Berlin, QCD, October 2009

M. Shifman
W.I. Fine Theoretical Physics Institute, University of Minnesota

Two most salient features of QCD:

(i) Quark Confinement;

(ii) Chiral Symmetry Breaking ($\chi$ SB)

and its consequences

# QCD is extremely rich:

- ☆ Nuclear Physics
  - ☆ Regge behavior
    - ☆ QGM: high-T/high $\mu$ (neutron stars)
      - ☆ Richness of the hadronic world:

- ☆ chiral;
- ☆ light & heavy quarkonia;
- ☆ glueballs & exotics;
- ☆ exclusive & inclusive phenomena;
- ☆ interplay between strong forces & weak interactions...

That's why I do not expect **FULL** analytic solution to QCD to be found

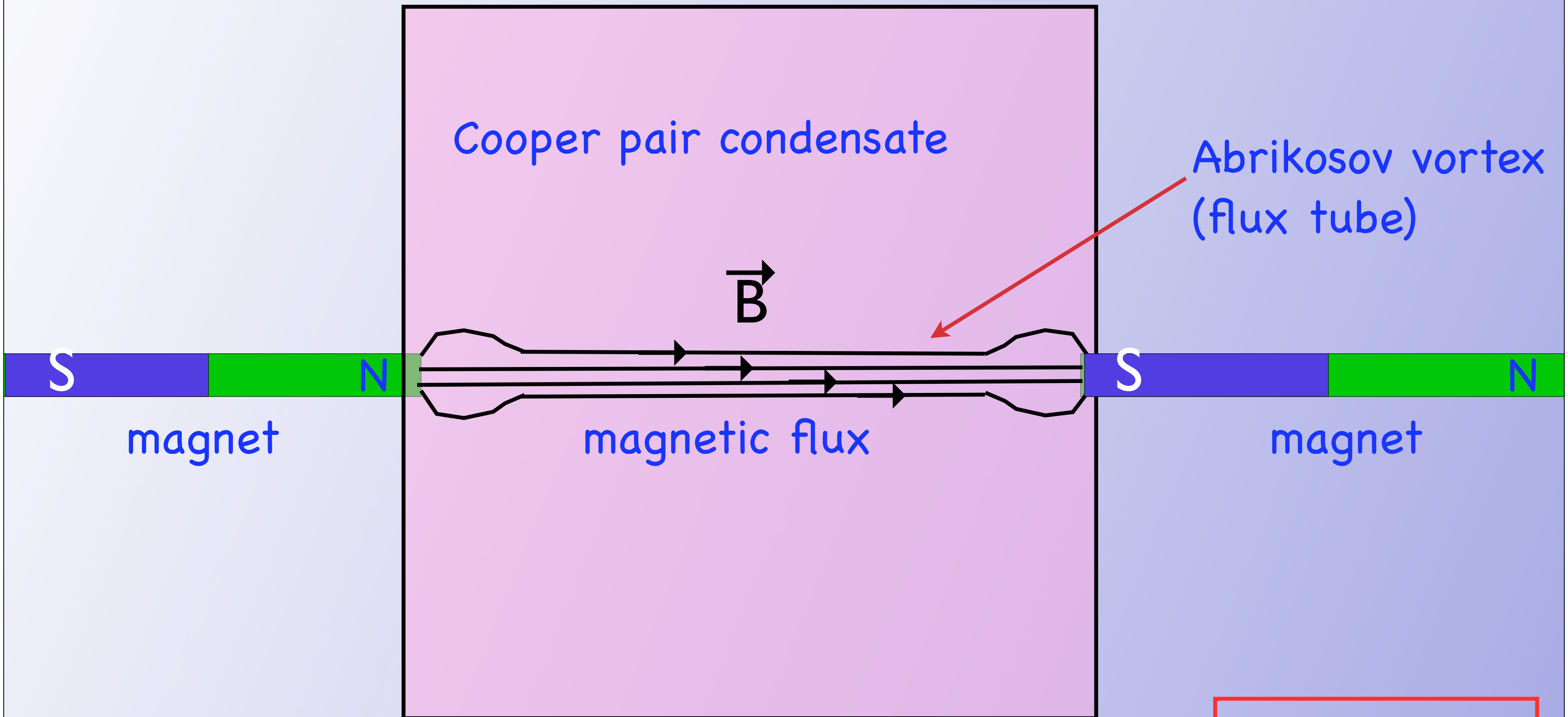
☆ BUT ... Advances are enormous!

The reference point: 1992 Aachen Conference “QCD, 20 Years Later”

Nothing of what I am going to discuss today was known then!

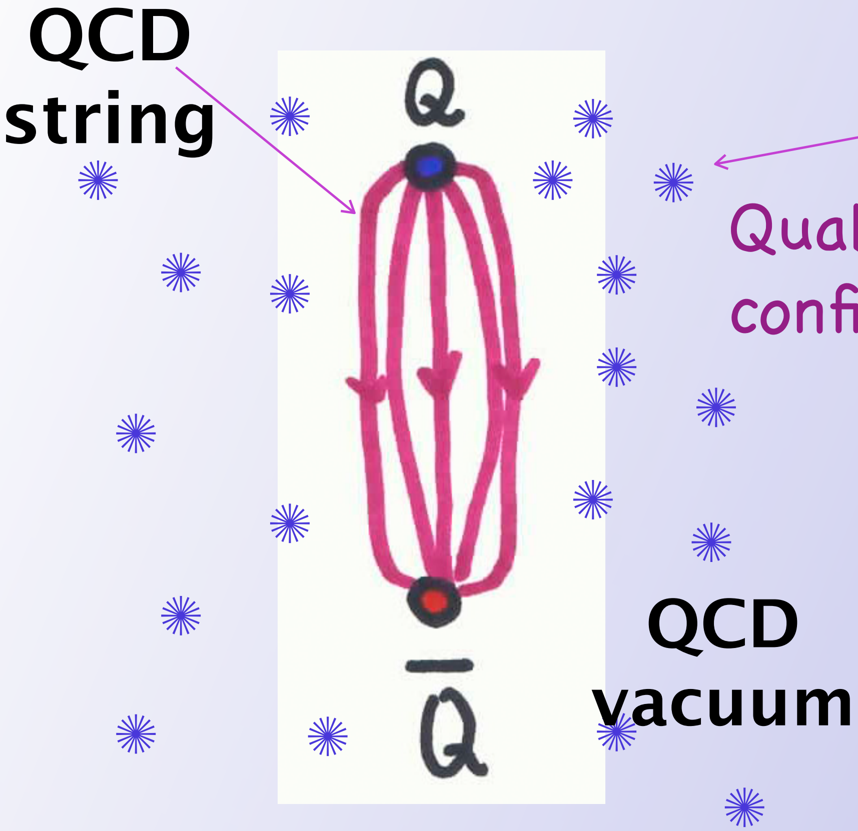
☞ Flux tubes: Are we aware of precedents?

☞ Yes, the Meissner effect! 1930s, 1960s




Superconductor of the 2nd kind

Abelian ☛




## Qualitative explanation of color confinement: Dual Meissner effect:

- 't Hooft, 1976
- Mandelstam, 1975
- Nambu, 1974

★ "...[monopoles] turn to develop a non-zero vacuum expectation value. Since they carry color-magnetic charges, the vacuum will behave like a superconductor for color-magnetic charges. What does that mean? Remember that in ordinary electric superconductors, magnetic charges are connected by magnetic vortex lines ... We now have the opposite: it is the color charges that are connected by electric flux tubes."
G. 't Hooft (1976)

- Seiberg & Witten, 1994, in N=2 SUSY confirmed monopole condensation and Dual Meissner effect!!!

  - SU(2) Gluons $A^a_\mu$, Dirac gluino in adjoint $\psi^a$, & adjoint complex scalar $a^a$;

    **In the vacuum $\langle a^a \rangle = v$**

    $SU(2) \to U(1)$

    - 't Hooft-Polyakov monopoles (heavy→light)

Duality & the power of N=2 SUSY

Electric charge ↔ Magnetic monopoles

Dual Meissner effect!

☆ Non-Abelian theory, but Abelian flux tube

☹

Extra scalar adjoint, NO $\chi$ SB !
(Casher's argument trashed)

☹

- Non-Abelian Strings, Confined monopoles

# ✱✱✱ Non-Abelian Strings ✱✱✱

- Abrikosov-Nielsen-Olesen string: Abelian
  Gauge group = U(1);
  Electric charge condenses;
  Magnetic flux is trapped in a tube and quantized;
  No internal degrees of freedom besides position of the tube center (e.g. Seiberg-Witten solution ⇒ ANO string)
- <u>Non-Abelian strings:</u> Assume the BULK theory has a global symmetry G unbroken in the vacuum.
  Assume G→H on the string;

→ Orientational moduli on the string world sheet;

→ G/H coset sigma model in 1+1 dimensions

Hanany-Tong; Auzzi et al.; Shifman-Yung

Step 1:

start from a simple Non-SUSY model: with 2 scalar quarks

Toy model? Yes, we can!

→ $U(2)$ gauge group, $SU(2)$ flavor

→ Scalar "quarks" in the fundamental (i.e. doublets)

→ 2 flavors $\varphi^A_i$ $(A=1,2;\ i=1,2)$

$$L = -\frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 - \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + \sum_{A=1,2}\left(\mathcal{D}_\mu\varphi^A\right)^*\left(\mathcal{D}^\mu\varphi^A\right) + \frac{g_2^2}{2}\left(\phi^*_A\,\frac{\tau^a}{2}\,\phi^A\right)^2 + \frac{g_1^2}{8}\left[\varphi^*_A\phi^A - 2v^2\right]^2$$

$$\phi = (\phi^1, \phi^2) = \begin{pmatrix} \phi^{11} & \phi^{12} \\ \phi^{21} & \phi^{22} \end{pmatrix}$$

In the vacuum

$$\Phi = v \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$SU(2)_{gauge} \rightarrow$ broken

$SU(2)_{flavor} \rightarrow$ broken

Global $SU(2)_{C+F}$ UNBROKEN

# String solutions $\underbrace{\pi_1(SU(2) \times U(1))}_{\text{Trivial or NOT?}}$

$$2\pi \text{ winding} = \underset{\substack{\uparrow \\ U(1)}}{\pi} + \underset{\substack{\uparrow \\ \tau_3 \text{ from } SU(2) \\ \underline{\text{Center}}}}{\pi}$$

$$\begin{bmatrix} \varphi^{11} e^{i(\alpha+\beta)} & 0 \\ 0 & \varphi^{22} e^{-i\alpha+i\beta} \end{bmatrix}$$

On the string

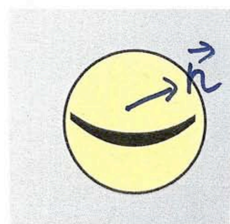
$$SU(2)_{C+F} \to U(1)$$

(rotations around $\tau_3$)

Hence the collective coordinates $SU(2)/U(1) = S_2$

$$\vec{n} = (n_1, n_2, n_3)$$

$$\vec{n}^2 = 1$$




$$S = \int d^2x \; \partial_\mu \vec{n} \, \partial^\mu \vec{n}$$

ASY free

Strongly coupled

Instantons, ....

Classically $\langle \vec{n} \rangle = \uparrow$. QM $\vec{n}$ wildly oscillates

In the bulk electric charges condense, theory weakly coupled.

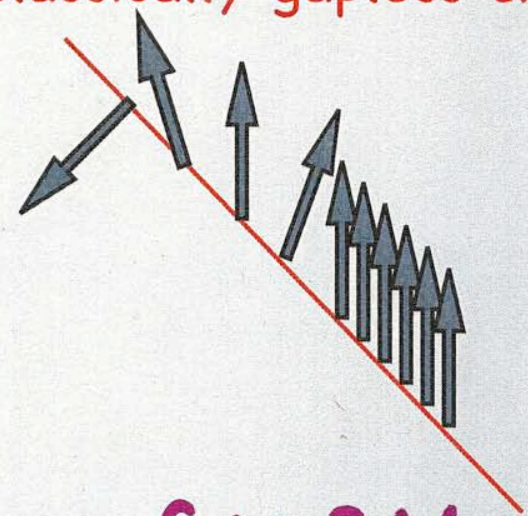
On the string strongly coupled

Add SUSY.

2 vacua + kink

# SU(2)/U(1) = CP(1)~O(3) sigma model

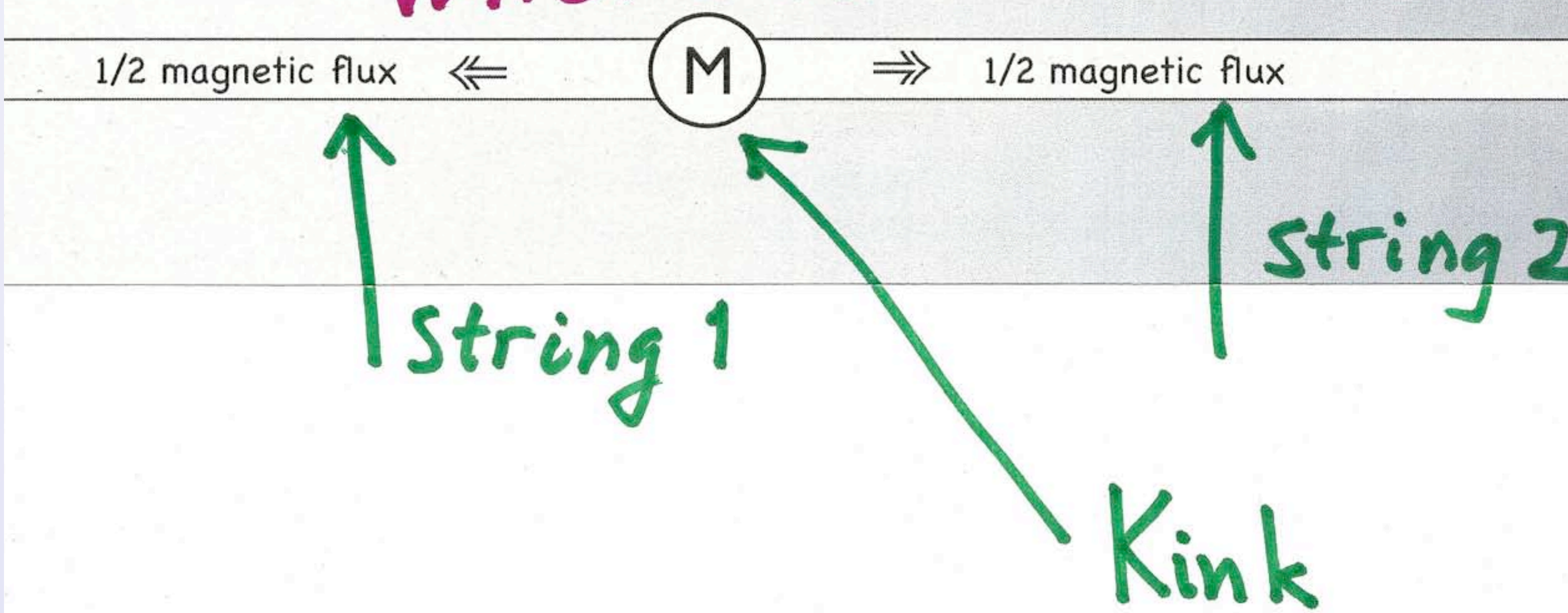
classically gapless excitation



$g^2$ of the bulk theory is matched by $g^2$ of the 2D sigma model, and so do $\Lambda$'s; 2D theory gets strongly coupled; mass gap generated; 2 vacua.

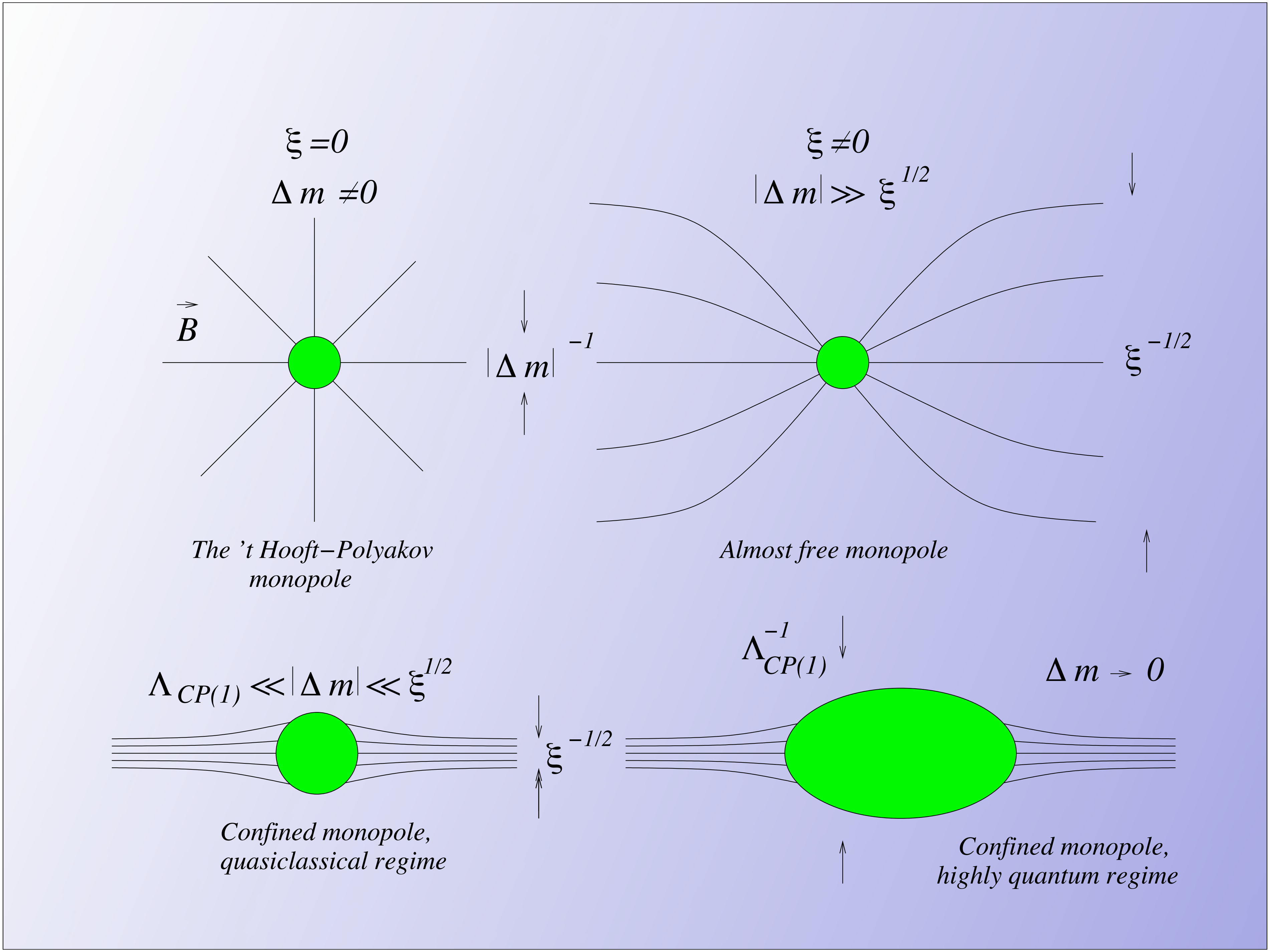
Kink = trapped monopole

Where is SUSY ?



Kink = Confined Monopole

Why ?


$\xi = 0$
$\Delta m \neq 0$
$\vec{B}$
$|\Delta m|^{-1}$
The 't Hooft−Polyakov monopole
$\xi \neq 0$
$|\Delta m| \gg \xi^{1/2}$
$\xi^{-1/2}$
Almost free monopole
$\Lambda_{CP(1)} \ll |\Delta m| \ll \xi^{1/2}$
$\xi^{-1/2}$
Confined monopole, quasiclassical regime
$\Lambda^{-1}_{CP(1)}$
$\Delta m \to 0$
Confined monopole, highly quantum regime

# Conclusions/projections

SUSY-based methods will proliferate

Closer relatives of QCD

Aspects of QCD per se

Other str-coupl. theories/cond.matter

Gap between "understood" and "realistic" gauge theories will narrow

Combination of SUSY and 1/N (or $g_{st}$) will become a quantitative tool

If time permits ...

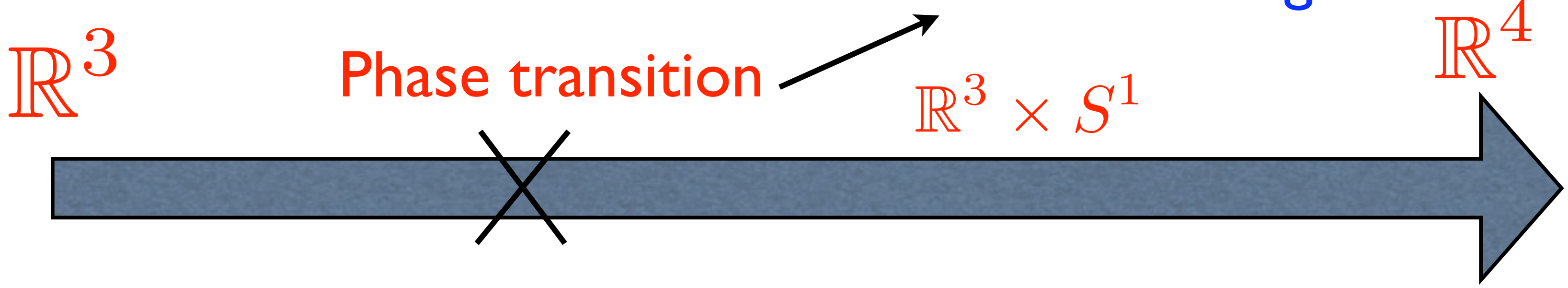
# A quick overview of main idea

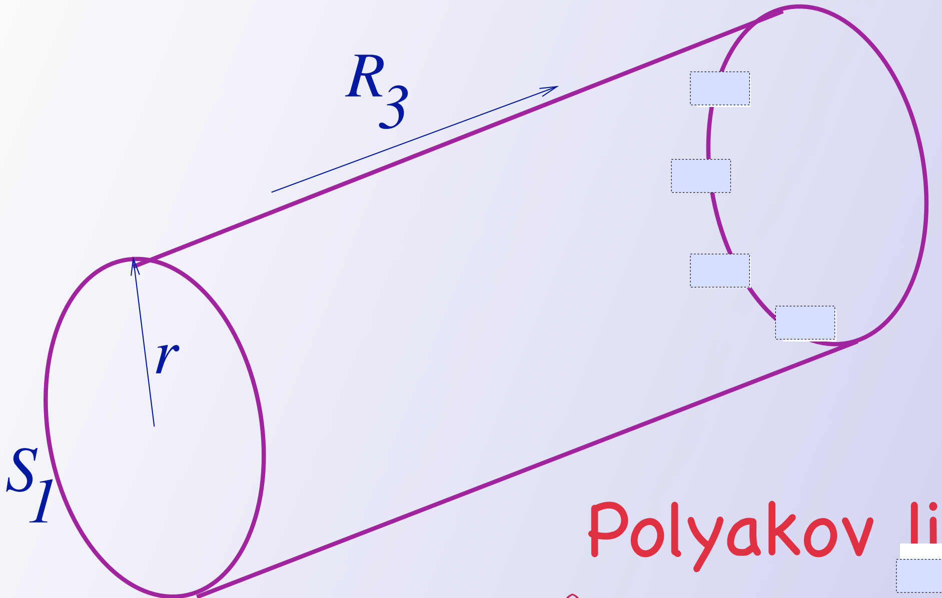
Theories on $\mathbb{R}^4$ (target theories , but hard)

Keep locally d=4 such as $\mathbb{R}^3 \times S^1$

Take advantage of circle (as control parameter).
Small circle, AF and weak coupling:

Traditional: thermal setting. Bad for our goal.






Polyakov line = $P \exp\{i\int dz\, A_z\} \equiv$ **U**

- Polyakov's criterion of confinement:

  Tr< **U** > = 0 (large r).

  Eigenvalues wildly fluctuate, the $Z_N$ center symmetry is unbroken.

  At small r, typically Tr< **U** >/N ≈ 1.

Center symmetry is spontaneously broken

Phase transition(s) at r ~ 1/Λ

## Double-trace deformations at small r :

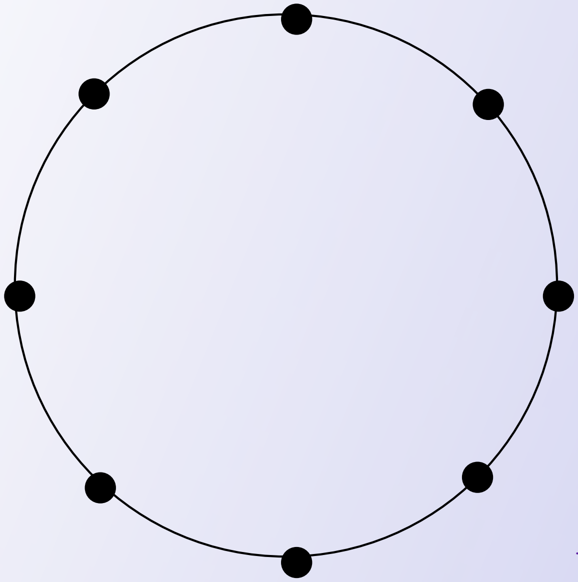
- stabilizes center symmetric vacuum;
- launches gauge symmetry Higgsing
  SU(N) → U(1)$^{N-1}$
- the latter makes the theory weakly coupled if r << 1/Λ.

$$P[U(\vec{x})] = \frac{2}{\pi^2 L^4} \sum_{n=1}^{[N/2]} d_n |TrU^n(\vec{x})|^2$$

eigenvalue repulsion

$$\Delta S = \int_{R_3} d^3x\, L\, P[U(x)]$$

# Eigenvalues of U at small r



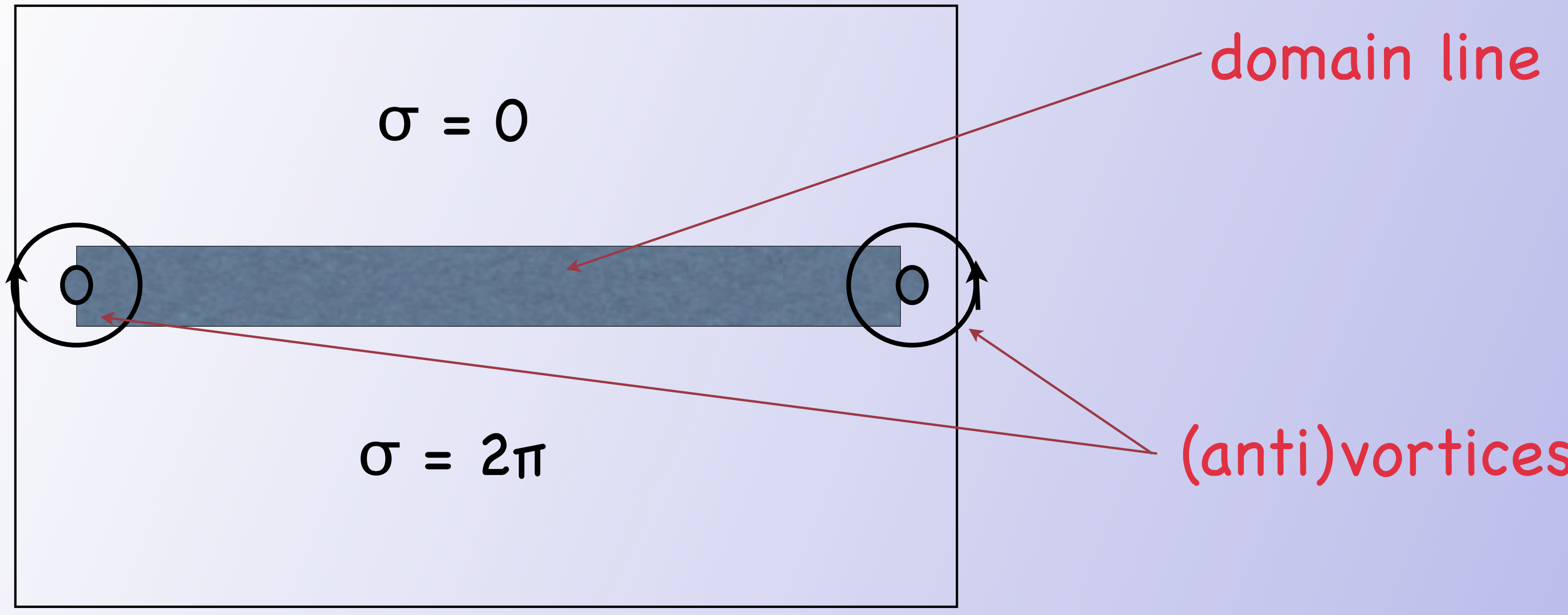
Polyakov confinement

ÜS 1-flavor chiral SB

$$\{La_i\} = \{-iL\ln v_i \;(mod\; 2\pi)\}$$

$$= \left\{-\frac{2\pi[N/2]}{N},\, -\frac{2\pi([N/2]-1)}{N},\, ....,\, \frac{2\pi[N/2]}{N}\right\}$$

The theory is Higgsed; W bosons are heavy, photons massless!




Domain line = Polyakov string; tension exponentially small; thickness exponentially large

✽ If massless quarks are switched on, fermion zero modes destroy the mass term for $\sigma$ and domain lines disappear. No Polyakov confinement in 3D with massless quarks!

# In 3D instanton≡monopole !!!! 'tH-P monopole & KK monopole

| | | |
|---|---|---|
| tH-P | magnetic 1 | topological 1/2 |
| KK | magnetic -1 | topological 1/2 |

**Instantole?**

## Long-distance 3d dual theory

$$S^{\mathrm{dual}} = \int_{\mathbb{R}^3} \Big[ \underbrace{\frac{1}{2L}\left(\frac{g}{2\pi}\right)^2 (\nabla\sigma)^2}_{\text{Maxwell term}} - \zeta \sum_{i=1}^{N} \underbrace{\cos(\alpha_i \cdot \sigma)}_{\text{Monopole Operator}} \Big] .$$

Abelian duality

$$\boxed{F_{\mu\nu}^{(j)} = \frac{g^2}{2\pi L}\,\epsilon_{\mu\nu\rho}\,\partial_\rho \sigma^j}$$

monopole due to
compactness of Higgs scalar

Lee, Yi, Kraan, vanBaal

Monopole charges $\Delta^0_{\mathrm{aff}} \equiv \{\underbrace{\alpha_1, \alpha_2, \dots, \alpha_{N-1}}_{\text{usual N-1 monopoles}}, \overline{\alpha_N}\}$ .

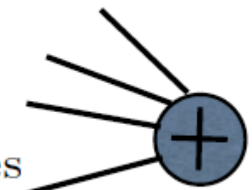
# Discrete Chiral Symmetry Breaking

✽ One flavor $\psi^{[ij]}$ or $\psi^{\{ij\}}$ or chiral e.g. SU(2) $\psi^{\{ijk\}}$

**Polyakov's confinement gone, $\chi$ SB occurs**

monopole operator has fermionic zero modes.

$$e^{-S_0} e^{i\sigma} \quad \underbrace{\psi \ldots \psi}_{\text{fermion zero modes}}$$

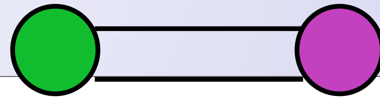


Hence, cannot generate confinement and mass gap.

2 AS / 2 S / 4 & 6 chiral

They DO generate chiral condensates $\langle\psi^2\rangle$, $\langle\psi^4\rangle$, $\psi^6$ at $e^{-S_0}$ level

In the nonchiral theories with $\psi^{[ij]}$ or $\psi^{\{ij\}}$ confining potential is due to Bions at the $e^{-2S_0}$ level.



tH-P $\overline{\text{KK}}$

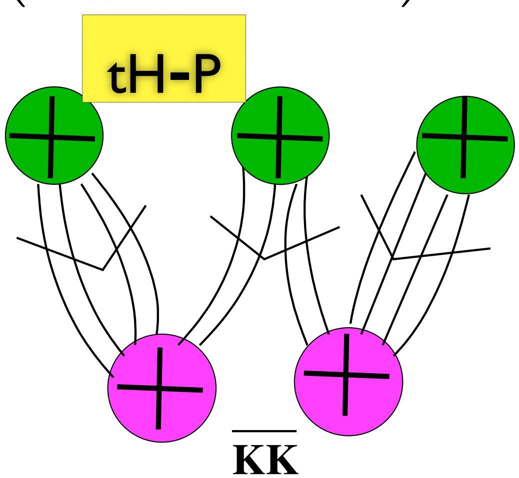
**chiral →**

In the absence of fermion zero modes, the constituents of the magnetic quintet interact repulsively.

$$[\mathcal{M}_1]^3[\overline{\mathcal{M}_2}]^2 \equiv [\text{tH-P}]^3[\overline{\text{KK}}]^2$$

$$\left(\int_{S^2_\infty} B, \quad \int F\widetilde{F}\right) = \left(\pm 5, \quad \pm\frac{1}{2}\right)$$

$e^{-5S_0}$ level




"The magnetic quintet" leading topological excitation that leads to confinement in non-susy chiral theory. (Testable on lattice)

# Conclusions:

- At small $r(S_1)$ nonperturbative dynamics is controllabe; various instanton/monopole molecules: monopoles, bions, , quintets, rings (classified by $\exp(-S_0)$)

- At small $r(S_1)$ we find linear confinement and discrete $\chi$SB in QCD-like and chiral theories;

**No phase transitions expected in passing to large $r(S_1)$**