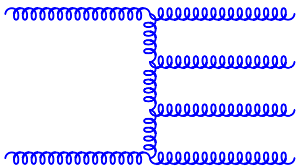
Very large uncertainty for multiparticle final states

- ✗ Large renormalisation scale uncertainty, magnified by the large amount of radiation e.g. a $\pm 10\%$ uncertainty in α_s leads to a $\pm 30\%$ uncertainty for $W + 3$ jets
- ✗ Large factorisation scale uncertainty
  higher factorisation scales deplete partons at large x - may increase or decrease cross section
- ✗ Both of these effects change the shapes of distributions
- ✓ Partly stabilised by going to NLO
- ✓ New channels open up at higher orders qg + large gluon PDF
- ✓ Increased phase space allows more radiation
- ✓ Large π^2 coefficients in s-channel $\Rightarrow$ large NLO corrections 30% - 100%

Anatomy of a NLO calculation



✓ one-loop $2 \rightarrow 3$ process
looks like 3 jets in final state



✓ tree-level $2 \rightarrow 4$ process
looks like 3 or 4 jets in final state

✓ plus method for combining the infrared divergent parts - dipole subtraction

Catani, Seymour; Dittmaier, Trocsanyi, Weinzierl, Phaf

✓ automated dipole subtraction

Gleisberg, Krauss (SHERPA); Hasegawa, Moch, Uwer; Frederix, Gehrmann, Greiner (MadDipole); Seymour, Tevlin

So far bottleneck has been one-loop matrix elements

LHC priority NLO wish list, Les Houches 2005/7*

process	background	status - mostly from Feynman diagram approach
$pp \to VV + 1$ jet	WBF $H \to VV$	WWj (07)
$pp \to t\bar{t} + b\bar{b}$	$t\bar{t}H$	$q\bar{q} \to t\bar{t}b\bar{b}$ (08)
$pp \to t\bar{t} + 2$ jets	$t\bar{t}H$	$t\bar{t}j$ (07), $t\bar{t}Z$ (08)
$pp \to VV + b\bar{b}$	WBF $H \to VV$, $t\bar{t}H$, NP	
$pp \to VV + 2$ jets	WBF $H \to VV$	WBF $pp \to VVjj$ (07)
$pp \to V + 3$ jets	NP	$W + 3$ jets (09)
$pp \to VVV$	SUSY trilepton	ZZZ (07), WWZ (07), WWW (08), ZZW (08)
$pp \to b\bar{b}b\bar{b}^*$	Higgs and NP	

✓ $pp \to H + 2$ jets via gluon fusion — Campbell, Ellis, Zanderighi, (06)

✓ $pp \to H + 2$ jets via WBF, electroweak and QCD corrections — Ciccolini, Denner, Dittmaier, (07)

✓ $pp \to H + 3$ jets via WBF, — Figy, Hankele, Zeppenfeld, (07)

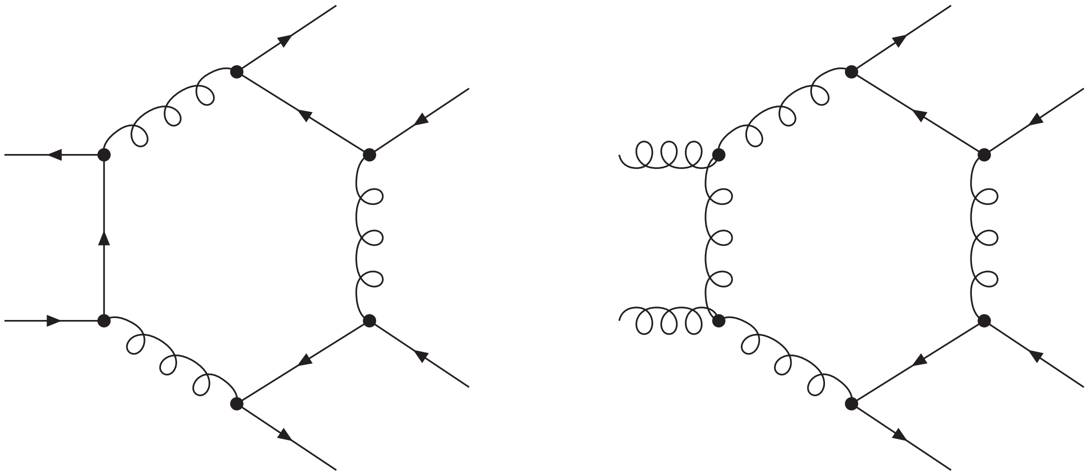
✓ ....

NLO example: Top pair plus bottom pair production

QCD corrections to $q\bar{q} \to t\bar{t}b\bar{b} + X$ and $gg \to t\bar{t}b\bar{b} + X$

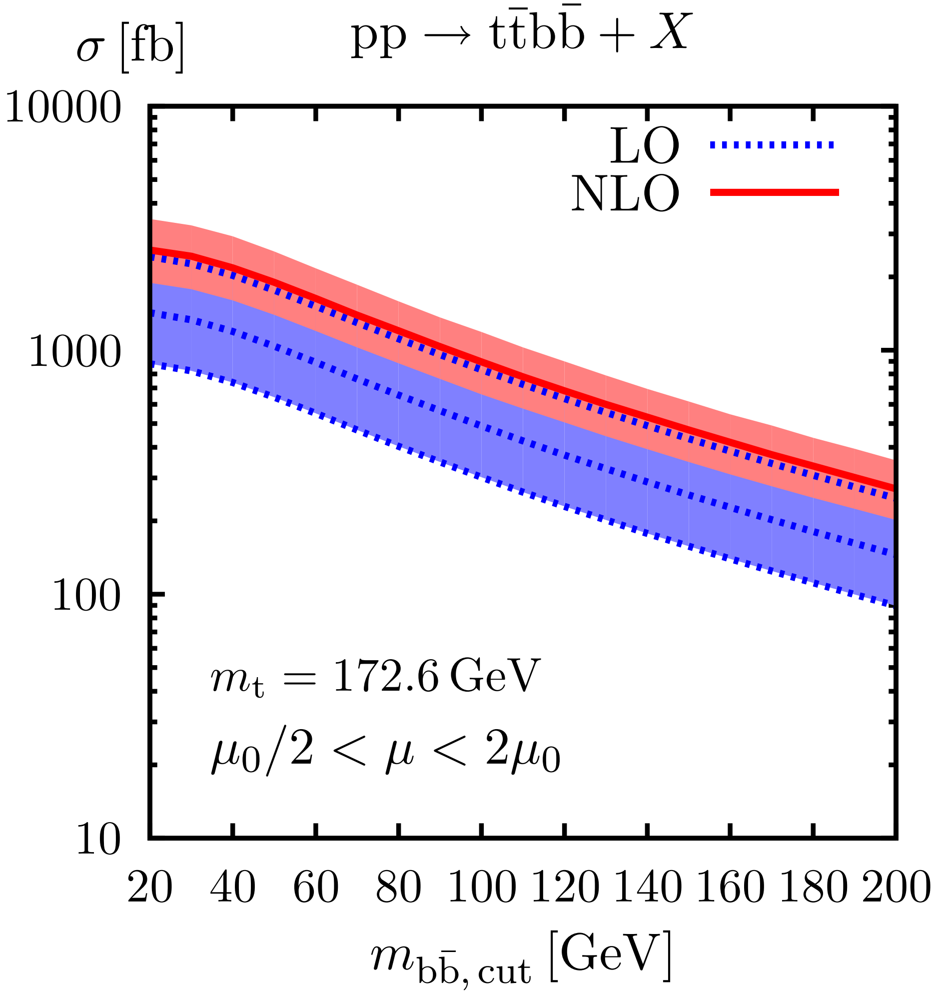
Bredenstein, Denner, Dittmaier, Pozzorini, (08,09)

✓ Background to the Higgs signal in $t\bar{t}H$ production where the Higgs decays into a bottom pair



✓ First successful demonstration of Feynman diagrammatic evaluation of $2 \to 4$ process at LHC

NLO example: Top pair plus bottom pair production



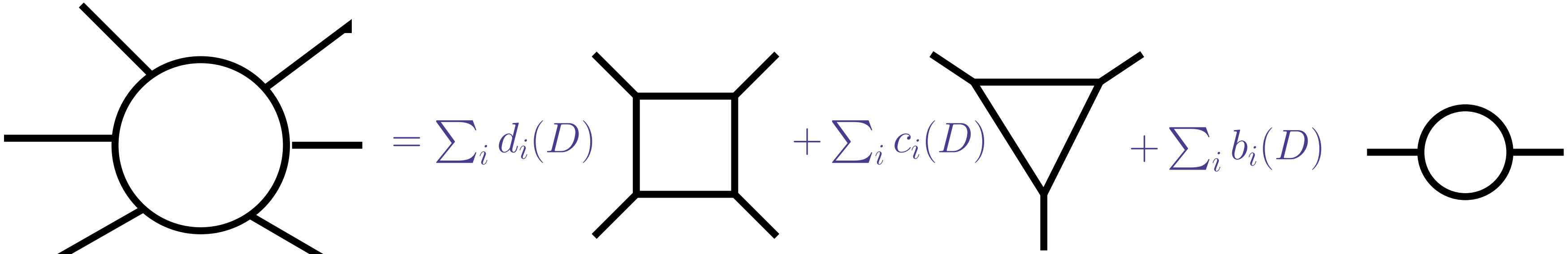
- ✓ NLO corrections appreciably reduce the unphysical scale dependence of the LO cross section
- ✓ but enhances the cross section by a K-factor of about 1.8 for the usual scale choice.
- ✓ the large correction factor is strongly affected by imposing a veto on hard jets.

- ✓ Similar results obtained by

G. Bevilacqua, M. Czakon, C. G. Papadopoulos, R. Pittau, M. Worek (09)

The one-loop problem

Any (massless) one-loop integral can be written as

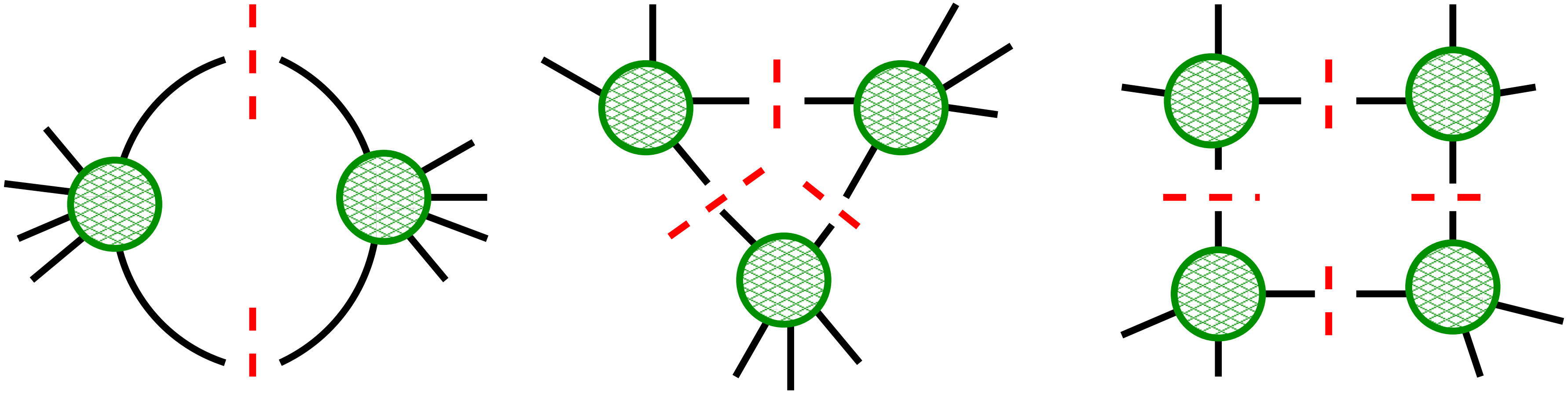


$$\mathcal{M} = \sum d(D)\text{boxes(D)} + \sum c(D)\text{triangles(D)} + \sum b(D)\text{bubbles(D)}$$

- ✓ higher polygon contributions drop out
- ✓ scalar loop integrals are known analytically around $D = 4$ Ellis, Zanderighi (08)
- ✓ need to compute the D-dimensional coefficients $a(D)$ etc.

The problem is complexity - the number of terms generated is too large to deal with, even with computer algebra systems, and there can be very large cancellations.

Breakthough idea - Generalised Unitarity

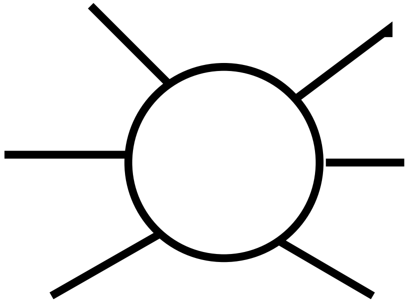
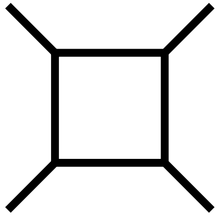
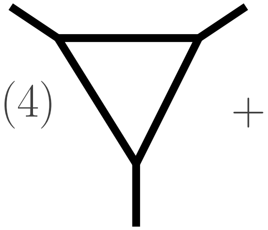
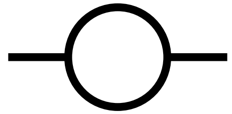


Bern, Dixon, Dunbar, Kosower (94); Britto, Cachazo, Feng (04)

- ✓ put internal propagators on-shell $\frac{1}{p^2+i0} \to -i\delta^+(p^2)$
- ✓ coefficient is product of tree-amplitudes with loop-momentum frozen
- ✓ can recycle tree-amplitudes in 4-D
- ✓ tree three-vertices do not vanish - complex momentum
- ✓ two-cut sensitive to box, triangle and bubble

4-dimensional unitarity

With 4-dimensional cuts - loop momentum in 4-dimensions and using 4-dimensional tree vertices

 $= \sum_i d_i(4)$  $+ \sum_i c_i(4)$  $+ \sum_i b_i(4)$  $+\mathcal{R}$

- ✓ $\mathcal{R}$ is a rational part that is generated by the D dependence of the coefficients $d_i(D)$ etc
- ✓ dimensionality of the loop momentum
- ✓ number of polarisation states of internal particles
- ✓ $\mathcal{R}$ can be computed with on-shell recursion (as for tree-diagrams)

Berger, Bern, Dixon, Forde, Kosower (06)

Analytic one-loop six gluon amplitude

$$A^{QCD} = A^{[1]} + \frac{n_f}{N} A^{[1/2]},$$

$$A^{[1]} = A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1} + A^{[0]}, \; A^{[1/2]} = A^{\mathcal{N}=1} - A^{[0]}$$

Amplitude	$\mathcal{N} = 4$	$\mathcal{N} = 1$	scalar(cut)	scalar (rat)
$--++++$	BDDK (94)	BDDK (94)	BDDK (94)	BDK (94)
$-+-+++$	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
$-++-++$	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
$---+++$	BDDK (94)	BDDK (94)	BBDI (05), BFM (06)	BBDFK (06), XYZ (06)
$--+-++$	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)
$-+-+-+$	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)

✓ Analytic computation

Bedford, Berger, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Buchbinder, Cachazo, Dixon, Dunbar, Feng, Forde, Kosower, Mastrolia, Perkins, Spence, Travaglini, Xiao, Yang, Zhu

A second breakthrough - OPP

Reducing full one-loop amplitudes to scalar integrals at the integrand level

Ossola, Papadopoulos, Pittau (06)

✓ systematic algebraic reduction at the integrand level

✓ integrand is decomposed by partial fractioning into linear combination of terms with 4-,3-,2,-1 denominator factors

$$\mathcal{A}(\ell) = \sum_{i_1,\ldots i_4} \frac{\overline{d_{i_1 i_2 i_3 i_4}}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{i_1,\ldots i_3} \frac{\overline{c_{i_1 i_2 i_3}}}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{i_1,\ldots i_2} \frac{\overline{b_{i_1 i_2}}}{d_{i_1} d_{i_2}}$$

✓ obtain numerators by taking residues; i.e. set inverse propagator $= 0$

$$\overline{d_{i_1 i_2 i_3 i_4}} = d_{i_1 i_2 i_3 i_4} + \tilde{d}_{i_1 i_2 i_3 i_4}, \qquad \text{etc.}$$

where $\tilde{d}_{i_1 i_2 i_3 i_4}$ integrates to zero

✓ Very algorithmic, can be automated.

NLO automation: HELAC/CutTools

Cafarella, van Hameren, Kanaki, Ossola, Papadopoulos, Pittau, Worek

- ✓ HELAC: off-shell recursion for the full Standard Model
- ✓ CutTools: fortan90 implementation of OPP recursion
- ✓ Specialized Feynman rules for rational parts
- ✓ Automatic 1-loop computation of amplitude at single phase-space point all 2→4 Les Houches wish-list processes

$$\begin{aligned} q\bar{q}, gg &\longrightarrow t\bar{t}b\bar{b}, b\bar{b}b\bar{b}, W^+W^-b\bar{b}, t\bar{t}gg \\ q\bar{q}' &\longrightarrow Wggg, Zggg \end{aligned}$$

van Hameren, Papadopoulos, Pittau (09)

- ✓ all masses, colours and helicities treated exactly
- ✓ Now combined with LO 2→ 5 processes using subtraction terms and efficient MC integration for $pp \to t\bar{t}b\bar{b}$

G. Bevilacqua, M. Czakon, C. G. Papadopoulos, R. Pittau, M. Worek (09)

NLO automation: BlackHat

Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre

- ✓ C++ implementation of on-shell technqiues for 1-loop amplitudes
- ✓ based on $D = 4$ unitarity - to generate all of the coefficients of loop integrals
- ✓ and on-shell recursion for the rational parts
- ✓ up to 8 gluon amplitudes numerically — Berger et al, (08)
- ✓ leading colour $Vq\bar{q}ggg$ numerically — Berger et al, (08)
- ✓ interfaced with SHERPA Monte Carlo for real radiation and infrared subtraction terms to produce complete NLO $W + 3$ jet cross sections

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

Berger et al, (09)

NLO automation: Rocket

Ellis, Giele, Kunzst, Melnikov, Zanderighi

a Fortran 90 package which fully automates the calculation of virtual amplitudes via tree level recursion + D-unitarity

- ✓ based on OPP and two different values of D
- ✓ off-shell recursion for tree-input
- ✓ up to 20 gluon amplitudes numerically

Giele, Zanderighi, (08)

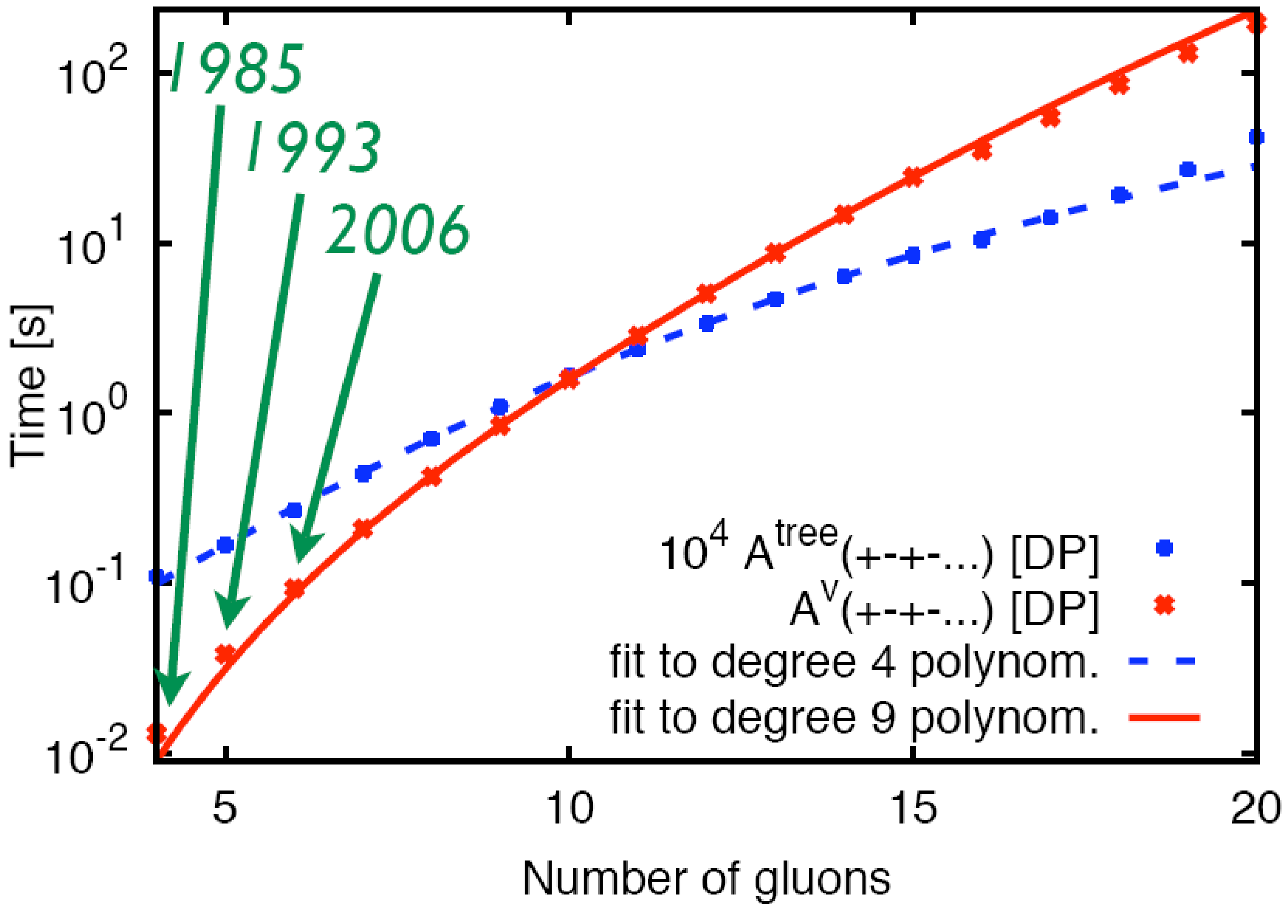
- ✓ all vector boson plus five parton processes numerically at single phase-space points

Ellis, Giele, Zanderighi, (08)

- ✓ physical $W+3$ jet cross section

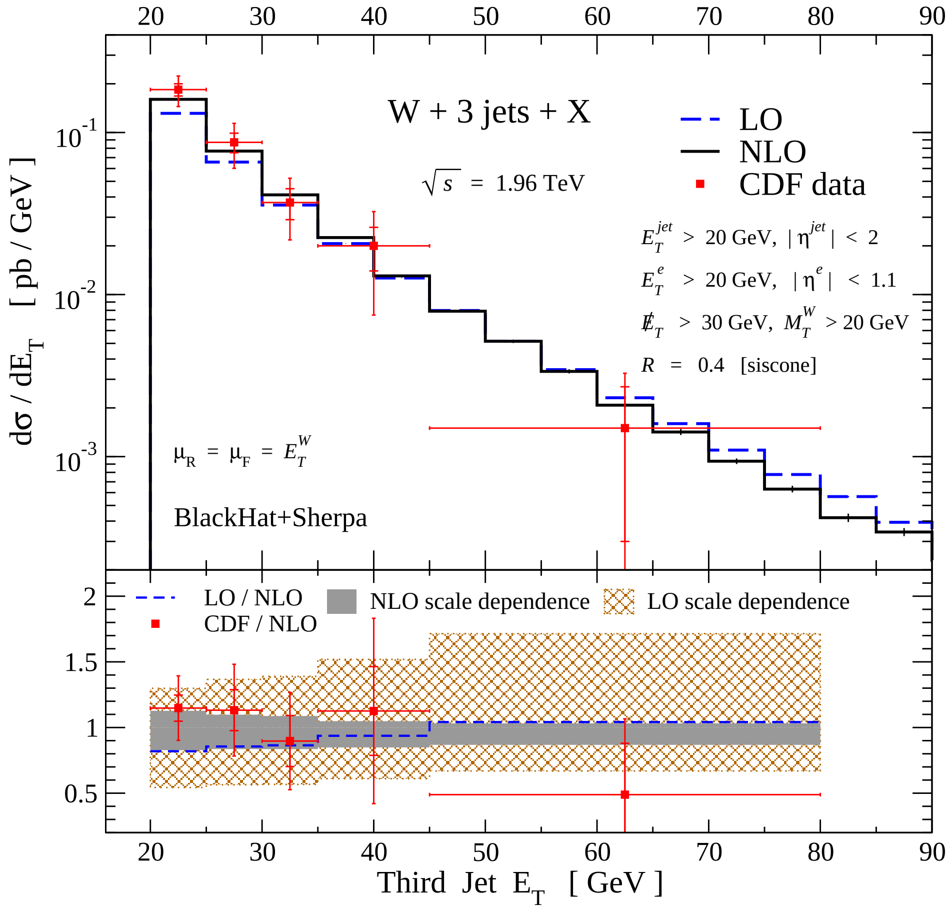
Ellis, Melnikov, Zanderighi, (09)

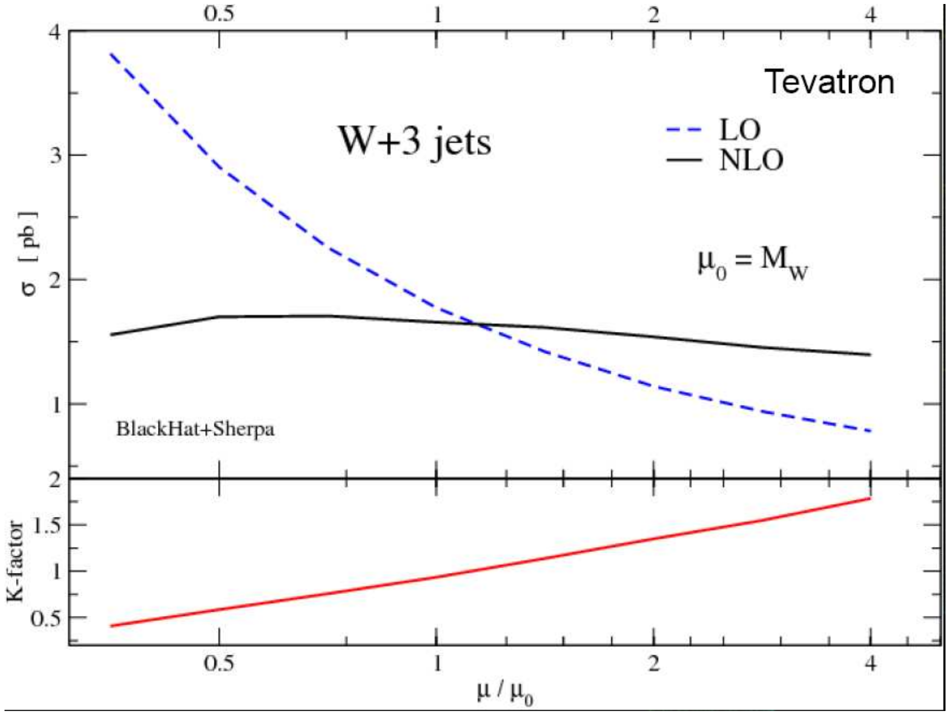
Massive leap forward: gg → (N-2)g at 1-loop



single colour ordering, single phase space point Giele, Zanderighi (08)
other numerical programs by Lazopoulos (08) and Giele, Winter (09)

Massive leap forward: W+3 jet at NLO

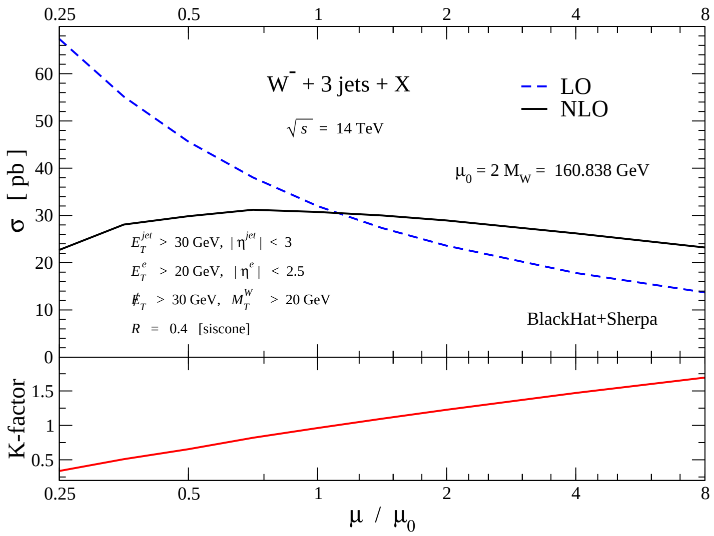




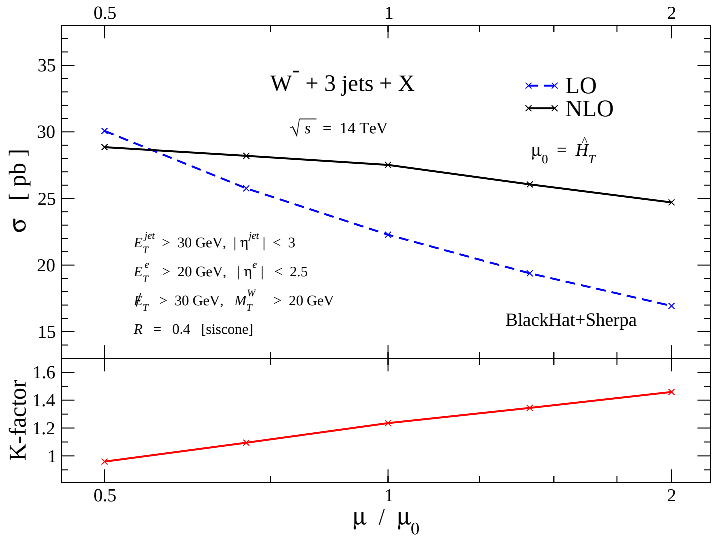
Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre, (09)

Scale dependence of observables

- ✓ Traditionally LO uncertainty estimated by varying scale around global scale like M_W or E_T^W
- ✓ At high energy, there are more different event structures and other scales are possible, that can dramatically affect the LO contribution and hence the K-factor



Fixed scale $\mu_0 = 2M_W$



Dynamical scale $\mu_0 = \hat{H}_T$

Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre, (09)

What can we hope for at NLO?

- ✓ Les Houches accord on standardisation of NLO computations and how to efficiently combine new virtual results with existing real radiation packages
  `http://www.lpthe.jussieu.fr/LesHouches09Wiki/index.php/Draft`

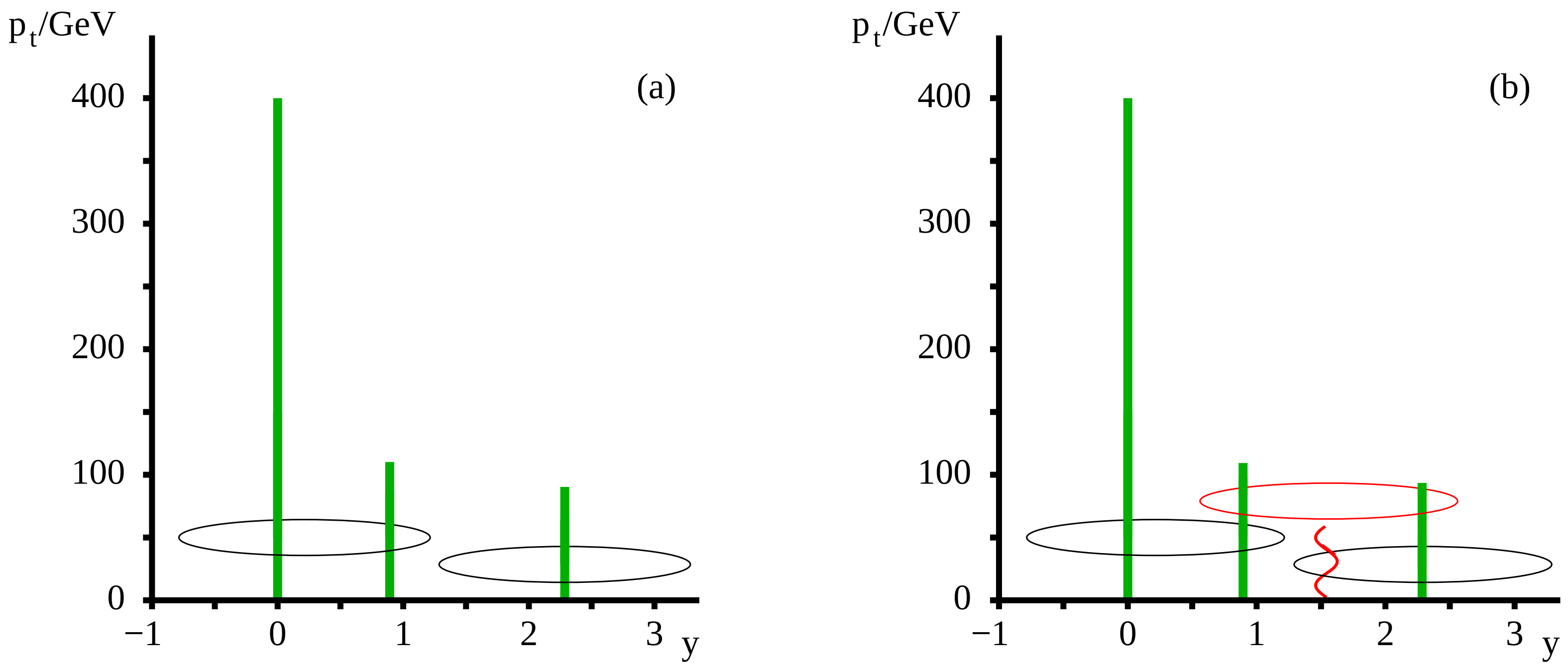
  Cannot do better than tree calculations..., at the moment processes with 7 or 8 particles in the final state.
- ✓ All 2 to 4 processes with both Feynman diagrammatic and newer unitarity/OPP based methods
- ✓ 2 to 5 and perhaps 2 to 6 processes with unitarity/OPP based methods
- ✓ hope for a better understanding of how to choose scale - possibly including more dynamic variables that depend on the event structure??

3. Jets

Jets: Cones vs Recombination

- ✓ Cone algorithms
  - ✓ Intutitive, clear jet structure
  - ✗ Complicated; problems with IR safety
  - ✓ Solved by SiSCone — Salam, Soyez, (07)
- ✓ Recombination algorithms (k_T etc)
  - ✓ Simple, IR safe
  - ✗ Messy jet structure
  - ✓ Solved by anti-k_T — Cacciari, Salam, Soyez, (08)

Cone algorithms and Infrared Safety



- ✗ Adding one extra soft particle changes the number of jets
- ✗ Soft emission changes the hard jets ⇒ algorithm is unsafe
- ✓ Solution: use seedless algorithm to find stable jet cones SISCone

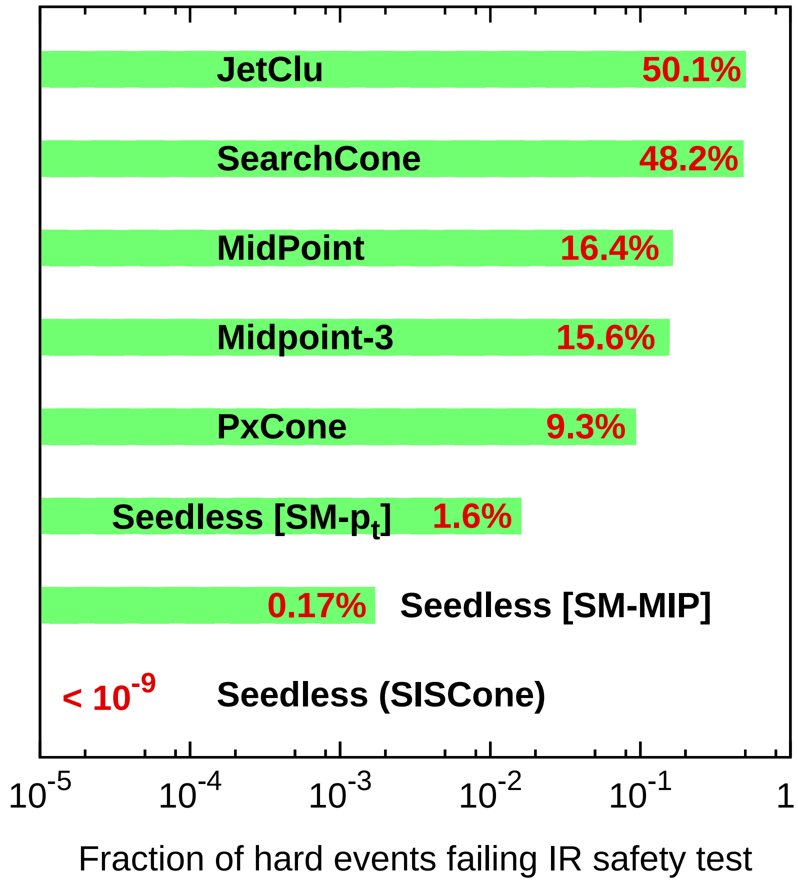
Salam, Soyez, (07)

Cone algorithms

Will find discrepancies between theory and experiment using the midpoint cone when more partons allowed in the event

Observable	problem at
Inclusive jet	NNLO
$V + 1$ jet	NNLO
3 jets	NLO
$V + 2$ jets	NLO
jet masses in 3 jets	LO
jet masses, V+2 jets	LO

Will have an impact on LHC physics



SISCone is IRC safe, similar complexity to midpoint algorithms

Salam, Soyez, (07)

Recombination algorithms

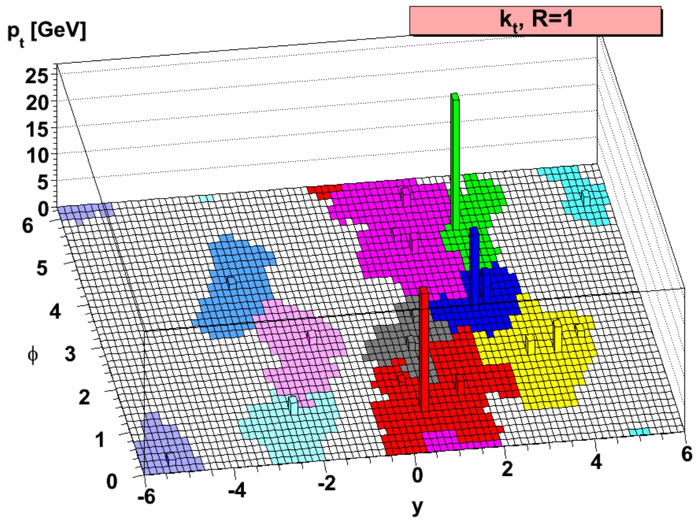
Compute the smallest "distance" d_{ij} or d_{iB} and either cluster i and j together or identify i as a jet

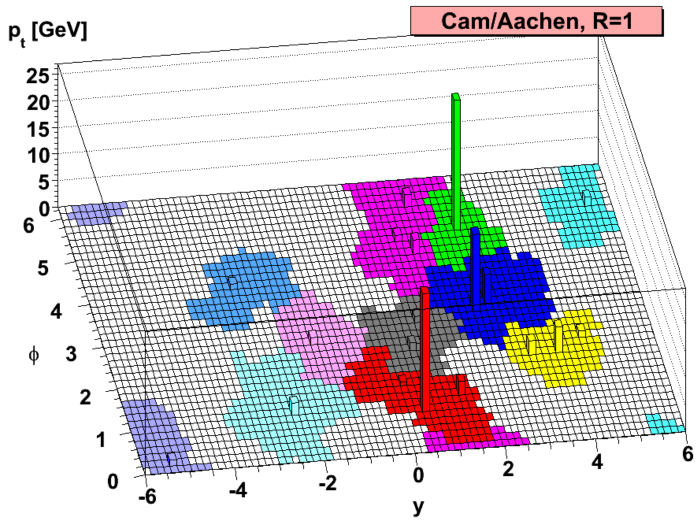
$$d_{ij} = \min\{k_{Ti}^p, k_{Tj}^p\}\Delta R_{ij}/R, \qquad d_{iB} = k_{Ti}^p$$

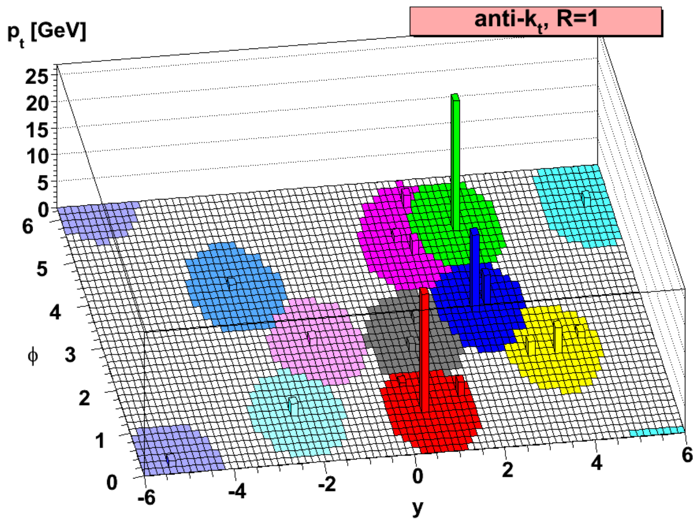
$$\Delta R^2 = \Delta\eta^2 + \Delta\phi^2$$

Algorithm	p	clusters first	comment
$\mathrm{k_T}$/Durham	> 0	softest	leads to very irregular jets includes a lot of underlying event hard to get jet energy scale right
Cambridge/Aachen	$= 0$	closest	still leads to very irregular jets similar problems to kT algorithm
anti-$\mathrm{k_T}$	< 0	hardest	shape of jet insensitive to soft particles ✓ cone-like jets ✓ may be easier to get jet energy scale right ✓

Recombination algorithms



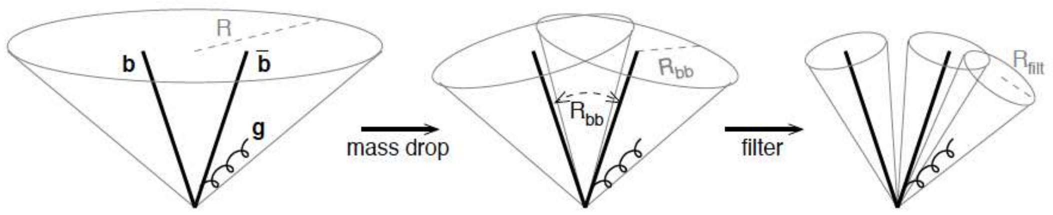




Visible benefits of anti-k_T algorithm!

Jet substructure

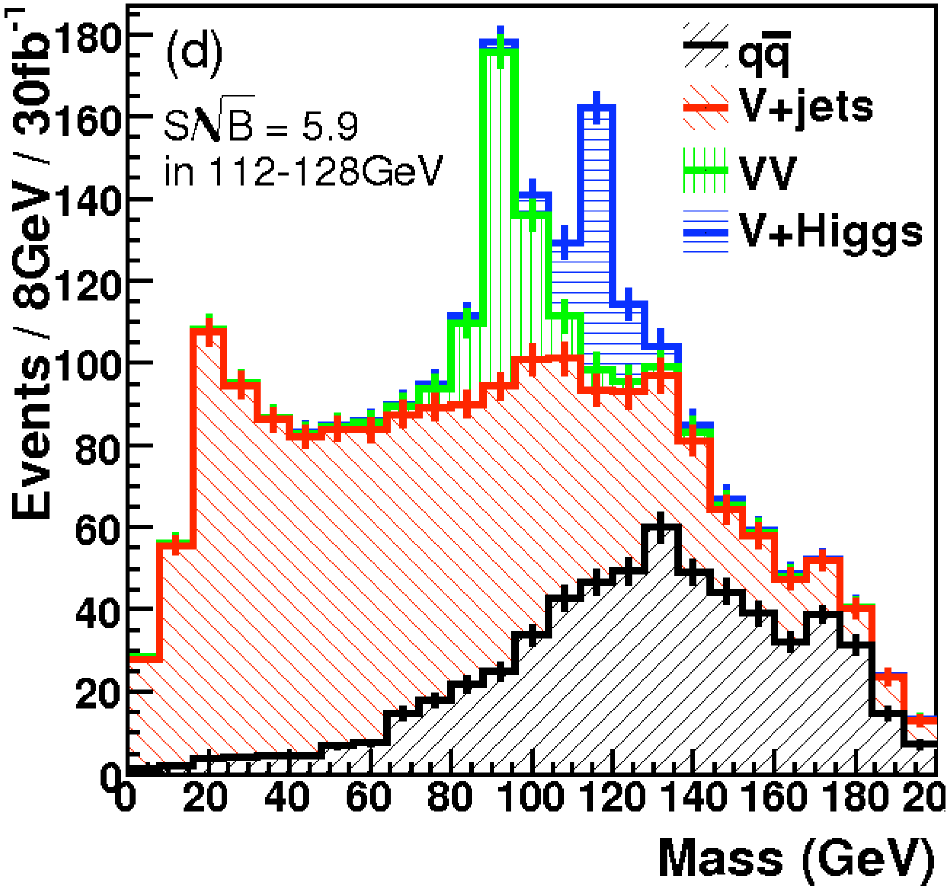
- ✓ The LHC is the first place where heavy (100 GeV) particles will be copiously produced well above threshold.
- ✓ They will often be boosted, and will often decay to hadrons.
- ✓ The decay products will often appear in a single jet.
- ✓ e.g. high p_T Higgs production with decay to $b\bar{b}$, looks like a single massive jet
- ✓ need to examine the substructure of massive jets to get the physics out.



- ✓ jet-finding adapted to identify the characteristic structure of Higgs decay into $b\bar{b}$ with small angular separation

Butterworth, Davison, Rubin, Salam (08)

Jet substructure: Z/W + H $(\to b\bar{b})$ rescued



✓ 5.9σ at 30 fb^{-1}: VH with H $\to$ bb recovered as one of the best discovery channels for light Higgs at LHC

Butterworth, Davison, Rubin, Salam (08)

✓ also used for high p_T top and top resonances

Kaplan, Rehermann, Schwartz, Tweedie (08)

✓ and R-parity violating three-jet decay of neutralino

Butterworth, Ellis, Raklev, Salam (09)

4. NNLO

NNLO

When is NNLO needed?

- ✓ When corrections are large - e.g. H production
- ✓ For benchmark measurements where experimental errors are small

What is known so far?

- ✓ Inclusive cross sections for W, Z and H production

  van Neerven, Harlander, Kilgore, Anastasiou, Melnikov, Ravindran, Smith;

- ✓ Semi-inclusive $2 \to 1$ distributions - W, Z and H rapidity distributions

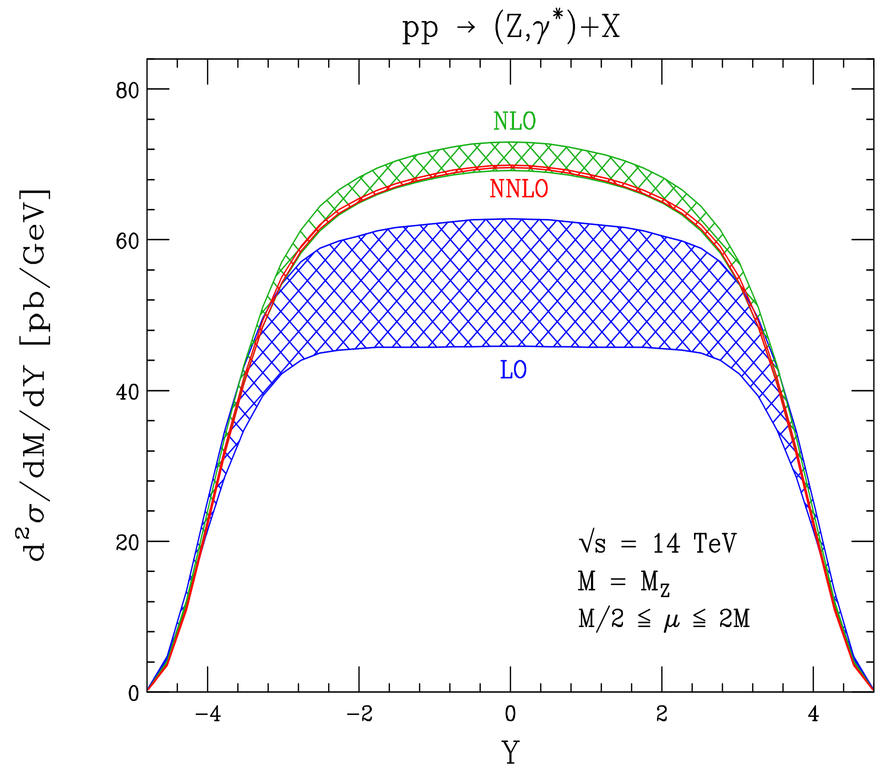
  Anastasiou, Dixon, Melnikov, Petriello

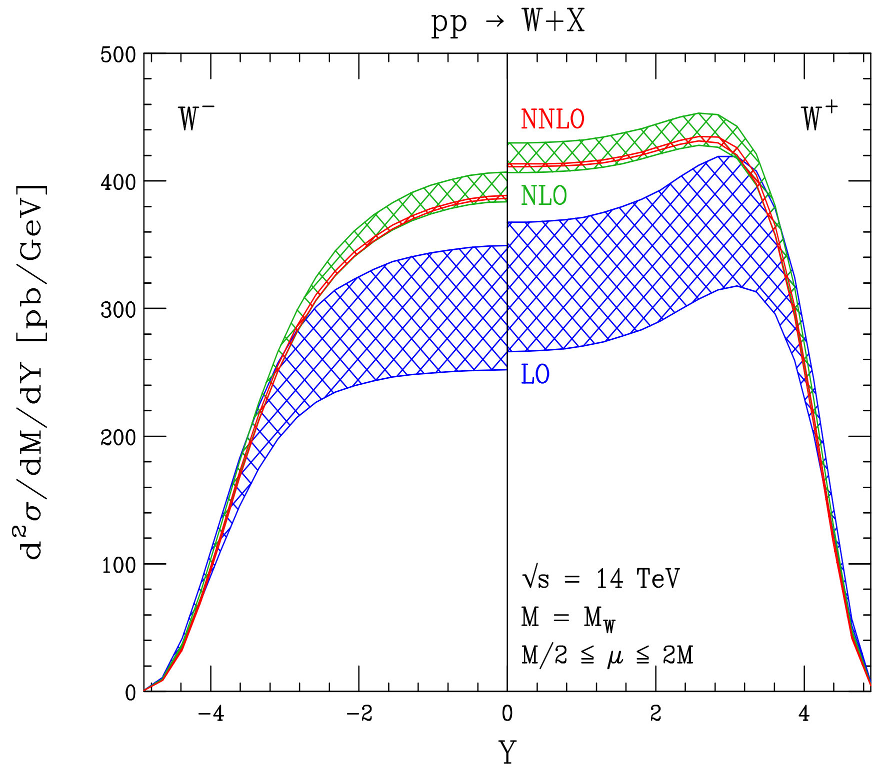
- ✓ Fully differential $pp \to H, W, Z + X$

  Anastasiou, Melnikov, Petriello; Catani, Cieri, de Florian, Grazzini

- ✓ DGLAP splitting kernels — Moch, Vermaseren, Vogt
- ✓ NNLO parton distributions — Martin, Stirling, Thorne, Watt

Gauge boson production at the LHC





Gold-plated process

Anastasiou, Dixon, Melnikov, Petriello (04)

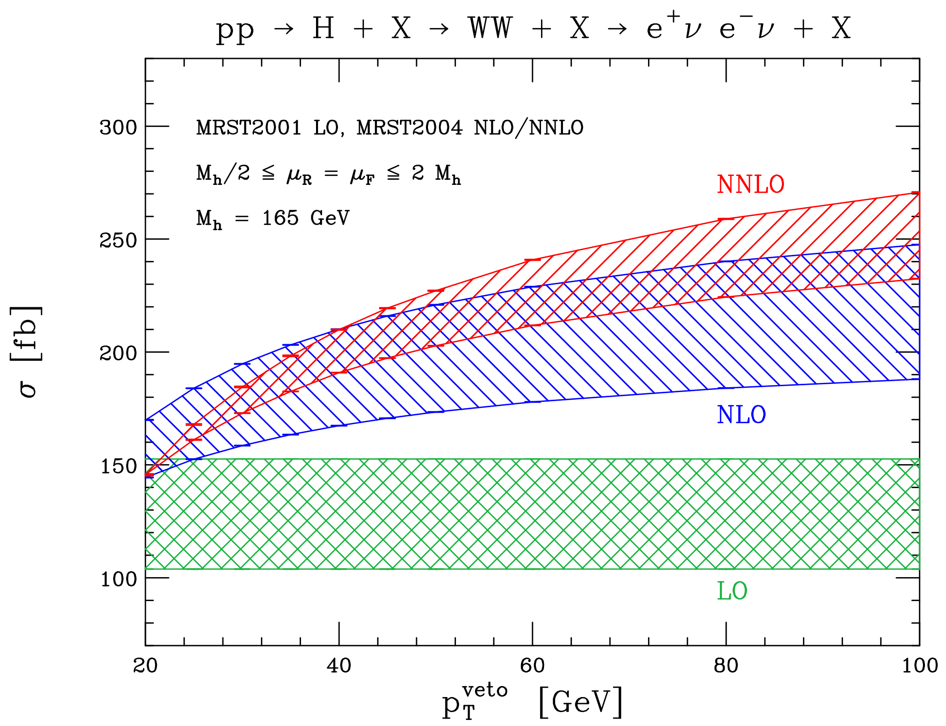
At LHC NNLO perturbative accuracy better than 1%

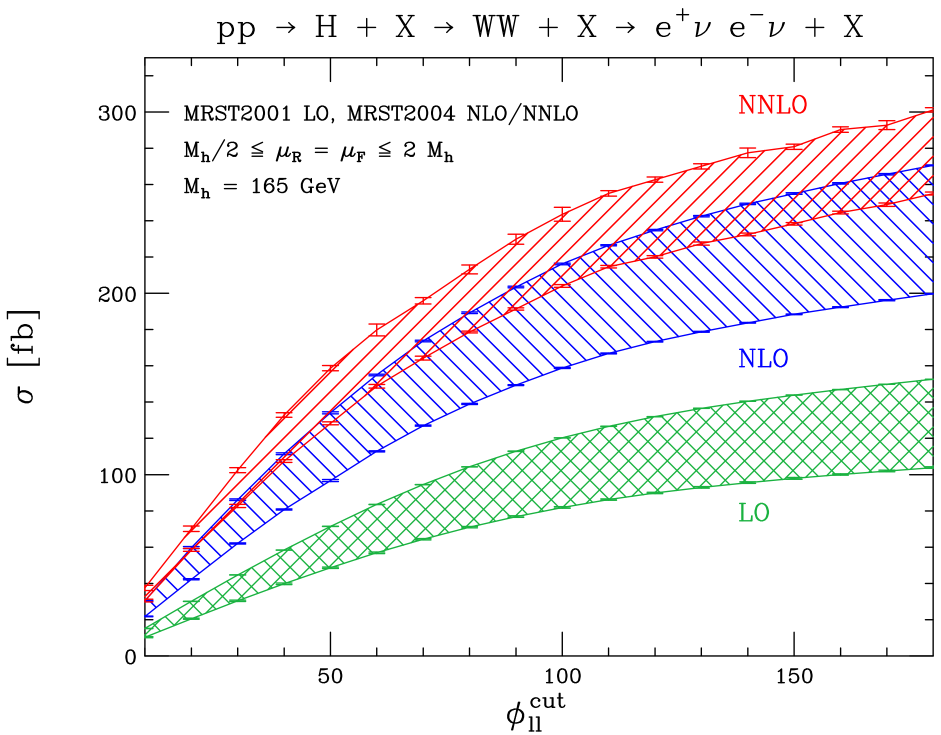
$\Rightarrow$ could use to determine parton-parton luminosities at the LHC

Higgs boson production at the LHC

- ✓ First study of fully exclusive $pp \to H \to WW \to \ell\nu\ell\nu$ with $m_H \sim 165$ GeV Anastasiou, Dissertori, Stöckli, (07) Catani, Grazzini (07)
- ✓ Experimental cuts to reduce backgrounds affect LO/NLO/NNLO cross sections differently e.g. jet-veto suppresses additional radiation,

$\Longrightarrow$ Absolutely vital to include cuts and decays in realistic studies





NNLO 3-jets in e^+e^-

- ✓ Motivation: error on α_s from jet-observables

$$\alpha_s(M_Z) = 0.121 \pm 0.001(\text{exp}) \pm 0.005(\text{th})$$

Bethke (06)

⇒ dominated by theoretical uncertainty

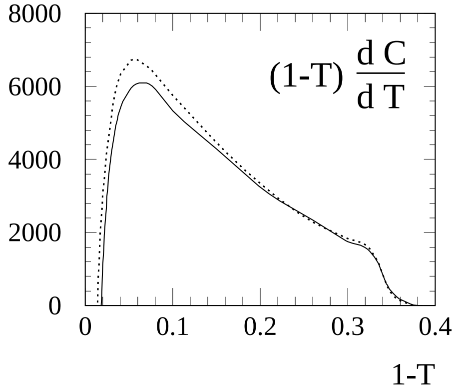
- ✓ First NNLO results for 3-jet event shapes in 2007

Gehrmann, Gehrmann-De Ridder, NG, Heinrich (07)

- ✓ Problem in the two-jet region identified in two colour structures

Becher, Schwartz (08); Weinzierl (08)

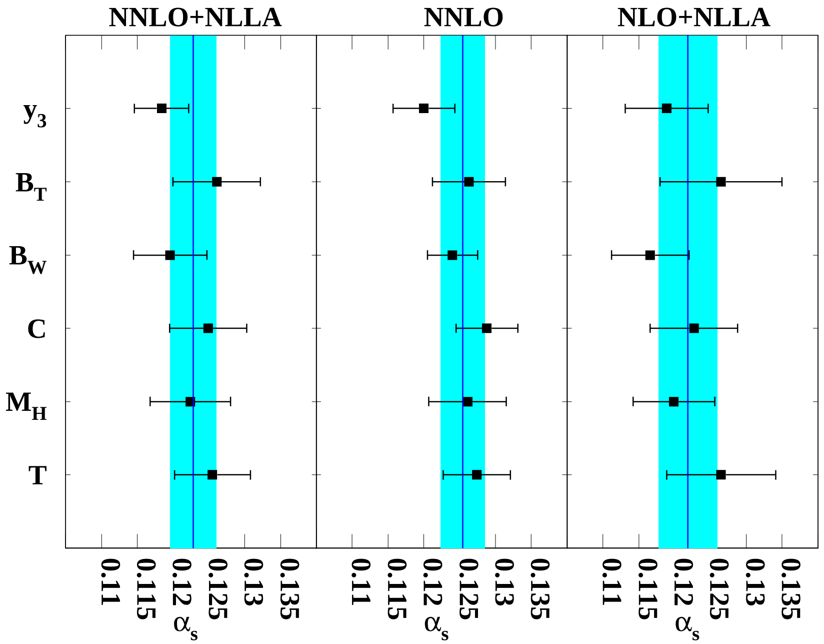
- ✓ over-subtraction of wide angle soft emission
- ✓ now fixed - minor correction in three-jet region



NNLO 3-jets in e^+e^-

✓ Application: extraction of α_s at NNLO+NLLA

Dissertori, Gehrmann, Gehrmann-De Ridder, NG, Heinrich, Luisoni, Stenzel (09),
Bethke, Kluth, Pahl, Schieck, the JADE Collaboration (08)



- ✓ clear improvements over NLO+NLLA
- ✓ spread of α_s determinations reduced
- ✓ renormalisation scale reduced
- ✓ theory error larger for NNLO+NLLA than NNLO because of mismatch in the cancellation of renormalisation scale logarithms

✓ fit to ALEPH data for six event shape observables yields

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009(stat) \pm 0.0009(exp) \pm 0.0012(had) \pm 0.0035(theo)$$

Other NNLO calculations on horizon

✓ $pp \to jet$ +X

- needed to constrain PDF's and fix strong coupling
- matrix elements known for some time — Anastasiou et al, Bern et al
- antenna subtraction terms worked out — Daleo, Gehrmann, Maitre

✓ $pp \to t\bar{t}$

- necessary for $\sigma_{t\bar{t}}$ and m_t determination
- matrix elements partially known

Czakon, Mitov, Moch; Bonciani, Ferroglia, Gehrmann, Studerus, Maitre

✓ $pp \to VV$

- signal: to study the gauge structure of the Standard Model
- background: for Higgs boson production and decay in the intermediate mass range
- large NLO corrections — Chachamis, Czakon, Eiras

Summary

- ✓ driven by the LHC needs, there has been a remarkable development in higher order QCD calculations
- ✓ first signs of automated mutiparticle NLO cross sections
- ✓ many new ideas for sophisticated jet definitions - and how to apply them to extract more information
- ✓ high precision NNLO calculations for standard candle processes on the way
- ✓ ready to take on the challenge of finding new physics at the LHC
- ✓ . . . apologies to those whose important work I have not (sufficiently) discussed