Some Insights from Studies of Variants of the Standard Model

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QCD Conference, Berlin-Brandenburgische Akademie der Wissenschaften, Berlin, Oct. 2009

Motivations

We know the gauge group

$$G_{SM} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

and fermion content of the Standard Model (SM). But there are many fundamental questions that are still unanswered about this theory. For example, what role does $N_c = 3$ play? What if $N_c = 5$ or another value? There is also an part of the model that is not yet verified, namely the Higgs mechanism for electroweak symmetry breaking. One can gain useful insights by studying the properties of variants of the SM, e.g., with $N_c \neq 3$ or without a conventional Higgs mechanism.

Outline

- Models with dynamical electroweak symmetry breaking (EWSB) as an alternate to the SM Higgs

- SM variants with general N_c

- SM variants with color-nonsinglet electroweak-singlet fermions

- SM variants in which QCD is the dominant source of EWSB

N.B.: Since the topic here is SM variants, I will not discuss our two recent papers;

S. Nussinov and RS, "On the π and K as $q\bar{q}$ Bound States and Approximate Nambu-Goldstone Bosons", Phys. Rev. D79, 016005 (2009), ArXiv:0811.3404.

S. Nussinov and RS, "Gluon-Glueball Duality and Searches for Glueballs", Phys. Rev. D80, 054003 (2009), ArXiv:0907.1577.

Dynamical Electroweak Symmetry Breaking

Origin of EWSB is an outstanding unsolved question in particle physics. Standard model Higgs mechanism is somewhat unsatisfying as an explanation since:

Spontaneous symmetry breaking is put in by hand: Higgs potential $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$ with $\mu^2 < 0$, so $\langle \phi \rangle = \binom{0}{v/\sqrt{2}}$, $\Rightarrow$ EWSB, but does not explain why μ^2 is negative when it could, a priori, have been positive.

SM accomodates, but does not explain, fermion masses and mixing, via Yukawa couplings, $m_f \simeq y_f v/\sqrt{2}$; y_f values and generational hierarchy put in by hand; some y_f's range down to 10^{-5} with no explanation.

Might history teach us something?

In both of two previous cases where fundamental scalar fields were used to model spontaneous symmetry breaking, the actual underlying physics did not involve fundamental scalar fields but instead bilinear fermion condensates:

- Superconductivity: Ginzburg-Landau free energy functional used complex scalar field ϕ with $V = c_2|\phi|^2 + c_4|\phi|^4$, $c_2 \propto (T - T_c)$, so for $T < T_c$, $c_2 < 0$ so $\langle \phi \rangle \neq 0$. But superconductivity is actually due to dynamical formation of a condensate of Cooper pairs $\langle ee \rangle$, as explained by the BCS theory.

- Gell-Mann Lévy σ model for spontaneous chiral symmetry breaking in hadronic physics, due to $\langle \sigma \rangle = f_\pi \neq 0$; in the actual underlying QCD, the SχSB is due to the dynamical formation of a $\langle \bar{q} q \rangle$ condensate.

QCD Breaks Electroweak Symmetry

We already know of a source of dynamical EWSB: QCD breaks electroweak symmetry. Consider, for simplicity, $N_f = 2$ QCD with massless u, d. The quark condensate $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R \rangle + \langle \bar{q}_R q_L \rangle$, transforms as $I_w = 1/2$, $|Y| = 1$. If this were the only source of EWSB, then the Nambu-Goldstone bosons (NGB's), $\pi^\pm$ and π^0, would be absorbed to become the longitudinal components of the $W^\pm$ and Z, giving them masses:

$$m_W^2 = \frac{g^2 f_\pi^2}{4}\,, \qquad m_Z^2 = \frac{(g^2 + g'^2) f_\pi^2}{4}$$

These satisfy the tree-level relation $\rho = 1$, where

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}$$

The value of f_π for massless u, d is slightly less than for $m_u,\ m_d \sim$ few MeV, but this is unimportant here; take $f_\pi \sim 90$ MeV. With this value, if one also takes the usual values of g, g', this yields $m_W \simeq 29$ MeV, $m_Z \simeq 33$ MeV.

While the scale here is too small by $\sim 10^3$ to explain the observed W and Z masses, it motivates a dynamical approach to EWSB, technicolor (TC) (Weinberg, Susskind, 1979).

In TC theories, one has a new vectorial gauge symmetry, technicolor, with a gauge group $G_{TC} = \mathrm{SU}(N_{TC})$, and a set of technifermions F subject to this gauge interaction and transforming nontrivially under EW interactions. For example, the "one-doublet" TC model uses

$$\begin{pmatrix} F_u^\tau \\ F_d^\tau \end{pmatrix}_L \qquad F_{uR}^\tau, \quad F_{dR}^\tau$$

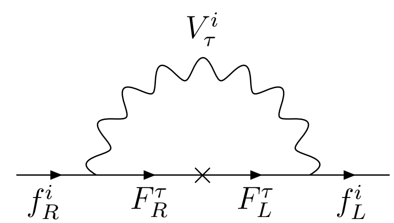
(TC indices τ) with $Y = 0$ for the SU(2)$_L$ technidoublet and $Y = \pm 1$ for the SU(2)$_L$ technisinglets. The one-family TC model uses one SM family of technifermions.

TC interaction is asymptotically free, gets strong at the scale Λ_{TC}, producing technifermion condensates $\langle \bar{F}_u F_u \rangle$, $\langle \bar{F}_d F_d \rangle$ transforming as $I_w = 1/2$, $|Y| = 1$, breaking EW symmetry. The resultant Nambu-Goldstone technipions are absorbed to give the W and Z masses:

$$m_W^2 \simeq \frac{g^2 f_{TC}^2 N_D}{4}, \qquad m_Z^2 \simeq \frac{(g^2 + g'^2) f_{TC}^2 N_D}{4}$$

where $f_{TC} \sim \Lambda_{TC}$ is TC analogue of f_π, $N_D =$ no. of SU(2)$_L$ technidoublets: 1 and $(N_c + 1) = 4$ for 1-doublet and 1-family TC. Again, $\rho = 1$.

Given asymptotic freedom of TC theory, condensate formation and hence EWSB are automatic, do not require any ad hoc parameter choice like $\mu^2 < 0$ in the SM. Because TC has no fundamental scalar field, there is no hierarchy problem.



To give masses to quarks and leptons, must communicate the EWSB in the TC sector to these SM fermions; hence, embed TC in a larger, extended technicolor (ETC) gauge theory with ETC gauge bosons V_τ^j with masses $M_{ETC,j}$ transforming (technisinglet) SM fermions of j'th generation into technifermions, Resultant SM fermion mass:

$$m_{f_j} \simeq \eta \frac{\Lambda_{TC}^3}{M_{ETC,j}^2}$$

Typical values: $M_{ETC,1} \sim 10^3$ TeV, $M_{ETC,2} \sim 10^2$ TeV, $M_{ETC,3} \sim$ few TeV. Current TC theories involve a gauge coupling that gets large but runs slowly over an extended interval of energies due to an approximate IR zero of the TC beta function; η is a resultant enhancement factor.

It follows that the running mass $m_{f_j}(p)$ is hard up to $M_{ETC,j}$ and has the power-law decay

$$m_{f_j}(p) \sim m_{f_j(0)} \frac{M_{ETC,j}^2}{p^2} \quad \text{for } p >> M_{ETC,j}$$

(Christensen and RS, PRL 94, 241801 (2005)).

How can TC/ETC explain neutrino masses without a GUT-scale seesaw? One can construct a possible mechanism for this (Appelquist and RS, PL B548, 204 (2002); PRL 90, 201801 (2003)).

Early concern with flavor-changing neutral current (FCNC) interactions, e.g., $\bar{K}^0 \leftrightarrow K^0$ (i.e., $s\bar{d} \leftrightarrow d\bar{s}$) and resultant $K_L - K_S$ mass difference $\Delta m_{K_L K_S}$. SM contribution consistent with experimental value $\Delta m_{K_L K_S}/m_K \simeq 0.7 \times 10^{-14}$.

Naive effective Lagrangian used in early studies without UV-complete ETC model: $\mathcal{L}_{eff} = c[s\gamma_\mu d]^2$ with coefficient $c \sim 1/M_{ETC}^2$. With a more UV-complete theory, we have analyzed this and shown that the coefficient is further suppressed by the approximate generational symmetry.

In terms of ETC eigenstates, an $s\bar{d}$ in a $\bar{K}^0$ produces a V_1^2 ETC gauge boson, but this cannot directly yield a $d\bar{s}$ in the final-state K^0; the latter is produced by a V_2^1. We have calculated ETC gauge boson mixing $V_1^2 \to V_2^1$:

Contribution from $V_1^2 \to V_2^1$:

$$c \sim \frac{1}{M_{ETC,1}^2}\,{}_2^1\Pi_1^2\,\frac{1}{M_{ETC,1}^2} \sim \frac{M_{ETC,3}^2}{M_{ETC,1}^4} << \frac{1}{M_{ETC,1}^2}$$

With above values, $M_{ETC,1} \sim 10^3$ TeV, $M_{ETC,3} \sim 3$ TeV, the suppression factor is $(M_{ETC,3}/M_{ETC,1})^2 \simeq 10^{-5}$. Hence, rather than the naive result $\Delta m_{K_L K_S}/m_K \sim \Lambda_{QCD}^2/M_{ETC,1}^2$, this yields the much smaller result

$$\frac{\Delta m_{K_L K_S}}{m_K} \sim \frac{M_{ETC,3}^2\,\Lambda_{QCD}^2}{M_{ETC,1}^4} \sim 10^{-18}$$

so this and other FCNC processes are not problematic as had been thought (Appelquist, Piai, RS, PR D69, 015002 (2004)).

We have studied fermion masses and mixing and constraints from FCNC's in a series of papers: Appelquist, Piai, RS, PL B593, 175 (2004); PL B595, 442 (2004); Appelquist, Christensen, Piai, RS, PRD 70, 093010 (2004)).

The approach in TC/ETC models is very ambitious, since it tries to explain dynamically not only EWSB but the pattern of fermion masses and mixings. Although one does not have fully realistic models, and it is challenging to try to satisfy all phenomenological constraints, e.g., large mass splitting between the t and b quarks, small contributions to electroweak precision quantities, etc., this approach serves as an instructive alternate to the SM and supersymmetric SM.

Modern TC theories use a slowly running ("walking") gauge coupling associated with an approximate IR fixed point of TC renorm. group. Approximate solutions of Dyson-Schwinger (DS) eqs. for a one-family SU(2)$_{TC}$ theory, which has $N_w(N_c+1)=8$ technifermions, plausibly yield this behavior. This has the potential to reduce TC corrections to precision EW quantities, such as S (Appelquist and Sannino, 1999; recent studies Kurachi and RS PR D74, 056003 (2006); Kurachi, RS, Yamawaki, PR D76, 035003 (2007)).

Several recent lattice studies (Appelquist et al., PRL 100, 171607 (2008); Deutzeman et al., PL B670, 41 (2008), Jin, Mawhinney, 2009) for $N_c=3$, N_f fermions in fund. rep. are consistent with suggestions, to within theoretical uncertainties, from approximate solutions of DS eqs. on $N_{f,cr}$ for boundary between phases with and without spontaneous chiral symmetry breaking (SχSB), which one needs to know to construct walking TC theories (also other work for higher reps.).

But there are significant theoretical uncertainties in the DS analysis, since it neglects higher-order semiperturbative gluon (technigluon) effects and instanton contributions, which enhance SχSB. Hence, one might have thought that it would underestimate the size of the SχSB phase.

We can understand how the DS prediction for $N_{f,cr}$ can nevertheless be roughly correct (Brodsky and RS, PL B666, 95 (2008), ArXiv:0806.1535).

To see this, recall that the DS eqs. also do not incorporate information on confinement, and the Euclidean loop integration takes k to 0. But because of confinement, the quarks and gluons (technifermions and technigluons) have nonzero bound state momenta of order Λ_{QCD} (Λ_{TC}), so the k integration should not really extend all the way to 0; the k integration measure is smaller.

Hence, in this respect, the DS analysis overestimates SχSB. The effects of these different physical features that are neglected are opposite and thus tend to cancel each other.

Some Insights Into the Role of $N_c = 3$ in the SM

To gain insight into the role of $N_c = 3$ in the SM, we study the N_c-Extended Standard Model (ESM) based on the gauge group

$$G_{ESM} = \mathrm{SU}(N_c) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

with various fermion contents. What are the properties of such theories? What condition(s) must be satisfied for them to be embedded in a grand unified theory? To keep baryons as fermions, one may require that N_c be odd.

The large-N_c expansion ('t Hooft, 1974; Witten, 1979...) has been very valuable as an analytic tool for studying QCD. In applications, a common practice has been to turn off the electroweak (EW) interactions and analyze the QCD sector by itself. Here we retain the EW sector and do not necessarily take $N_c \to \infty$.

First, consider the fermion content consisting of

$$Q_L^a = \begin{pmatrix} u^a \\ d^a \end{pmatrix}_L \ : \ (N_c, 2)_{Y_{Q_L}}$$

$$u_R^a \ : \ (N_c, 1)_{Y_{u_R}}$$

$$d_R^a \ : \ (N_c, 1)_{Y_{d_R}}$$

$$L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \ : \ (1, 2)_{Y_{L_L}}$$

$$\nu_R \ : \ (1, 1)_{Y_{\nu_R}}$$

$$e_R \ : \ (1, 1)_{Y_{e_R}}$$

for first generation, with N_g in all. Here, $Q = T_3 + (Y/2)$, so $q_u = q_d + 1$, $q_\nu = q_e + 1$.

The SU$(N_c)^2$ U$(1)_Y$ anomalies vanish automatically. The vanishing of the SU$(2)^2$ U$(1)_Y$ anomaly implies

$$N_c Y_{Q_L} + Y_{\mathcal{L}_L} = 0$$

i.e.,

$$N_c(2q_d + 1) + (2q_e + 1) = 0$$

This also yields zero $U(1)_Y^3$ gauge anomaly.

General solution:

$$q_d = -\frac{1}{2}\left[1 + \frac{Y_{\mathcal{L}_L}}{N_c}\right] = -\frac{1}{2}\left[1 + \frac{1}{N_c}(2q_e + 1)\right]$$

or equivalently,

$$q_e = -\frac{1}{2}\left[1 + N_c Y_{Q_L}\right] = -\frac{1}{2}\left[1 + N_c(2q_d + 1)\right]$$

Thus, for a given N_c, anomaly cancellation yields a one-parameter family of solutions for the fermion charges. The values of these charges are in $\mathbb{R}$, not, in general, in $\mathbb{Q}$, so that electric and hypercharge are not quantized, although if one is rational, then they all are, since $Y_{\mathcal{L}_L}/Y_{Q_L} = -N_c$, so $Y_{\mathcal{L}_L} \in \mathbb{Q} \Longleftrightarrow Y_{Q_L} \in \mathbb{Q}$ and similarly for Y_{f_R}.

case	q_d	(q_u, q_d)	Y_{Q_L}	$Y_{\mathcal{L}_L}$
$C1_q$	> 0	$(+,+)$	> 1	$< -N_c$
$C2_q$	$-1 < q_d < 0$	$(+,-)$	$-1 < Y_{Q_L} < 1$	$-N_c < Y_{\mathcal{L}_L} < N_c$
$C2_{q,sym}$	$-1/2$	$(1/2,-1/2)$	0	0
$C3_q$	< -1	$(-,-)$	< -1	$> N_c$
$C4_q$	0	(1,0)	1	$-N_c$
$C5_q$	-1	$(0,-1)$	-1	N_c

case	q_e	(q_ν, q_e)	$Y_{\mathcal{L}_L}$	Y_{Q_L}
$C1_\ell$	> 0	$(+,+)$	> 1	$< -1/N_c$
$C2_\ell$	$-1 < q_e < 0$	$(+,-)$	$-1 < Y_{\mathcal{L}_L} < 1$	$-1/N_c < Y_{Q_L} < 1/N_c$
$C2_{\ell,sym}$	$-1/2$	$(1/2,-1/2)$	0	0
$C3_\ell$	< -1	$(-,-)$	< -1	$> 1/N_c$
$C4_\ell$	0	(1,0)	1	$-1/N_c$
$C5_\ell$	-1	$(0,-1)$	-1	$1/N_c$

Tables listing classes of solutions for quark and lepton charges

Question: Can one embed the theory with G_{ESM} in a simple group G_{GUT}? This is desirable since it

- unifies quarks and leptons, extending the link already contained in the condition for the vanishing of the anomaly in gauged currents
- predicts the relative sizes of gauge couplings in the factor groups
- quantizes charges, since Q is a generator of G_{GUT}

Clearly, the rank $\quad rk(G_{GUT}) \geq rk(G_{ESM}) = N_c + 1$

Use standard procedure for constructing a GUT:

- put fermions in complex representations to avoid bare mass terms that would produce masses of order M_{GUT} for all fermions
- require absence of gauge anomalies

Try to place all fermions of each generation into one representation, which requires that Tr$(Y) = \sum_f Y_f = 0$ (for each generation) since the hypercharge Y is a generator of G_{GUT}. This condition is satisfied with the fermion content shown:

$$\mathrm{Tr}(Y) = 2(N_c Y_{Q_L} + Y_{\mathcal{L}_L}) + N_c(Y_{u^c_L} + Y_{d^c_L}) + Y_{e^c_L} + Y_{\nu^c_L} = 0$$

Since the exceptional groups have bounded ranks, they cannot satisfy the rank condition above for arbitrary N_c; instead, use a G_{GUT} that can satisfy this condition for arbitrary N_c.

It is natural to try to embed G_{ESM} in an SO($4k+2$) GUT, since this group has complex reps. and, for $k \geq 2$, is anomaly-free. This would be a generalization, to higher N_c, of the embedding of G_{SM} in SO(10) (Fritzsch-Minkowski, Georgi). (Recall that SO(N) has only real representations for odd N and for $N = 0$ mod 4.)

Focus on odd N_c. Since rank $rk(\mathrm{SO}(4k+2)) = 2k+1$, the condition $rk(G_{GUT}) \geq rk(G_{ESM})$ is $2k \geq N_c$, but, since N_c is odd, this is $2k \geq N_c + 1$, so the $G_{GUT} = \mathrm{SO}(2N_c+4)$ with rank $N_c + 2$. The chiral spinor rep. of this group has dimension 2^{N_c+1}. Each generation has $4(N_c+1)$ Weyl fermions. The condition that these fit into the spinor is

$$4(N_c+1) = 2^{N_c+1}$$

But the only solution of this equation is $N_c = 3$. Similar argument for even N_c. So the N_c-extended SM can only be grand-unified in this way for $N_c = 3$. This may give insight into the special role that $N_c = 3$ plays in our world. (RS, Phys. Rev. D53, 6465 (1996)).

Next, consider grand unification with an N_c-extended SM with modification of the fermion content (RS, Phys Rev. D76, 055010 (2007),ArXiv:0704.3464).

Focus on a different minimal GUT, with $rk(G_{GUT}) = rk(G_{ESM}) = N_c + 1$, namely $G_{GUT} = \mathrm{SU}(N)$ with $N = N_c + 2$.

Study examples both with and without supersymmetry. Consider two types of (anomaly-free) chiral matter fermion content. First, consider N_g copies ($\sim$ flavors) of chiral superfields transforming as

$$[2]_N + (N-4)[\bar{1}]_N$$

where $[k]_N$ denotes antisymmetric rank-k tensor rep. of SU(N) ($[1]_N$, $[2]_N$ equiv. to notation $\square$, $\begin{smallmatrix}\square\\\square\end{smallmatrix}$, etc.), and fermion fields are written left-handed here. This is a chiral gauge theory, so no fermion mass terms at GUT scale.

Assign Y for the fundamental representation so that

$$Y = \mathrm{diag}(-2/N_c ..., -2/N_c, 1, 1)$$

and hence

$$Q = \mathrm{diag}(-1/N_c, ..., -1/N_c, 1, 0)$$

With respect to G_{ESM}, the (matter) fermions transform as $N-4=N_c-2$ copies of

$$[\bar{1}]_N : \ (\bar{N}_c, 1)_{2/N_c} + (1,2)_{-1}$$

with corresponding fermions fields

$$d^c_{a,p,L}\ , \qquad L_{p,L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{p,L}$$

(where $1 \leq p \leq N-4$ is the copy index), and

$$[2]_N : \quad ([2]_{N_c}, 1)_{-4/N_c} + (N_c, 2)_{1-(2/N_c)} + (1,1)_2$$

with fields

$$\xi^{ab}_L\ , \qquad \begin{pmatrix} u^a \\ d^a \end{pmatrix}_L , \qquad e^c_L$$

Charges:

$$q_d = -\frac{1}{N_c}\ , \qquad q_u = 1 - \frac{1}{N_c}\ , \qquad q_\xi = -\frac{2}{N_c}$$

If and only if $N_c = 3$, then ξ_L is a $\bar{3}$ and is u^c_L; for larger N_c, it is a distinct (antisymmetric rank-2) representation of SU(N_c), $[2]_{N_c}$.

Moreover, since for $N_c = 3$, this set of fermions reduces to the SM-nonsinglet fermions discussed above for the N_c-extended SM with $q_\nu = 0$, it follows that the charges coincide. For example,

$$q_d = -\frac{1}{N_c} = -\frac{1}{2}\left[1 - \frac{1}{N_c}\right] \Leftrightarrow N_c = 3$$

Each generation contains $N_d = 2(N_c - 1)$ SU(2)$_L$ doublets of matter fermions, of which N_c are color-nonsinglets and $N_c - 2$ are color-singlets (leptons). We exclude the value $N_c = 2$ because the resulting theory would not have any leptonic SU(2)$_L$ doublets.

The electromagnetic U(1)$_{em}$ gauge interaction is vectorial if and only if the charges of the (left-handed) fermions can be written as a set of equal and opposite pairs together with possible zero values. It suffices to consider a single generation.

For the charges of the fermions in the $[2]_N$ we have

- $q_\xi = -2/N_c$, with multiplicity $N_c(N_c - 1)/2$
- $q_u = 1 - (1/N_c)$ with multiplicity N_c
- $q_d = -1/N_c$ with multiplicity N_c
- $q_{e^c} = 1$, multiplicity 1

The charges of the fermions in the $N - 4 = N_c - 2$ copies of the $[\bar{1}]_N$ are:

- $q_{d^c} = 1/N_c$ with multiplicity $(N_c - 2)N_c$
- $q_e = -1$ with multiplicity $N_c - 2$
- $q_\nu = 0$ with multiplicity $N_c - 2$

These charges constitute a vectorial set, with nonzero charges in equal and opposite pairs, if and only if $N_c = 3$. Hence, for $N_c \geq 4$, U(1)$_{em}$ is an (anomaly-free) chiral gauge interaction.

Since the SU(N_c) gauge interaction is asymptotically free, as the energy decreases below the GUT scale, α_s grows and, at the scale Λ_{QCD}, causes condensation of color nonsinglet matter fermions. If $N_c \neq 3$, this condensation breaks U(1)$_{em}$ and gives the photon a mass.

Illustrate this in the simplest case, $N_c = 4$, i.e., an SU(6) GUT. Here the matter fermions for a given generation transform as

$$[2]_6 + 2([\bar{1}]_6)$$

and hence, with respect to $\mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, as

$$\{([2]_4, 1)_{-1} + ([1]_4, [1]_2)_{1/2} + (1, 1)_2\} + \{2([\bar{1}]_4, 1)_{1/2} + 2(1, 2)_{-1}\}$$

with charges $q_u = 3/4,\ \ q_d = -1/4,\ \ q_\xi = -1/2$ (and $q_\nu = 0,\ \ q_e = -1$). Note that $[k]_N \approx [\bar{k}]_N$ iff $k = N/2$, so $[2]_4 \approx [\bar{2}]_4$.

The most attractive channel (MAC) for color-nonsinglet matter fermion condensation is $([2]_4, 1)_{-1} \times ([2]_4, 1)_{-1} \to (1,1)_{-2}$, with the condensate

$$\langle \epsilon_{abrs} \xi_L^{ab\ T} C \xi_L^{rs} \rangle$$

This is invariant under SU(4)$_c$ but has charge $q = -1$, hypercharge $y = 2q = -2$ and hence breaks not only U(1)$_Y$ but also U(1)$_{em}$.

For $N_c = 5$, i.e., an SU(7) GUT, SU(5)$_c$ is a chiral gauge interaction and the MAC for the color-nonsinglet matter fermion condensation is

$$([2]_5, 1)_{-4/5} \times ([2]_5, 1)_{-4/5} \to ([\bar{1}]_5, 1)_{-8/5}$$

with condensate $\langle \epsilon_{abrsv} \xi_L^{ab\ T} C \xi_L^{rs} \rangle$, which breaks not only U(1)$_{em}$ and U(1)$_Y$, but also self-breaks color SU(5)$_c$ to SU(4)$_c$ (the SU(4)$_c$ theory is vectorial and does not break further). In this theory, $q_u = 4/5$, $q_d = -1/5$, $q_\xi = -2/5$.

For odd $N_c = 2m-1$, a second choice for the left-handed matter fermion content in the SU(N) GUT with $N = N_c + 2 = 2m+1$ is $N_g = 3$ copies of the set

$$\sum_{\ell=1}^{m} [2\ell]_N$$

Since $[N-k]_N \approx [\bar{k}]_N$, the $\ell = m$ term is $[N-1]_N \approx [\bar{1}]_N$. For $N_c = 3$, this fermion content is $[2]_5 + [4]_5 \approx [2]_5 + [\bar{1}]_5$ and thus coincides with the first set and comprises the fermion content of the Georgi-Glashow SU(5) GUT. For other values of N_c it constitutes a second type of N_c-generalization for a GUT.

The group SU(N) with $N = 2m+1$ has an embedding in SO($4m+2$) given by $\mathrm{SU}(2m+1) \times \mathrm{U}(1)_X \subset \mathrm{SO}(4m+2)$, where U(1)$_X$ is an additional U(1) symmetry. The total number of chiral matter fermions in this set is

$$\sum_{\ell=1}^{m} \binom{2m+1}{2\ell} = 2^{2m} - 1$$

Adding an SU(N)-singlet field to this set thus yields $2^{2m} = 2^{N_c+1}$ chiral fermions, which fit in the spinor representation of SO($4m+2$).

Because of the profusion of fields for $N_c \geq 5$, the SU(N_c) color sector loses asymptotic freedom in the energy interval $1~\mathrm{TeV}~\leq E \leq M_{GUT}$.

A SM Variant with Color-Nonsinglet, Electroweak-Singlet Fermions

The SM has fermions that are (i) color-nonsinglets and electroweak-nonsinglets (the quarks) and (ii) color-singlets and EW-nonsinglets (the leptons), but no fermions that are color-nonsinglets and EW-singlets. What would be the properties of a SM variant with color-nonsinglet, electroweak-singlet (matter) fermions?

A study of such an SM variant reveals that it would have a number of unusual properties (RS, Phys. Rev. D78, 076009 (2008), ArXiv:0809.0087).

One example: consider a SM variant with SU(2)$_L$ doublets for quarks and leptons (for each generation):

$$Q^a_L = \begin{pmatrix} u^a(1/2) \\ d^a(-1/2) \end{pmatrix}_L \quad : \quad Y_{Q_L} = 0$$

$$L_L = \begin{pmatrix} \ell_1(1/2) \\ \ell_2(-1/2) \end{pmatrix}_L \text{‘} : \quad Y_{L_L} = 0$$

and a set of SU(2)$_L$ singlets $\{f_R\}$ with

$$Y_{f_R} = q_{f_R} = 0 \quad \forall \ f_R$$

To keep SU(3)$_c$ vectorial, the set $\{f_R\}$ includes two color triplets for each generation, denoted η_R^a and $\eta_R'^a$.

The remainder of the set $\{f_R\}$ is comprised of (two or some other number of) G_{SM}-singlets.

The SU(3)$_c$ color interaction confines and spontaneously breaks chiral symmetry. The most attractive channel, $3 \times \bar{3} \to 1$, yields the condensates

$$\langle \bar{u}_{a,L}\, f_R^a \rangle \ , \quad \langle \bar{d}_{a,L}\, f_R^a \rangle$$

where f_R refers to η_R or η_R'. Since $q_{u_L} = 1/2$, $q_{d_L} = -1/2$, and $q_{\eta_R} = q_{\eta_R'} = 0$, these condensates break not just SU(2)$_L$, but also U(1)$_{em}$. Other examples analyzed in RS, Phys. Rev. D78, 076009 (2008).

A Standard Model Variant with Primary Electroweak Symmetry Breaking due to QCD

As noted, the QCD quark condensate $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R \rangle + \langle \bar{q}_R q_L \rangle$, transforms as an $I_w = 1/2$, $|Y| = 1$ operator and breaks electroweak gauge symmetry at the scale of $\Lambda_{QCD} \sim 10^2$ MeV. In the real world, this breaking is very small compared to the main EWSB at the scale 250 GeV.

How would the world be different if the QCD-induced EWSB were dominant? A study of models of this type reveals many striking differences with the real world (C. Quigg and RS, Phys. Rev. D79, 096002 (2009), ArXiv:0901.3958). For this study, we mainly keep $N_c = 3$.

So consider reduced Standard Models (RSM's) in which the real-world 250 GeV-scale EWSB is absent. We focus first on two types of models:

1. RSM1, with no bare mass terms for the quarks and leptons,
2. RSM2, with bare mass terms for quarks and leptons, explicitly violating EW symmetry, restricted to be sufficiently small that the model serves as a reasonable low-energy effective field theory up to energy scales well above the QCD scale.

We also study models with augmented electroweak symmetry groups.

We consider variable N_g and first take $N_g = 1$. The SU(3)$_c$, SU(2)$_L$, and U(1)$_Y$ gauge couplings g_s, g, and g' are taken to have approximately their actual values.

As noted, the would-be Nambu-Goldstone bosons $\pi^\pm$ and π^0 are absorbed to become the longitudinal components of the $W^\pm$ and Z, with masses (marked here with bars to distinguish them from the real-world m_W and m_Z)

$$\bar{m}_W^2 = \frac{g^2 N_D f_\pi^2}{4}\,, \qquad \bar{m}_Z^2 = \frac{(g^2+g'^2) N_D f_\pi^2}{4}$$

where $N_D = N_g =$ number of quark SU(2)$_L$ doublets. Focus on $N_D = 1$ first, for which $m_W \sim 29$ MeV, $m_Z \sim 33$ MeV.

There is a new type of unification of weak and residual strong interactions here, resulting from the absorption of the would-be π's to make the electroweak vector bosons W and Z.

The strength of charged- and neutral-current weak interactions is given by the Fermi coupling

$$\frac{\bar{G}_F}{\sqrt{2}} = \frac{g^2}{8\bar{m}_W^2} = \frac{1}{2f_\pi^2} = \frac{g^2 + g'^2}{8\bar{m}_Z^2}$$

This is much larger than in the real world:

$$\frac{\bar{G}_F}{G_F} = \frac{v^2}{f_\pi^2} \simeq 0.7 \times 10^7$$

where $v = 246$ GeV is the real-world EW scale. This produces a number of interesting differences between this hypothetical world and our real world.

For weak decays and charged-current (CC) and neutral-current (NC) cross sections with momentum transfers small compared with $\bar{m}_W$ and $\bar{m}_Z$, the effective strength of weak interactions $\propto G_F^2$, is a factor of $\sim 10^{13}$ larger than in the real world and is much closer to the strength of the residual strong interactions than in the real world.

For example, if, as is plausible, the masses of the nucleons p and n differ by a few MeV, then the heavier nucleon beta-decays to the lighter with a lifetime

$$\tau \sim \left(\frac{f_\pi}{v}\right)^4 \tau_n \sim 10^{-11}\,\mathrm{sec}$$

where $\tau_n \simeq 0.9 \times 10^3$ s.

The long-range component of the residual strong interactions between color-singlet hadrons in the real world is mediated by pion exchange, with range $\sim 1/m_\pi = 1.4$ fm. Here there are no pions, as such, in the hadron spectrum, but instead there are the very low-mass W and Z. Pion exchange is replaced by the weak CC and NC exchange of the W and Z, with the greater range $1/\bar{m}_{W,Z} \sim 6$ fm. These interactions violate P and C.

Effect on nucleon binding to form nuclei: A simple description of NN binding uses a solution of the Schrödinger equation in a square-well potential. Let the radial size of the square well be a and the depth V_0. The occurrence and number of bound states is determined by the dimensionless parameter

$$\xi = \frac{2\mu V_0 a^2}{\hbar^2 \pi^2}$$

where μ is the reduced mass, i.e., $M_N/2$ for the NN system. When $\xi > O(1)$ a first bound state appears, and as ξ increases, more bound states appear in the spectrum. A figure of merit is $V_0 a^2$. Now

$$\frac{\bar{a}}{a_{rw}} = \frac{(1/\bar{m}_{W,Z})}{(1/m_\pi)} = \frac{2m_\pi}{g f_\pi} \simeq 4.5$$

and since $V_0 \sim$ amplitude for π or $W,\ Z$ exchange, we have

$$\frac{\bar{V}_0}{(V_0)_{rw}} = \frac{g^2/(8\bar{m}_W^2)}{g_{\pi NN}^2/m_\pi^2} = \frac{1/(2f_\pi^2)}{g_{\pi NN}^2/m_\pi^2}$$

so

$$\frac{\bar{\xi}}{\xi_{rw}} = \frac{2m_\pi^4}{g_{\pi NN}^2 g^2 f_\pi^4} \sim \frac{1}{6}$$

Thus, the fact that the coupling g^2 for the $W,\ Z$ exchange in this world is smaller than the coupling $g_{\pi NN}$ for π exchange in the real world is partially cancelled by the greater range, $\bar{a} > a_{rw}$.

Since the masses of the nucleons p and n in the real world are mainly due to confinement energy of the quarks and gluons (as shown in bag models) or equivalently to the dynamically generated constituent quark masses, but not to the very small current-quark masses of a few MeV for u and d in the real world, taking the latter to zero only reduces m_N slightly.

The absence of a Higgs boson means that the perturbatively calculated partial wave amplitudes (PWA's) for $W,\ Z$ scattering exceed unitarity at a scale of few $\times \Lambda_{QCD}$, reflecting the formation of hadronic bound states - ρ, etc. This is understandable, since perturbation theory should not hold in the presence of strong interactions that produce bound states. The full amplitudes including the effect of the resonances do, of course, obey unitarity; the resonances "unitarize" the amplitudes.

For $s >> \bar{m}_W^2,\ \bar{m}_Z^2$, the matrix of $J = 0$ partial wave amplitudes for $2 \longrightarrow 2$ scattering of the normalized states $|W^+W^-\rangle$ and $|ZZ/\sqrt{2}\rangle$ is

$$a_0 = \frac{s}{32\pi N_g f_\pi^2} \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$$

The larger eigenvalue is $a_{0,max} = s/(16\pi N_g f_\pi^2)$. Imposing the unitarity condition $|a_0| \leq 1$ yields the inequality $\sqrt{s} < 4\sqrt{N_g \pi}\, f_\pi$, i.e., $\simeq 640$ MeV for $N_g = 1$, for the perturbatively calculated PWA's to hold. This makes sense, since the nonperturbative bound state, ρ, with mass comparable to its real-world value, 775 MeV, is just above this. Other contributions to $2 \to 2$ W and Z scattering would include the analogue of the $S = 1$, $L = 1$, $J = 2$ $f_2(1270)$ meson, etc.

The decay width of the ρ in this world would be similar to that of the real-world ρ_{rw}:

$$\Gamma(\rho \to W^+W^-) \simeq \Gamma(\rho_{rw} \to \pi^+\pi^-)$$

In the real world, the difference $m_d - m_u$ is important in counterbalancing the greater Coulombic self-energy of the proton and producing the difference $m_n - m_p = 1.3$ MeV. Here, with $m_u = m_d = 0$, this effect is absent.

However, there are important corrections to the p and n propagators due to emission and reabsorption of virtual W's and Z's. The W contributions cancel in the difference $m_n - m_p$, while estimates of the Z exchange contributions are of comparable size and, for $\sin^2\theta_W < 1/2$, of opposite sign, relative to the electromagnetic contributions, so the estimate of the sign of $m_n - m_p$ is model-dependent.

The EWSB by the $\langle \bar{q}q \rangle$ condensate does not directly give masses to the SM fermions. These fermion masses are dependent on the UV completion of the theory. If one assumes nothing further, then there is an infrared pathology due to the presence of the massless charged unconfined electron. Among other things, this leads to the collapse of the vacuum into a plasma due to the production of an avalanche of e^+e^- pairs by arbitrarily soft photons and arbitrarily weak electric fields.

If one assumes a a GUT UV completion of this theory, a tree-level process that contributes to proton decay is $u + u \longrightarrow X \longrightarrow e^+ d^c$, where X is a GUT-mass vector boson. The four-fermion operator for this transition also has a transition matrix element between $|e^+\rangle$ and $|p\rangle$, with size $\epsilon \sim \Lambda_{QCD}^3/M_{GUT}^2 \sim 10^{-34}\Lambda_{QCD}$. Diagonalizing the mass mixing matrix involving $|e^+\rangle$ and $|p\rangle$,

$$M_{e^+,p} = \begin{pmatrix} 0 & \epsilon \\ \epsilon & m_p \end{pmatrix}$$

gives $m_e = \epsilon^2/m_p$. For $M_{GUT} \sim 10^{16}$ GeV, this gives a nonzero but extremely small value for m_e, so the infrared pathology and vacuum instability via e^+e^- pair creation remains.

If one considers $N_g \geq 2$ generations of (massless) fermions, the breaking of the global $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ chiral symmetry to vectorial $\mathrm{SU}(N_f)_V$, where $N_f = 2N_g$, by the $\langle \bar{q}q \rangle$ condensate leads to $N_f^2 - 1 = 4N_g^2 - 1$ Nambu-Goldstone bosons (NGB's). Of these, there are (i) $2N_g^2$ with charges ± 1, (ii) $2N_g(N_g - 1)$ electrically neutral non-selfconjugate NGB's, and (iii) $2N_g - 1$ self-conjugate NGB's. Of these, two of (i) and one of (iii) get absorbed by the $W^\pm$ and Z; the residual $2N_g^2 - 2$ charged NGB's produce a further IR pathology.

In view of the IR pathology of the RSM1 model, consider a second model, RSM2, with bare SM fermion mass terms. This has the advantage of being free of any IR pathology and still allowing a study of the properties of a model in which EWSB can be dominantly due to the QCD condensate, for small enough m_f.

The bare SM fermion mass terms explicitly violate the electroweak symmetry, and this is reflected in a growth in perturbatively calculated partial wave amplitudes for $f\bar{f} \to VV$, $V = W, Z$ like $G_F m_f E$. These PWA's exceed the unitarity bounds if $\sqrt{s} > 8\pi c N_g f_\pi^2 / m_f$, where $c \sim O(1)$.

We keep the bare SM fermion mass terms sufficiently small so that the SMR2 is a good low-energy effective field theory up to energies where VV scattering becomes strong, $\sqrt{s} \simeq 640\sqrt{N_g}$ MeV. This allows $m_f \lesssim \sqrt{N_g} f_\pi$. For small nonzero m_f in this range, the QCD $\langle \bar{q} q \rangle$ condensate remains the dominant source of EWSB; as m_f approaches the upper bound, both QCD and explicit EWSB-breaking fermion mass terms are important.

One can also generalize N_c in these theories. One has $f_\pi \propto N_c^{1/2}$. Recall the 't Hooft limit $N_c \longrightarrow \infty$ with $g_s^2 N_c$ fixed, i.e., $g_s \propto N_c^{-1/2}$. To control the size of EW versus QCD interactions in this limit, one also requires that $g, \ g' \propto N_c^{-1/2}$. Hence,

$$\bar{m}_W^2 = \frac{g^2 N_D f_\pi^2}{4} \propto (N_c)^0$$

and similarly $\bar{m}_Z^2$ is independent of N_c, so that

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\bar{m}_W^2} = \frac{g^2 + g'^2}{8\bar{m}_Z^2} = \frac{1}{2 N_D f_\pi^2} \propto N_c^{-1}$$

Variants of Models with Augmented Electroweak Gauge Groups

It is also instructive to analyze a model with an augmented EW gauge group,

$$G_{LR} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$$

and fermions transforming, for each generation, as

$$Q^a_L : \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad (3,2,1)_{1/3} \ , \qquad Q^a_R : \quad \begin{pmatrix} u \\ d \end{pmatrix}_R \quad (3,1,2)_{1/3}$$

$$L_L : \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad (1,2,1)_{-1} \ , \qquad L_R : \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_R \quad (1,1,2)_{-1}$$

This has the appeal that the electric charge operator takes the elegant form

$$Q = T_{3L} + T_{3R} + \frac{1}{2}(B - L)$$

where B and L are baryon and lepton number. We focus on $N_g = 1$. Here the QCD $\langle \bar{q} q \rangle$ condensate breaks the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ EW gauge symmetry to the diagonal vectorial subgroup SU(2)$_V$. The gauge bosons corresponding to the three axial weak isospin generators pick up a common mass given by

$$m_A^2 = \frac{(g_L^2 + g_R^2) f_\pi^2}{4}$$

The corresponding three gauge bosons corresponding to the vectorial weak isospin generators remain massless and couple with gauge coupling

$$g_V = \frac{g_L g_R}{\sqrt{g_L^2 + g_R^2}}$$

Below the scale Λ_{QCD}, the theory has the gauge symmetry $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_V \times \mathrm{U}(1)_{B-L}$, and color-nonsinglets are confined, so the effective gauge symmetry is $\mathrm{SU}(2)_V \times \mathrm{U}(1)_{B-L}$. In terms of the conserved generators, the electric charge $Q = T_{3V} + (B - L)/2$.

The spin 1/2 fermions include $N = \binom{p}{n}$ and $L = \binom{\nu}{e}$, vectorially coupled and transforming as $(1, 2)_1$ and $(1, 2)_{-1}$. The leptons are massless at this stage.

Since the SU(2)$_V$ gauge interaction is asymptotically free, as the energy scale decreases to a much smaller value Λ_V, g_V grows sufficiently large to produce the condensate

$$\langle \bar{L}L \rangle = \langle \bar{e}e + \bar{\nu}\nu \rangle$$

This is invariant under the $\mathrm{SU}(2)_V \times \mathrm{U}(1)_{B-L}$ gauge symmetry and gives the e and ν dynamical masses of order Λ_V. Because of the logarithmic running of g_V, $\Lambda_V << \Lambda_{QCD}$; for example, with $g_L = g_R = g$ at the EW level, $\Lambda_V \sim 10^{-24}\Lambda_{QCD}$.

The confining SU(2)$_V$ interaction produces a set of leptonic bound states, including leptonic mesons of the form $\bar{\nu}\nu + \bar{e}e$ with $L = 0$, leptonic baryons νe with $L = 2$, and SU(2)$_V$ glueballs, with masses $\sim \Lambda_V$.

An interesting feature of this theory is that it has two quite different scales of confinement, chiral symmetry breaking, and bound-state masses, Λ_{QCD} and Λ_V.

Below the scale Λ_V, since SU(2)$_V$ is confined, the surviving abelian gauge symmetry is not the full U(1)$_{em}$, but only the $(B-L)/2$ part of it.

The condensate $\langle \bar{L}L \rangle$ spontaneously breaks the leptonic global chiral symmetry $\mathrm{SU}(2)_L^{(\ell)} \times \mathrm{SU}(2)_R^{(\ell)}$ to $\mathrm{SU}(2)_V^{(\ell)}$, giving rise to three massless leptonic NGB's. These are neutral under the only surviving deconfined gauge interaction, U(1)$_{B-L}$, so they do not cause any IR pathology.

We have also studied a theory based on the gauge group

$$G_{422} = \mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$

with fermions

$$\mathcal{F}_L = \begin{pmatrix} u^a & \nu \\ d^a & e \end{pmatrix}_L \quad : \quad (4, 2, 1)$$

$$\mathcal{F}_R = \begin{pmatrix} u^a & \nu \\ d^a & e \end{pmatrix}_R \quad : \quad (4, 1, 2)$$

($N_g = 1$ for simplicity). Here the $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{B-L}$ is embedded in the Pati-Salam group SU(4)$_{PS}$. This theory has the appeal that electric charge is quantized: $Q = T_{3L} + T_{3R} + \sqrt{2/3}T_{PS,15} = T_{3L} + T_{3R} + (1/6)\mathrm{diag}(1, 1, 1, -3)$.

The SU(4)$_{PS}$ interaction is asymptotically free and confines and breaks chiral symmetry at the scale Λ_{PS} via the formation of the condensate $\langle \bar{\mathcal{F}}_L \mathcal{F}_R \rangle$, i.e.,

$$\langle \bar{u}u + \bar{\nu}\nu \rangle = \langle \bar{d}d + \bar{e}e \rangle$$

These preserve SU(4)$_{PS}$ and break $\mathrm{SU(2)}_L \times \mathrm{SU(2)}_R$ to the diagonal vectorial SU(2)$_V$, giving a common mass to the gauge bosons corresponding to the axial generators, as before. This condensation produces a common dynamical mass $\sim \Lambda_{PS}$ for u, d, ν, and e. The SU(4)$_{PS}$-singlet bound states at this scale include mesons, (bosonic) baryons, and SU(4)$_{PS}$ glueballs. Since SU(4)$_{PS}$ confines, effectively, $Q = T_{3V}$.

Below Λ_{PS} the active gauge symmetry is SU(2)$_V$. There are no light fermions below this scale. The SU(2)$_V$ gauge interaction is asymptotically free and becomes strong at the much lower scale $\Lambda_V << \Lambda_{PS}$, producing a spectrum of glueballs with masses $\sim \Lambda_V$. There is no residual exact abelian gauge symmetry and hence no long-range force in the low-energy effective field theory at this scale.

Conclusions

We hope to have illustrated how one can gain useful insights into the Standard Model by constructing and analyzing variants of it. We have discussed several such variants:

- Models with dynamical EWSB provide an instructive alternative to the SM Higgs. The LHC should elucidate the source of EWSB
- SM variants with general N_c show the special role of $N_c = 3$
- SM variants with color-nonsinglet electroweak-singlet fermions
- SM variants in which QCD-induced EWSB is the dominant source of EWSB