High Energy Scattering and the AdS/CFT Correspondence

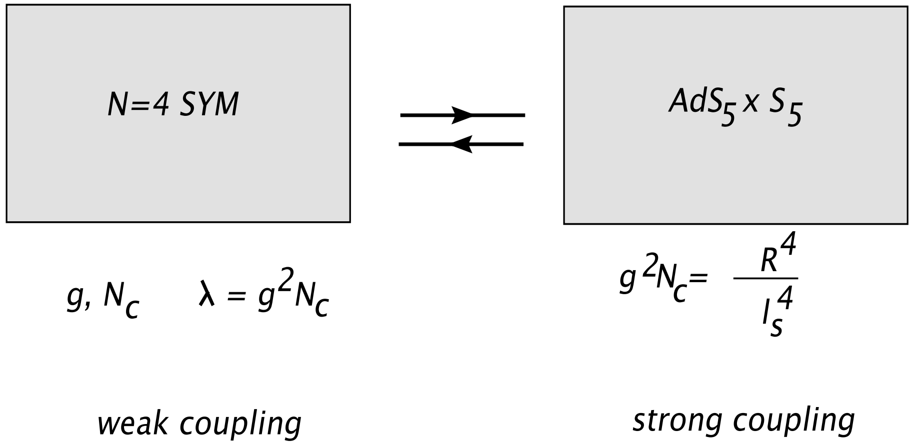
Jochen Bartels
II.Inst.f.Theor.Physik, Univ.Hamburg

QCD: The Modern View of the Strong Interactions
Berlin, 4 - 9 October

- Introduction: realistic expectations
- High energy scattering of planar amplitudes
- The Pomeron in AdS/CFT
- Conclusions

Introduction

Frame of this talk is the AdS/CFT correspondence hypothesis:



On both sides expansion in $1/N_c$ (expansion in toplogy).

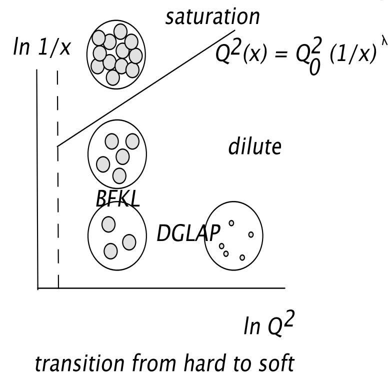
History: Regge limit stimulated string theory (Veneziano amplitudes),
interesting to analyse high energy scattering amplitudes within the AdS/CFT duality.

This talk: two steps

(a) scattering amplitudes in the planar limit (compare with Veneziano amplitudes).
Main interest: n point amplitudes in $N = 4$, guide for multiloop/multileg amplitudes in QCD, BDS formula.
Is $N = 4$SYM soluble: integrability?

(b) Vacuum exchange (Pomeron, cylinder):
(Soft) Pomeron in hadron-hadron scattering is non-pertubative: need methods other the pQCD.
But: (Soft) Pomeron is also sensitive to low-energy features of QCD (slope α': chiral dynamics).

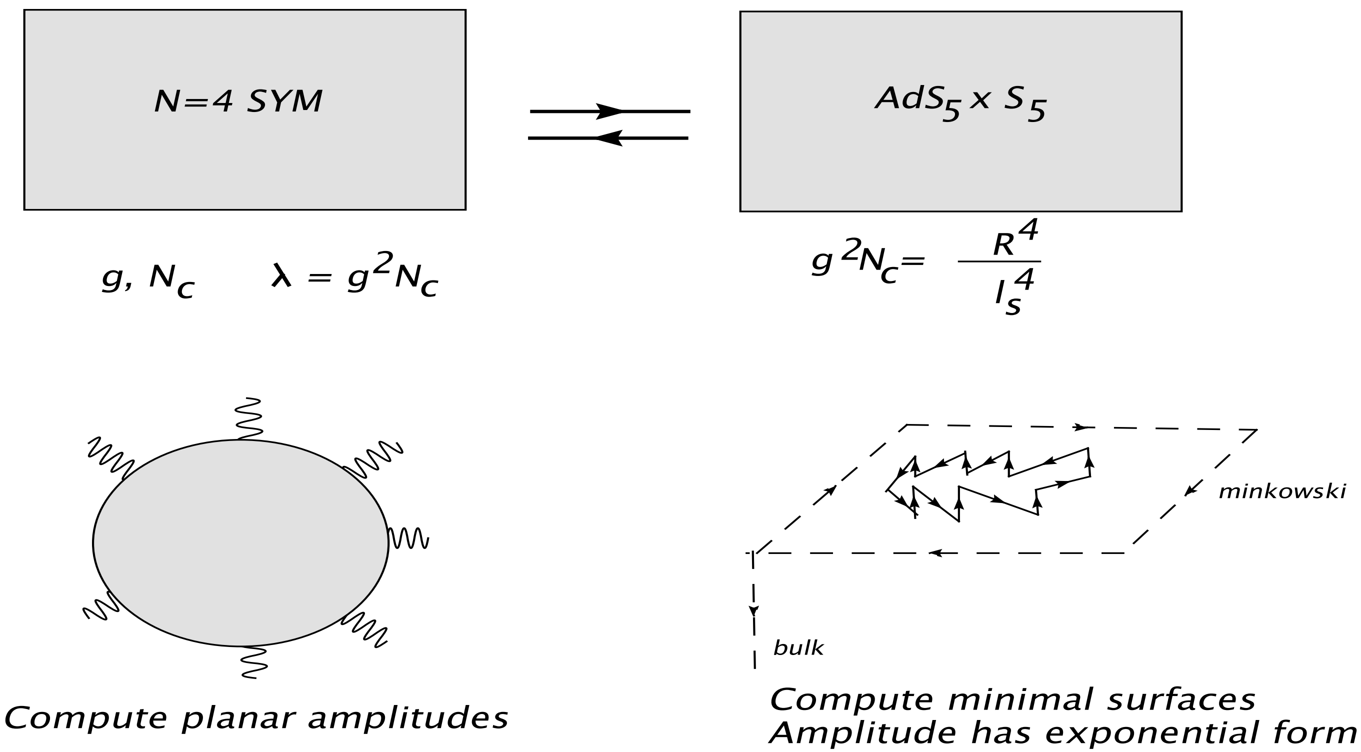
Hard Pomeron: in scattering of small-size projectiles (virtual photon)
Soft Pomeron: in hadron-hadron scattering
Transition in deep inelastic scattering (saturation, unitarization)



AdS/CFT correspondence: first the hard Pomeron, unitarization.
For soft Pomeron: need more sophisticated geometry on the string theory (modelling).

Planar scattering amplitudes at high energies

$N = 4$, MHV amplitudes. Duality:



Gauge theory side: enormous activity in two loop calculations, beyond MHV.
String theory side: mimimal surfaces are hard to compute, a few cases are known (Alday, Maldacena).

Most remarkable: Bern-Dixon-Smirnow (BDS) formula for planar n-gluon scattering amplitude:

Remove color factors, factor out tree amplitude, IR singular:

$$tr(T^{a1}...T^{an}) + noncycl.perm, \quad A_n = A_n^{tree} \cdot M_n(\epsilon)$$

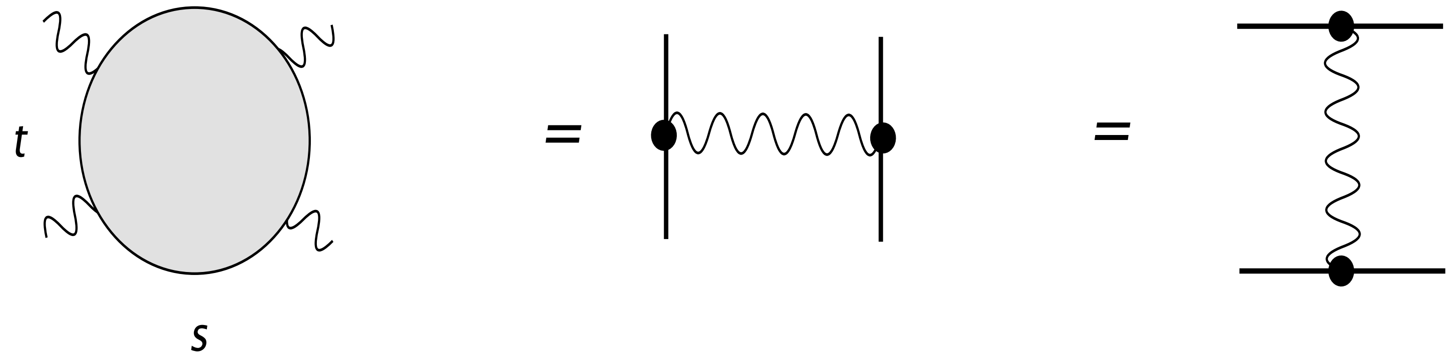
$$\ln M_n = \sum_l a^l \Big[\Big(f^{(l)}(\epsilon) I_n(l\epsilon) + F_n(0) \Big) + C^{(l)} + E_n^{(l)}[\epsilon] \Big]$$

$$a = \frac{N_c \alpha}{2\pi} (4\pi e^{-\gamma})^\epsilon, \; d = 4 - 2\epsilon$$

Based upon: universality of IR singularities (=poles in ϵ), and 1-loop calculation.

Several tests (Alday,Maldacena; Drummond,Korchemsky,Sokatchev; JB,Lipatov,Sabio-Vera):
partly successful ($n \leq 5$, partly disagreement $n \geq 6$).
This talk: high energy limit (Regge limit) of BDS formula:

Four-point function:

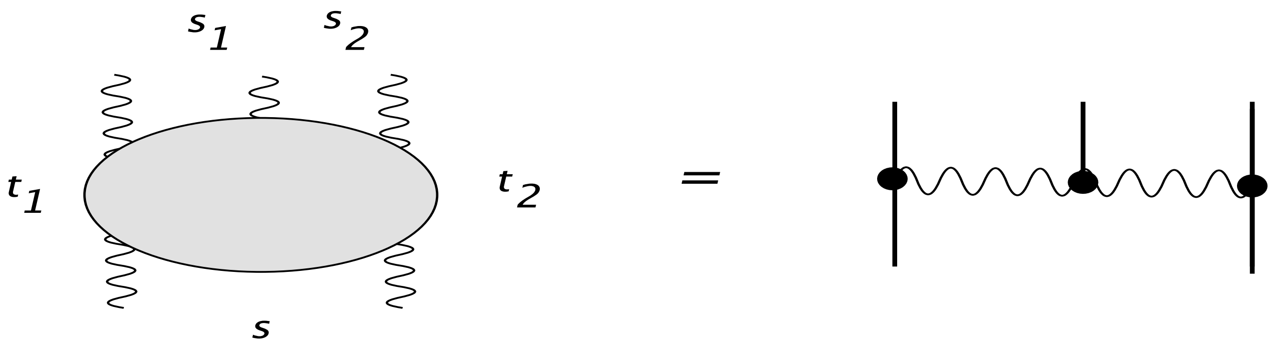


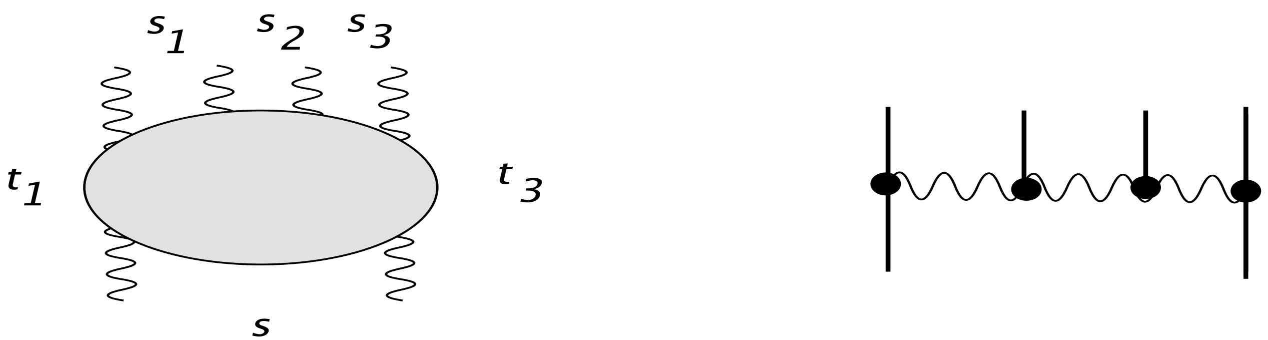
$$A_4(s,t) = \Gamma(t)\left(\frac{-s}{\mu^2}\right)^{\omega(t)}\Gamma(t) = \Gamma(s)\left(\frac{-t}{\mu^2}\right)^{\omega(s)}\Gamma(s)$$

All order gluon trajectory function, vertex function.

Comparison with Veneziano amplitude $B_4(s,t)$.

Five, six point functions:



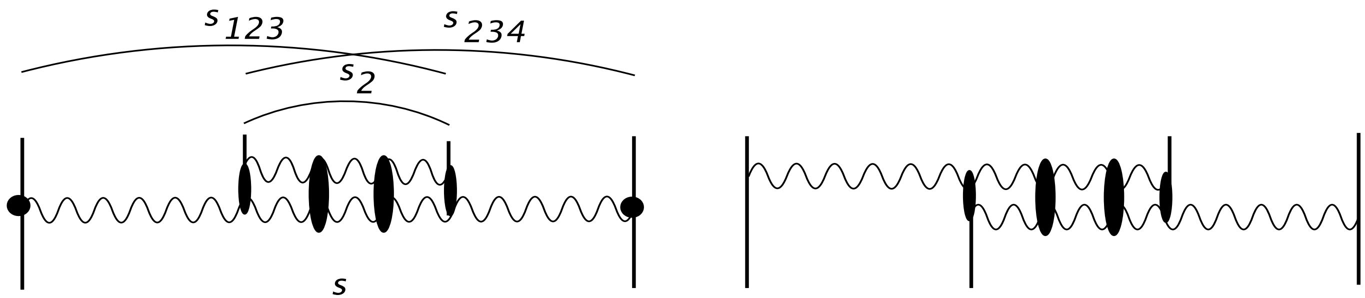


Same trajectory, vertex function, production vertex:

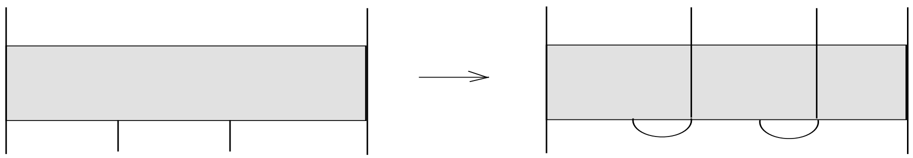
all seems to be consistent. But for $n \geq 6$:

Analytic structure:
scattering amplitudes = functions of several complex-valued variables: Steinmann relations

Comparison with leading-log calculations in QCD (JB, Lipatov,Sabio-Vera):
disagreement for $2 \longrightarrow 4$, $3 \longrightarrow 3$, ...:
piece is missing (beyond one loop) (absent also in Veneziano amplitudes).

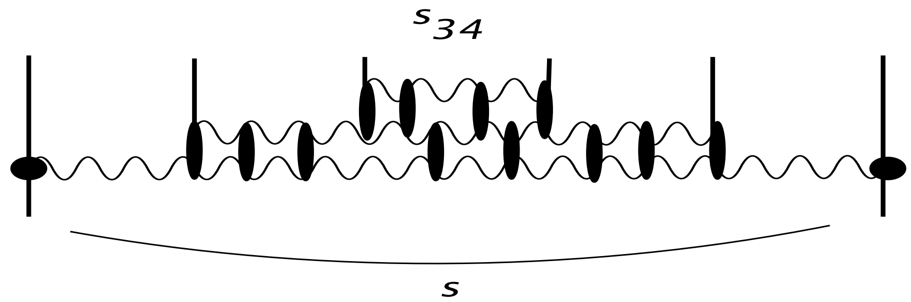


Visible in energy discontinuity or in another physical region:
$s, s_2 > 0$, $s_{123}, s_{234} < 0$:



Special feature of this extra piece: integrability.

Go to multi-leg amplitudes $n > 8$, e.g.

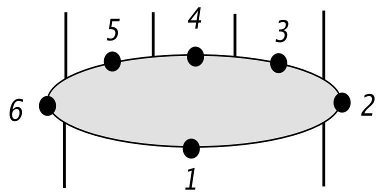


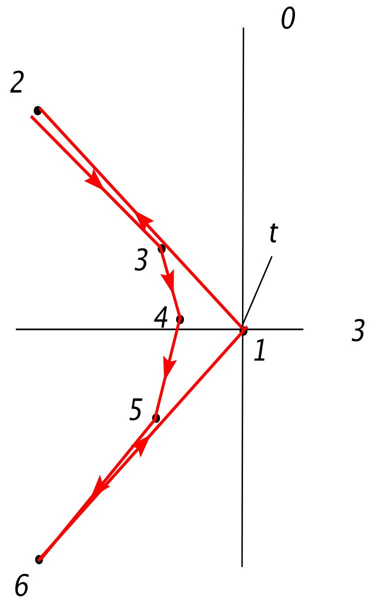
This Regge-cut piece, again, is visible in (double) energy discontinuities or in special physical regions. Dependence upon s_{34}:

$$A_8 \sim s_{34}^{-E_3}, \qquad \text{where} \qquad H_{3,open}\psi = E_3\psi$$

is the lowest energy of the BKP Hamiltonian describing the rapidity evolution in the t_3 channel.
In the planar limit the t_3 channel is in a octet state: open chain
$H_{3,open}$ is integrable (Lipatov).

On the string side:
High energy limit contours on the string side have characteristic spike





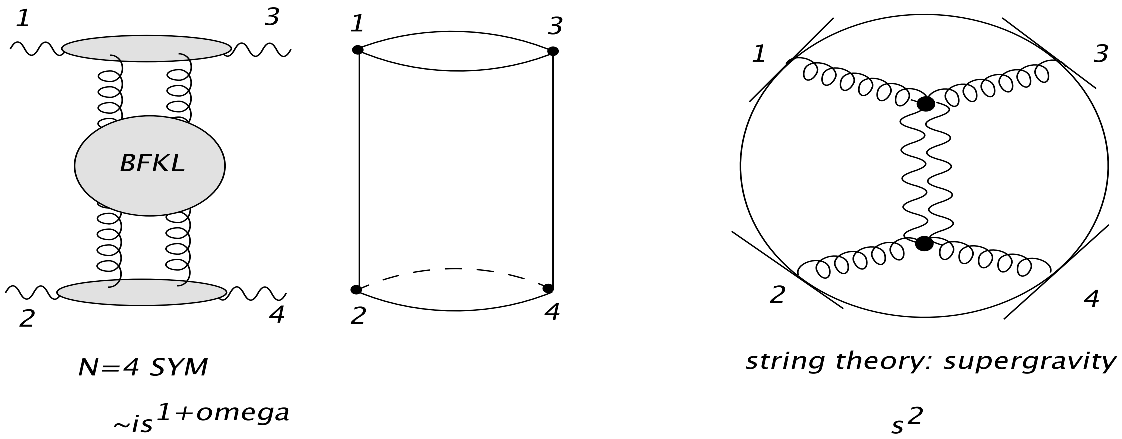
Surfaces not known fo general n.
Analytic continuation of kinematic regions $\leftrightarrow$ deformations of contours and minimal surfaces.

Study of these deformations might provide some guidance.

The Pomeron in AdS/CFT

A. The 'conservative' approach:

Testing gound: correlator of R-currents (global $SU(4)$ symmetry): analogue of $\gamma^*\gamma^*$-scattering in QCD.
Elastic scattering: $< R_{\mu_1}(x_1) R_{\mu_2}(x_2) R_{\mu_3}(x_3) R_{\mu_4}(x_4) >$.



Basic message: BFKL in $N = 4$ SYM is dual to the graviton in AdS_5

In more detail:
on the weak coupling side the BFKL amplitude

$$A(s,t) = is \int \frac{d\omega}{2\pi i} \left(\frac{s}{kk'}\right)^{\omega} \Phi_1(Q_A^2, k, q-k) \otimes G_\omega(k, q-k; k', q-k') \otimes \Phi_2(Q_B^2, k', q-k')$$

Impact factors (for scalar currents) (Balitski; Cornalba et al.), characteristic BFKL function (Lipatov et al)
known in NLO:

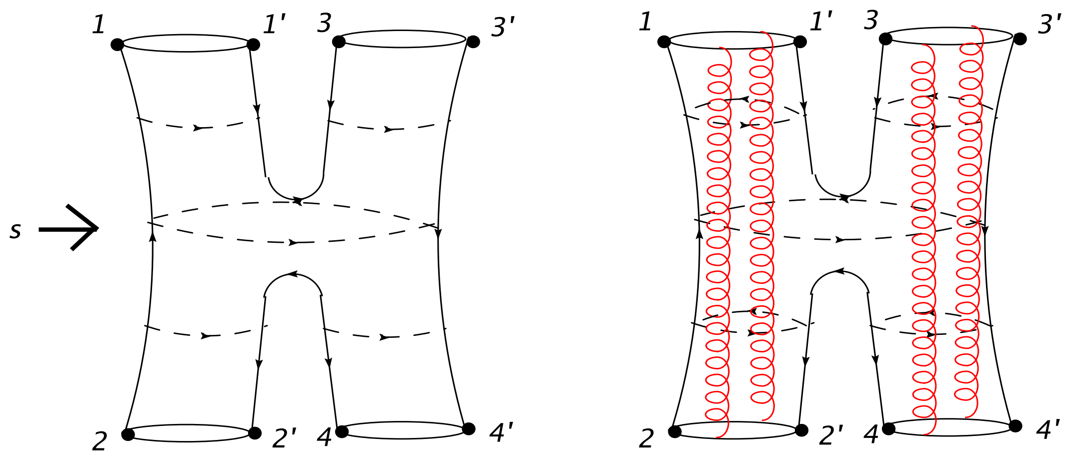
$$G_\omega(k, q-k; k', q-k') \sim \frac{1}{\omega - \chi(n,\nu})$$

Impact factors for R-currents in $N=4$ SYM known in LO.

Connection between small x-limit and short distance limit (DIS):
leading twist anomalous dimension near $\omega = j-1 \approx 0$

$$A(s,t=0) \sim \frac{is}{Q^2} \int \frac{d\omega}{2\pi i} \left(\frac{s}{Q_1^2}\right)^{\omega} \int \frac{d\nu}{2\pi i} \left(\frac{Q_1^2}{Q_2}\right)^{\nu} \Phi_1(n,\nu) \frac{1}{\omega - \chi(\nu,0)} \Phi_2(n,\nu)$$

Important feature of BFKL: generalize from 2 to $n > 2$ gluons,
(LO) Hamiltonian of BKP states is integrable for large N_c.
Where to find large-N_c BKP states: in multi-leg amplitudes, e.g. eight point correlator for $4 \longrightarrow 4$:
(such an amplitude is not quite academic: heavy ion collisions)

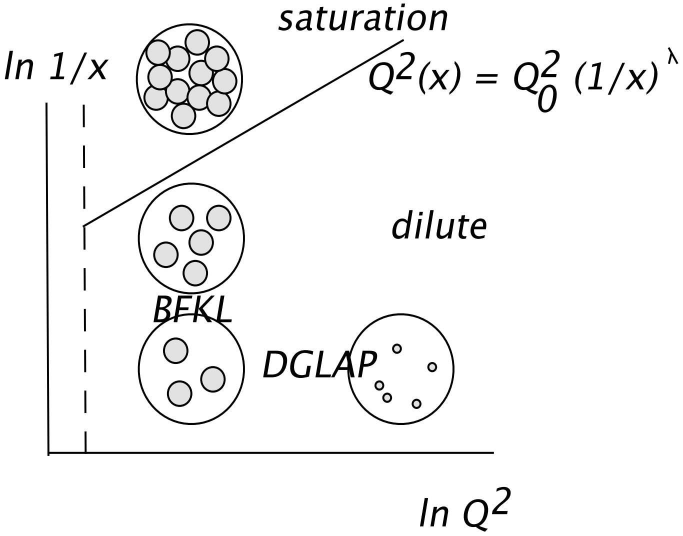


$A_8 \sim s^{1-E}$, where $H_{4,closed}\psi = E\psi$

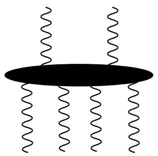
E is the lowest eigenvalue of the energy spectrum of the 4 gluon BKP Hamiltonian (closed chain).
Expect: when combined with short distance limit $\longrightarrow$ anomalous dimension of twist four operators.

Unitarization problem: as old as strong interactions.

Best understood in deep inelastic scattering:



transition from hard to soft



From large x, Q^2 to small x, Q^2, three regions:

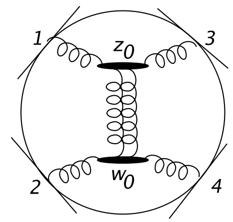
dilute (hard), saturation (dense),
strong interaction (soft Pomeron).

Near the saturation region:
vital role of triple Pomeron vertex (BK-kernel)
Appealing physical picture

The strong coupling side:

the leading term (in $1/\lambda$) is given by supergravity (Witten diagram): graviton exchange. Calculation (JB et al) gives:

$$T_2 = s^2 \int dz_0 \int dw_0 \Phi(p_1^2, p_3^2, z_0) \Sigma(t, z_o, w_o) \Phi(p_2^2, p_4^2, z_0)$$



Limit of $p_1^2 = p_3^2 \gg p_2^2 = p_4^2$: dominant region close to the boundary (z_0 small, r large): 'graviton $\leftrightarrow$ hard Pomeron' lives close to the boundary'

First correction (Lipatov et al; Polchinski et al)

$$j = 2 - \frac{2}{\sqrt{\lambda}} + O(\frac{1}{\lambda})$$

Diffusion in $\ln r$ (Polchinski et al).

Unitarization:

problem worse than BFKL: single $\sim s^2$, double $\sim s^3$,...
Saturation (see below)?
Triple reggeized-graviton vertex: need string calculation, Witten diagrams too simple.

Integrability:

in analogy with weak coupling, study higher correlators $(n \geq 8)$, take combined short distance and h
energy limit.

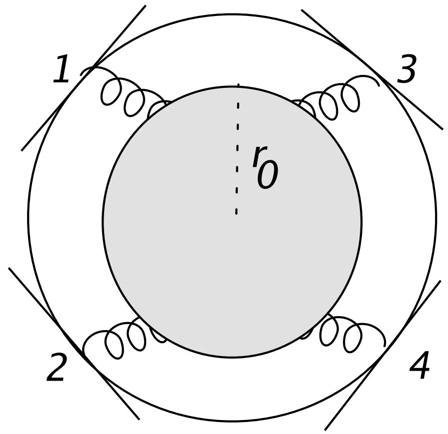
Conclusion for this part:

- intercept: function $j(\lambda)$ interpolates between weak and strong coupling: $1 < j(\lambda) < 2$.
  We know the first two corrections for $\lambda \to 0$, first correction at $\lambda \to \infty$.
  Connection with anomalous dimension.
- impact factor: we know the first two terms at $\lambda \to 0$, the first term at $\lambda \to \infty$
- interactions of (reggeized) gravitons?

B. A more ambitious approach: a 'soft' Pomeron in a 'confining' theory (Polchinski et al)

Observation: 'soft' Pomeron comes from larger values of fifth coordinate z_0. (smaller r):

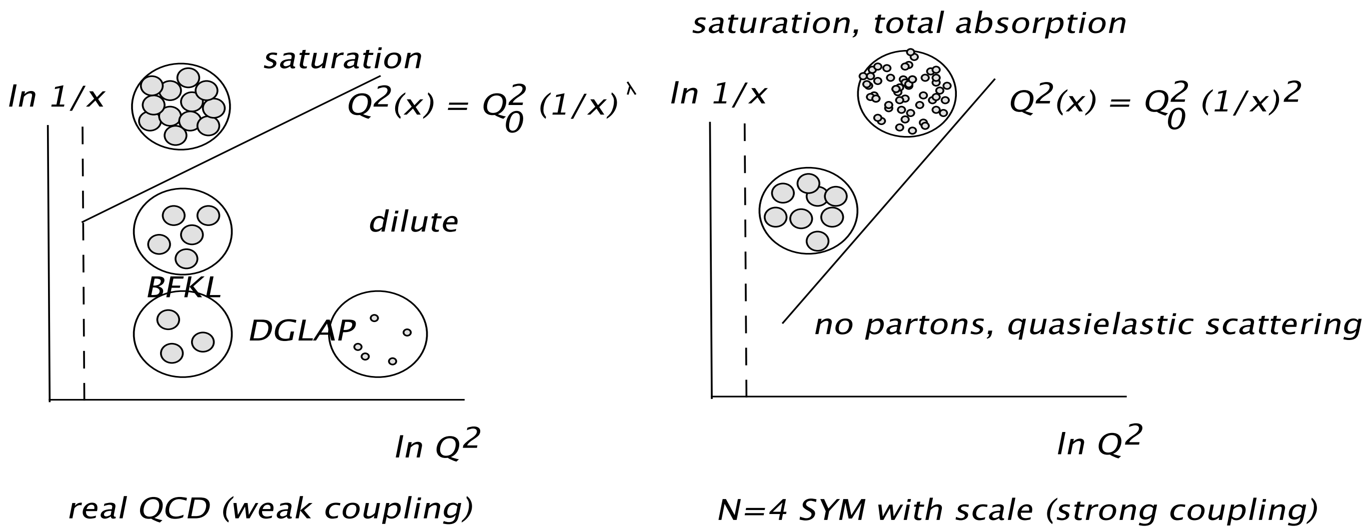
Modify the $AdS_5 \times W$: boundary $\longrightarrow$ scale.
Compute glueball, continue in t.
Obtain slope parameter.



Questions:
how to connect this soft 'Pomeron' with the hard Pomeron (=reggeized graviton)? Is there 'saturation'?

C. Deep Inelastic scattering (Polchinski et al; Mueller,Hatta,Iancu)

Goal: deep inelastic scattering for all x.
Framework: $N = 4$ DIS on hot plasma, or DIS on dilaton field



Most striking results:
- no partons at finite x
- saturation line $Q_s^2 \sim (T/x)^2$ (multiple graviton exchange).

Conclusions

We are at the beginning of exciting investigations.

A few tasks:

- Planar amplitudes:
  'Islands' of integrability; BDS formula
  hope that study of Regge limit will help to get correct expression
- Pomeron (1): how does integrability on the gauge theory side translate to the string side?
  Interpolation from strong to weak coupling?
  Pomeron (2): Unitarization. 'Saturation' in DIS: multiple graviton exchange?
  Pomeron (3): soft Pomeron needs modelling, dual analogue of QCD.