

Gauge boson binaries amidst gluonic mesons a QCD odyssey

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Topics

Introductory words

- 1 **The two central anomalies : scale and chiral U1**
- 2 **The Cartan subalgebra injection and vacuum condensates**
- 3 **Consequences from the gauge boson bilinear vacuum expected value**

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List of contents

Introductory words	4
1-1 QCD – the two central anomalies : scale and chiral U1	6
The two central anomalies alongside : scale- or trace- and U1-axial anomaly	17
2-1 Consolidation and development of (new) methods adapted to the perturbatively accessible region – in a nutshell	19
2-2 The Cartan subalgebra injection and vacuum condensates	26
Consequences from the gauge boson bilinear vacuum expected value	30
R References	32

Fig-1

List of figures

Fig A21	$\alpha_s(Q) = 4\pi \kappa \bar{\mu} = Q$ from ref. [A215-2009] .	20
Fig A22	$\alpha_s(Q) = 4\pi \kappa \bar{\mu} = Q$ compared with Fig. A21 .	21
Fig A23	$m_q(Q) / m^*$ with fixed ratio of rescaled quark masses $m_u : \frac{1}{2}(m_d + m_u) : m_d = 3 : 4 : 5$.	22
Fig R22	The two representative Regge trajectories	26
Fig 4	boundary conditions are important	29

ΟΔΥΣΣΕΙΑ

Ἄνδρα μοι ἔννεπε, Μοῦσα, πολύτροπον, ὃς μάλα πολλὰ
 πλάγχθη, ἐπεὶ Τροίης Ἱερὸν πτολίεθρον ἔπερσε·
 πολλῶν δ' ἀνθρώπων ἦδεν ἄστεα καὶ νόον ἔγνω,
 πολλὰ δ' ὅ γ' ἐν πόντῳ πάθεν ἄλγεα ὃν κατὰ θυμόν,
 ἀρνύμενος ἦν τε ψυχὴν καὶ νόστον ἔταιρων.
 ἀλλ' οὐδ' ὡς ἐτάρους ἐρρύσατο, ίέμενός περ·
 αὐτῶν γὰρ σφετέρησιν ἀτασθαλίησιν ὅλοντο,
 νήπιοι, οἵ κατὰ βοῦς Ὑπερίονος Ἡελίοιο
 ἥσθιον· αὐτὰρ ὁ τοῖσιν ἀφείλετο νόστιμον ἥμαρ.

Homer's Odyssey : the beginning in original greek .

**Tell me, Muse, of that man, so ready at need, who wandered
 far and wide, after he had sacked the sacred citadel of
 Troy, and many were the men whose towns he saw and whose
 mind he learnt, yea, and many the woes he suffered in his
 heart upon the deep, striving to win his own life and the
 return of his company. Nay, but even so he saved not his
 company, though he desired it sore. For through the
 blindness of their own hearts they perished, fools, who
 devoured the oxen of Helios Hyperion:
 but the god took from them their day of returning.**

**Of these things,goddess,daughter of Zeus, whencesoever thou hast heard thereof,
 declare thou even unto us.**

Translation into english prose : S. H. BUTCHER, M.A. and A. LANG, M.A, Project Gutenberg 1999 .



I2

Sage mir Muse die Taten des vielgewanderten Mannes,
Welcher so weit geirrt, nach der heiligen Troia Zerstörung,
Vieler Menschen Städte gesehn und Sitte gelernt hat,
Und auf dem Meer so viel' unnennbare Leiden erduldet,
Seine Seele zu retten und seiner Freunde Zurückkunft.

Aber die Freunde rettet' er nicht, wie eifrig er strebte;
Denn sie bereiteten selbst durch Missetat ihr Verderben;
Toren! welche die Rinder des hohen Sonnenbeherrschers
Schlachteten; siehe, der Gott nahm ihnen den Tag der Zurückkunft.
Sage hievon auch uns ein weniges, Tochter Kronions.

Translation into german by Johann Heinrich Voss , Birkhäuser Verlag, Basel 1781 .

1-1

1-1 QCD – the two central anomalies : scale and chiral U1

Premises

We face the theoretical abstraction of QCD with $N_{fl} = 6$, representing strong interactions – adaptable to two or three light flavors (u, d, s) of quarks and antiquarks. \leftrightarrow

quarks : color is counted in $\pi^0 \rightarrow \gamma\gamma$

spin and flavor are clearly seen in $q\bar{q}$ and $3q$, $3\bar{q}$ spectroscopy.

$$(1) \quad \mathcal{L} = \left[\begin{array}{c} \bar{q} \dot{c}' \dot{\mathcal{B}} \dot{f} \left\{ \begin{array}{c} \frac{i}{2} \overleftrightarrow{\partial}_\mu \delta_{c'c} \\ - v_\mu^A \left(\frac{1}{2} \lambda^A \right)_{c'\dot{c}} \end{array} \right\} \gamma^\mu_{\dot{\mathcal{B}}\mathcal{A}} q^c_{\mathcal{A}f} \\ - m_f \bar{q} \dot{c} \dot{\mathcal{A}} \dot{f} q^c_{\mathcal{A}f} \\ - \frac{1}{4} g^2 F^{\mu\nu A} F^A_{\mu\nu} + \Delta \mathcal{L} \end{array} \right]$$

quarks : c' , $c = 1, 2, 3$ **color**, $f = 1, \dots, 6$ **flavor**

$\mathcal{B}, \mathcal{A} = 1, \dots, 4$ **spin**, m_f **mass**



1-2

gauge bosons :

$$\begin{aligned}
 F_{\mu\nu}^A &= \partial_\nu v_\mu^A - \partial_\mu v_\nu^A - f_{ABC} v_\nu^B v_\mu^C \\
 A, B, C &= 1, \dots, \dim(G = SU3_c) = 8 \\
 \text{Lie algebra labels, } [\frac{1}{2} \lambda^A, \frac{1}{2} \lambda^B] &= i f_{ABC} \frac{1}{2} \lambda^C
 \end{aligned}
 \tag{2}$$

perturbative rescaling :

$$v_\mu^A = g v_{\mu \text{ pert}}^A, F_{\mu\nu}^A = g F_{\mu\nu \text{ pert}}^A$$

Degrees of freedom are seen in jets , in (e.g.) the energy momentum sum rule in deep inelastic scattering but not clearly in spectroscopy.

Completing $\Delta \mathcal{L}$ in Fermi gauges

$$\begin{aligned}
 \Delta \mathcal{L} &= \left\{ \begin{array}{l} -\frac{1}{2\eta g^2} (\partial_\mu v^\mu{}^A)^2 \\ + \partial^\mu \bar{c}^A (D_\mu c)^A \end{array} \right\}; \quad \eta : \text{gauge parameter} \\
 \text{ghost fermion fields : } c, \bar{c} &; \quad (D_\mu c)^A = \partial_\mu c^A - f_{ABD} v_\mu^B c^D \\
 \text{gauge fixing constraint : } C^A &= \partial_\mu v^\mu{}^A
 \end{aligned}
 \tag{3}$$



Gauge boson binary bilocal and adjoint string operators

One goal is, to identify – not just some candidate resonance – gluonic mesons, binary and higher modes, and to relate them to the base quantities within QCD.

$$\begin{aligned}
 & B_{[\mu_1 \nu_1], [\mu_2 \nu_2]}(x_1, x_2) = \\
 (4) \quad & = F_{[\mu_1 \nu_1]}(x_1; A) U(x_1, A; x_2, B) F_{[\mu_2 \nu_2]}(x_2; B) \\
 & A, B, \dots = 1, \dots, 8 ; \text{ no flavor but spin}
 \end{aligned}$$

$F_{[\mu \nu]}(x; A)$ denote the color octet of field strengths.

The quantity $U(x, A; y, B)$ in eq. (4) denotes the octet string operator, i. e. the path ordered exponential over a straight line path \mathcal{C} from y to x

$$\begin{aligned}
 (5) \quad U(x, A; y, B) &= P \exp \left(\int_y^x \Big|_{\mathcal{C}} dz^\mu \frac{1}{i} v_\mu(z, D) \mathcal{F}_D \right)_{AB} \\
 (\mathcal{F}_D)_{AB} &= i f_{ADB}
 \end{aligned}$$

with the local limit



1-4

$$(6) \quad B_{[\mu_1 \nu_1], [\mu_2 \nu_2]}(x_1 = x_2 = x) = (:) F_{[\mu_1 \nu_1]}^A(x) F_{[\mu_2 \nu_2]}^A(x) (:)$$

no flavor but spin

The same procedure involving a triplet string applies to $\bar{q} q$ bilinears

$$(7) \quad B_{[\dot{\mathcal{A}} f_1, \mathcal{B} f_2]}^q(x_1, x_2) = \bar{q}_{\mathcal{B} \dot{f}_2}^{\dot{c}_1}(x_1) U(x_1, c_1; x_2, \dot{c}_2) q_{\dot{\mathcal{A}} f_1}^c(x_2)$$

$$U(x_1, c_1; x_2, \dot{c}_2) = P \exp \left(\int_y^x \Big|_{\mathcal{C}} dz^\mu \frac{1}{i} v_\mu(z, D) \frac{1}{2} \lambda_D \right)_{c_1 \dot{c}_2}$$

flavor and spin

with the local limit

$$(8) \quad B_{[\dot{\mathcal{B}} \dot{f}_2, \mathcal{A} f_1]}^q(x_1 = x_2 = x) = (:) \bar{q}_{\mathcal{B} \dot{f}_2}^{\dot{c}}(x) q_{\dot{\mathcal{A}} f_1}^c(x) (:)$$

The symbols $(:)$ in eqs. 6 and 8 should indicate that normal ordering or regulating the local limits is required and further that such normal ordering is *not unique*, and dependent on quark masses in the case of the $\bar{q} q$ bilinears .

→

1-5

The U1-axial central anomaly involves the local chiral current projections from $B_{[\dot{\mathcal{B}} f_2, \mathcal{A} f_1]}^q(x)$ in eq. 8

$$\begin{aligned}
 (j_\mu^\pm)_{\dot{f}_2 f_1}(x) &= B_{[\dot{\mathcal{B}} f_2, \mathcal{A} f_1]}^q(x) (\gamma_\mu \frac{1}{2} (\not{\epsilon} \pm \gamma_5))_{\dot{\mathcal{B}} \dot{\mathcal{A}}} \\
 (9) \quad &= (:) \bar{q}_{\dot{f}_2}^c \gamma_\mu^\pm q_{f_1}^c (x) (:)
 \end{aligned}$$

$$\gamma_5 = \gamma_{5R} = \frac{1}{i} \gamma_0 \gamma_1 \gamma_2 \gamma_3 ; \quad \gamma_\mu^\pm = \gamma_\mu \frac{1}{2} (\not{\epsilon} \pm \gamma_5)$$

The equations of motion for the fermion fields are and superficially imply (upon $f_1 \leftrightarrow f_2$)

$$\begin{aligned}
 \partial^\mu q_{f_2}^c &= \frac{1}{i} \left(\not{\epsilon}^c \dot{c}' + \delta^{c'} \dot{c} m_{f_2} \right) q_{f_2}^{c'} \\
 \overleftarrow{\bar{q}_{\dot{f}_1}^c} \overrightarrow{\partial} &= \bar{q}_{\dot{f}_1}^c \frac{1}{i} \left(-\not{\epsilon}^{c'} \dot{c} - \delta^{c'} \dot{c} m_{f_1} \right) ; \quad \text{no sums over } \dot{f}_1, f_2 \rightarrow \\
 \partial^\mu (j_\mu^\pm)_{\dot{f}_1 f_2} &= \frac{1}{2i} \left((m_{f_2} - m_{f_1}) S_{\dot{f}_1 f_2} \mp (m_{f_2} + m_{f_1}) P_{\dot{f}_1 f_2} \right) \\
 S_{\dot{f}_1 f_2} &= (:) \bar{q}_{\dot{f}_1}^c q_{f_2}^c (:), \quad P_{\dot{f}_1 f_2} = (:) \bar{q}_{\dot{f}_1}^c \gamma_5 q_{f_2}^c (:)
 \end{aligned}$$

(10)

In eq. 10 m_f denotes the real, nonnegative quark mass for flavor f.

→

From eq. 10 the relations for vector and axial vector currents superficially follow

$$\begin{aligned}
 (j_\mu)_{\dot{f}_1 f_2} &= (j_\mu^+)_{\dot{f}_1 f_2} + (j_\mu^-)_{\dot{f}_1 f_2} \\
 (j_\mu^5)_{\dot{f}_1 f_2} &= (j_\mu^+)_{\dot{f}_1 f_2} - (j_\mu^-)_{\dot{f}_1 f_2} \\
 \partial^\mu (j_\mu)_{\dot{f}_1 f_2} &= \frac{1}{i} (m_{f_2} - m_{f_1}) S_{\dot{f}_1 f_2} \\
 \partial^\mu (j_\mu^5)_{\dot{f}_1 f_2} &= (m_{f_2} + m_{f_1}) i P_{\dot{f}_1 f_2}
 \end{aligned} \tag{11}$$

As it follows from the original derivation by Adler and Bell and Jackiw [1-1-1969] in QED, the vector current Ward identities in eq. 11 can be implemented also in QCD, leaving the axial current ones reduced to the flavor non-singlet case, leaving the U(1) axial current divergent anomalous

$$\begin{aligned}
 \partial^\mu (j_\mu)_{\dot{f}_1 f_2} &= \frac{1}{i} (m_{f_2} - m_{f_1}) S_{\dot{f}_1 f_2} \quad \checkmark \\
 \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{\dot{f}_1 f_2}^{NS} &= \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{\dot{f}_1 f_2} - \frac{1}{N_{fl}} \delta_{\dot{f}_1 f_2} \sum_f \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{\dot{f}_f}
 \end{aligned} \tag{12}$$

→

and similarly

$$(13) \quad \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{f_1 f_2}^S = \sum_f \left\{ \begin{array}{c} j_\mu^5 \\ P \end{array} \right\}_{ff}$$

Quark masses and splittings : m_f and $\Delta m_f = m_f - \langle m \rangle$

In the subtitle above $\langle m \rangle$ stands for the mean quark mass

$$(14) \quad \langle m \rangle = \frac{1}{N_{fl}} \sum_f m_f$$

The identities for vector currents in eqs. 11 and 12 can be extended separating the contributions proportional to Δm_f and $\langle m \rangle$

$$(15) \quad \begin{aligned} \partial^\mu (j_\mu)_{f_1 f_2} &= \frac{1}{i} (\Delta m_{f_2} - \Delta m_{f_1}) S_{f_1 f_2} \quad \checkmark \\ \partial^\mu (j_\mu^5)_{f_1 f_2}^{NS} &= (\Delta m_{f_2} + \Delta m_{f_1}) i P_{f_1 f_2}^{NS} \quad \checkmark \\ \partial^\mu (j_\mu^5)_{f_1 f_2}^S &= 2 \langle m \rangle i P^S \quad \cancel{\checkmark} [\longrightarrow +\delta_5] \\ \delta_5 &= (2 N_{fl}) \frac{1}{32\pi^2} F_{\mu\nu}^A \tilde{F}^{\mu\nu A} \Big|_{\rightarrow ren.gr.inv} ; \quad \tilde{F}_{\mu\nu}^A = \frac{1}{2} \varepsilon_{\mu\nu\sigma\tau} F^{\mu\nu A} \end{aligned}$$

a

→

^a δ_5 was – as far as I know – introduced by Murray Gell-Mann in lectures ∼ 1970 in Hawaii.

The singlet axial current anomaly

We shall return to the question of how the local operator $ch_2(F) \equiv \frac{1}{32\pi^2}(:) F_{\mu\nu}^A \tilde{F}^{\mu\nu A}(:)$ is to be normalized and rendered renormalization group invariant [1-2-1991]. Here we just assume this to have been achieved and denote the U1-axial anomaly, the first of the central two, in its general form (eq. 15)

$$(16) \quad \left. \begin{aligned} \left\{ \partial^\mu (j_\mu^5)^S = 2 \langle m \rangle i P^S + \delta_5 \right\} (x) \\ \delta_5 = (2 N_{fl}) \frac{1}{32\pi^2} (:) F_{\mu\nu}^A \tilde{F}^{\mu\nu A}(:) \Big|_{\rightarrow ren.gr.inv} \end{aligned} \right.$$

From here it is conceptually clear how the scale- (or trace-) anomaly arises but strictly within QCD. The renormalizability of a field theory in the limit of uncurved space-time gives rise to a local, symmetric and conserved energy momentum tensor, implying exact Poincaré invariance

$$(17) \quad \begin{aligned} \{ \vartheta_{\mu\nu} = \vartheta_{\nu\mu} \} (x) \\ \partial^\nu \vartheta_{\mu\nu} = 0 \end{aligned}$$

In connection with the normal ordering questions it is important to admit in the precise form of the energy momentum tensor a nontrivial vacuum expected value, which



in view of exact Poincaré invariance must be of the form

$$(18) \quad \left\{ \begin{array}{l} \langle \Omega | \vartheta_{\mu\nu}(x) | \Omega \rangle = \frac{1}{4} \eta_{\mu\nu} \tau \\ \eta_{\mu\nu} = \text{diag} (1, -1, -1, -1) \\ \tau \end{array} \right\} \xrightarrow{\text{independent of } x}$$

$$\Delta \vartheta_{\mu\nu}(x) = \vartheta_{\mu\nu}(x) - \langle \Omega | \vartheta_{\mu\nu}(x) | \Omega \rangle \times \left\{ \begin{array}{l} \hat{\Psi} \\ \text{or } |\Omega\rangle\langle\Omega| \end{array} \right\}$$

$$\text{with } \partial^\nu \Delta \vartheta_{\mu\nu}(x) = 0 ; \quad \langle \Omega | \Delta \vartheta_{\mu\nu}(x) | \Omega \rangle = 0$$

In eq. 18 $\hat{\Psi}$ denotes the unit operator in the entire Hilbert space of states , while $P_\Omega = |\Omega\rangle\langle\Omega|$ stands for the projector on the ground state .

Furthermore from the two local , conserved tensors in eq. 18 only $\Delta \vartheta_{\mu\nu}(x)$ with vanishing vacuum expected value is acceptable as representing the conserved 4 momentum operators in the integral form

$$(19) \quad \widehat{P}_\mu = \int_t d^3 x \Delta_{\mu 0}(t, \vec{x})$$



All these arguments notwithstanding to subtract any eventual vacuum expected values of local operators , often put forward as mathematical prerequisites , it is wise *not to do so* in the presence of spontaneous parameters , the dynamical origin of spontaneous symmetry breaking, e.g. chiral symmetries in the limit or neighbourhood of some $m_f \rightarrow 0$.

Using the (classical) equations of motion pertaining to the Lagrangean in eqs. 1 - 3

$$\begin{aligned}
 (D_\nu F^{\mu\nu})^A &= j^\mu{}^A(\bar{q}, q) ; \quad F \rightarrow F_{pert} \\
 (D_\varrho F^{\mu\nu})^A &= \partial_\varrho F^{\mu\nu A} - f_{ABD} v_\varrho^B F^{\mu\nu D} \\
 (20) \quad j_\mu^A(\bar{q}, q) &= g \bar{q} \dot{\bar{q}}_{\dot{A}\dot{f}} (\gamma_\mu)_{\dot{A}\dot{B}} \frac{1}{2} (\lambda^A)_{c\dot{c}'} q_{\dot{A}f}^{c'} \\
 i (\gamma^\mu D_\mu q)^c_{\dot{A}f} &= m_f q_{\dot{A}f}^c \text{ and } q \rightarrow \bar{q} \\
 (D_\mu q)^c_{\dot{A}f} &= [\partial_\mu \delta_{c\dot{c}'} + i g v_\mu^D \frac{1}{2} (\lambda^D)_{c\dot{c}'}] q_{\dot{A}f}^{c'}
 \end{aligned}$$

the associated form of the energy momentum becomes



$$(21) \quad \vartheta_{\mu\nu}^{(cl)} = \left[F_{\mu\rho}^A F_{\nu}^{\rho A} - \frac{1}{4} \eta_{\mu\nu} F_{\sigma\rho}^A F_{\rho}^{\sigma A} + \right. \\ \left. + \frac{1}{2} \left[\bar{q}_f \gamma_{\mu} \frac{i}{2} \vec{D}_{\nu} q_f + \mu \leftrightarrow \nu \right] \right]$$

and using once more the fermion part of the equations of motion the trace of the classical energy momentum tensor becomes

$$(22) \quad \vartheta^{\mu}_{\nu}^{(cl)} = \sum_f m_f S_{\dot{f}f} \\ S_{\dot{f}_1 f_2} = (:) \bar{q}_{\dot{f}_1}^c q_{f_2}^c (:)$$

The scale- or trace- anomaly

From the classical soft fermionic contribution to the trace of the energy momentum tensor there is a clear conjecture , also by Murray Gell-Mann , of the anomalous contribution , which subsequently became the scale- or trace- anomaly within QCD

$$(23) \quad \vartheta^{\mu}_{\mu} = \sum_f m_f S_{\dot{f}f} + \delta_0 \\ \delta_0 = (-2\beta(g)/g^3) \left[-\frac{1}{4} (:) F_{\mu\nu}^A F^{\mu\nu A}(:) \right] \rightarrow ren.gr.inv$$



The two central anomalies alongside : scale- or trace- and U1-axial anomaly

We collect the two anomalous identities in eqs. 23 and 16

$$\begin{aligned}
 & \left\{ \vartheta^\mu_\mu = \sum_f m_f S_{f f} + \delta_0 \right\} (x) \\
 & \left\{ \partial^\mu (j_\mu^5)^S = 2 \langle m \rangle i P^S + \delta_5 \right\} (x) \\
 \delta_0 &= - (-2 \beta(g) / g^3) \left[\frac{1}{4} (:) F_{\mu\nu}^A F^{\mu\nu A} (:) \right] \xrightarrow{\text{ren.gr.inv}} \\
 (24) \quad \delta_5 &= (2 N_{fl}) \frac{1}{8\pi^2} \left[\frac{1}{4} (:) F_{\mu\nu}^A \tilde{F}^{\mu\nu A} (:) \right] \xrightarrow{\text{ren.gr.inv}}
 \end{aligned}$$

$$-\beta/g^3 = \frac{1}{16\pi^2} b_0 + O(Y); \quad Y = g^2 / (16\pi^2)$$

$\beta(g)$: Callan-Symanzik rescaling function in QCD

The qualification 'central' for the anomalies in eq. 24 stands for the property that in rendering the square coupling constant and the associated ϑ – parameter in the gauge boson renormalized Lagrangean density x dependent

$$\begin{aligned}
 (25) \quad \mathcal{L}_{g.b.} &= -\frac{1}{g^2} \frac{1}{4} (:) F_{\mu\nu}^A F^{\mu\nu A} (:) + \vartheta \frac{1}{8\pi^2} \frac{1}{4} (:) F_{\mu\nu}^A \tilde{F}^{\mu\nu A} \longrightarrow \\
 g^2 &\rightarrow g^2(x); \quad \vartheta \rightarrow \vartheta(x)
 \end{aligned}$$

maintains perturbative renormalizability and acts together with suitable boundary conditions



as external sources for the scalar and pseudoscalar local field strength bilinears

$$(26) \quad \frac{1}{4} (:) F_{\mu\nu}^A F^{\mu\nu A} (:) , \quad \frac{1}{4} (:) F_{\mu\nu}^A \tilde{F}^{\mu\nu A}$$

We will use the following definitions relative to the rescaling function β

$$-\beta/g = X B(X) ; \quad B(X) = b_0 A(X)$$

$$B(X) \sim \sum_{n=0}^{\infty} b_n X^n , \quad A(X) \sim \sum_{n=0}^{\infty} a_n X^n$$

$$\kappa = g^2 / (16\pi^2) \text{ generic } X, Y$$

$$(27) \quad b_0 = \frac{1}{3} (33 - 2N_{fl}) , \quad a_0 = 1 , \quad a_n = b_n / b_0$$

$$b_1 = \frac{2}{3} (153 - 19N_{fl})$$

$$b_2 = \frac{1}{54} (77139 - 15099N_{fl} + 325N_{fl}^2)$$

$$b_3 \sim 29243 - 6946.3N_{fl} + 405.089N_{fl}^2 + 1.49931N_{fl}^3$$

2-1 Consolidation and development of (new) methods adapted to the perturbatively accessible region – in a nutshell ^a

Three figures subsuming the work of many authors including those quoted below shall be *shortly* discussed



^a see the talks at this conference by D. Gross, J. Stirling, S. Frixione, W. Vogelsang, S. Bethke, N. Glover, J. Vermaseren, J. Kühn, K. Chetyrkin, S.-O. Moch, M. Steinhauser, M. Czakon, P. Uwer, Z. Bern, L. Dixon, T. Gehrmann and M. Neubert .

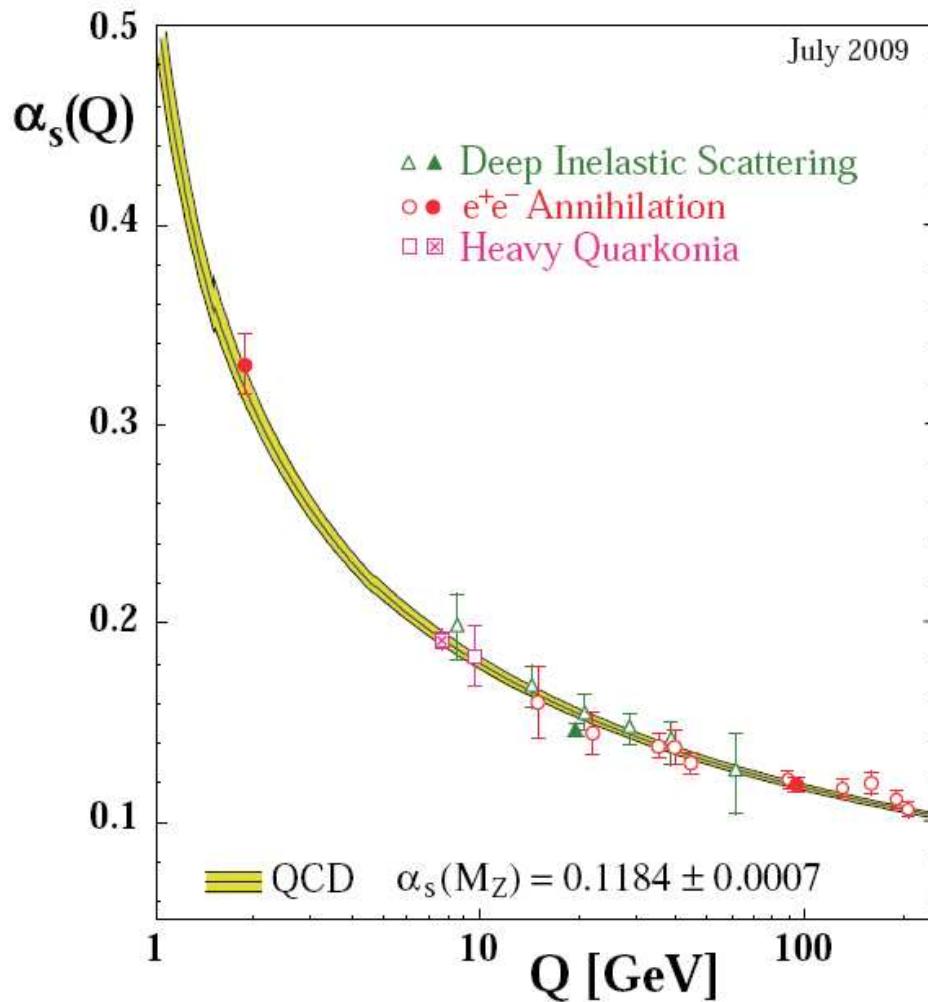


Fig A21 : $\alpha_s(Q) = 4\pi \kappa \bar{\mu} = Q$ from ref. [A215-2009].

A2-44b

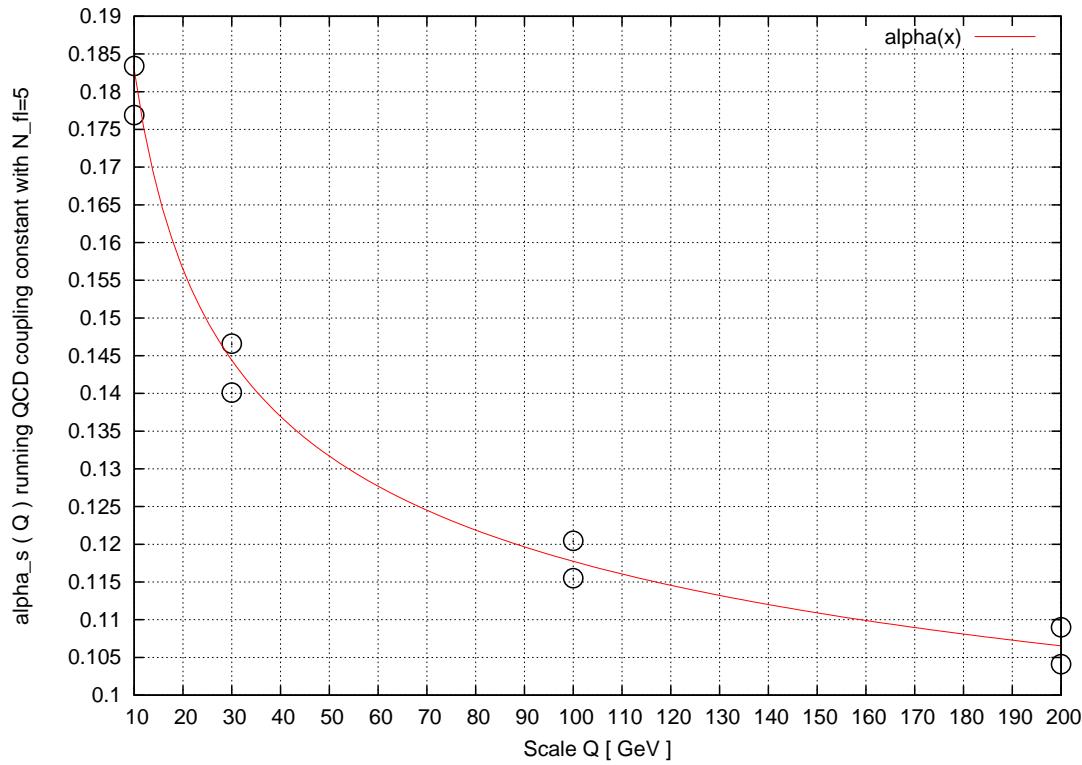


Fig A22 : $\alpha_s(Q) = 4\pi \kappa_{\bar{\mu}} = Q$ **compared with Fig. A21 .**



A2-44c

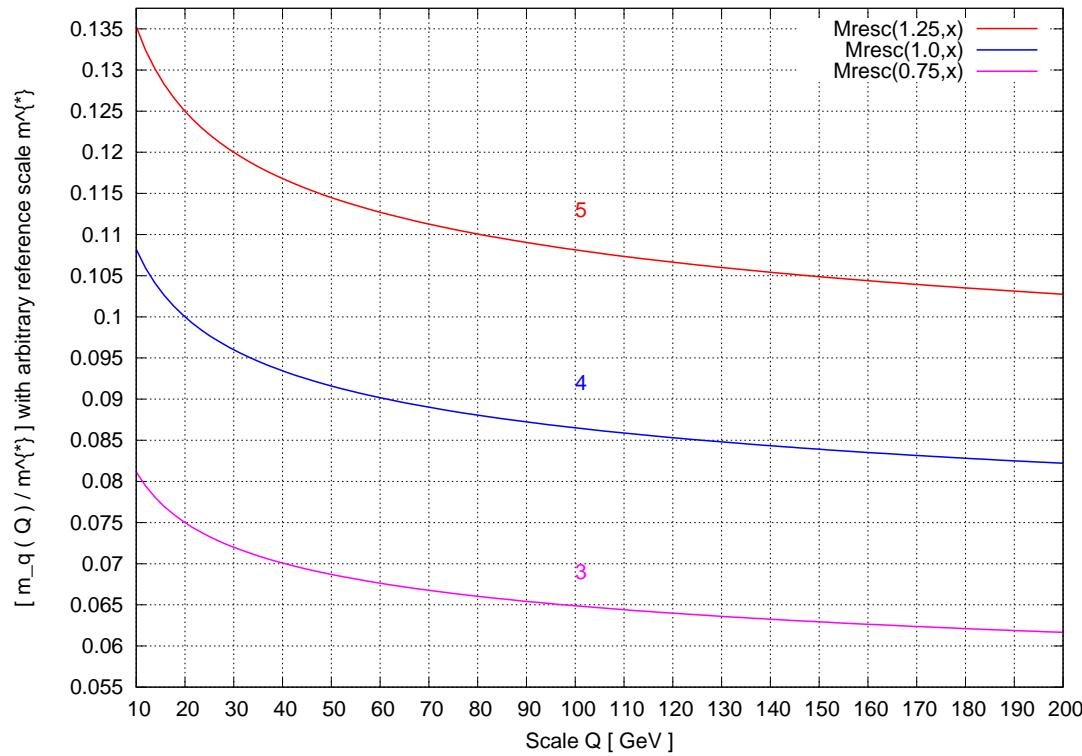


Fig A23 : $m_q(Q) / m^*$ with fixed ratio of rescaled quark masses

$$m_u : \frac{1}{2} (m_d + m_u) : m_d = 3 : 4 : 5 .$$



To Fig A21 : In the four loop evaluation of the running coupling constant in ref. [A215-2009] the renormalization group invariant quantity is obtained in the \overline{MS} renormalization scheme

$$(28) \quad \Lambda_5^{(4)} = 213 \pm 9 \text{ MeV} \longleftrightarrow \alpha_s(m_Z) = 0.1184 \pm 0.0007$$

$$\Lambda_5^{(4)} = 213 \text{ MeV} \rightarrow \begin{cases} \Lambda_4^{(4)} = 296 \text{ MeV} \\ \Lambda_3^{(4)} = 338 \text{ MeV} \end{cases}$$

The matching between $N_{fl} = 5 \rightarrow 4 \rightarrow 3$ in ref. [A215-2009] involves the modeling of the b- and c-flavor associated thresholds through the perturbatively assigned b- and c-quark pole-masses $m_b = 4.7 \text{ GeV}$, $m_c = 1.5 \text{ GeV}$. This is a nonuniversal way to rescale quark masses , and thus does not follow the strict quark mass rescaling at zero quark mass , used here . As a comparison in determining up and down quark masses at an \overline{MS} scale of 2 GeV , Dominguez, Nasrallah, Röntsch and Schilcher [A223-2008] use

$$(29) \quad \Lambda_3^{(4)} = 381 \pm 16 \text{ MeV} \leftrightarrow \alpha_s(m_\tau) = 0.344 \pm 0.009$$

and adopting the scheme of quark mass rescaling at zero mass obtain for the u,d,s quark mass ratios

$$(30) \quad \begin{array}{ccccccccc} m_u & : & \frac{1}{2}(m_d + m_u) & : & m_d & : & m_s \\ 2.9 \pm 0.2 & : & 4.1 \pm 0.2 & : & 5.3 \pm 0.5 & : & 102 \pm 8 \end{array}$$



To Fig A22 : The following value was used : $\alpha_s(M_Z) = 0.184$ corresponding – for the two loop running – to $\Lambda_5^{(2)} \sim 408 \text{ MeV}$. The so determined running coupling constant is compared with the $1 - \sigma$ limits of the same quantity as determined in 4 loop order in ref. [A215-2009] confirming the validity of the two loop approximation in the range $10 \text{ GeV} \leq Q \leq 200 \text{ GeV}$ within the accuracy claimed in ref. [A215-2009].

To Fig A23 : Here the strength and weakness of the mass rescaling at zero mass within the perturbatively accessible region is illustrated using as a guide *only* the ratio of u,d quark masses

$$(31) \quad \begin{array}{c} m_u : \frac{1}{2}(m_d + m_u) : m_d \\ 3 : 4 : 5 \end{array}$$

It seems appropriate to me to refer to the *in principle* approach of rescaling in a universal way the coupling constant and quark masses *initially* restricting all analysis to the perturbatively accessible region , citing (adapting) the pertinent comment by Murray Gell-Mann : 'Rising when last (first) seen .'

On the other hand the progress achieved in transgressing the perturbatively accessible region , using universal mass rescaling , in refs. [A223-2008] , [A221-2006] and references cited therein, is significant, based on improved treatment of finite energy sum rules pioneered by Shifman , Vainshtain and Zakharov [A224-1979] .



A2-47

To Fig A23 *continued* : It is worth noting that the value of the gauge boson condensate, found in ref. [A223-2008] , approximated as

$$(32) \quad \langle \Omega | \frac{\alpha_s}{\pi} : F_{\mu\nu}^A F^{\mu\nu A} : | \Omega \rangle \rightarrow 0.06 \text{ GeV}^4$$

is 5 times larger , than its original estimate in ref. [A224-1979] .

The basics of chiral expansions in assessing ratios of the u,d,s quark masses continue to provide additional benchmarks at low hadron energies [A225-2001] and references cited therein, while fine details of these ratios can be subject to improvement . Finally the validity of chiral expansions as guidelines for lattice calculations present another *strategy in principle* [A226-2008] .

We add here a few representative determination of $\alpha_s (m_Z)$

	$\alpha_s (m_Z)$	processes	source	authors
(33)	0.1176 ± 0.0020	average	[A227-2008]	PDG
	0.1172 ± 0.0022	thrust distributions at LEP	[A228-2008]	Becher , Schwartz
	0.1184 ± 0.0007	average	[A215-2009]	Bethke

2-2 The Cartan subalgebra injection and vacuum condensates

The two representative Regge trajectories and their challenge

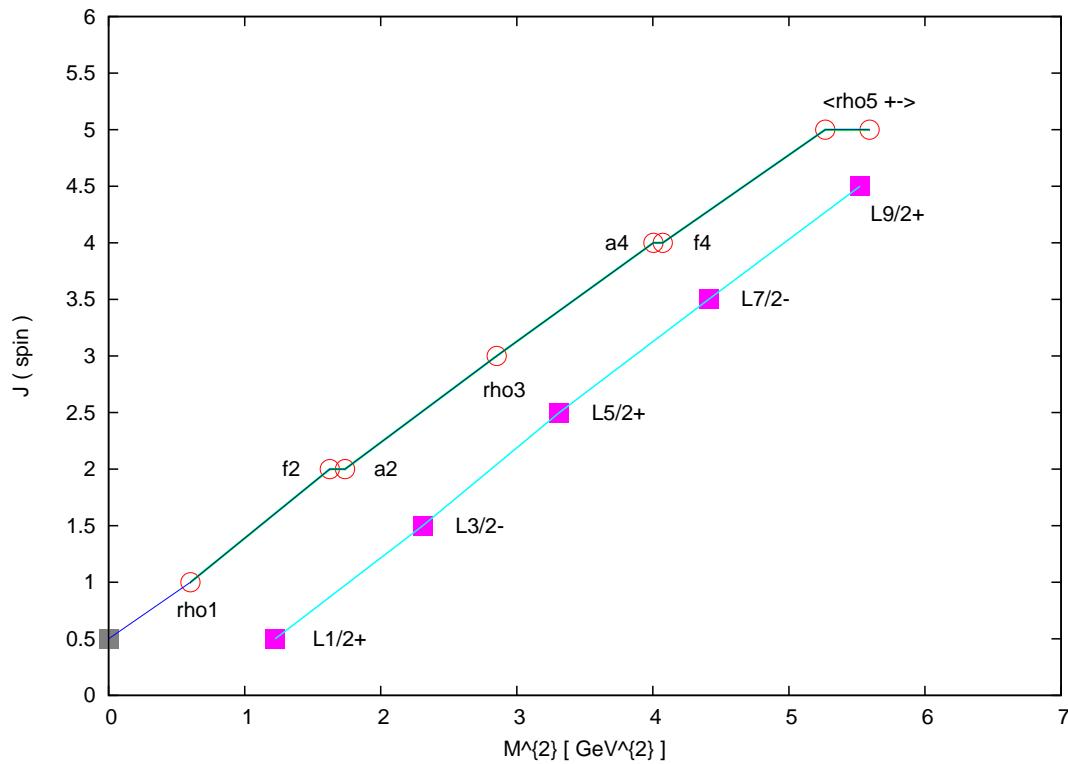


Fig R22 : The two representative Regge trajectories : $\varrho = f/a$; $J^{PC} (1,3,5) ---$ and $J^{PC} (2,4) ++$ together with Λ ; $J^P (\frac{1}{2}, \frac{5}{2}, \frac{9}{2})^+$ and $J^P (\frac{3}{2}, \frac{7}{2})^-$.



2-2-1

The challenge has been met and invokes the search for 'harmonic analysis' in the framework of QCD and pertaining to $q \bar{q}$ and $3 q$ configurations . These configurations shall con-form with the (apparently) universal spectroscopic relation as shown in figure R22 and idealized to

$$A(s) = A' s + A_0 \leftrightarrow s = (A')^{-1} [A(s) - A_0]$$

(34) $A(s) = J \rightarrow m_J^2 = (A')^{-1} J + m_0^2 ; = M^2 J + m_0^2$

$M^2 = (A')^{-1} \sim 1.12 \text{ GeV}^2$ for $q_1 \bar{q}_2$ and $q_1 q_2 q_3$ $m_q \rightarrow 0$ configurations

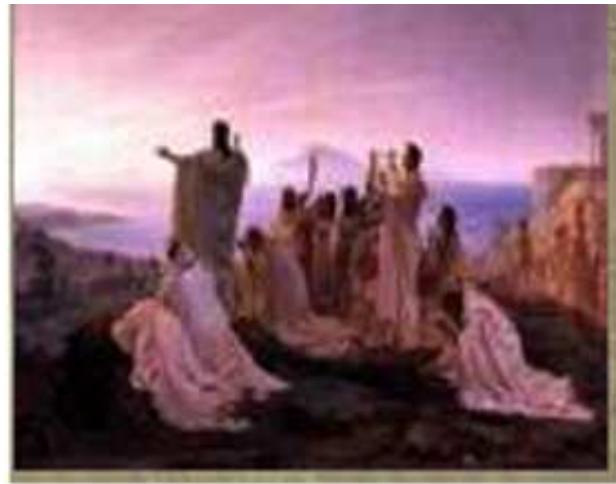


Fig 3 : 'the Pythagoreans'



2-2-2

Let me repeat the central anomalies here (eq. 24) and transform to euclidean space time

$$\begin{aligned}
 \left\{ \vartheta^\mu{}_\mu = \sum_f m_f S_{\dot{f} f} + \delta_0 \right\} (x) \\
 \left\{ \partial^\mu (j_\mu^5)^S = 2 \langle m \rangle i P^S + \delta_5 \right\} (x) \\
 \delta_0 = - (-2\beta(g)/g^3) \left[\frac{1}{4} (:) F_{\mu\nu}^A F^{\mu\nu A} (:) \right] \xrightarrow{\text{ren.gr.inv}} \\
 (35) \quad \left[\frac{1}{4} (:) F_{\mu\nu}^A F^{\mu\nu A} (:) \right] \xrightarrow{\text{ren.gr.inv}} \equiv \frac{1}{2} (:) \left[(\vec{\mathcal{B}}^A)^2 + (\vec{\mathcal{E}}^A)^2 \right] (:) \Big|_{\text{eucl}} \\
 \left[\frac{1}{4} (:) F_{\mu\nu}^A \tilde{F}^{\mu\nu A} (:) \right] \xrightarrow{\text{ren.gr.inv}} \equiv (:) \left[\vec{\mathcal{B}}^A \vec{\mathcal{E}}^A \right] (:) \Big|_{\text{eucl}}
 \end{aligned}$$

$(\vec{\mathcal{B}}^A, \vec{\mathcal{E}}^A)_{\text{eucl}}$: **euclidean magnetic- and electric field strengths**

$(:),_{\text{eucl}}$: **to be suppressed in the following**

This brings up the question – for me long ‘undecidable’ until the seminal work of Shifman, Vainshtein and Zakharov [A224-1979] induced a new try to the affirmative [in refs. [2-21-1978] and [2-22-1980]] – whether the positive parity operator $\frac{1}{4} F_{\mu\nu}^A F^{\mu\nu A}$ could develop a vacuum expected value . \longrightarrow

2-2-3

Indeed

$$(36) \quad \begin{aligned} B^* \langle \Omega | \frac{1}{4} F_{\mu\nu}^A F^{\mu\nu A} | \Omega \rangle_{B^*} &= p \equiv (B^*)^2 > 0 \\ B^* = \frac{1}{2} M^2 &\sim m_\varrho^2 \sim 0.60 \pm 0.007 \text{ GeV}^2 \quad \rightarrow \\ p = \frac{1}{4} M^4 &\sim m_\varrho^4 \sim 0.36 \pm 0.01 \text{ GeV}^4 \end{aligned}$$

The estimate from charmonium sum rules (ref. [A224-1979]) was $p \sim 0.12 \text{ GeV}^4$, while in a recent evaluation of the u-d flavor currents and (pseudo-) scalar densities $S_{f_1 f_2}$ and $P_{f_1 f_2}$ by Dominguez, Nasrallah, Röntsch and Schilcher in ref. [A223-2008] approximated as (eq. 32)

$$(37) \quad \langle \Omega | \frac{\alpha_s}{\pi} : F_{\mu\nu}^A F^{\mu\nu A} : | \Omega \rangle = \frac{1}{\pi^2} p \rightarrow 0.06 \text{ GeV}^4$$

yields $p \sim 0.59 \text{ GeV}^4$, (almost) 5 times larger than the original estimate .

The fields $(V_\mu^a)_\infty$, $(V_{\mu\nu}^a)_\infty$ in eq. (1) are extrema of the classical action subject to the constraint

$$\int \left[\frac{1}{4} V_{\mu\nu}^a V_{\mu\nu}^a - p \right] d^4x = \text{finite},$$

$$p = \text{constant} = 4(R_\infty)^2. \quad (4)$$

The constant p in eq. (4) with the dimension of pressure [i.e., (mass)⁴] is related to the gluon pairing strength—a spontaneous parameter of the ground state:

$$\langle \Omega | (: V_{\mu\nu}^a(x) V_{\alpha\beta}^b(x) :)_{\text{ph}} | \Omega \rangle = \frac{1}{3} \delta_{ab} \left\{ [g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}] \frac{1}{3} p - \epsilon_{\mu\nu\alpha\beta} \frac{1}{3} q \right\}. \quad (5)$$

Fig 4 : boundary conditions are important (from ref. [2-22-1980] with $V ; a,b \rightarrow F ; A,B$)

and gauge group $SU2_c \rightarrow$

2-2-4

Consequences from the gauge boson bilinear vacuum expected value (eq. 36)

$$(38) \quad B^* \langle \Omega | \frac{1}{4} F_{\mu\nu}^A F^{\mu\nu A} | \Omega \rangle_{B^*} = p \equiv (B^*)^2 > 0$$

- 1) $p > 0$ in eq. 38 represents a local, gauge invariant Bose *field pair*- condensation, coupled with the full Poincaré invariance pertaining to the ground state.
- 2) *single field quantum*- Bose condensation is forbidden by exact gauge invariance.
- 3) the symmetries including C, P, T of the ground state in 1) and 2) enforce the spin structure

$$\begin{aligned}
 & B^* \langle \Omega | \frac{1}{4} F_{\mu\nu}^A F^{\sigma\tau A} | \Omega \rangle_{B^*} = X_{\mu\nu}^{\sigma\tau} = \frac{1}{3} (\delta_{\mu}^{\sigma} \delta_{\nu}^{\tau} - \delta_{\mu}^{\tau} \delta_{\nu}^{\sigma}) p \\
 & Y_{\mu}^{\sigma} = X_{\mu\varrho}^{\sigma\varrho} = \delta_{\mu}^{\tau} p, \quad Z = Y_{\mu}^{\mu} = 4p; \quad q = 0 \quad \rightarrow \\
 & \vartheta_{\mu}^{\sigma} = (\vartheta_{\mu}^{\sigma} - \frac{1}{4} \delta_{\mu}^{\sigma} \vartheta_{\varrho}^{\varrho}) + \frac{1}{4} \delta_{\mu}^{\sigma} \vartheta_{\varrho}^{\varrho} \\
 & B^* \langle \Omega | \vartheta_{\mu}^{\sigma} - \frac{1}{4} \delta_{\mu}^{\sigma} \vartheta_{\varrho}^{\varrho} | \Omega \rangle_{B^*} = 0 \\
 (39) \quad & \vartheta_{\mu}^{\sigma} |_{spin \ 0} = \frac{1}{4} \delta_{\mu}^{\sigma} \vartheta_{\varrho}^{\varrho} = - \frac{1}{\pi^2} \frac{b_0}{8} \delta_{\mu}^{\sigma} \left[\frac{1}{4} F_{\alpha\beta}^A F^{\alpha\beta A} \right] \quad \rightarrow \\
 & B^* \langle \Omega | \vartheta_{\mu\nu} | \Omega \rangle_{B^*} = (-\eta_{\mu\nu}) \frac{1}{4\pi^2} \frac{b_0}{8} p = \begin{cases} \varepsilon \text{ for } \mu = \nu = 0 \\ \tilde{p} \text{ for } \mu = \nu = 1, 2, 3 \end{cases}
 \end{aligned}$$

with $\tilde{p} = -\varepsilon > 0$



2-2-5

- 3) (continued) – the last two relations in eq. 39 constitute a cosmological term with *positive pressure and negative energy density* , defying a response from gravity , whence restricted to four space-time dimensions , yet consistent with the ground state belonging to *the minimum of energy density* .
- 4) the embedding of chiral symmetry depends in a nontrivial way on the strength of the *field pair*- Bose condensate (in eq. 39) as does the phase structure of QCD and the excitation of binary and higher gauge boson compounds ('glueballs') . The experimental verification of their spectral properties however is reminiscent of the 'odyssey' – and thus shall be continued in due time .

— Thank you —

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