Tests of General Relativiy using Gravitational Waves

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1 Motivation

- GW's observed by LIGO come from merging BH's, i.e. from a phenomenon involving strong gravitational fields.
- In this regime one may find several phenomena modifying the standard GW emission, e.g. creation of many weakly interaction light particles such as axions could modify the standard expected waveform (see e.g. [1]) or *modifications to GR* could also change the waveform.
- First time we can learn from possible modifications to GR in the strong gravitational field regime.
- **In this talk**: focus on possible modifications to GR and the subsecuent modification to the standard GW emission. We consider that *only gravitons are involved* in this completion, no new DoF (see e.g. [2] for a paper where other DoF are included to modify gravity). We follow [3].
- **Approach**:
  1. Construct EFT action UV completing Einstein-Hilbert up to a cutoff scale Λ_c. We only include operators providing the most important corrections.
  2. Study the observable implications of the new operators.
  3. Compare the new operators with those from GR with enough precision.

2 The Modified Gravity EFT

We want to find a UV completion for the Einstein-Hilbert action

$$\mathcal{S}_{EH}[g_{\mu\nu}] = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\; R \tag{2.1}$$

involving only gravitons. We will add higher derivative terms to this action *a la* EFT, and the corresponding EFT will be valid (in the perturbative regime) for energies below a cutoff scale $E < \Lambda_c$.

- We require: new operators respect same symmetries as GR (Diff...), locality, causality, not excluded by other experiments, testable.
- Organize new operators/interactions order by order in E/Λ_c, so most important ones: those with less derivatives.
- We are modifying gravity below some energy scale Λ_c. But we perform experiments at high energies and see no deviation from GR. Problem? No: we consider that the theories gets 'resolved' at higher energies (operators become small at higher energies). This happens e.g. in String Theory with 4-graviton amplitudes. A similar mechanism forbids operators involving metric and matter fields.

- Because we are perturbing GR, we must be consitent with $\Lambda_c \to \infty$, i.e. we consider new operators that do not vanish on shell in GR limit. From Einstein's equations on the vacuum:

$$R_{\mu\nu} = 0 \tag{2.2}$$

  In other words, these operators can 'dissappear' by redefining the metric and never appear at observables (see e.g. [4]).

2.1 New operators

As said above, we proceed using EFT ideas and order operators order by order in derivatives. because of (2.2), we can only use Riemann tensors.

- At the 2 derivative level we have the Einstein-Hilbert action (2.1).
- 4 derivatives: there are two operators

$$\mathcal{C} \equiv R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \qquad \& \qquad \tilde{\mathcal{C}} \equiv \varepsilon_{\mu\nu\sigma\rho} R^{\mu\nu\alpha\beta} R_{\alpha\beta}{}^{\sigma\rho} \ . \tag{2.3}$$

  But $\tilde{\mathcal{C}}$ is a total derivative and for $R_{\mu\nu} = 0$, $\mathcal{C} = 0$ is a total derivative as well. So no 4 derivative term present.

- 6 derivatives:

$$\mathcal{S}_{eff}[g_{\mu\nu}] = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left(R + c_3 \frac{R_{\mu\nu\sigma\rho} R^{\mu\nu\alpha\beta} R_{\alpha\beta}{}^{\sigma\rho}}{\Lambda_c^4} + \tilde{c}_3 \frac{\varepsilon_{\mu\nu\omega\delta} R^{\omega\delta}{}_{\sigma\rho} R^{\mu\nu\alpha\beta} R_{\alpha\beta}{}^{\sigma\rho}}{\Lambda_c^4} \right) \ .$$

  These operators violate causality [5] unless $c_3 = \tilde{c}_3 = 0$ or $c_3 \sim \Lambda_c^2/M_p^2$ and we include an infinite tower of higher spin particles coupled gravitationally to the Standard Model with a mass $m \sim \sqrt{\Lambda_c M_p}$.

- 8 derivatives:

$$\mathcal{S}_{eff}[g_{\mu\nu}] = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left(R + c \frac{\mathcal{C}^2}{\Lambda_c^6} + \tilde{c} \frac{\tilde{\mathcal{C}}^2}{\Lambda_c^6} + c_- \frac{\tilde{\mathcal{C}}\mathcal{C}}{\Lambda_c^6} \right) \ . \tag{2.4}$$

  So far, no reason why these operators are forbidden, so this will be our effective action.

Causality set constrains on coefficients of (2.4) [6]. Resulting EoM is

$$R^{\mu\alpha} - \frac{1}{2} g^{\mu\alpha} R = -\frac{c}{\Lambda_c^6} \left(8 R^{\mu\nu\alpha\beta} \nabla_\nu \nabla_\beta \mathcal{C} + \frac{1}{2} g^{\mu\alpha} \mathcal{C}^2 \right) - \frac{\tilde{c}}{\Lambda_c^6} \left(8 \varepsilon^{\mu\nu\sigma\rho} R_{\sigma\rho}{}^{\alpha\beta} \nabla_\nu \nabla_\beta \tilde{\mathcal{C}} + \frac{1}{2} g^{\mu\alpha} \tilde{\mathcal{C}}^2 \right)$$
$$- \frac{c_-}{\Lambda_c^6} \left(4 \varepsilon^{\mu\nu\sigma\rho} R_{\sigma\rho}{}^{\alpha\beta} \nabla_\nu \nabla_\beta \tilde{\mathcal{C}} + 4 R^{\mu\nu\alpha\beta} \nabla_\nu \nabla_\beta \mathcal{C} + \frac{1}{2} g^{\mu\alpha} \mathcal{C}\tilde{\mathcal{C}} \right) \ , \tag{2.5}$$

where $R_{\mu\nu} = 0$ was used on the RHS. This gives a dispersion relation for high energy gravitons in transverse traceless gauge

$$-k^2 = \frac{64}{\Lambda_c^6} \left[c (k_\mu k_\nu R^{\alpha\mu\nu\beta} e_{\alpha\beta})^2 + \tilde{c} (k_\mu k_\nu \varepsilon^{\alpha\mu\sigma\rho} R_{\sigma\rho}{}^{\nu\beta} e_{\alpha\beta})^2 + c_- (k_\mu k_\nu R^{\alpha\mu\nu\beta} e_{\alpha\beta})(k_\mu k_\nu \varepsilon^{\alpha\mu\sigma\rho} R_{\sigma\rho}{}^{\nu\beta} e_{\alpha\beta}) \right]$$

Causality implies that $k^2 \leq 0$, so for different polarizations $e_{\alpha\beta}$ and momenta k^μ one finds

$$c, \ \tilde{c} \geq 0 \qquad ; \qquad c_-^2 \leq 4 c \tilde{c} \ .$$

- Here we neglect higher order operators.

3 The 2 Black Hole system

To study the GW emission in this modified theory of gravity, one should include matter and them solve (2.5) with $T_{\mu\nu} \neq 0$. This requires numerical methods and is hard...

Instead, we consider the 2BH system in the Post-Newtonian (PN) regime ($v \ll 1$).

$$\mathcal{S}_{pp} = \int d^4x \left[\delta^{(3)}(\vec{x}-\vec{x}_1)\left(m_1(1+\frac{\vec{v}_1^2}{2}) + d_1\sqrt{g_{\alpha\beta}\dot{x}_1^\alpha \dot{x}_1^\beta}\mathcal{C} + \ ...\right) + \delta^{(3)}(\vec{x}-\vec{x}_2)\left(m_2(1+\frac{\vec{v}_2^2}{2}) + ...\right)\right] .$$

Here for simplicity $d_i = 0$, but $d_i \neq 0$ can lead to interesting phenomena [3]. So the total action we start with is

$$\mathcal{S}_{tot} = \mathcal{S}_{eff} + \mathcal{S}_{pp} \ . \tag{3.1}$$

We will once again use an EFT approach to deal with the 2BH system, ignoring matter and gravitational perturbations of $\lambda < r$ and treat the system as an extended object with internal degrees of freedom [7]. On the center of mass frame this gives an action:

$$\mathcal{S}_{ext.obj.} = \int dt \left[m_1 + m_2 + \frac{\mu(t)}{2}\vec{v}_{12}^2 - V(r(t)) + \frac{1}{2}Q_{ij}(t)R^{i0j0} + \ ...\ \right] .$$

with $\mu(t)$ the reduced mass, $Q_{ij}(t)$ the mass quadrupole that in the Newtonian limit is

$$Q_{ij}^{(N)}(t) = \sum_k m_k(x_k(t)^i x_k(t)^j - \frac{1}{3}x_k(t)^2\delta^{ij}) \ . \tag{3.2}$$

3.1 Intuitive effect of modifying gravity

Before studying the effect of the new operators, recall that the angular velocity of the 2BH system, and thus the frequency of the emitted GW's, depends on the gravitational potential

$$\omega \sim \sqrt{\frac{1}{r}\frac{dV}{dr}} \ . \tag{3.3}$$

Also, the quadrupole formula (recall Thibaud's lecture) tells that

$$\left[h_{ij}^{TT}(t,\vec{x})\right]_{quad} = \frac{2G}{R}\Lambda_{ij,kl}\ddot{Q}_{kl}(t-R) = -\frac{8G\omega^2}{R}\Lambda_{ij,kl}Q_{kl}(t-R) \tag{3.4}$$

for a quasi-circular orbit.

The effect of modifying gravity will be to change the gravitational potential $V(r(t))$ (and thus the frequency ω). It will also provide new channels of graviton emission, modifying the mass quadrupole and therefore the amplitude h_{ij}^{TT}.

4 Quantitative effect of the modification

The new terms in (2.4) correspond to 4-graviton vertices. These leads to new Feynman diagrams modifying the gravitational potential between both BH's and providing new emission channels. Let us see how this works by deriving Newton's potential.

4.1 Newtonian potential from Feynman diagrams

We first need the corresponding Feynmann rules. The propagator of a graviton in GR is

$$-i\frac{\eta_{\nu\beta}\eta_{\mu\alpha}+\eta_{\mu\beta}\eta_{\alpha\nu}-\eta_{\mu\nu}\eta_{\alpha\beta}}{2k^2}\delta(t_1-t_2)(2\pi)^3\delta^{(3)}(\vec{q}+\vec{k})$$

where $\vec{k}$ and $\vec{q}$ are outgoing momenta. We can compute the potential graviton (H_{00}) coupling to point particles Feymann rules from the point particle action

$$\mathcal{S}_{pp}=\int dt\left(-\frac{2mH_{00}}{M_p}-\frac{2mH_{0i}v^i}{M_p}-...\right)\ .$$

In the PN limit this implies that the vertex rule is [7]

$$-i\delta_0^\alpha\delta_0^\beta\frac{2m}{M_p}e^{-i\vec{k}\cdot\vec{x}}$$

Using these rules we compute the exchange of a potential graviton between two objects

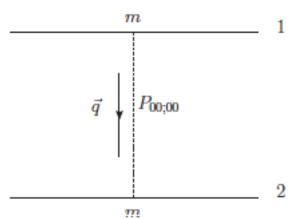

Figure 1: Virtual (potential) graviton exchange between two point particles

which gives rise to the Newtonian potential as follows:

$$\int dt_1\int dt_2\int_{\vec{q},\vec{k}}\left[-i\frac{\eta_{\nu\beta}\eta_{\mu\alpha}+\eta_{\mu\beta}\eta_{\alpha\nu}-\eta_{\mu\nu}\eta_{\alpha\beta}}{2k^2}\delta(t_1-t_2)(2\pi)^3\delta^{(3)}(\vec{q}+\vec{k})\right]\left[-i\delta_0^\alpha\delta_0^\beta\frac{2m_1}{M_p}e^{-i\vec{k}\cdot\vec{x}_1}\right]\left[-i\delta_0^\mu\delta_0^\nu\frac{2m_2}{M_p}e^{-i\vec{q}\cdot\vec{x}_2}\right]$$

$$=-i\frac{2m_1m_2}{M_p}\int dt\int_{\vec{k}}\frac{d^{3-\varepsilon}k}{(2\pi)^{3-\varepsilon}}\frac{e^{-i\vec{k}\cdot\vec{x}_{12}}}{k^2}=i\int dt\frac{Gm_1m_2}{x_{12}}=i\int dt(-V_N)=i\mathcal{S}_{ext.obj.}$$

4.2 Corrections from the $\mathcal{C}^2$ term

In a similar way, one can compute the relevant diagrams modifying the above potential or the gravitational wave emission. Each of the new operators is a 4 graviton vertex, we will give some details about how the $\mathcal{C}^2$ affects, then just give the results for the other operators.

Contribution to the potential

The leading order diagrams are those with a unique 4 graviton vertex. There are 2 such diagrams with all gravitons ending up on the two objects at leading order

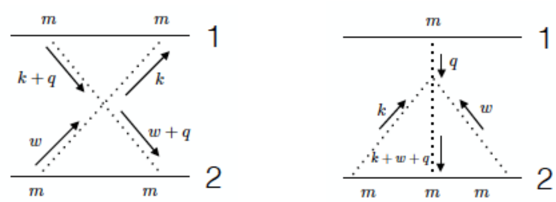

Figure 2: Leading processes involving the $\mathcal{C}^2$ vertex giving rise to corrections to the gravitational potential

but the first diagram is zero on the PN limit. The second one has a contribution

$$i\int dt\left(\frac{-64c^6 m_1 m_2^3}{\pi^3\Lambda_c^6 M_p^6}\frac{1}{r(t)^9}\right)$$

Including also the diagram where $1\leftrightarrow 2$, the leading order contribution to the potential is

$$\Delta V_{\mathcal{C}^2}=\frac{8}{\pi^6}\frac{Gm_1m_2}{r}\left(\frac{2\pi c}{\Lambda_c r}\right)^6\frac{G^2(m_1^2+m_2^2)}{r^2}$$

Note that the perturbation parameter is

$$\frac{2\pi c}{\Lambda_c r}\ll 1\ . \tag{4.1}$$

This modification of the gravitational potential will result in a change of the angular velocity of the 2BH system

$$\Delta\omega_{\mathcal{C}^2}=-\frac{9}{2}\frac{\Delta V_{\mathcal{C}^2}}{r^2\omega}\qquad\Rightarrow\qquad\frac{\Delta\omega_{\mathcal{C}^2}}{\omega}=-\frac{9}{\pi^6}\frac{4G^2(m_1^2+m_2^2)}{r^2}\left(\frac{2\pi c}{\Lambda_c r}\right)^6$$

Contribution to the emission amplitude

Now the leading diagrams have one 4 graviton vertex and one outgoing graviton. There is only one in this case:

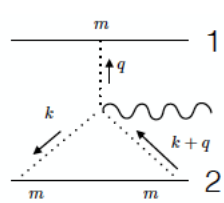

Figure 3: Leading contribution from $\mathcal{C}^2$ operator correcting graviton emission.

The corresponding amplitude is

$$i\frac{m_1m_2^2c^6}{\Lambda_c^6M_p^4}\frac{7\cdot 16}{\pi^2}\int dt\left(\frac{3r^ir^j}{r^8}-\frac{\delta^{ij}}{r^6}\right)R^{0i0j}(\bar{h})$$

where $\bar{h}$ represents the outgoing on-shell graviton. This diagram together with the one where $1 \leftrightarrow 2$ give

$$\mathcal{S}_{\mathcal{C}^2,rad} = \int dt \frac{21}{\pi^6}\left(\frac{2\pi c}{\Lambda_c r}\right)^6 \left(\frac{G(m_1+m_2)}{r}\right)^2 \frac{m_1 m_2}{m_1+m_2}\left(r^i r^j - \frac{r^2\delta^{ij}}{3}\right) R^{0i0j}(\bar{h}) \tag{4.2}$$

or equivalently modify

$$Q_{ij} \to \left(1 + \frac{42}{\pi^6}\left(\frac{2\pi c}{\Lambda_c r}\right)^6 \left(\frac{G(m_1+m_2)}{r}\right)^2\right) Q_{ij}^{(N)}$$

4.3 Contributions from other vertices

The other 2 operators also give rise to modifications of the gravitationalk potential and emission amplitude. We skip the details here, which can be found in [3].

5 Observable consecuences

Let us consider how to measure the above effects. We restrict to quasi circular orbits where $\omega \gg \dot{r}/r$

We first observe that since both the frequency and the mass quadrupole get affected, plugging in both changes in (3.4) to leading order in c (only the $\mathcal{C}^2$ operator effects)

$$\left[\Delta h_{ij}^{TT}(t,\vec{x})\right]_{\mathcal{C}^2} = \left(\frac{2\pi c}{\Lambda_c r}\right)^6 \frac{8G^2}{\pi^6 r^2}\left(\frac{21(m_1+m_2)^2}{4} - 9(m_1^2+m_2^2)\right)\frac{2G}{R}\Lambda_{ij,kl}\ddot{Q}_{kl}(t-R) \tag{5.1}$$

where the first part comes from the new emission channel and the second one from the frequency change.

Let's see how it scales

$$\Delta h_{\mathcal{C}^2} \sim h\frac{\Delta\omega_{\mathcal{C}^2}}{\omega} \sim h\frac{1}{(\Lambda_c r)^6}\left(\frac{Gm}{r}\right)^2 \sim h\frac{1}{(\Lambda_c r)^6} v^4 \tag{5.2}$$

This is small due to v and also to $\Lambda_c r$. How can we know if this term is present? Compute the PN correction to Δh in GR of larger order than v^4 such that the higher orders can compensate for the $\Lambda_c r$ powers. Then we can trust this contribution. Specially when $\Lambda_c r \sim 1$.

Similarly we can perform the same analysis for magnitudes other $\Delta h/h$ and see how they scale in powers of v to compare them to the GR expansion in the PN regime. We can also do the same including the effect of the 2 other vertices. The different scalings are given in this table

Operator	$\mathcal{C}^2$	$\tilde{\mathcal{C}}^2$	$\mathcal{C}\tilde{\mathcal{C}}$
$\Delta V/V$	v^4	$v^7 a$	$v^6 a$
$\Delta\omega/\omega$	v^4	$v^7 a$	$v^6 a$
$\Delta Q_{ij}/Q_{ij}$	v^4	$-$	v^5
$\Delta J_{ij}/J_{ij}$	$-$	v^4	v^3
$\Delta P/P$	v^4	v^6	$v^6 a$
$\Delta h/h$	v^4	v^5	v^4

where J_{ij} is the current quadrupole, P the emitted power and a is related to the spin of the BH's.

According to [3], the current experiments that could provide more information about possible modifications of gravity are X-ray binaries, where there is also a rather strong gravitational field. The modified gravitational potential implies a modification deviating from Schwarzschild/Kerr metric that may be observable.

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