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Gravitational Waves from (p)reheating

1. Intro: Gravitational waves in the early Universe

↝ Inflation : • tensor primordial quantum fluctuations generated during expansion
↳ low amplitude ($\Omega_{GW} \sim 10^{-15}$) & red spectrum.
Maybe detectable with Big Bang Observatory or DECIGO.
See Kenichi's talk

• With active sources – spectator fields
– PBHs
↳ Strongly constrained, as GW generation is associated to P_J & f_{NL} generation.
See 1610.06481.

↝ Preheating : • Sourcing inhomogeneities (this talk)
• Formation of solitons (see Enrico's talk)
& pseudo-solitons (this talk)

↝ Other cosmic defects (see Enrico's talk)

↝ Phase transitions : related to bubble nucleation
See Geraldine & Christophe's paper hep-ph/0607107
↳ May be detectable by LISA

for a classic reference on reheating hep-ph/9704452

2. Perturbative Reheating

→ After inflation ends, the inflaton ϕ starts oscillating around the minimum of its potential, and most of its energy is kinetic $\frac{1}{2}\dot{\phi}^2$. This energy needs to be converted into Standard Model degrees of freedom.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) - g^2\phi^2\chi^2$$

$$V(\phi) = \frac{1}{2}m^2(\phi-\sigma)^2 + \ldots$$ expanding around minim.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \underbrace{2g^2\sigma\phi\chi^2}_{\Gamma_{\phi\to\chi\chi}} - \underbrace{g^2\phi^2\chi^2}_{\Gamma_{\phi\phi\to\chi\chi}}$$

e.o.m. : $$\boxed{\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + m^2\phi = 0}$$

for example: for $2g^2\sigma\phi\chi^2$, $\Gamma_{\phi\to\chi\chi} = \frac{g^4\sigma^2}{8\pi m}$

→ Particle decay can be described as an extra friction term.

⚠ PROBLEMS WITH THIS DESCRIPTION:

It doesn't include the backreaction of the classical inflaton ϕ on the quantum produced χ-particles. This will cause resonance & radically change the scenario.

3. Parametric Resonance & Preheating

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \quad \longrightarrow \quad \phi(t) \simeq \Phi(t)\sin(mt) \qquad \Phi(t) = \frac{M_P}{mt}$$

$$\hat{\chi}(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left(\hat{a}_k \chi_k(t) e^{-i\vec{k}\cdot\vec{x}} + \hat{a}_k^{\dagger} \chi_k^*(t) e^{i\vec{k}\cdot\vec{x}} \right)$$

with $$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\chi_k = 0 \quad (*)$$

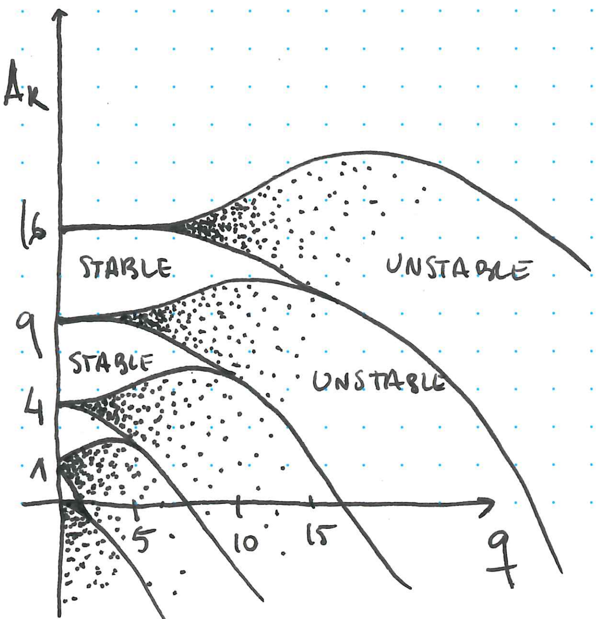
ignoring the expansion of the Universe (we'll come to this):

$$\ddot{\chi}_k + \left(k^2 + g^2\Phi^2 \sin(mt)\right)\chi_k = 0$$

with $z = mt$
$$\boxed{\chi_k'' + \left(A_k - 2q\cos(2z)\right)\chi_k = 0}$$

Mathieu eq.

where $A_k \equiv \frac{k^2}{m^2} + 2q$ & $q \equiv \frac{g^2\Phi^2}{4m^2}$, new eff. mass.



STABILITY / INSTABILITY CHART:

UNSTABLE RESONANT BANDS:

$$\chi_k \propto \exp(\mu_k z)$$

$$n_k = |\chi_k|^2 \propto \exp(2\mu_k z)$$

μ_k Floquet exponent (instability parameter)

(*) $m_\chi \equiv 0$ but this could be relaxed, and it wouldn't change much.

When does resonance dominate over perturbative decays?

Heuristic: $\ddot{\chi}_k + 3H\dot{\chi}_k + \Gamma_{\chi\to\phi}\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\chi_k = 0$

$\frac{k^2}{a^2}$: $\ll 1$, superhorizon

$$(3H + \Gamma)\dot{\chi} < (g^2\phi^2)\chi$$

$$\frac{\chi}{\dot{\chi}} \sim \frac{1}{m} \Rightarrow \boxed{3H + \Gamma < q\,m}$$

- How does χ_k look like on stable bands?

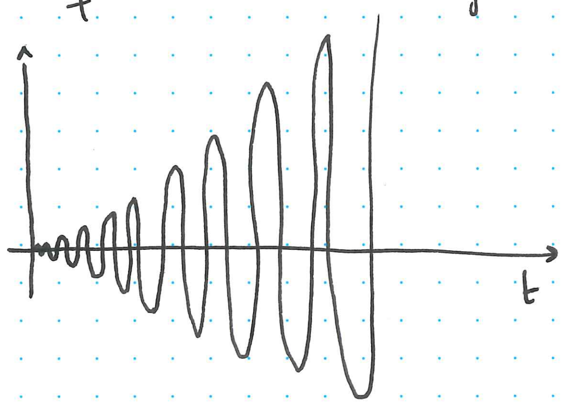
(sin modulated by sin)

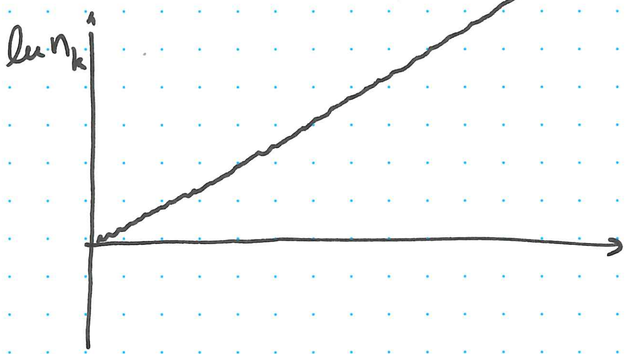
- How does χ_k behave on unstable bands?

3.1 $q \ll 1$: Narrow resonance

resonance occurs for $A_k^{(n)} \simeq n^2$ & $\Delta k^{(n)} \sim m q^n$

for $q < 1$ the first band is the most important.





→ Expansion of the Universe:
Decreases the width of the resonance band $\Delta k \propto 1/t$ and redshifts modes out of the band.
→ Rescattering of χ-particles also redshifts modes out of the band.
So narrow resonance is rather delicate process.

3.2 $g \gg 1$: Broad resonance.

↳ Production of massive χ.

Particle production occurs when the adiabatic condition is violated:

$$R \equiv \frac{|\dot{\omega}|}{\omega^2} > 1 \qquad \text{with} \quad \omega(t) = \sqrt{k^2 + g^2\phi^2}$$

note that for $k \to 0$, $R = \frac{\dot{\phi}}{g\phi^2}$ → diverges at $\phi \to 0$ (see stability chart)

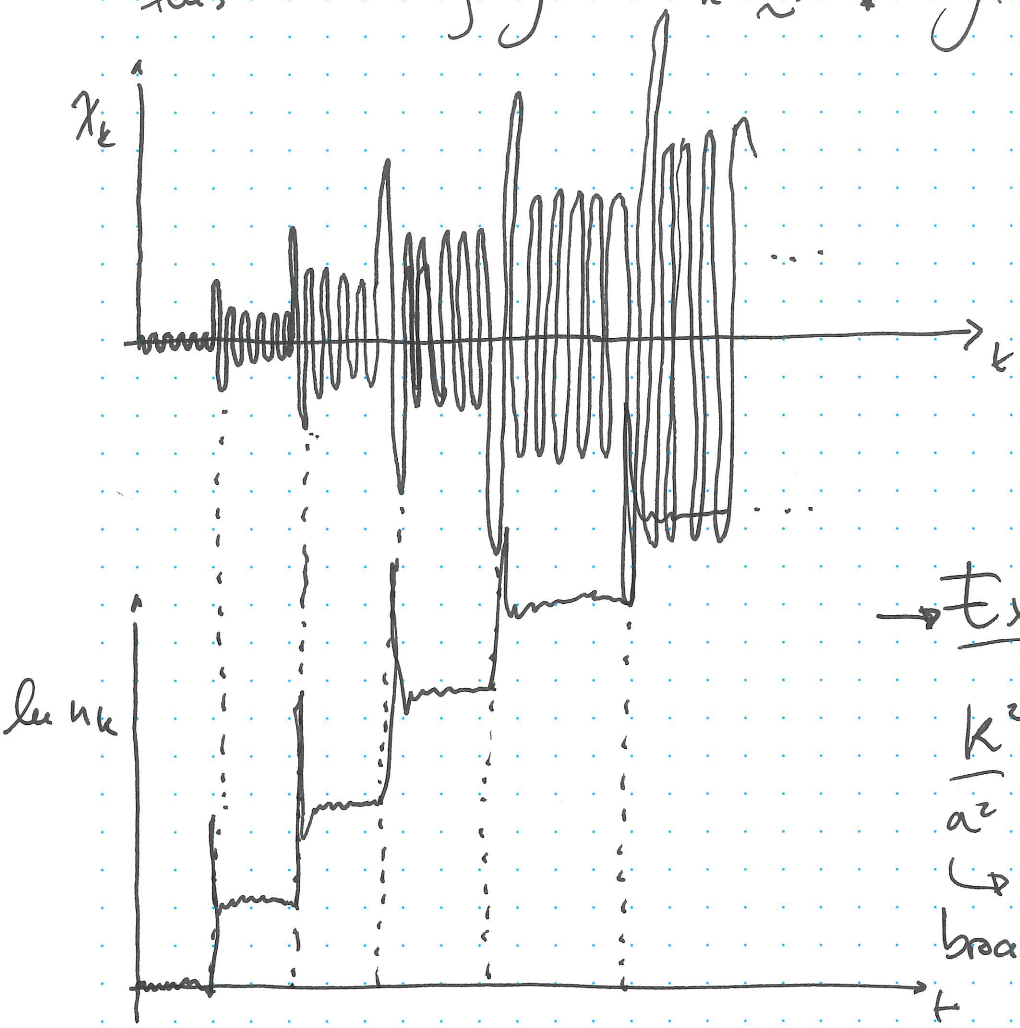
for finite k, the condition for resonant particle production is

$$R = \frac{g^2\phi\dot{\phi}}{(k^2+g^2\phi^2)^{3/2}} > 1$$

$\dot{\phi} = m\Phi\cos(mt) \sim m\Phi$

$$\boxed{k^2 \lesssim (g^2\phi m\Phi)^{2/3} - g^2\phi^2 \equiv f(\phi)}$$

The maximum range of wavenumbers satisfy this condition at ϕ_* where $f'(\phi_*)=0$;
this is roughly $k^2 \lesssim k_*^2 \equiv g m \Phi$



→ Expansion of the Universe

$\frac{k^2}{a^2} \lesssim k_*^2 = g m \Phi$

↪ Redshifting makes broad resonance more efficient!

3.3 Termination of preheating

Backreaction of χ-particles into ϕ ultimately terminate preheating

$$\Delta m^2 > m^2 \quad \text{where} \quad \Delta m^2 = g^2 \langle \chi^2 \rangle$$

Rigorously studying backreaction requires numerical analysis.

We are left with decay products and inflaton particles that survived preheating. The Universe is inhomogeneous. These particles will rescatter in a highly non-perturbative and non-linear process, leading to thermalization (hence the "pre"-heating).

$\rightsquigarrow$ NON LINEAR PHYSICS $\Rightarrow$ PRODUCTION OF G.W.?

4. Production of gravitational waves during/~~after~~ preheating

4.1 G.W. from inhomogeneous modes (χ or ϕ)

$$T_{\mu\nu} = \partial_\mu \phi_a \partial_\nu \phi_a - g_{\mu\nu}\left(\frac{1}{2} g^{\rho\sigma} \partial_\rho \phi_a \partial_\sigma \phi_a + V\right)$$

$\hookrightarrow$ energy-momentum tensor for ϕ_a inhomogeneous fields

these can act as sources for G.W.

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})\, dx^i dx^j\right]$$

in fourier space & redefining $h_{ij} \to a h_{ij}$

$$h''_{ij}(k) + \left(k^2 - \frac{a''}{a}\right) h_{ij}(k) = 16\pi G a^3 \Pi^{TT}_{ij}(k)$$

where $\Pi^{TT}_{ij} = \dfrac{T_{ij}}{a^2} - \langle p \rangle g_{ij}$, $\langle p \rangle$ background pressure.

$$\Pi^{TT}_{ij} \supseteq \partial_i \chi \, \partial_j \chi + \dots$$

→ Solve this numerically (non-perturbative)
Results are model dependent, however typically we find frequencies in the MHz or GHz range, so outside detection from planned detectors.

See 0707.0875
astro-ph/0612294, 1309.1148

For an example of decay of SM Higgs after preheating see 1602.03085.
In this model χ is the SM Higgs which thermalizes into the rest of SM species.
Lattice simulation → GW produced after parametric resonance.

But $\Omega_{GW} h^2 < 10^{-29} - 10^{-16}$ (by boosting the ~~[illegible]~~ amplitude with a kination stage)

$$f_p \lesssim 10^{11}\ \text{Hz}.$$

4.2 G.W. from pseudo-solitons: oscillons

During the non-perturbative history of preheating and just after, many interesting phenomena may take place, including the formation of topological solitons.

As Enrico talked about cosmic strings last week, I will try to talk about quasi or pseudo-solitons: oscillons.

Oscillons are spatially localized, temporarilly oscillating configurations of a non-linear field theory. They are not stricktly stable, but they are long lived (they survive many thousand oscillations).

A helpful way to think about the highly non-linear physics after preheating is to picture the inflaton background fragment into clumps, becoming non-homogeneous (non-gaussian).

An oscillon is a configuration of such clumps which is quasi-stable.

- <u>How do</u> they <u>arrise</u>?

Consider the potential around the minimum

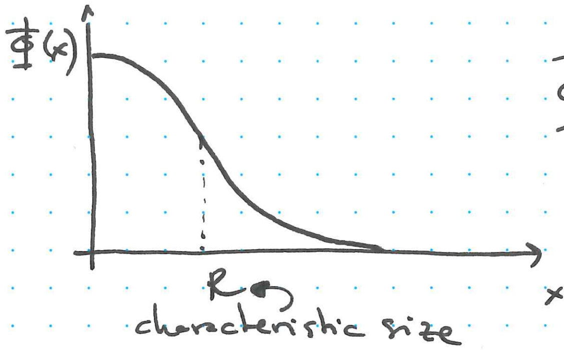
$$V(\phi) = \frac{\mu^2 \phi^2}{2} + V_{nl}(\phi)$$

for small oscillations

$$\ddot{\phi} - \nabla^2 \phi + \mu^2 \phi + V'_{nl}(\phi) = 0$$

Oscillon solutions have a dominant oscillation frequency so we can start by using the ansatz $\phi(x) = \Phi(x)\cos(\omega t)$

where $\Phi(x)$ is a positive localized function that falls from the centre. for example, the gaussian profile:



$$\Phi(x) = (\Phi_R - \Phi_0)^{-x^2/R^2} + \Phi_0$$

$$\Phi_0 = \Phi(0)$$

$$\Phi_R = \Phi(R)$$

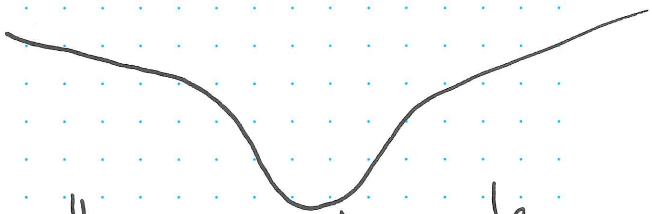
Imposing boundary conditions:

- $\nabla^2\phi^2$ for large x must be positive
- $\nabla^2\phi^2$ for $x=0$ must be negative and $\Phi'(0)=0$.

(these conditions are equivalent to asking for stability of this gaussian profile)

for these conditions we find a constraint on V_{nl}

$$\boxed{V_{nl}(\phi) < 0}$$



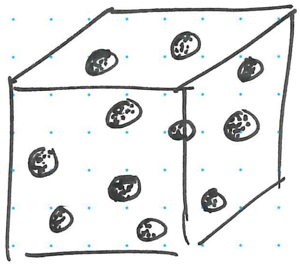
the potential needs to "open up" away from the minimum.

Physically, this is saying that a trade-off between dissipative quadratic terms and an attractive force generated by V_{nl}, ensures that the formation of an oscillon is energetically favourable.

This is very interesting as this "shallower" potentials are preferred by data. They are also common in string-pheno: monodromy.

In fact, 1106.3335 argues that most energy after inflation is stored into oscillons in many models.

For $$V(\phi) = \frac{m^2 M^2}{2\alpha}\left[\left(1+\frac{\phi^2}{M^2}\right)^{\alpha} - 1\right]$$



Analytical expectations:

- Oscillons should have spherical symmetry as that is energetically favourable $\to$ no G.W.
- Interactions unimportant as they are localized and well separated.

See 1304.6094
1006.3075
and contradiction
1607.01314

Numerical results:

- GW produced as oscillons get produced $\Rightarrow$ equivalent to preheating signal (high frequencies)
- No GW produced once oscillons are "stable".

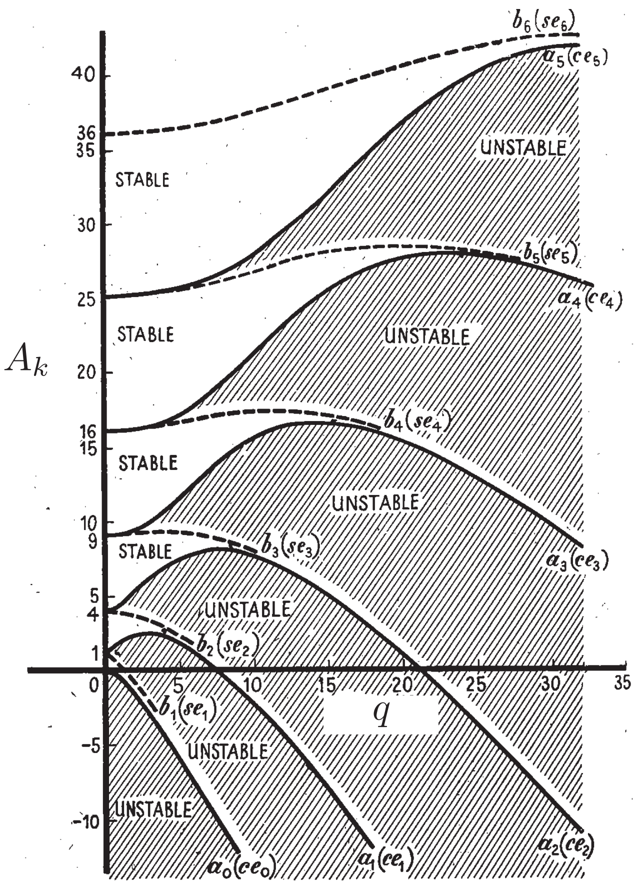


Figure 1: *Instability bands of the Mathieu equation.*

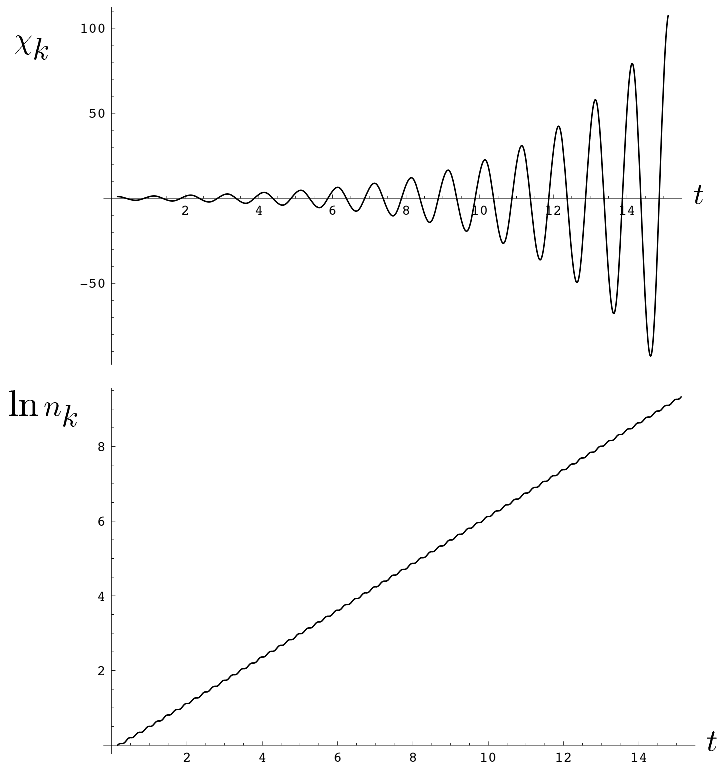


Figure 2: (Reproduced from Kofman et al.) *Narrow parametric resonance for the field χ in the theory $\frac{1}{2}m^2\phi^2$ in Minkowski space for $q \sim 0.1$. Here, time is plotted in units of $[m/2\pi]^{-1}$.*

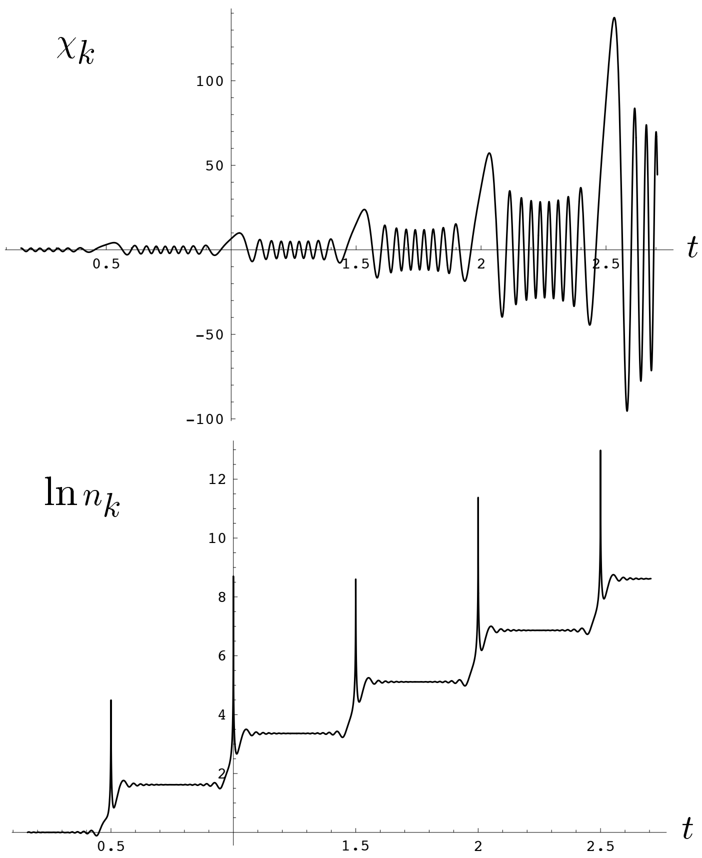


Figure 3: (Reproduced from Kofman et al.) *Broad parametric resonance for the field χ in Minkowski space for $q \sim 2 \times 10^2$ in the theory $\frac{1}{2}m^2\phi^2$.*

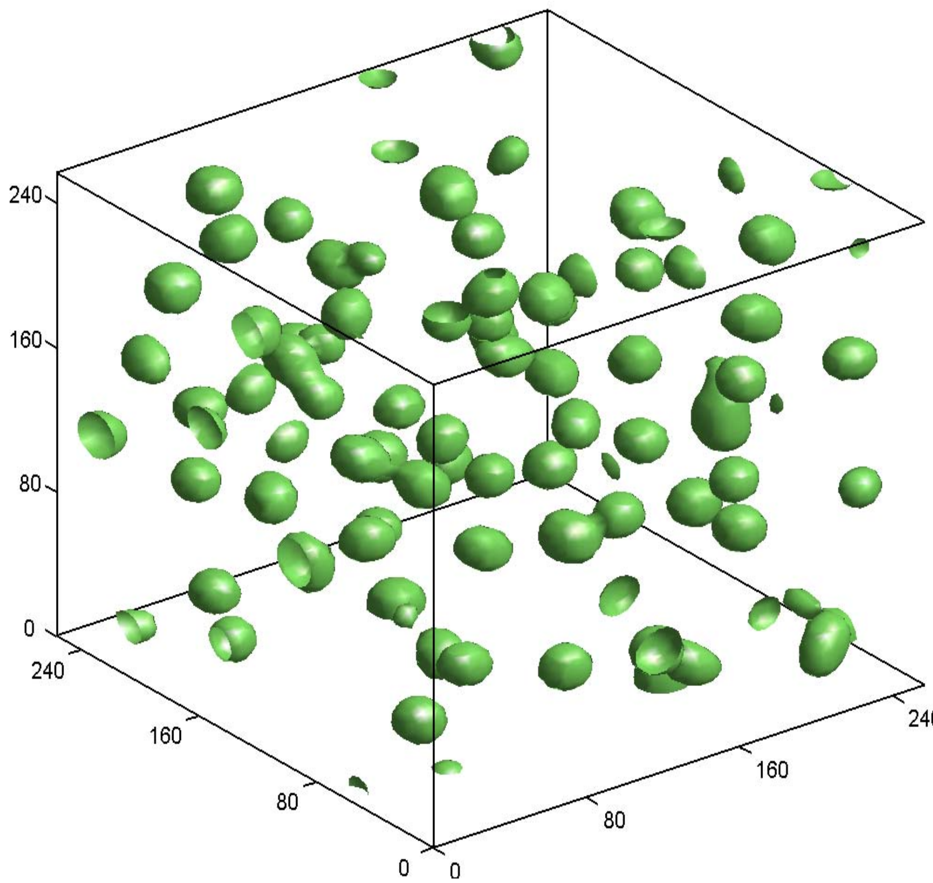


Figure 1. A late time snap shot of the energy density in oscillon preheating. The model is that of Eq. (2.12) with $\alpha = 1/2$ and $M = 0.01M_P$. The box size is $L = 50/m$ and the energy density isosurface is taken at a value 5 times the average energy density.