Non-perturbative Heavy Quark Effective Theory

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NIC, DESY, A Research Centre of the Helmholtz Association

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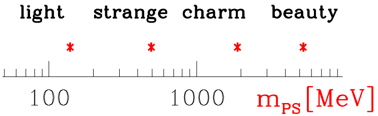
  HQET is a great help

Lattice QCD is a challenge

multiple scale problem
always difficult
for a numerical treatment



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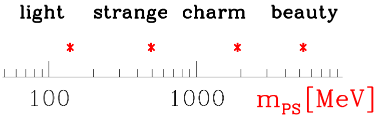

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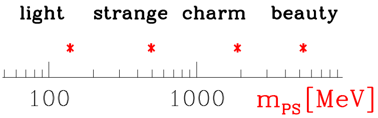

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$$\Lambda_{\mathrm{UV}} = a^{-1}$$
$$\Lambda_{\mathrm{IR}} = L^{-1}$$

$$L^{-1} \ll m_\pi\,,\ \ldots\ , m_{\mathrm{D}}\,, m_{\mathrm{B}} \ll a^{-1}$$

$$\mathrm{O}(\mathrm{e}^{-Lm_\pi}) \qquad\qquad m_{\mathrm{D}}\, a \lesssim 1/2$$

$$\downarrow \qquad\qquad \downarrow$$

$$L \gtrsim 4/m_\pi \sim 6\,\mathrm{fm} \qquad\qquad a \approx 0.05\,\mathrm{fm}$$

$$L/a \gtrsim 120$$

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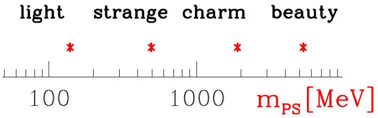

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lattice cutoffs:

$$\begin{aligned} \Lambda_{\rm UV} &= a^{-1} \\ \Lambda_{\rm IR} &= L^{-1} \end{aligned}$$

$$\begin{array}{ccc} L^{-1} & \ll \; m_\pi \,, \;\ldots\;, m_{\rm D}\,, m_{\rm B} \; \ll & a^{-1} \\ {\rm O}({\rm e}^{-L m_\pi}) & & m_{\rm D}\, a \lesssim 1/2 \\ \downarrow & & \downarrow \\ L \gtrsim 4/m_\pi \sim 6\,{\rm fm} & & a \approx 0.05\,{\rm fm} \end{array}$$

$$L/a \gtrsim 120$$

beauty not accomodated: need an effective theory, $\Lambda_{\rm QCD}/m_{\rm b}$ expansion

Mass dependence in QCD

heavy light current $A_\mu(x) = \overline{\psi}_{\rm b}(x)\gamma_\mu\gamma_5\psi_{\rm l}(x)$

matrix element $\Phi(m_{\rm b}) = \langle\beta,{\rm b}|A_\mu(x)|\alpha\rangle$

(non-relativistic, mass independent normalization of states)

Interested in the behaviour at large $m = m_{\rm b}$; no other large scale.

some massless scheme:

$(\overline{m}(\mu),\ \bar{g}(\mu))$ $\quad \mu\frac{\partial\bar{g}}{\partial\mu} = \beta(\bar{g})\ ,\quad \frac{\mu}{\overline{m}}\frac{\partial\overline{m}}{\partial\mu} = \tau(\bar{g})$

fix the scale: $\quad \mu = m_\star = \overline{m}(m_\star)\,,\quad g_\star = \bar{g}(m_\star)\,,$

mass dependence

$$\frac{m_\star}{\Phi}\frac{\partial\Phi}{\partial m_\star} = \gamma^{\Phi}_{\rm match}(g_\star) \overset{m_\star\to\infty}{\sim} -g_\star^2 \underbrace{\gamma_0}_{=-1/(4\pi^2)} + {\rm O}(g_\star^4)$$

[Shifman & Voloshin]

"factorization", effective theory:

$$\gamma^{\Phi}_{\rm match}(g_\star) = \underbrace{\gamma_{\rm match}(g_\star)}_{\text{universal for all ME}} + {\rm O}(\Lambda/m_\star)$$

Mass dependence in QCD

$\frac{m_\star}{\Phi}\frac{\partial\Phi}{\partial m_\star} = \gamma_{\rm match}(g_\star) + \mathrm{O}(\Lambda/m_\star)$ depends on the scheme

$\to$ switch to RGI's Λ, M

$$\Lambda = m_\star \exp\left\{-\int^{g_\star} \mathrm{d}x \frac{1}{\beta(x)}\right\}, \qquad M = m_\star \exp\left\{-\int^{g_\star} \mathrm{d}x \frac{\tau(x)}{\beta(x)}\right\},$$

then

$$\frac{M}{\Phi}\frac{\partial\Phi}{\partial M} = \gamma_{A,M}(M/\Lambda) + \mathrm{O}(\Lambda/M)$$

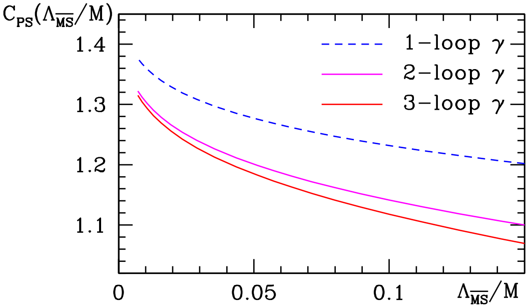
$$\gamma_{A,M}(M/\Lambda) = \frac{\gamma_{\rm match}(g_\star(M/\Lambda))}{1-\tau(g_\star(M/\Lambda))}$$

Integrate:

$$\Phi(M,\Lambda) = C_{\rm PS}(M/\Lambda)\,\Phi_{\rm RGI} + \mathrm{O}(\Lambda/M), \qquad C_{\rm PS} = \exp\left\{\int^{g_\star(M/\Lambda)} \mathrm{d}x \frac{\gamma_{A,M}(x)}{\beta(x)}\right\}$$

$\Phi_{\rm RGI}$ unambiguous, computable in the effective theory, mass-independent

Perturbative conversion functions



B-physics: $\Lambda_{\overline{\rm MS}}/M_{\rm b} \approx 0.04$

$\gamma_0 = -1/(4\pi^2)$

[Shifman & Voloshin, 87; Politzer & Wise, 88]

γ_1 from

[Ji & Musolf, 91; Broadhurst & Grozin, 91; Gimenez, 92]

γ_2 from

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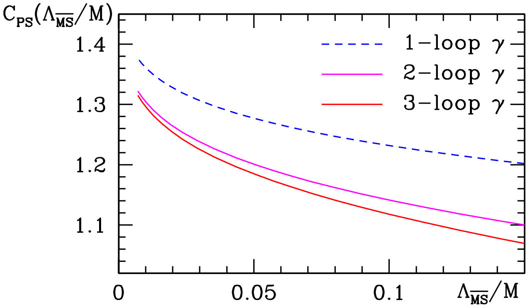

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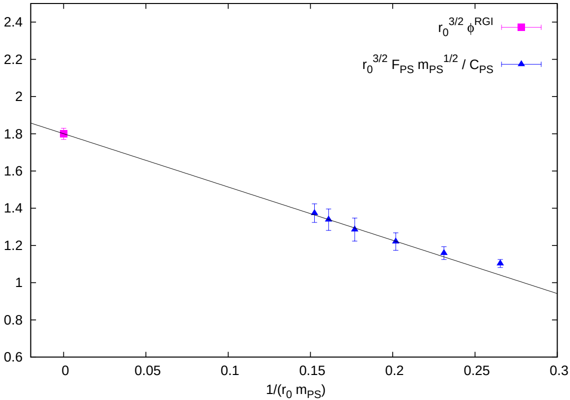
B-physics: $\Lambda_{\overline{\mathrm{MS}}}/M_{\mathrm{b}} \approx 0.04$

For a large range of $1/m$ the functions are almost linear!

Application of perturbative conversion functions



Example of an interpolation between a static result and results with $m_{\rm h} < m_{\rm b}$.
Continuum extrapolations are done before the interpolation. The point at $1/r_0 m_{\rm PS} = 0$ is given by $r_0^{3/2}\Phi_{\rm RGI}$.
This quenched computation is done for validating and demonstrating the applicability of HQET.

The effective theory: HQET

[Eichten; Isgur & Wise; Georgi]

QCD: $$\mathcal{L}_{\rm QCD} = -\frac{1}{2g_0^2}\,\mathrm{tr}\,\{F_{\mu\nu}F_{\mu\nu}\} + \sum_f \overline{\psi}_f [D_\mu\gamma_\mu + m_f]\psi_f$$

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HQET: in the rest frame of a B-meson

$$\begin{aligned}
\overline{\psi}_{\rm b}[D_\mu\gamma_\mu + m_{\rm b}]\psi_{\rm b} \quad &\rightarrow \quad \mathcal{L}_{\rm stat} + \mathcal{L}^{(1)} + \mathrm{O}(1/m_{\rm b}^2)\,, \\
\mathcal{L}_{\rm stat} \quad &= \quad \overline{\psi}_{\rm h}\, D_0\, \psi_{\rm h} \\
\tfrac{1}{2}(1+\gamma_0)\psi_{\rm h} \quad &= \quad \psi_{\rm h}\,, \qquad \text{“large” components} \\
\mathcal{L}^{(1)} \quad &= \quad \frac{1}{2m_{\rm b}}\overline{\psi}_{\rm h}(-\sigma\cdot\mathbf{B} - \tfrac{1}{2}\mathbf{D}^2)\psi_{\rm h}
\end{aligned}$$

easily derived for smooth (classical) fields (FWT trafo)

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easily derived for smooth (classical) fields (FWT trafo)

(*) and $E_{\mathrm{QCD}} = E^{\mathrm{stat}} + m_{\mathrm{b}}$ (universal energy shift)
Similarly: $1/m_{\mathrm{b}}$-terms for composite fields, e.g.

$$A_0^{\mathrm{HQET}} = Z_{\mathrm{A}}^{\mathrm{HQET}}\{\overline{\psi}_{\mathrm{l}}\gamma_0\gamma_5\psi_{\mathrm{h}} + c_{\mathrm{A}}^{\mathrm{HQET}}\overline{\psi}_{\mathrm{l}}\gamma_k\overleftarrow{D}_k\gamma_5\psi_{\mathrm{h}}\}$$

Remarks on the effective theory

- "Derivation" is essentially classical
  high momentum fluctuations assumed to produce only local terms
- Renormalizable by looking at engineering dimensions of the terms in the Lagrangian $\quad \rightarrow \quad$ continuum limit exists
  iff $1/m_{\mathrm{b}}$-terms are treated as insertions
- Lattice: static Lagrangian is automatically $\mathrm{O}(a)$ improved [M. Kurth & R.S., 2001]
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- Verification of these properties:
  1 loop PT (lattice reg.) [Eichten & Hill, 90; Boucaud et al; Flynn & Hill, 91; M. Kurth & R.S., 01]
  1-3 loop (dim.reg.) renormalization [Eichten & Hill, 90; Flynn & Hill, 91; Ji & Musolf, 91; Broadhurst & Grozin, 91; Gimenez, 92, Chetyrkin & Grozin, 2003; ...]
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- Lattice: precise computations are possible ...

... precise lattice computations are possible

after some developments

- a suitable choice of action [Della Morte, Shindler, S, 2005]
- "all-to-all" propagators [Foley, Juge, O'Cais, Peardon, Ryan, Skullerud, 05]
- GEVP method [Blossier, Della Morte, von Hippel, Mendes, S, 2008]

Generalized EigenValue Problem

matrix of correlation functions on an infinite T, finite L, lattice

$$C_{ij}(t) \;=\; \langle O_i(0) O_j(t) \rangle = \sum_{n=1}^{\infty} \mathrm{e}^{-E_n t} \psi_{ni} \psi_{nj} \,, \quad i,j = 1,\ldots,N$$

$$\psi_{ni} \equiv (\psi_n)_i = \langle n | \hat{O}_i | 0 \rangle = \psi_{ni}^* \quad E_n \leq E_{n+1}$$

the GEVP is

$$C(t)\, v_n(t,t_0) = \lambda_n(t,t_0)\, C(t_0)\, v_n(t,t_0) \,, \quad n = 1,\ldots,N \quad t > t_0,$$

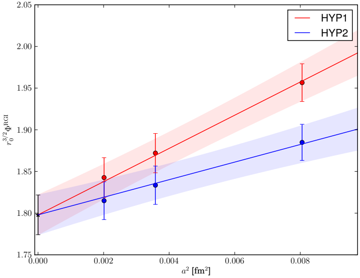
prooven that λ_n and v_n can be combined to make a creation operator for the state $|n\rangle$ [Blossier, Della Morte, von Hippel, Mendes, S, 2008]

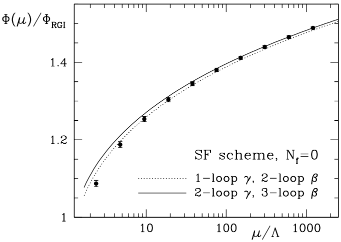
$$A_n(t_0) = \sum_i (t_0) c_i O_i(t_0) \,, \quad A_n(t_0)|0\rangle = |n\rangle + \mathrm{O}(\mathrm{e}^{-(E_{N+1}-E_n)\, t_0})$$

fast convergence for the ground state: $(\mathrm{O}(\mathrm{e}^{-(E_{N+1}-E_1)\, t_0}))$
access to excited state matrix elements!

The static B-meson decay constant: $\Phi = F_{\rm B}\sqrt{m_{\rm B}}$

in lowest order of HQET (static)





[Heitger, Kurth, S.]

[Blossier et al., in prep.]

$$\Phi_{\rm RGI} = \lim_{\mu\to\infty} [2b_0\bar{g}^2(\mu)]^{-\gamma_0/2b_0}\Phi_{\rm R}(\mu)$$

$$= \frac{Z^{\rm stat}_{\rm A,RGI}}{Z^{\rm stat}_{\rm A}(\mu,0)} \lim_{a\to 0}\left[Z^{\rm stat}_{\rm A}(\mu,a) \qquad \underbrace{\Phi(a)}_{\text{bare decay constant}} \right]$$

$1/m$ corrections

$$F_{\rm B}\sqrt{m_{\rm B}} = \Phi(M_{\rm b}/\Lambda) = \underbrace{C_{\rm PS}(M_{\rm b}/\Lambda)}_{\Uparrow} \times \Phi_{\rm RGI} \quad + \quad \underbrace{{\rm O}(1/m_{\rm b})}_{\Uparrow}$$

$$\text{in PT: error} \sim \alpha(m_{\rm b})^n \sim \left\{ \frac{1}{2b_0 \ln(m_{\rm b}/\Lambda)} \right\}^n \quad \overset{m_{\rm b}\to\infty}{\gg} \quad \frac{\Lambda}{m_{\rm b}}$$

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- general problem: clean computation of power corrections needs full non-perturbative treatment of the leading term

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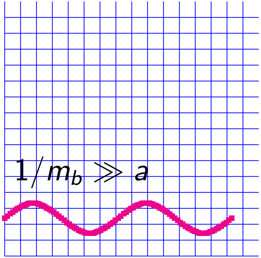
- general problem: clean computation of power corrections needs full non-perturbative treatment of the leading term
- in addition, on the lattice: $1/a^n$ power divergences need to be removed non-perturbatively (otherwise $a \to 0$ does not exist)
- do everything on the lattice, including matching – renormalization factor of axial curent
  – coefficients in the Lagrangian, e.g. $\omega_{\rm spin}\,\overline{\psi}_{\rm h}\sigma\cdot\mathbf{B}\psi_{\rm h}$

Non-perturbative matching of HQET and QCD [Heitger, S., 2001]

- The trick: start in small volume, $L = L_1 \approx 0.4\,\mathrm{fm}\,, \quad a = 0.01\,\mathrm{fm}$

Φ_k finite volume masses, decay constants ...

QCD

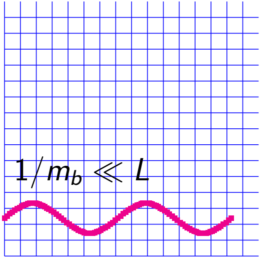


$$\boxed{\Phi_k^{\mathrm{QCD}} = \Phi_k^{\mathrm{HQET}}}$$

$k = 1, 2, \ldots, N_{\mathrm{HQET}}$

$N_{\mathrm{HQET}} =$
$\#$ of parameters

HQET



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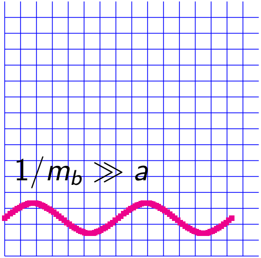

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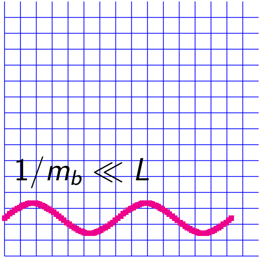


$\rightarrow$ HQET-parameters from QCD-observables in small volume

– at small lattice spacing $\quad L^{-1} \ll m_{\mathrm{b}} \ll a^{-1}$

power divergences subtracted non-perturbatively

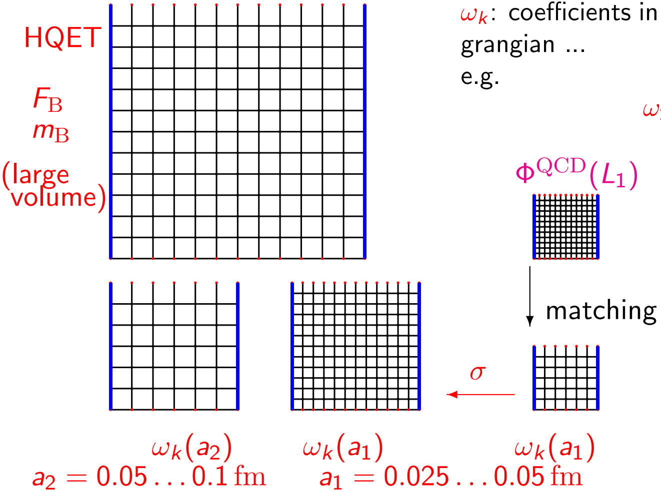
The HQET strategy: first view

[Heitger, S., 2001]

ω_k: coefficients in effective Lagrangian ...

e.g. $\omega_2\, \bar{b}\, \sigma \cdot \mathbf{B}\, b$

$\omega_2 \sim 1/(2m_\mathrm{b})$



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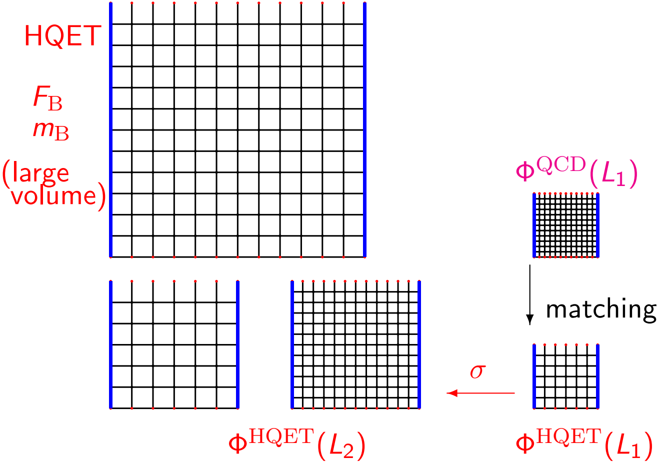

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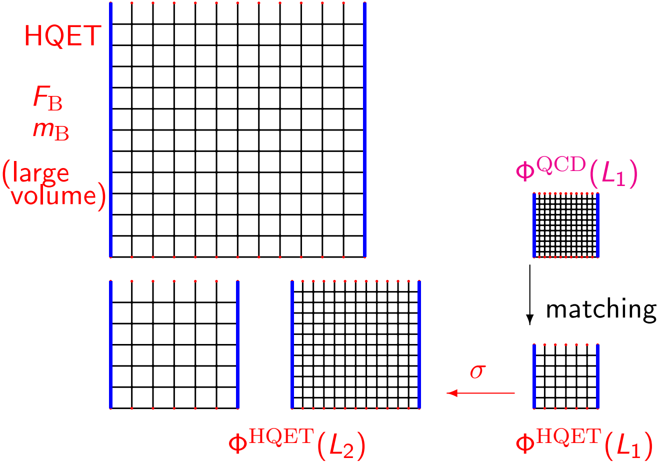
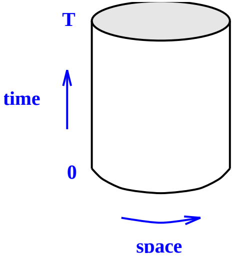


- continuum limit can be taken in all steps

Schrödinger functional toolbox for finite volume

$T \times L^3$, Euclidean
Dirichlet boundary conditions in time

[Lüscher, Narayanan, Weisz, Wolff, 92; Sint, 93; ALPHA Collaboration, 92-...]



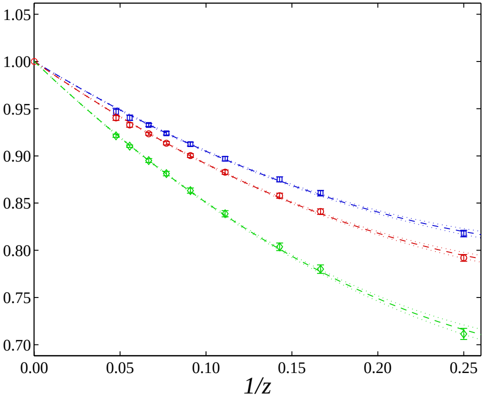
- no zero-modes
- boundary fields for gauge invariant quark correlation functions
- running coupling ...

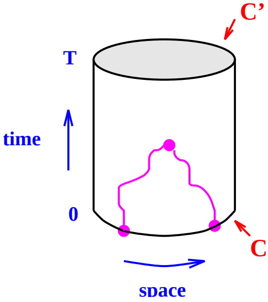
Tests of HQET

An example with $N_\mathrm{f} = 2$ dynamical fermions is [ALPHA Collaboration, 2008]

$$R_{AV} = \frac{-f_\mathrm{A}(L/2)}{k_\mathrm{V}(L/2)} = C_\mathrm{PS}/C_\mathrm{V} + \mathrm{O}(1/z) \qquad z = ML$$

$N_\mathrm{f} = 2,\ L \approx 0.5\,\mathrm{fm}$

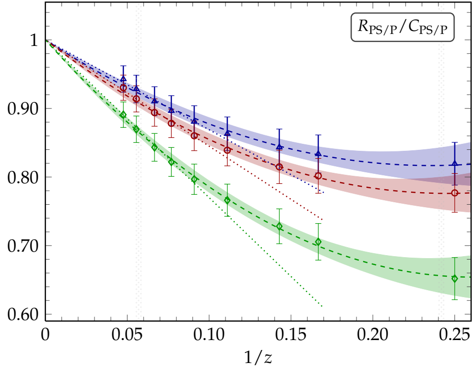


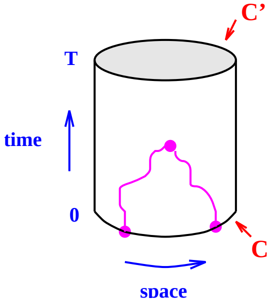


Tests of HQET

$$R_{AP} = \frac{-f_{\rm A}(L/2)}{f_{\rm P}(L/2)} = C_{\rm PS}/C_{\rm P} + {\rm O}(1/z)$$

The ratio R_{AP} from Patrick Fritzsch with $N_{\rm f} = 2$.





Tests of HQET

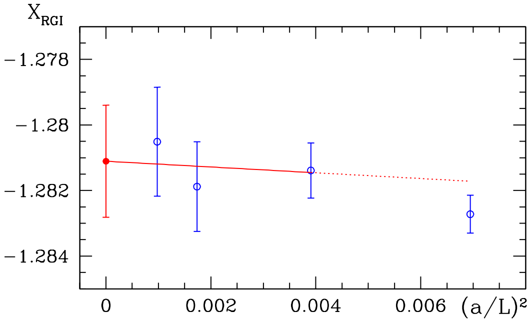
[Heitger, Jüttner, S & Wennekers, '04]

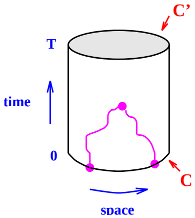
axial current matrix element, quenched

$$Y_{\rm PS}(L,M_{\rm b})/C_{\rm PS}(M_{\rm b}/\Lambda) = X_{\rm RGI} + {\rm O}(1/z)\,, \quad z = M_{\rm b}L\,,$$

Continuum extrapolation of X_{RGI} (static approx.)

quenched, $m_{\rm l} = 0$, $L \approx 0.2$ fm



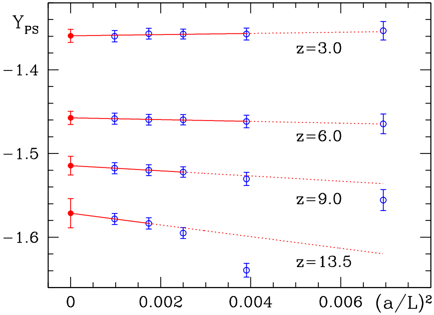


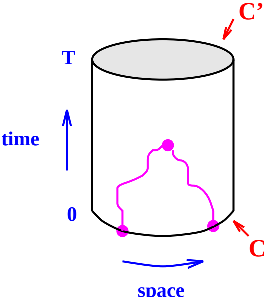
Tests of HQET

$$Y_{\mathrm{PS}}(L, M_{\mathrm{b}})/C_{\mathrm{PS}}(M_{\mathrm{b}}/\Lambda) = X_{\mathrm{RGI}} + \mathrm{O}(1/z)\,, \quad z = M_{\mathrm{b}}L\,,$$

Continuum extrapolation of Y_{RGI} (QCD)

quenched, $m_{\mathrm{l}} = 0$, $L \approx 0.2$ fm





Tests of HQET

$$Y_{\rm PS}(L, M_{\rm b})/C_{\rm PS}(M_{\rm b}/\Lambda) = X_{\rm RGI} + {\rm O}(1/z)\,, \quad z = M_{\rm b} L\,,$$

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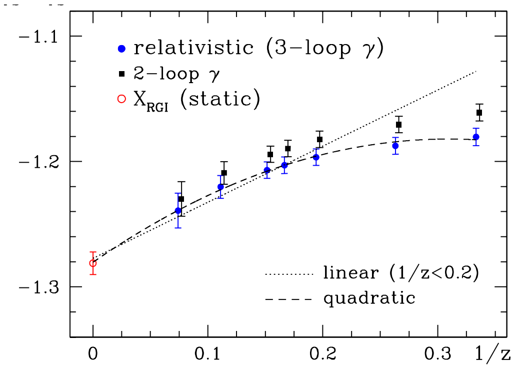
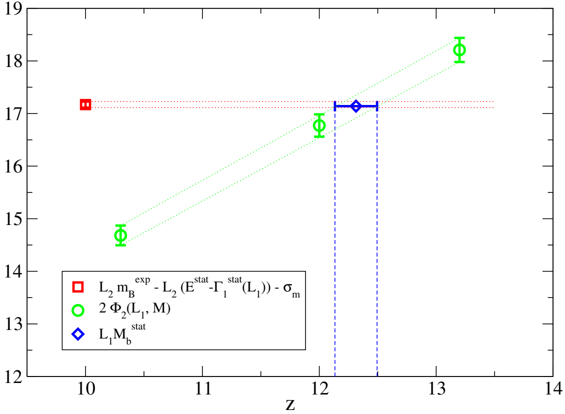


Large volume quenched results

... test HQET strategy, see achievable precision
... not for phenomenology

$$m_{\rm B} = \lim_{a\to 0}[E^{\rm stat} - \Gamma^{\rm stat}(L_2,a)] + \lim_{a\to 0}[\Gamma^{\rm stat}(L_2,a) - \Gamma^{\rm stat}(L_1,a)] + \frac{1}{L_1}\lim_{a\to 0}\Phi_1(L_1,M_{\rm b},a)$$

After continuum extrapolations



Quenched results

	$r_0 M_{\rm b}^{(0)}$	$r_0(M_{\rm b}^{(0)} + M_{\rm b}^{(1)})$		
		$(\theta_1, \theta_2) = (0, 0.5)$	$(\theta_1, \theta_2) = (0.5, 1)$	$(\theta_1, \theta_2) = (1, 0)$
$\theta_0 = 0$	17.15 ± 0.25	17.45 ± 0.26	17.45 ± 0.26	17.45 ± 0.26
$\theta_0 = 0.5$	17.11 ± 0.26	17.43 ± 0.27	17.43 ± 0.27	17.43 ± 0.27
$\theta_0 = 1$	16.93 ± 0.28	17.39 ± 0.30	17.39 ± 0.30	17.39 ± 0.30

Table: Interpolated b-quark mass, obtained from the spin averaged $B_{\rm s}$ meson, for the different values of the θ angles.

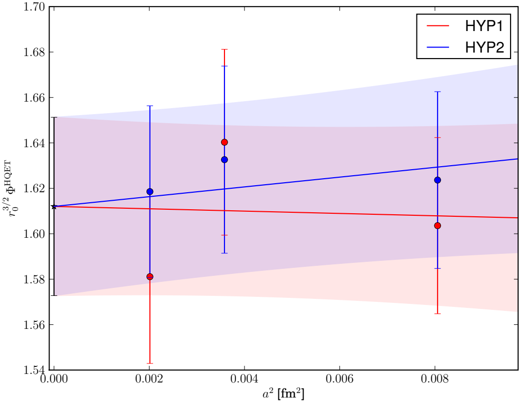
spread in $r_0(M_{\rm b}^{(0)}$: $\quad \mathrm{O}(1/(r_0 M_{\rm b}))$

$r_0(M_{\rm b}^{(0)} + M_{\rm b}^{(1)})$: $\quad \mathrm{O}(1/(r_0 M_{\rm b})^2)$: very small

Quenched results (unpublished) including $1/m_{\rm b}$ terms

[M. Della Morte, N. Garron, G. von Hippel T. Mendes, H. Simma & R.S.]

Continuum extrapolation of $\Phi^{\rm HQET} = r_0^{3/2} F_{\rm B} \sqrt{m_{\rm B}}$



Quenched results

Decay constants in MeV, using $r_0 = 0.5\,\mathrm{fm}$.

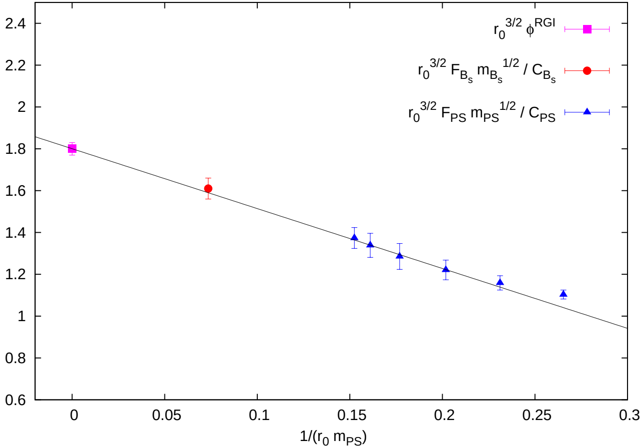
	$f^{(0)}_{\mathrm{B_s}}$	$f^{(0)}_{\mathrm{B_s}} + f^{(1)}_{\mathrm{B_s}}$		
		$(\theta_1,\theta_2)=(0,0.5)$	$(\theta_1,\theta_2)=(0.5,1)$	$(\theta_1,\theta_2)=(1,0)$
$\theta_0 = 0$	230.16 ± 8.14	214.45 ± 7.95	214.50 ± 7.54	214.49 ± 7.54
$\theta_0 = 0.5$	226.41 ± 7.97	213.52 ± 7.77	213.39 ± 7.51	213.42 ± 7.51
$\theta_0 = 1$	215.37 ± 7.53	213.51 ± 7.99	212.70 ± 7.74	212.86 ± 7.78

	$f^{(0)}_{\mathrm{B^*{}_s}}$	$f^{(0)}_{\mathrm{B^*{}_s}} + f^{(1)}_{\mathrm{B^*{}_s}}$		
		$(\theta_1,\theta_2)=(0,0.5)$	$(\theta_1,\theta_2)=(0.5,1)$	$(\theta_1,\theta_2)=(1,0)$
$\theta_0 = 0$	234.24 ± 8.28	216.11 ± 7.99	216.62 ± 7.65	216.51 ± 7.65
$\theta_0 = 0.5$	233.76 ± 8.26	215.47 ± 7.86	215.69 ± 7.65	215.65 ± 7.65
$\theta_0 = 1$	232.44 ± 8.23	216.79 ± 8.35	215.88 ± 8.02	216.10 ± 8.08

$f^{(0)}_{\mathrm{B^*{}_s}} + f^{(1)}_{\mathrm{B^*{}_s}}$ and $f^{(0)}_{\mathrm{B_s}} + f^{(1)}_{\mathrm{B_s}}$: $\mathrm{O}(1/(r_0 M_\mathrm{b})^2)$: 1% level

Quenched results: B_s decay constant

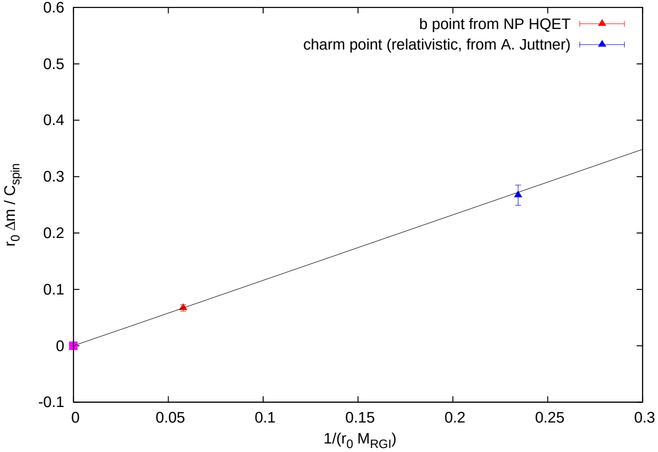
Static results together with results with $m_\mathrm{h} < m_\mathrm{b}$ and an HQET computation with $1/m_\mathrm{b}$ corrections included. C_PS at 3-loop.



at charm: -30% $1/m_\mathrm{c}$ correction, but where is $1/m_\mathrm{c}^2$?

Quenched results: spin splitting

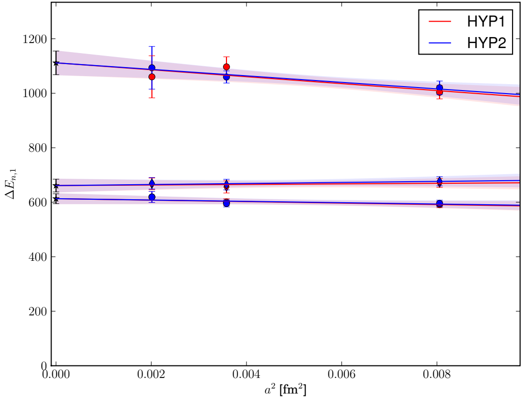
Static results together with results with $m_{\rm h} < m_{\rm b}$ and an HQET computation with $1/m_{\rm b}$ corrections included.



where is $1/m_{\rm c}^2$?

Quenched results: excited pseudo scalars

Continuum extrapolation of pseudoscalar energy levels in HQET. From bottom to top:
2s – 1s splitting static
2s – 1s splitting static + $1/m_{\rm b}$
3s – 1s splitting static



Concluding remarks

- Non-perturbative confirmation of HQET as an effective theory
  [very nice but: numerics does of course not provide a proof]
- indications that asymptotic convergence extends to charm:
  linearity in $1/m$ seems to extend to $1/m_{\rm c}$
  are $1/m_{\rm c}^2$ terms really so small?
  suggests to carry out a direct HQET computation for charm quarks
- $N_{\rm f} = 2$ computations are on the way
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  ... once dynamical fermion updating of topological sectors is under control