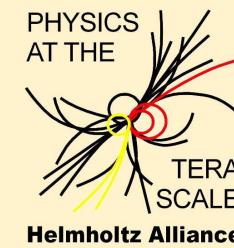
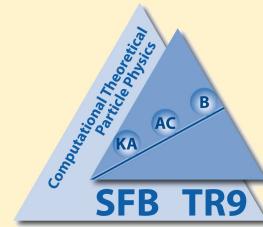


Two-loop QCD corrections to B meson decays

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Outline

- Introduction and theoretical framework of non-leptonic B decays
- Technical details of two-loop calculation
- Results on non-leptonic B decays
- Semi-leptonic B decays: determination of V_{ub} at NNLO

Introduction

- Non-leptonic B decays offer a rich and interesting phenomenology
 - Large data sets from B -factories, in the future from LHCb, possibly SuperB
 - $\mathcal{O}(100)$ final states. Numerous observables: BR, CP asymmetries, polarisations . . .
 - Test of CKM mechanism (CP violation), New Physics?

Theory (here QCDF)

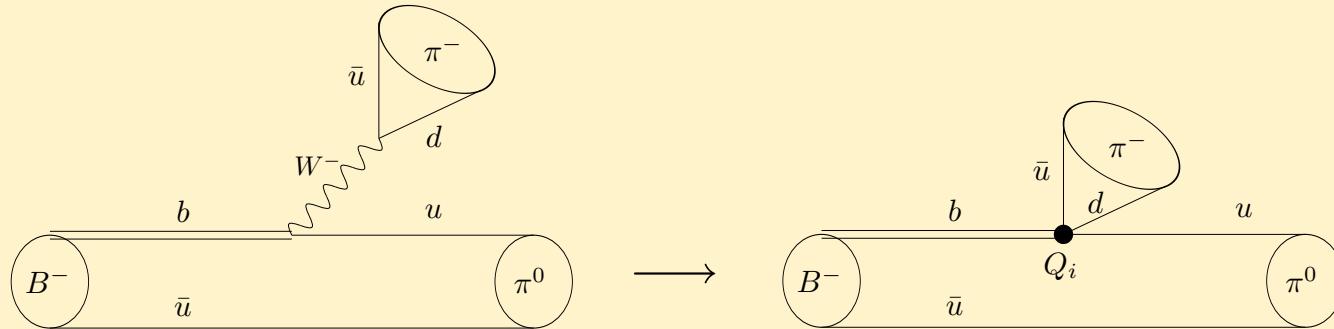
$$\begin{aligned}\mathcal{B}(B^- \rightarrow \pi^-\pi^0) &= (5.5 \pm 1.0) \times 10^{-6} \\ \mathcal{B}(\bar{B}^0 \rightarrow \pi^+\pi^-) &= (5.0 \pm 1.2) \times 10^{-6} \\ \mathcal{B}(\bar{B}^0 \rightarrow \pi^0\pi^0) &= (0.73 \pm 0.54) \times 10^{-6} \\ &\quad [Beneke, Jäger '05] \\ \mathcal{B}(\bar{B}^0 \rightarrow \rho^0\rho^0) &= (0.9 \pm 1.4) \times 10^{-6} \\ &\quad [Beneke, Rohrer, Yang '06] \\ \mathcal{A}_{CP}(\bar{B}^0 \rightarrow \pi^+\pi^-) &= 0.103 \\ \mathcal{A}_{CP}(\bar{B}^0 \rightarrow \pi^0\pi^0) &= -0.190 \\ &\quad [Beneke, Neubert '03]\end{aligned}$$

Experiment

$$\begin{aligned}\mathcal{B}(B^- \rightarrow \pi^-\pi^0) &= (5.7 \pm 0.5) \times 10^{-6} \\ \mathcal{B}(\bar{B}^0 \rightarrow \pi^+\pi^-) &= (5.13 \pm 0.24) \times 10^{-6} \\ \mathcal{B}(\bar{B}^0 \rightarrow \pi^0\pi^0) &= (1.62 \pm 0.31) \times 10^{-6} \\ \mathcal{B}(\bar{B}^0 \rightarrow \rho^0\rho^0) &= (1.1 \pm 0.4) \times 10^{-6} \\ \mathcal{A}_{CP}(\bar{B}^0 \rightarrow \pi^+\pi^-) &= 0.38 \pm 0.07 \\ \mathcal{A}_{CP}(\bar{B}^0 \rightarrow \pi^0\pi^0) &= 0.48 \pm 0.30 \\ &\quad [PDG '08]\end{aligned}$$

- Problems with “colour-suppressed” tree-dominated decays (e. g. $\bar{B}^0 \rightarrow \pi^0\pi^0$).

Effective theory for B decays



- Effective Hamiltonian:

[Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Münz '98]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^6 C_k Q_k + C_8 Q_8 \right] + h.c.$$

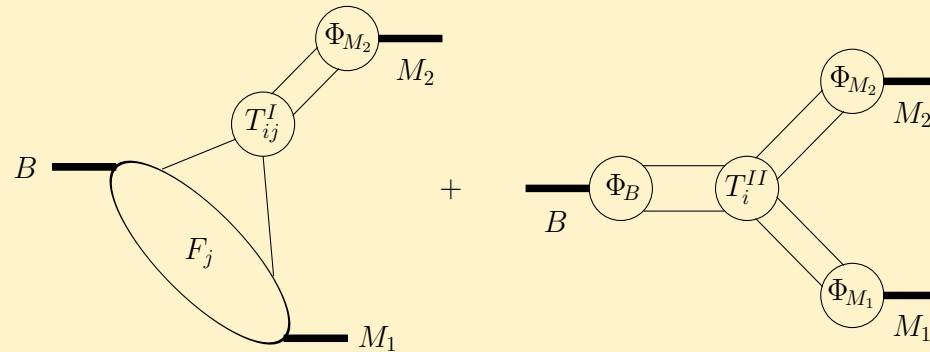
$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \quad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \sum_q (\bar{q} \gamma_\mu T^a q) \quad Q_8 = -\frac{g_s}{16\pi^2} m_b \bar{d}_L \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \quad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \sum_q (\bar{q} \gamma_\mu q) \quad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \quad \lambda_p = V_{pb} V_{pd}^*$$

- To be supplemented by evanescent operators (vanish in 4 dim., but not in D dim.)
 - Required to make the system closed under renormalisation
- Can use naïvely anticommuting γ_5 in dim. reg. in CMM basis

QCD factorisation



- Theoretical description of non-leptonic B decays difficult due to complicated QCD effects in the purely hadronic final state
- Simplification in the limit $m_b \gg \Lambda_{\text{QCD}}$

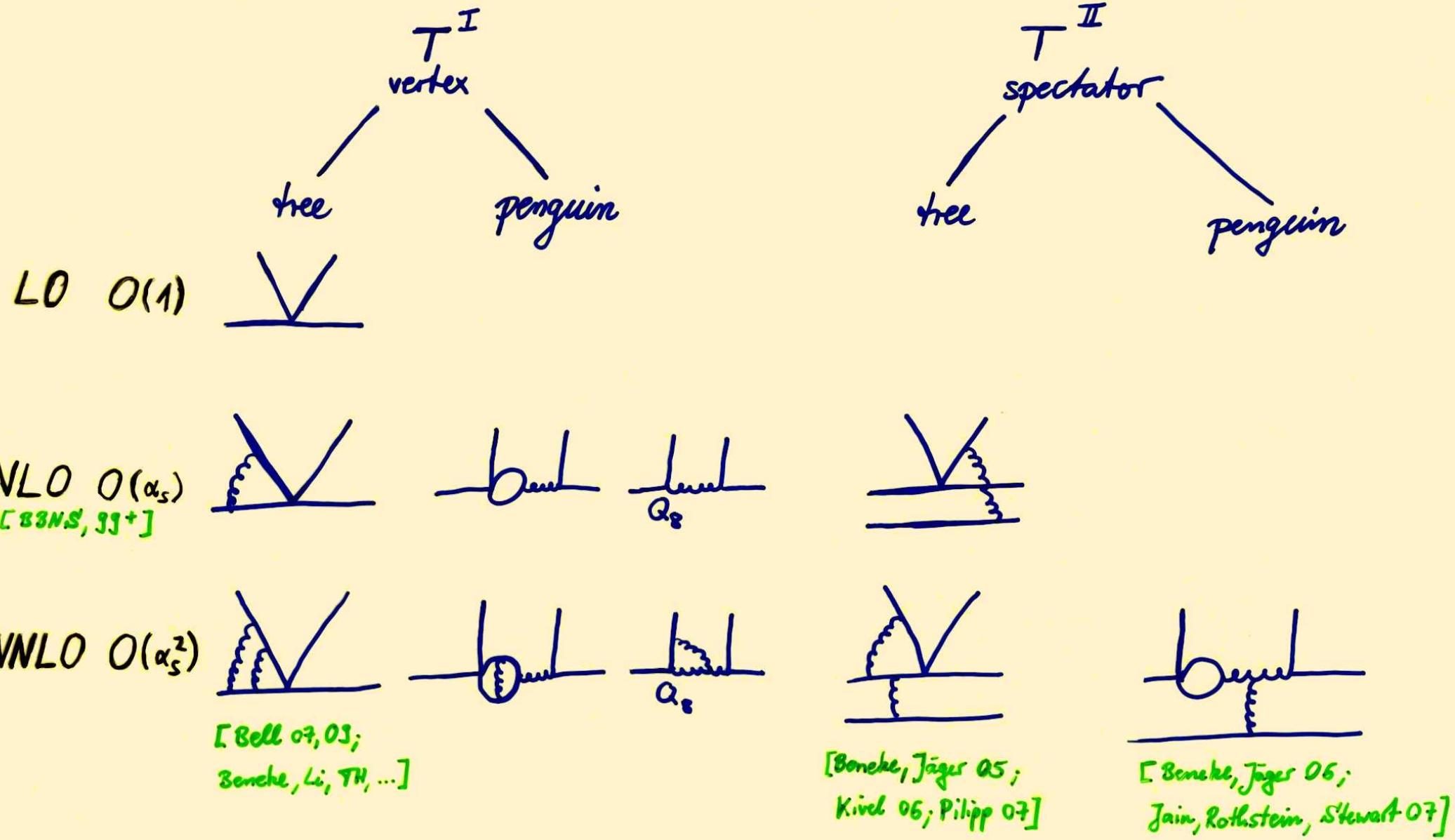
[Beneke, Buchalla, Neubert, Sachrajda '99-'04]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 F_+^{B \rightarrow M_1}(0) f_{M_2} \int_0^1 du \ T_i^I(u) \phi_{M_2}(u)$$

$$+ f_B f_{M_1} f_{M_2} \int_0^1 d\omega dv du \ T_i^{II}(\omega, v, u) \phi_B(\omega) \phi_{M_1}(v) \phi_{M_2}(u)$$

- $T^{I,II}$: Hard scattering kernels, perturbatively calculable. $T^{II} = \mathcal{O}(\alpha_s)$
- F_+ : $B \rightarrow M$ form factor
- f_i : decay constants
- ϕ_i : light-cone distribution amplitudes

QCD factorisation



moreover: „right“ vs. „wrong“ insertion

QCD factorisation, motivation for NNLO

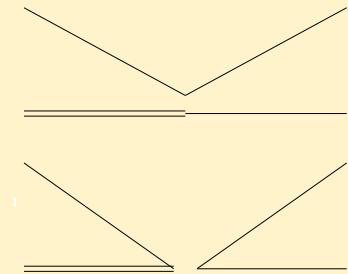
$$\sqrt{2} \langle \pi^- \pi^0 | \mathcal{H}_{eff} | B^- \rangle = \lambda_u [\alpha_1(\pi\pi) + \alpha_2(\pi\pi)] A_{\pi\pi}$$

$$\langle \pi^+ \pi^- | \mathcal{H}_{eff} | \bar{B}^0 \rangle = \{\lambda_u [\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi)] + \lambda_c \alpha_4^c(\pi\pi)\} A_{\pi\pi}$$

$$- \langle \pi^0 \pi^0 | \mathcal{H}_{eff} | \bar{B}^0 \rangle = \{\lambda_u [\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi)] - \lambda_c \alpha_4^c(\pi\pi)\} A_{\pi\pi}$$

[Beneke, Neubert '03]

- α_1 : colour-allowed tree amplitude, “right insertion”
- α_2 : colour-suppressed tree amplitude, “wrong insertion”



$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010i]_{NLO} - \left[\frac{r_{sp}}{0.485} \right] \left\{ [0.015]_{LOSp} + [0.009]_{tw3} \right\} = 1.008^{+0.034}_{-0.072} + (0.010^{+0.025}_{-0.051})i$$

$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{NLO} + \left[\frac{r_{sp}}{0.485} \right] \left\{ [0.123]_{LOSp} + [0.072]_{tw3} \right\} = 0.236^{+0.228}_{-0.135} + (-0.077^{+0.115}_{-0.082})i$$

[Beneke, Buchalla, Neubert, Sachrajda '99, '01; Beneke, Neubert '03; Beneke, Jäger '05, '06; Kivel '06; Pilipp '07; Bell '07]

[Hill, Becher, Lee, Neubert '04; Becher, Hill '04; Kirilin '05; Beneke, Yang '05]

- Large cancellation in LO + NLO in α_2 , particularly sensitive to NNLO
- Direct CP asymmetries start at $\mathcal{O}(\alpha_s)$, NNLO is only the first correction
- Q: Does factorization hold? Does NNLO QCDF tend toward the right direction?
- Goal: $\mathcal{O}(\alpha_s^2)$ vertex corrections to α_1 and $\alpha_2 \Leftrightarrow$ 2-loop matrix elements of Q_1, Q_2

Matching of QCD onto SCET

- Consider matrix element of $Q_1 = (\bar{d}_L \gamma^\mu T^a u_L)(\bar{u}_L \gamma_\mu T^a b_L)$. One needs

$$\langle Q_1 \rangle = \sum_a H_{1a} O_a \quad \text{if} \quad M_2 = [\bar{d}u]$$

$$\langle Q_1 \rangle = \sum_a \tilde{H}_{1a} \tilde{O}_a \quad \text{if} \quad M_2 = [\bar{u}u]$$

- O_1 is the only physical SCET operator. It factorizes into form factor times LCDA
- Fierz(\tilde{O}_1) = O_1 in $D = 4$
- $O_{2,3}$, $\tilde{O}_{2,3}$ and $\tilde{O}_1 - O_1$ are evanescent and must be renormalized to zero

Right insertion

$$O_1 = [\bar{\chi} \frac{\not{q}_-}{2} (1 - \gamma_5) \chi] [\bar{\xi} \not{\eta}_+ (1 - \gamma_5) h_v]$$

$$O_2 = [\bar{\chi} \frac{\not{q}_-}{2} (1 - \gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi] [\bar{\xi} \not{\eta}_+ (1 - \gamma_5) \gamma_\beta^\perp \gamma_\alpha^\perp h_v]$$

$$O_3 = [\bar{\chi} \frac{\not{q}_-}{2} (1 - \gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi] [\bar{\xi} \not{\eta}_+ (1 - \gamma_5) \gamma_\delta^\perp \gamma_\gamma^\perp \gamma_\beta^\perp \gamma_\alpha^\perp h_v]$$

Wrong insertion

$$\tilde{O}_1 = [\bar{\xi} \gamma_\perp^\alpha (1 - \gamma_5) \chi] [\bar{\chi} (1 + \gamma_5) \gamma_\alpha^\perp h_v]$$

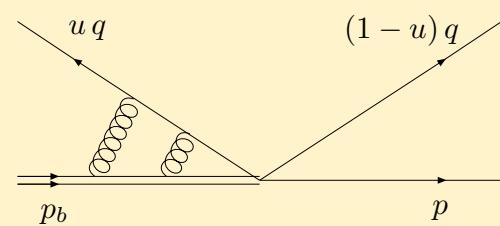
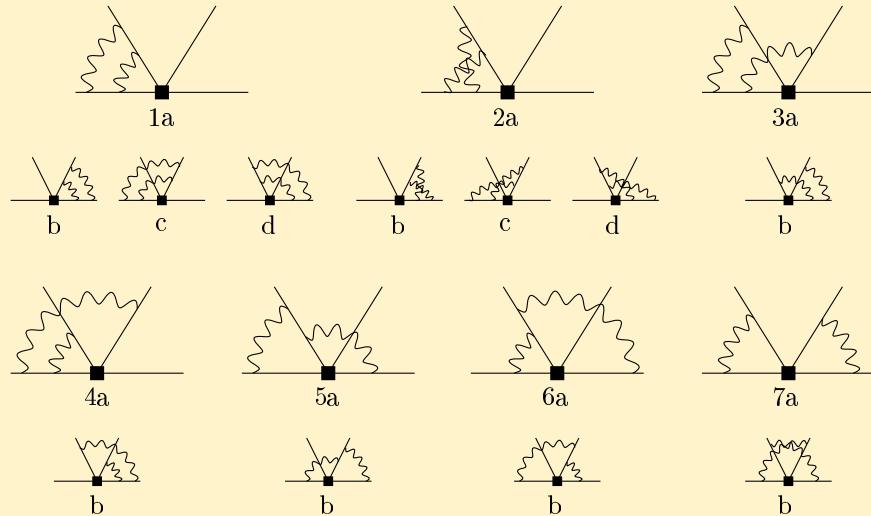
$$\tilde{O}_2 = [\bar{\xi} \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma (1 - \gamma_5) \chi] [\bar{\chi} (1 + \gamma_5) \gamma_\alpha^\perp \gamma_\gamma^\perp \gamma_\beta^\perp h_v]$$

$$\tilde{O}_3 = [\bar{\xi} \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \gamma_\perp^\epsilon (1 - \gamma_5) \chi] [\bar{\chi} (1 + \gamma_5) \gamma_\alpha^\perp \gamma_\epsilon^\perp \gamma_\delta^\perp \gamma_\gamma^\perp \gamma_\beta^\perp h_v]$$

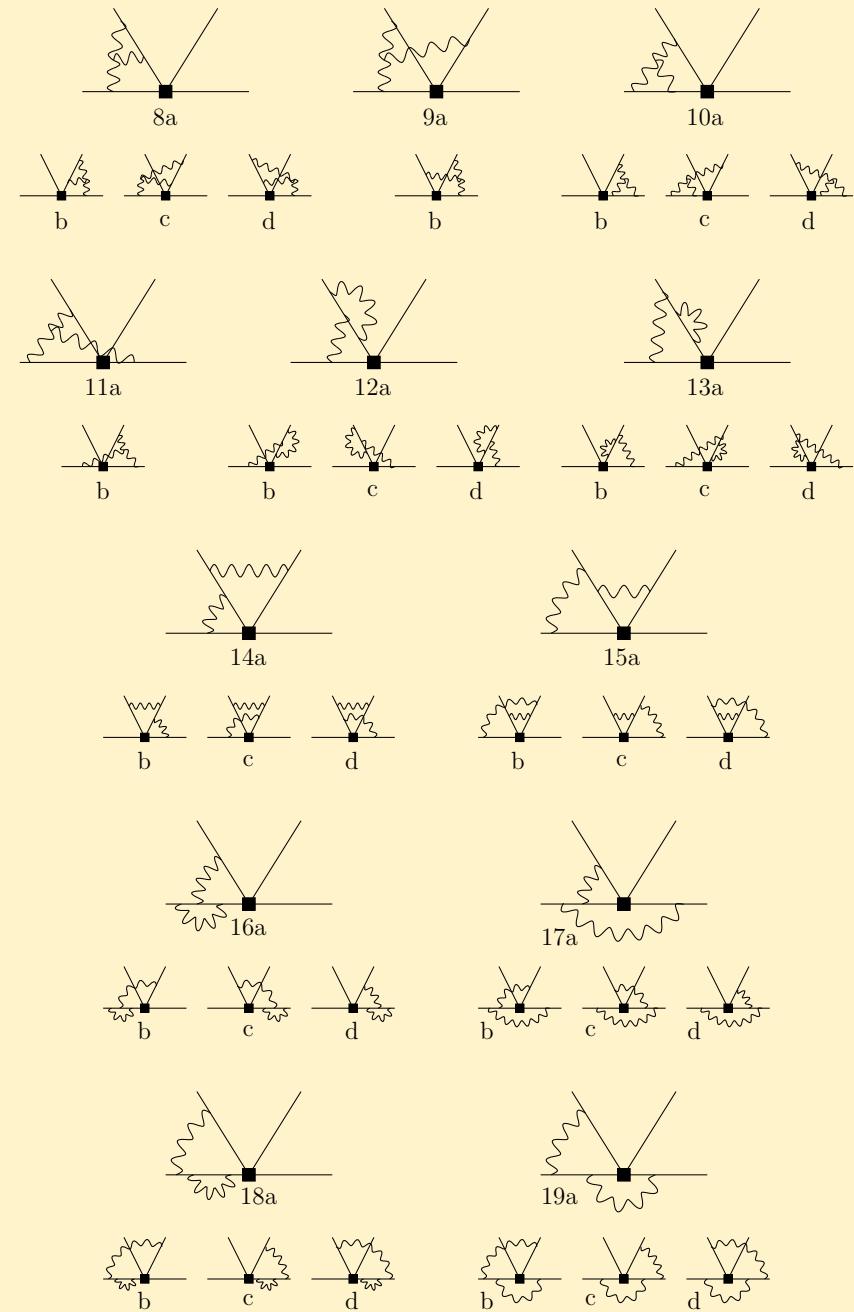
Two-loop diagrams

- Non-factorizable two-loop diagrams for non-leptonic B -decays

[Beneke, Buchalla, Neubert, Sachrajda '00]



- Kinematics: $p_b^2 = m_b^2$, $q^2 = 0$,
 $p^2 = 0$ or $p^2 = m_c^2$



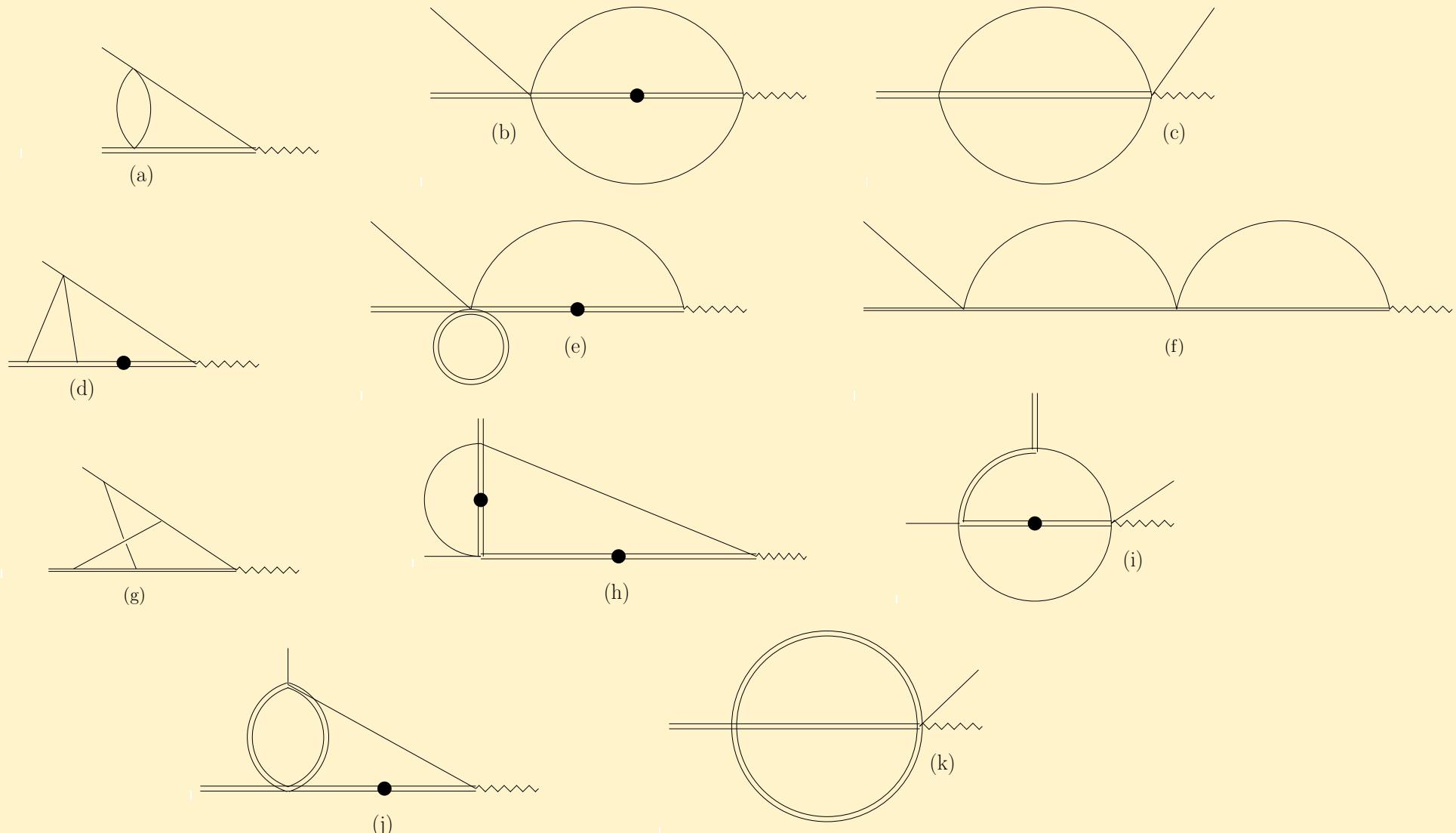
Reduction methods

- Dimensional regularisation with $D = 4 - 2\epsilon$ regulates UV and IR. Poles up to $1/\epsilon^4$.
- Passarino-Veltman reduction to scalar integrals
(in general with irreducible scalar products in the numerator) [Passarino, Veltman '79]
- Integration-by-parts (IBP) identities, 8 per diagram [Tkachov '81; Chetyrkin, Tkachov '81]
- Lorentz-Invarianz (LI) identities, 1 per diagram [Gehrman, Remiddi '99]
- Solve system of equations with Laporta algorithm [Laporta '01; Anastasiou, Lazopoulos '04; Smirnov '08]
- Obtain scalar integrals as a linear combination of master integrals

$$\text{Diagram} = \frac{(8 - 3D)(7uD - 8D - 24u + 28)}{3(D - 4)^2 m_b^4 u^3} \text{Diagram} - \frac{2[u^2(D - 4) + (16D - 56)(1 - u)]}{3(D - 4)^2 m_b^2 u^3} \text{Diagram}$$

- Reduction is carried out for $m_c = 0$ ($B \rightarrow \pi\pi$) and $m_c \neq 0$ ($B \rightarrow D\pi$)

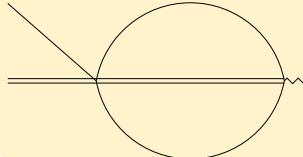
Sample master integrals



- Double lines are massive, single lines are massless
- Dots on lines denote squared propagators

Master Integrals

- Reduction yields 42 master integrals for $m_c = 0$. For finite m_c , this roughly doubles.
- Poles up to $1/\epsilon^4$. Analytic calculation of coefficient functions for $m_c = 0$.
Harmonic polylogarithms up to weight 4 of argument u or $1 - u$. [Remiddi, Vermaseren '99]
- Several calculations in agreement [Bell'07; Bonciani, Ferroglia '08; Asatrian, Greub, Pecjak '08; Beneke, Li, TH '08]
- Applied techniques
 - Hypergeometric functions


$$= \frac{(m_b^2)^{1-2\epsilon}}{(4\pi)^{4-2\epsilon}} \frac{\Gamma^2(1-\epsilon)\Gamma(\epsilon)\Gamma(2\epsilon-1)}{\Gamma(2-\epsilon)} {}_2F_1(\epsilon, 2\epsilon-1; 2-\epsilon; 1-u)$$

* ϵ -expansion: XSummer (Form), HypExp (Mathematica) [Moch, Uwer '05; Maitre, TH '05, '07]

– Differential equations [Kotikov '91; Remiddi '97]

$$\frac{\partial}{\partial u} \text{MI}_i(u) = f(u, \epsilon) \text{MI}_i(u) + \sum_{j \neq i} g_j(u, \epsilon) \text{MI}_j(u)$$

* Requires result of Laporta reduction.

* Boundary condition in $u = 0$ or $u = 1$ from Mellin-Barnes representation

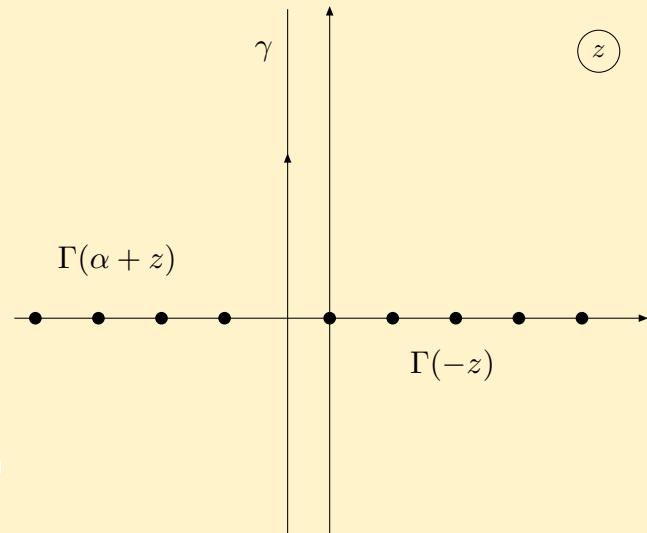
Master Integrals (cont'd.)

- Applied techniques (cont'd.)
 - Mellin-Barnes representation [Smirnov'99; Tausk'99]

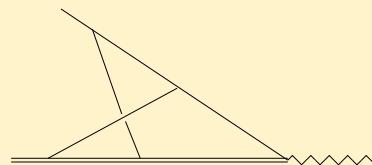
$$\frac{1}{(A_1 + A_2)^\alpha} = \int_{\gamma} \frac{dz}{2\pi i} A_1^z A_2^{-\alpha-z} \frac{\Gamma(-z) \Gamma(\alpha+z)}{\Gamma(\alpha)}$$

- * partially automated
- * Numerical cross checks possible

[Czakon'05; Gluza, Kajda, Riemann'07]



- Most difficult master integral:

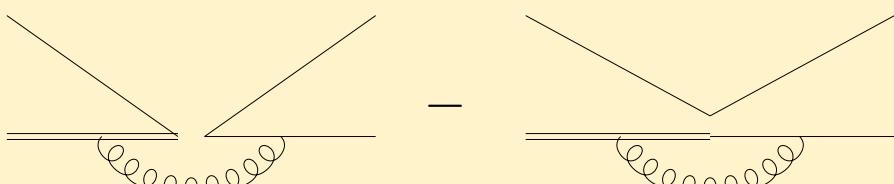


[TH'09]

- Solved with differential equations technique
- Possesses a three-fold Mellin-Barnes integral at $u = 1$

Master formula, hard scattering kernel for α_2

$$\begin{aligned}
\tilde{T}_i^{(1)} &= \tilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)} \tilde{A}_{j1}^{(0)} \\
&\quad + \underbrace{\tilde{A}_{i1}^{(1),f} - A^{(1),f} \tilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\tilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \tilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} \\
\tilde{T}_i^{(2)} &= \tilde{A}_{i1}^{(2),nf} + Z_{ij}^{(1)} \tilde{A}_{j1}^{(1)} + Z_{ij}^{(2)} \tilde{A}_{j1}^{(0)} + Z_\alpha^{(1)} \tilde{A}_{i1}^{(1),nf} \\
&\quad + (-i) \delta m^{(1)} \tilde{A}_{i1}'^{(1),nf} \\
&\quad + (Z_{ext}^{(1)} + \xi_{45}^{(1)}) [\tilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)} \tilde{A}_{j1}^{(0)}] \\
&\quad - \tilde{T}_i^{(1)} [C_{FF}^{(1)} + \tilde{Y}_{11}^{(1)}] - \sum_{b>1} \tilde{H}_{ib}^{(1)} \tilde{Y}_{b1}^{(1)} \\
&\quad + [\tilde{A}_{i1}^{(2),f} - A^{(2),f} \tilde{A}_{i1}^{(0)}] \\
&\quad + (-i) \delta m^{(1)} [\tilde{A}_{i1}'^{(1),f} - A'^{(1),f} \tilde{A}_{i1}^{(0)}] \\
&\quad + (Z_\alpha^{(1)} + Z_{ext}^{(1)} + \xi_{45}^{(1)}) [\tilde{A}_{i1}^{(1),f} - A^{(1),f} \tilde{A}_{i1}^{(0)}] \\
&\quad - C_{FF}^{(1)} \tilde{A}_{i1}^{(0)} [\tilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] - [\tilde{Y}_{11}^{(2)} - Y_{11}^{(2)}] \tilde{A}_{i1}^{(0)}
\end{aligned}$$



$$\begin{aligned}
T_1^{(2),re} &= \frac{(47u^5 - 278u^4 + 1223u^3 - 2316u^2 + 2036u - 652) \ln^4(1-u)}{162(u-1)^2 u^3} \\
&\quad - \frac{(2u^3 + 4u^2 + 173u + 16) \ln^3(1-u)}{81u} - \frac{(4u^3 - 61u^2 - 436u + 16) \ln^2(1-u)}{54u^2} \\
&\quad + \frac{2(73u^5 + 38u^4 - 1103u^3 + 2316u^2 - 2036u + 652) \ln(u) \ln^3(1-u)}{81(u-1)^2 u^3} \\
&\quad - \frac{(17u^3 + 300u^2 - 1098u + 978) \ln^2(u) \ln^2(1-u)}{27u^3} \\
&\quad - \frac{\pi^2 (9u^5 + 166u^4 - 1167u^3 + 2316u^2 - 2036u + 652) \ln^2(1-u)}{81(u-1)^2 u^3} \\
&\quad + \frac{(2u^5 - 20u^3 + 125u^2 - 76u - 52) \ln(u) \ln^2(1-u)}{27(u-1)^2 u} + \frac{2}{9} \ln^3(u) \ln(1-u) \\
&\quad + \frac{7(u-2)^2 \ln(2-u) \ln^2(1-u)}{9(u-1)^2} + \frac{16}{9} \text{Li}_2(u) \ln^2(1-u) \\
&\quad + \frac{(2u^6 + 4u^5 - 191u^4 - 167u^3 + 1022u^2 - 646u - 6) \ln^2(u) \ln(1-u)}{27(u-1)u^3} \\
&\quad - \frac{\pi^2 (2u^5 + 355u^3 - 623u^2 + 385u - 140) \ln(1-u)}{81(u-1)^2 u} \\
&\quad - \frac{(4u^4 - 638u^3 + 1487u^2 - 1597u + 664) \ln(u) \ln(1-u)}{27(u-1)u^2} \\
&\quad + \frac{14(u-2)^2 \text{Li}_2(u-1) \ln(1-u)}{9(u-1)^2} + \frac{16(6u^2 - 16u - 5) \text{Li}_3(u) \ln(1-u)}{27(u-1)^2} \\
&\quad - \frac{2(94u^3 - 271u^2 + 166u + 32) \text{Li}_2(u) \ln(1-u)}{27(u-1)^2 u} + \frac{(1601u - 1172) \ln(1-u)}{54u} \\
&\quad + \frac{4(4u^3 - 50u^2 + 183u - 163) \ln(u) \text{Li}_2(u) \ln(1-u)}{27u^3} \\
&\quad + \frac{(2u^3 - 436u^2 + 657u - 332) \ln^2(u)}{27(u-1)u} - \frac{8(3u^2 - 14u - 19) \zeta(3) \ln(1-u)}{27(u-1)^2} \\
&\quad + \frac{2(20u^5 - 94u^4 + 292u^3 - 579u^2 + 509u - 163) \text{Li}_2(u)^2}{27(u-1)^2 u^3} \\
&\quad + \frac{\pi^2 (4u^4 - 435u^3 + 3174u^2 - 5346u + 2688)}{162(u-1)u^2} + \frac{64}{9} \text{Li}_3(1-u) \ln(1-u) \\
&\quad + \dots \text{ (five pages)}
\end{aligned}$$

Numerical Results

- Convolution of hard scattering kernels with pion LCDA yields topological tree amplitudes $\alpha_1(\pi\pi)$ and $\alpha_2(\pi\pi)$ to NNLO

- Have expressions for $\alpha_1(\pi\pi)$ and $\alpha_2(\pi\pi)$ completely analytically, including m_c dependence

$$\begin{aligned} \alpha_1(\pi\pi) \supset & \dots + 8194\zeta_5 - 2028\pi^2\zeta_3 - \ln^3\left(\frac{4z}{(\sqrt{4z+1}+1)^2}\right) \\ & - 12\text{Li}_3\left(\frac{4z}{(\sqrt{4z+1}+1)^2}\right) + 2\text{Li}_3\left(\frac{2\sqrt{z}}{\sqrt{z}+1}\right) + \dots \quad (3 \text{ pages}) \end{aligned}$$

- We find complete agreement (numerically) with G. Bell

[G. Bell'09]

$$\begin{aligned} \alpha_1(\pi\pi) = & 1.009 + [0.023 + 0.010i]_{\text{NLO}} + [0.026 + 0.028i]_{\text{NNLO}} \\ & - \left[\frac{r_{\text{sp}}}{0.485} \right] \left\{ [0.015]_{\text{LO}_{\text{Sp}}} + [0.037 + 0.029i]_{\text{NLO}_{\text{Sp}}} + [0.009]_{\text{tw3}} \right\} \\ = & 1.00 + 0.01i \quad \rightarrow \quad 0.93 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}}) \end{aligned}$$

$$r_{\text{sp}} = \frac{9f_{M_1}\hat{f}_B}{m_b\lambda_B f_+^{B\pi}(0)}$$

$$\begin{aligned} \alpha_2(\pi\pi) = & 0.220 - [0.179 + 0.077i]_{\text{NLO}} - [0.031 + 0.050i]_{\text{NNLO}} \\ & + \left[\frac{r_{\text{sp}}}{0.485} \right] \left\{ [0.123]_{\text{LO}_{\text{Sp}}} + [0.053 + 0.054i]_{\text{NLO}_{\text{Sp}}} + [0.072]_{\text{tw3}} \right\} \\ = & 0.26 - 0.07i \quad \rightarrow \quad 0.51 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}}) \end{aligned}$$

- NNLO corrections to vertex and spectator terms significant but tend to cancel! ☹

Factorisation test

$$\frac{\Gamma(B^- \rightarrow \pi^-\pi^0)}{d\Gamma(\bar{B}^0 \rightarrow \pi^+\ell^-\bar{\nu})/dq^2|_{q^2=0}} = 3\pi^2 f_\pi^2 |V_{ud}|^2 |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2$$

- From semi-leptonic data

[cf. Becher, Hill'05; Ball'06; BaBar'06]

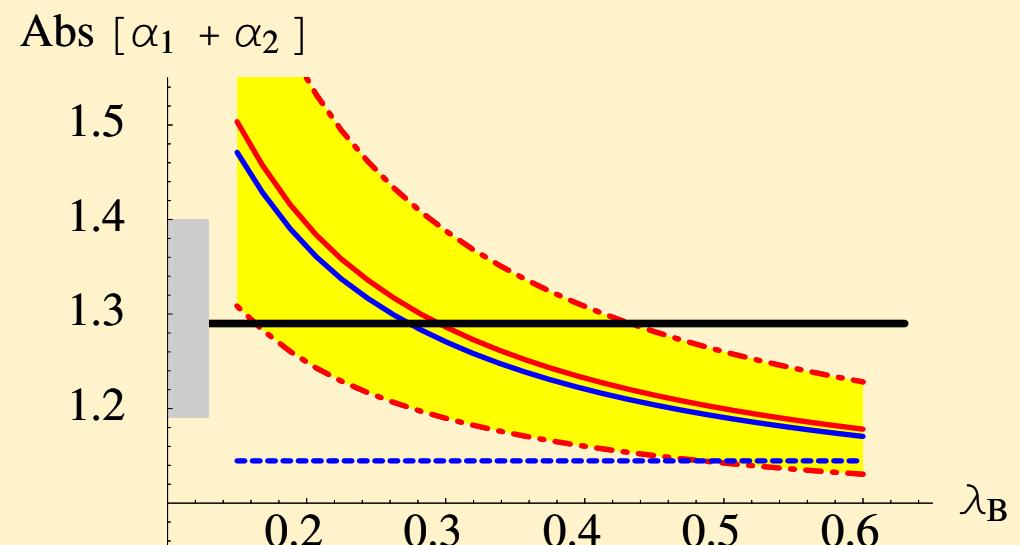
$$|V_{ub}|f_+(0) = (9.1 \pm 0.7) \times 10^{-4}$$

equivalent to

$$|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|_{\text{exp}} = 1.29 \pm 0.11$$

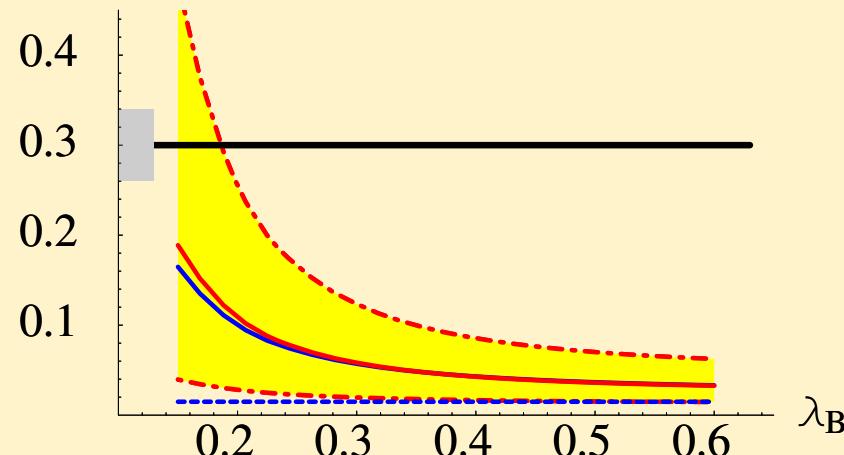
- Leading uncertainties: λ_B (B LCDA), a_2^π (pion LCDA), power corrections, μ_{hc}

$$\lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega, \mu)$$

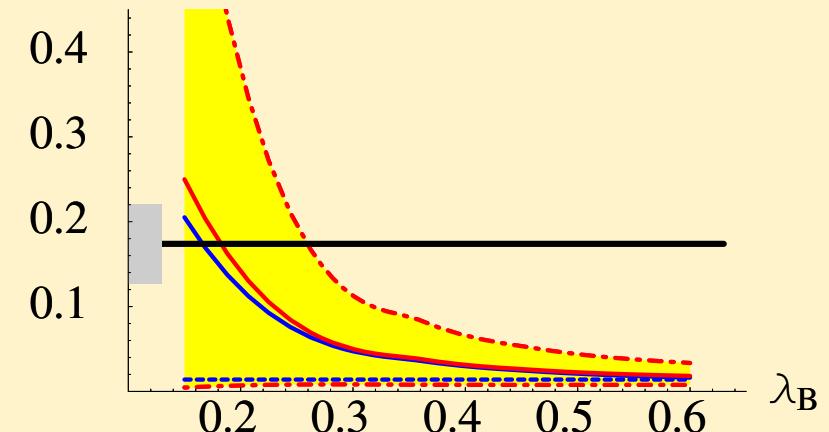


Colour-suppressed vs. colour-allowed decays

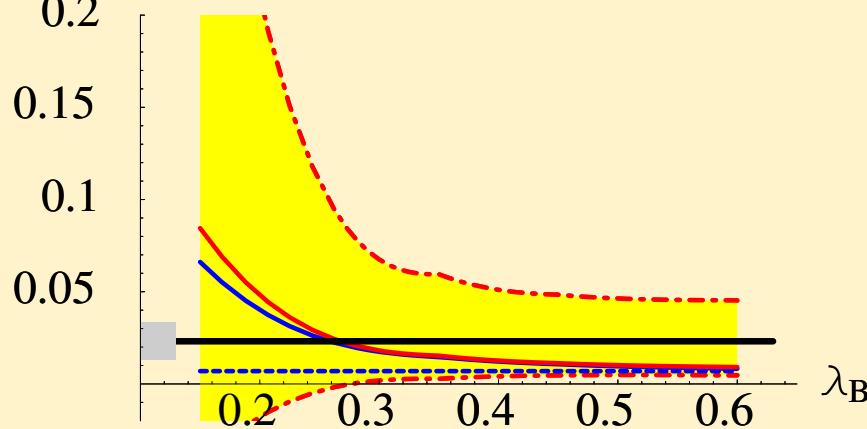
$$\text{Br} (\pi^0 \pi^0) / \text{Br} (\pi^+ \pi^-)$$



$$\text{Br} (\pi^0 \rho^0) / \text{Br} (\pi^{+/-} \rho^{-/+})$$



$$\text{Br} (\rho^0 \rho^0) / \text{Br} (\rho^+ \rho^-)$$



Preference for small λ_B , i.e. strong spectator scattering, as already found at NLO in [Beneke, Neubert '03]

Semi-leptonic $b \rightarrow u \ell \nu$ decays and V_{ub}

- V_{ub} is an important parameter in quark flavour physics,
 - governs strength of $b \rightarrow u$ transition
 - determines the side of the unitarity triangle opposite to β
- Determination of $|V_{ub}|$ from inclusive vs. exclusive semi-leptonic $b \rightarrow u \ell \nu_\ell$ modes:

[M. Antonelli et. al., flavour review, 07/2009]

$$|V_{ub}|^{\text{incl.}} = (4.11^{+0.27}_{-0.28}) 10^{-3} \quad [\text{BLNP, GGOU, DGE}]$$

$$|V_{ub}|^{\text{excl.}} = (3.38 \pm 0.36) 10^{-3} \quad [B \rightarrow \pi \ell \nu_\ell, \text{lattice. J. Bailey et. al.}]$$

- Inclusive $\bar{B} \rightarrow X_u \ell \nu_\ell$ transition,
kinematic regions require different theoretical treatment.

- kinematic variables $p_X^\pm = E_X \mp |\vec{p}_X|$

- Local OPE region: $\Lambda_{\text{QCD}} \ll p_X^+ \sim p_X^-$

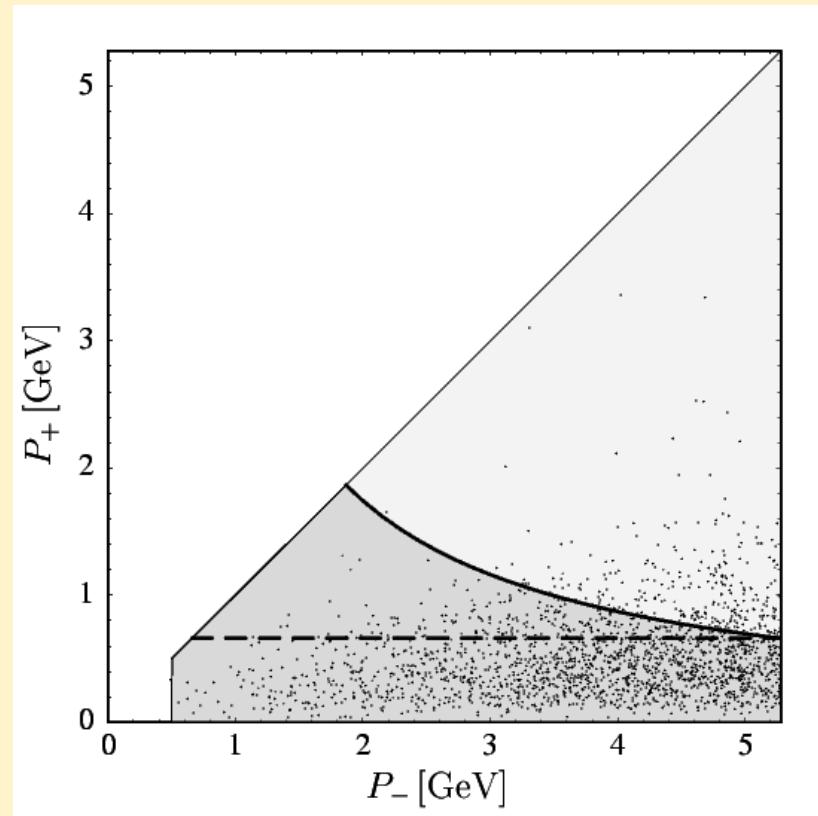
- SCET region: $\Lambda_{\text{QCD}} \sim p_X^+ \ll p_X^-$

- Shape-function OPE: $\Lambda_{\text{QCD}} \ll p_X^+ \ll p_X^-$

Semi-leptonic $b \rightarrow u \ell \nu$ decays and V_{ub}

- Experimental determination of $|V_{ub}|$ requires kinematic cuts, due to elimination of charm background

- $\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell$ events distributed over PS
- Shaded regions: $s_X = p_X^+ p_X^- \gtrless M_D^2$
- Dashed line:
 $E_\ell \geq (M_B^2 - M_D^2)/(2 M_B)$
implies $p_X^+ \leq M_D^2/M_B$



[Bosch, Lange, Neubert, Paz '04, '05]

- Even after cuts still many events left in the shape-function region of small p_X^+ and large p_X^-

Semi-leptonic $b \rightarrow u \ell \nu$ decays and V_{ub}

- Triple differential decay rate in regions where $p_X^+ \ll p_X^-$

$$\frac{d^3\Gamma(\bar{B} \rightarrow X_u \ell \bar{\nu}_\ell)}{dE_\ell \, dp_X^+ \, p_X^-} = \Gamma_{0u} H_u(E_\ell, p_X^+, p_X^-, \mu_i) \int d\omega (n_- \cdot p) J((n_- \cdot p)\omega, \mu_i) S(p_X^+ - \omega, \mu_i)$$

[Korchemsky, Sterman; Bauer, Pirjol, Stewart]

- Jet function J and perturbative part of soft function S known to two loops

[Becher, Neubert '05, '06]

- Two-loop QCD corrections to H_u , matching from QCD onto SCET

$$\langle u(p) | \bar{\psi} \gamma^\mu (1 - \gamma_5) Q | b(p_b) \rangle = \sum_{i=1}^3 F_i(u) \bar{u}(p) \Gamma_i^\mu u(p_b) = \sum_{i=1}^3 Z_J \textcolor{blue}{C}_i(u) \bar{u}_{n_-} \Gamma'_i{}^\mu u_v$$

$$\Gamma_1^\mu = \gamma^\mu (1 - \gamma_5)$$

$$\Gamma_2^\mu = \frac{p_b^\mu}{m_b} (1 + \gamma_5)$$

$$\Gamma_3^\mu = \frac{m_b p^\mu}{p_b \cdot p} (1 + \gamma_5)$$

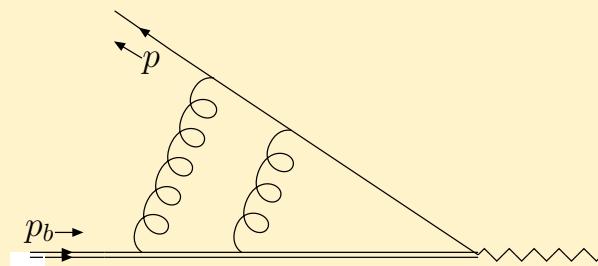
$$\Gamma'_1{}^\mu = \gamma^\mu (1 - \gamma_5)$$

$$\Gamma'_2{}^\mu = v^\mu (1 + \gamma_5)$$

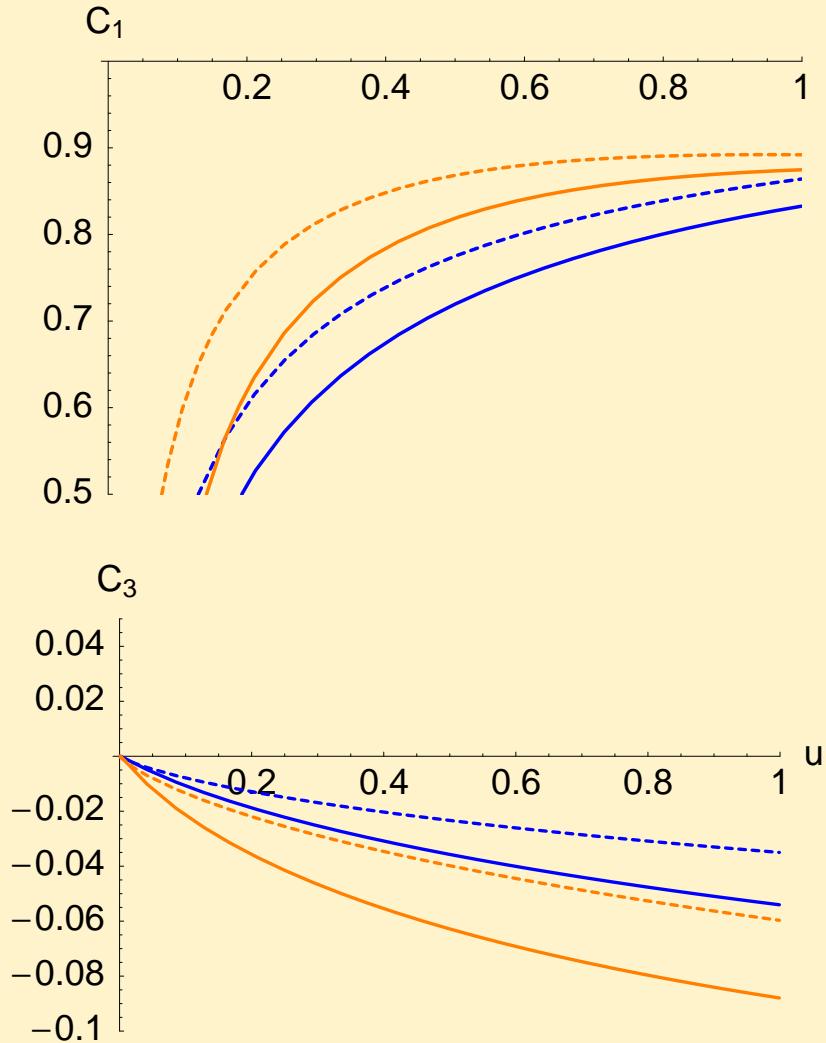
$$\Gamma'_3{}^\mu = n_-^\mu (1 + \gamma_5)$$

- $\textcolor{blue}{C}_i(u) = Z_J^{-1} F_i(u)$ depend

only on $u \equiv \frac{2(p_b \cdot p)}{m_b^2}$



Matching coefficients to NNLO



- $\mu = m_b$ vs. $\mu = 1.5$ GeV
- Dashed: NLO
- Solid: NNLO
- Several calculations in full agreement

[Bonciani, Ferroglia '08; Asatrian, Greub, Pecjak '08]

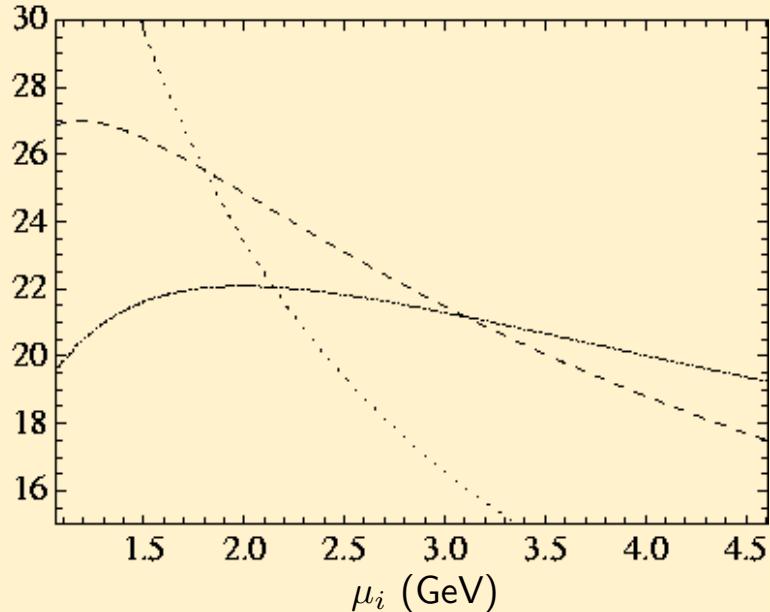
[Beneke, Li, TH '08; Bell '08]

Implications on $|V_{ub}|$

[Greub, Neubert, Pecjak'09]

- NNLO shift on NLO partial decay rates in the BLNP framework for the jet scale $\mu_i = 1.5$ GeV in resummed PT is -15% to -20% .
- For higher values of μ_i and in fixed order PT the shifts are more moderate
- Large dependence of NLO rates on μ_i reduced but still significant at NNLO
- NNLO corrections raise NLO value of $|V_{ub}|$ by $\lesssim 10\%$

$$\Gamma_u^{(0)}(E_l > E_0)$$



Method	$\Delta \mathcal{B}^{\text{exp}} [10^{-4}]$	$ V_{ub} [10^{-3}]$	
		NLO	NNLO
$E_l > 2.1$ GeV	$3.3 \pm 0.2 \pm 0.7$	$3.56 \pm 0.40^{+0.48+0.31}_{-0.27-0.26}$	$3.81 \pm 0.43^{+0.33+0.31}_{-0.21-0.26}$
$E_l > 2.0$ GeV	$5.7 \pm 0.4 \pm 0.5$	$3.97 \pm 0.22^{+0.37+0.26}_{-0.23-0.25}$	$4.30 \pm 0.24^{+0.26+0.28}_{-0.20-0.27}$
$E_l > 1.9$ GeV	$8.5 \pm 0.4 \pm 1.5$	$4.27 \pm 0.39^{+0.32+0.25}_{-0.19-0.22}$	$4.65 \pm 0.43^{+0.27+0.27}_{-0.18-0.24}$
$M_X < 1.7$ GeV	$12.3 \pm 1.1 \pm 1.2$	$3.55 \pm 0.24^{+0.22+0.21}_{-0.13-0.19}$	$3.87 \pm 0.26^{+0.21+0.21}_{-0.13-0.19}$
$M_X < 1.55$ GeV	$11.7 \pm 0.9 \pm 0.7$	$3.67 \pm 0.18^{+0.29+0.26}_{-0.17-0.24}$	$3.96 \pm 0.19^{+0.20+0.26}_{-0.13-0.24}$
$P_+ < 0.66$ GeV	$11.0 \pm 1.0 \pm 1.6$	$3.56 \pm 0.31^{+0.30+0.27}_{-0.17-0.23}$	$3.84 \pm 0.33^{+0.21+0.26}_{-0.13-0.22}$
$P_+ < 0.66$ GeV	$9.4 \pm 1.0 \pm 0.8$	$3.30 \pm 0.23^{+0.27+0.25}_{-0.16-0.22}$	$3.55 \pm 0.24^{+0.19+0.24}_{-0.13-0.21}$

central values: $\mu_i = 2.0$ (GeV), $\mu_h = 4.25$ (GeV)

[Greub, Neubert, Pecjak'09]

Conclusion

- The colour-allowed and colour-suppressed tree amplitudes have been computed completely analytically to NNLO
- The two-loop computation requires sophisticated computational techniques
- The NNLO corrections are very small. Accidental cancellation between vertex and spectator term
- To do: Two-loop penguin amplitudes, CP asymmetries at NLO
- NNLO Corrections to inclusive semi-leptonic $\bar{B} \rightarrow X_u \ell \nu$ decays tend to increase the difference between $|V_{ub}|$ determined from inclusive vs. exclusive decay modes.