

HOMs and Mode Rotation in the ILC Main Linacs

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- Injected emittance: $\epsilon_{x,y} = 8e-6, 2e-8$ m.rads
- Injection beam energy 5 GeV \Rightarrow O/P 500 GeV c.m. energy
- $\gamma_{i,f} \sim 9782$, and 489,000
- All simulations are made with the code LIAR.
- 500 bunches in all simulations.
- Initial vertical and horizontal offset $\sim 10 \mu\text{m}$ and $400 \mu\text{m}$.
- c.f. initial beam $\sigma_{y,x} (= [\beta \epsilon_n / \gamma]^{1/2}) \sim 10.1 \mu\text{m}, 270 \mu\text{m}$.
- $\sigma'_{x,y} (= [(1 + \alpha^2) / \beta \epsilon_n / \gamma]^{1/2}) \sim 5.1, 0.26 \mu\text{rads}$
- Initial $\beta_{x,y} \sim 89.3, 50.7$ m



Thanks to *Steve Molloy, Nicoleta Baboi, Chris Adolphsen, Joe Frisch, Roger Miller, Tor Raubenheimer, Mark Ross, Peter Tenenbaum.*

Why?

- A perfectly elliptical TESLA cavity has identical (rotated) x-y fields; their frequencies are degenerate.
- In reality the geometry of the vertical plane will be slightly different from the horizontal plane.
- Thus, the freq. degeneracy is split quite naturally as a consequence of quite relaxed tolerances in making several thousand cavities.
- The plane of the eigenmodes will be shifted.



Overview

Make several injection offset simulations to investigate issue of emittance dilution.

1. Force the angle of all couplers relative to the dipole field to be static.
2. Allow complete azimuthal randomization of couplers (0 to 2π).
3. Limit the azimuthal spread of the couplers.

- For *detailed* simulations consider a single machine for the above cases.
- For general emittance dilution consider 200 separate machines.

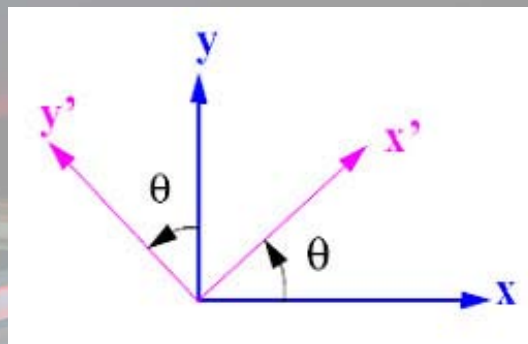
Two cases are investigated:

1. ‘Properly’ damped cavities (all modes damped according to the design)
2. ‘Rogue’ mode miss-damped. What is the impact on the emittance dilution?

$$V_{x'}(t) = V_x(0) \cos \theta$$

$$V_{y'}(t) = -V_x(0) \sin \theta$$

$$\begin{pmatrix} V_x(t) \\ V_y(t) \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} V_{x'}(t) \\ V_{y'}(t) \end{pmatrix}$$



Now, the frequency degeneracies of the modes are split and this gives a horizontal and vertical mode frequency so that:

$V_{x'}(t) \propto \exp(i\omega_x t)$ and $V_{y'}(t) \propto \exp(i\omega_y t)$. Thus, in cartesian coords:

$$\begin{pmatrix} V_x(t) \\ V_y(t) \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} V_x(0) \cos \theta \exp(i\omega_x t) \\ -V_x(0) \sin \theta \exp(i\omega_y t) \end{pmatrix}$$

$$\Rightarrow V_y(t) = iV_x(0) \sin 2\theta \exp(i\langle \omega \rangle t) \sin(\Delta\omega t / 2)$$

$$V_x(t) = V_x(0) \exp\left[i\left(\langle \omega \rangle - \Delta\omega / 2\right)t\right] \left[1 + 2i \cos^2 \theta \exp(i\Delta\omega t / 2) \sin(\Delta\omega t / 2)\right]$$

where the splitting of the degeneracy is given by: $\Delta\omega = (\omega_x - \omega_y)$ and the average

frequency $\langle \omega \rangle = (\omega_x + \omega_y) / 2$.

$$V_y = iV_x(0) \sin 2\theta \exp(i\langle\omega\rangle t) \sin(\Delta\omega t/2)$$

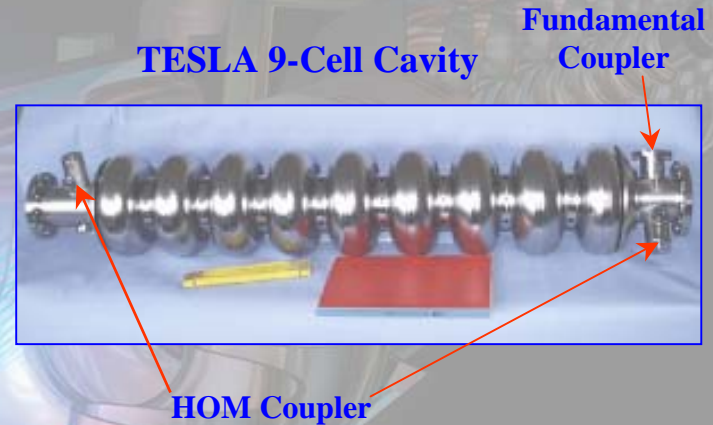
Thus for $\theta = 0$ or $\pi/2$ $V_y(0) = 0$ as expected!

The additional kick to the beam is of the form:

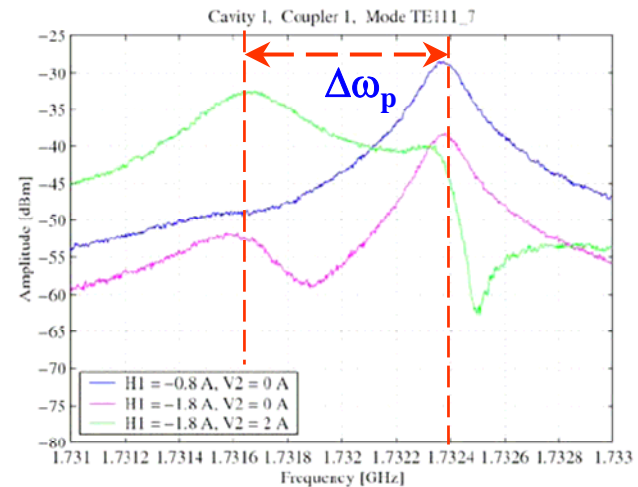
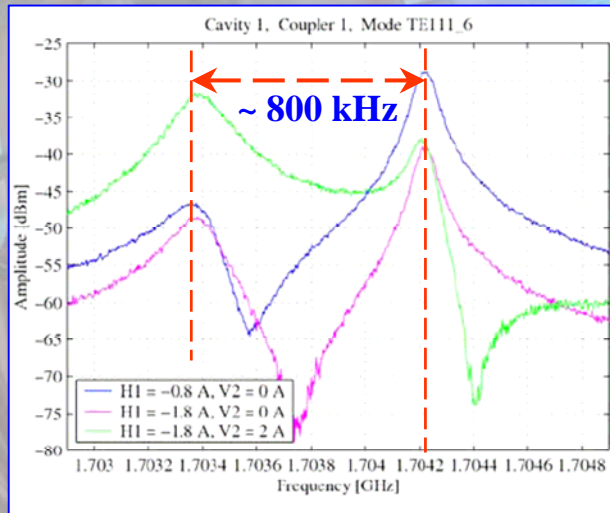
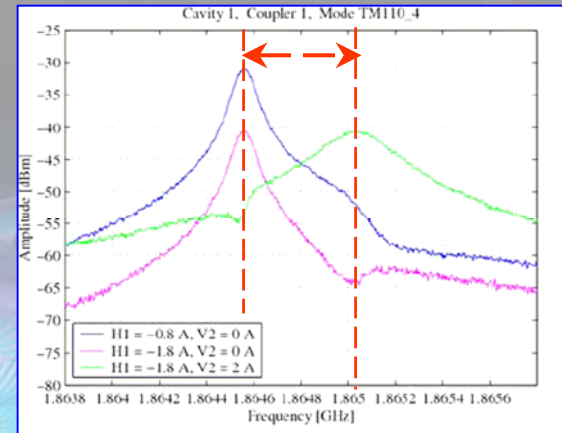
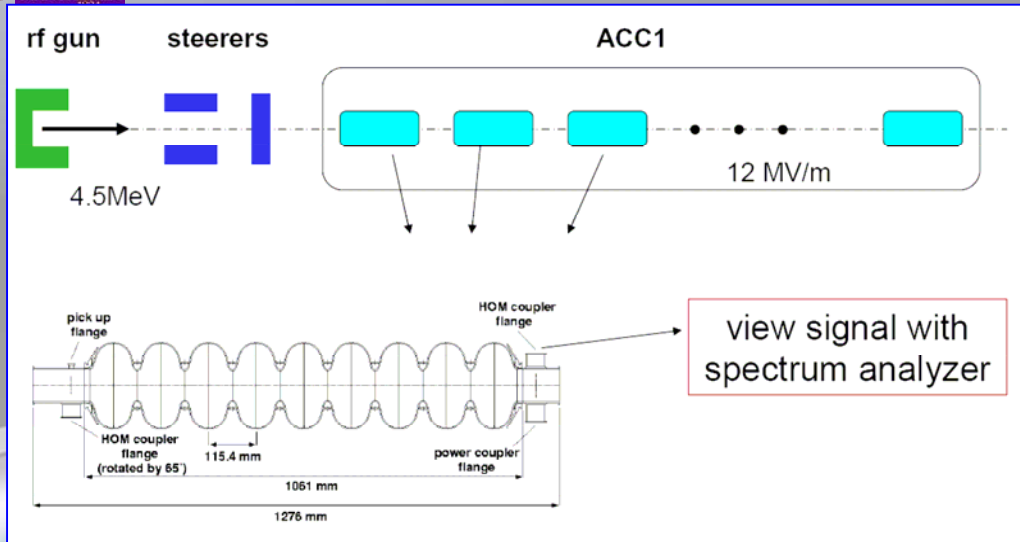
$$V(t) = \sum K_n \sin(\omega_p t) \exp(-\omega_p t/2Q_p) U(t) \hat{y} + \sum K'_n \cos(\langle\omega_p\rangle t) \sin(\Delta\omega_p t/2) \exp(-\langle\omega_p\rangle t/2Q_p) U(t) \hat{x}$$

Where the sum is taken over all modes of interest and $U(t)$ is the unit step function

=> Recent TTF experiments (Molloy, Ross, Baboi & Napoly) suggest that this frequency splitting varies by ~ 400 kHz to 800 kHz. In all simulations presented here we use 600 kHz frequency splitting for all modes.



Measurement of HOMs at TTF



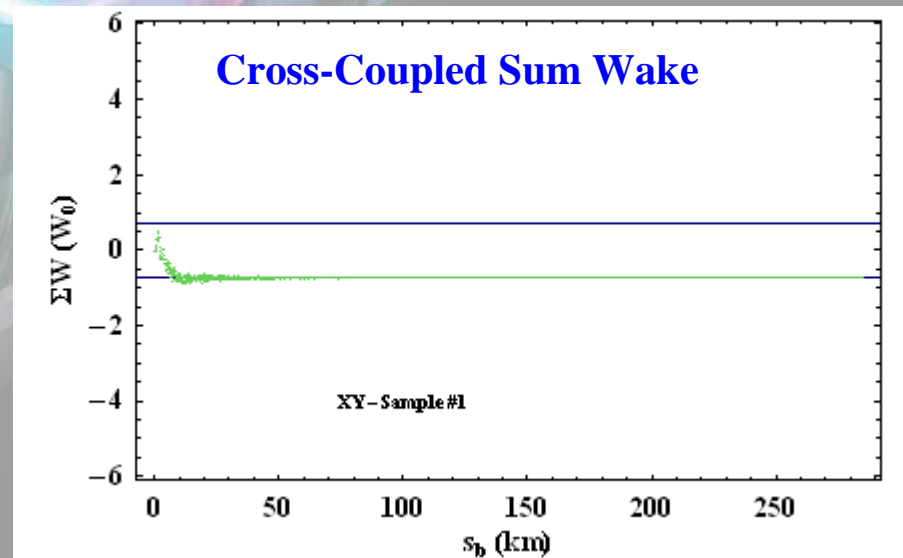
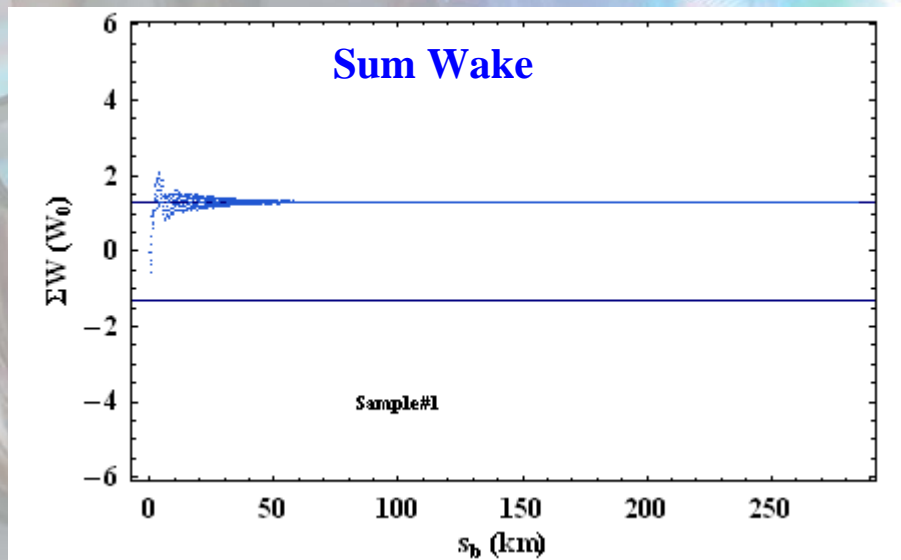
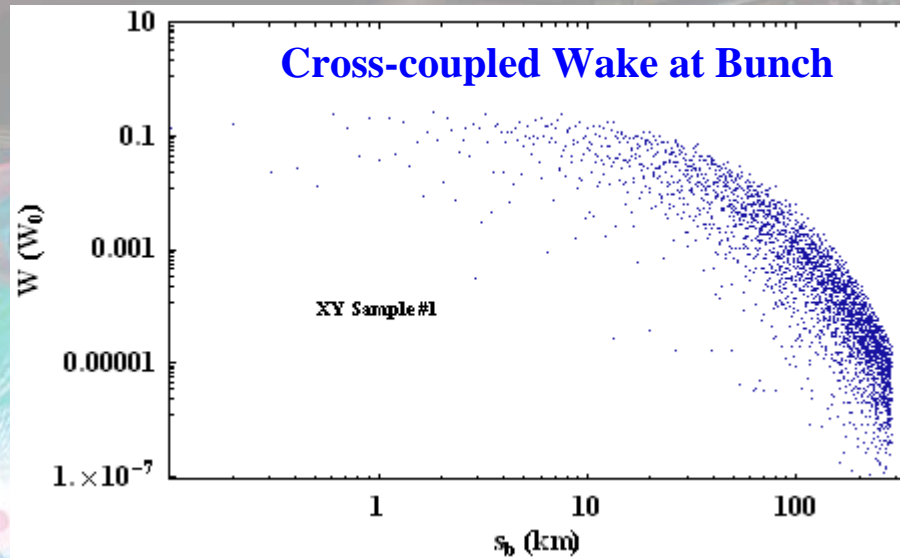
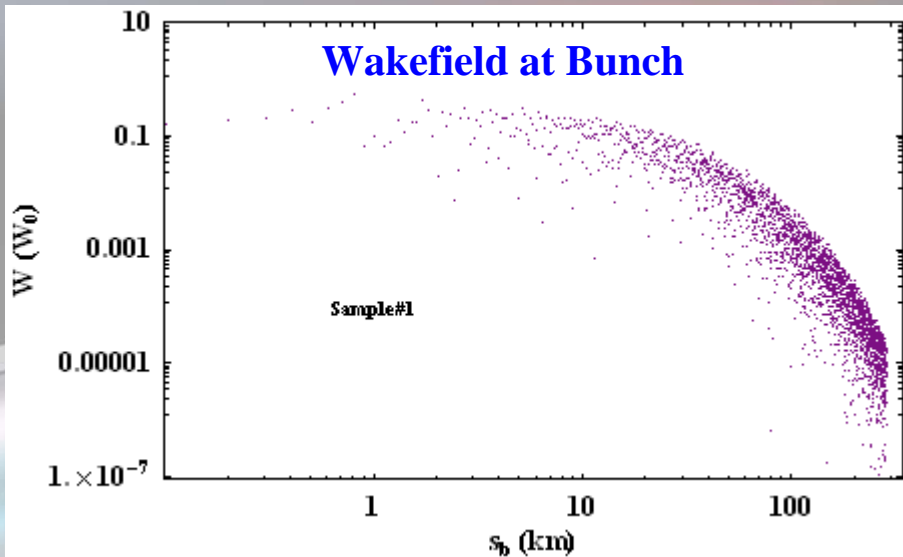
Taken from Ross, Baboi, Napoly et al. See Linac04 paper & SLAC-PUB 11190.



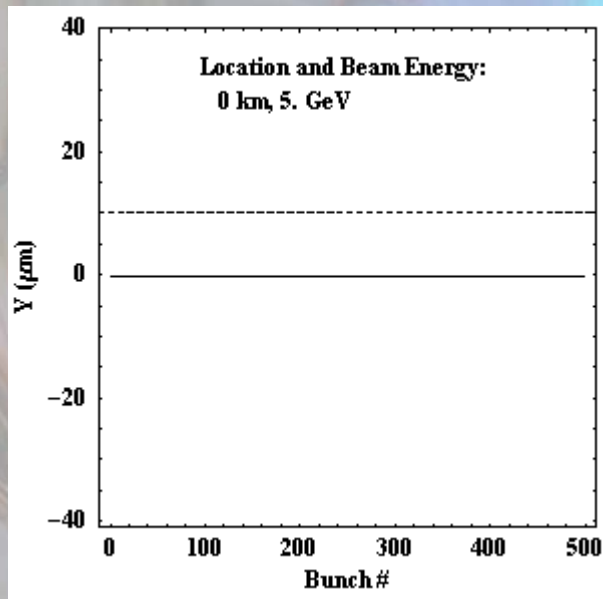
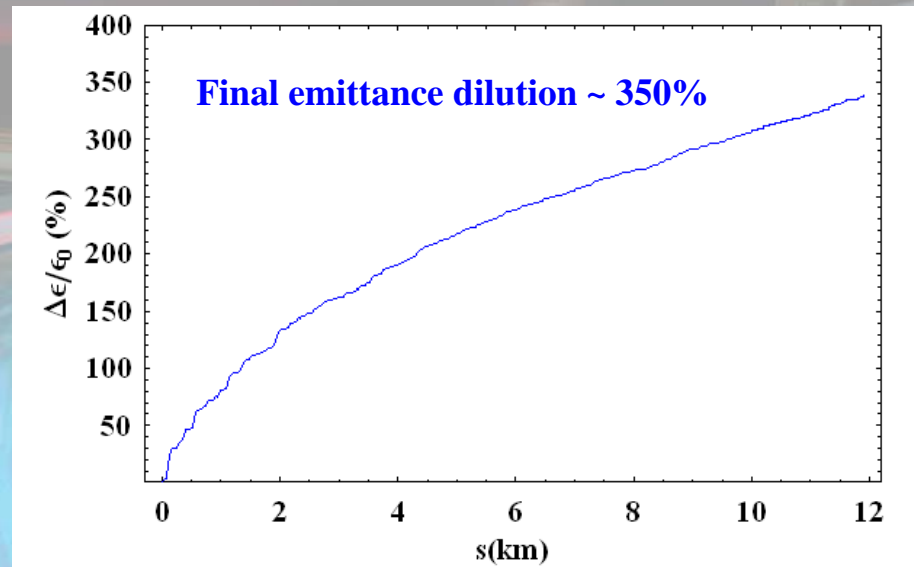
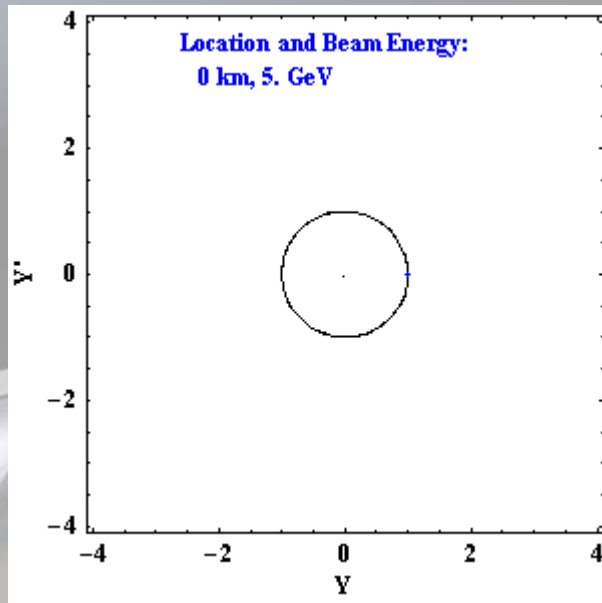
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Transverse Wakefields in TESLA-like Cavities.

$$W_y(t) = \Sigma K_n \sin(\omega_p t) \exp(-\omega_p t / 2Q_p) U(t), \quad W_{xy} = \Sigma K_n \cos(\langle \omega_p \rangle t) \sin(\Delta \omega_p t / 2) \exp(-\langle \omega_p \rangle t / 2Q_p) U(t).$$

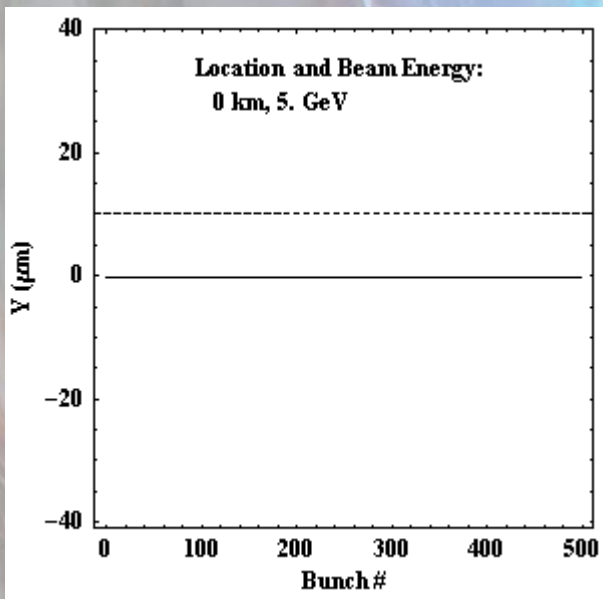
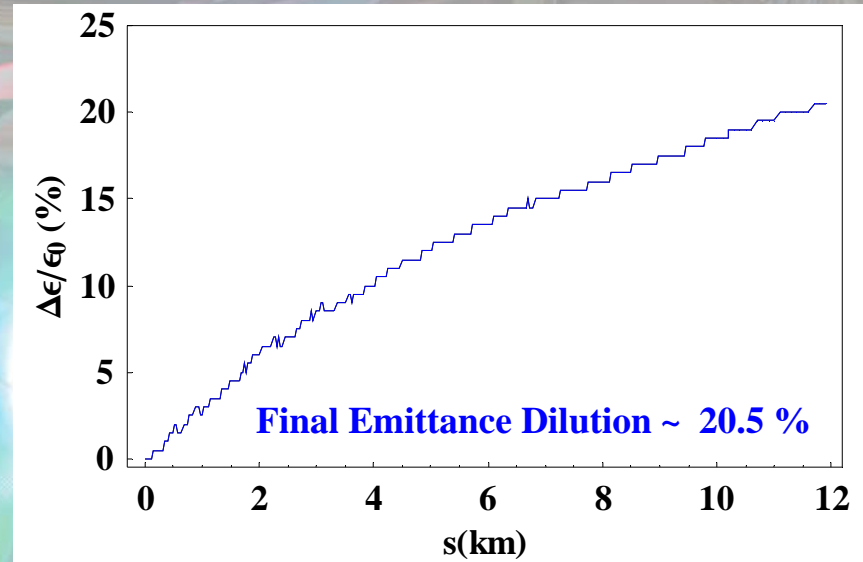
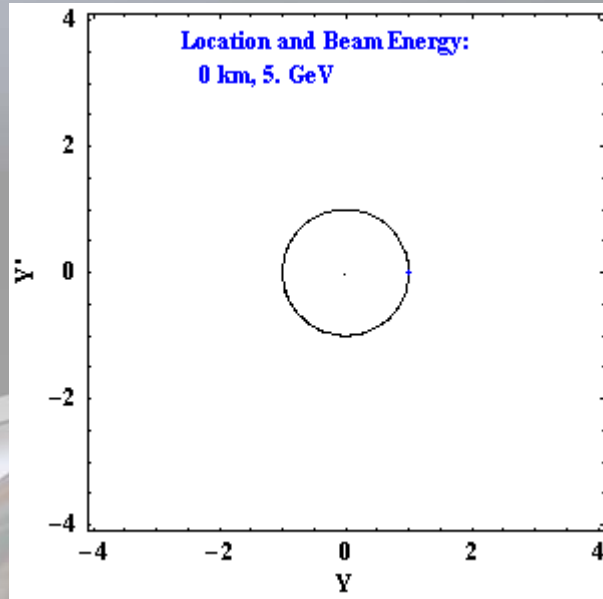


Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$).



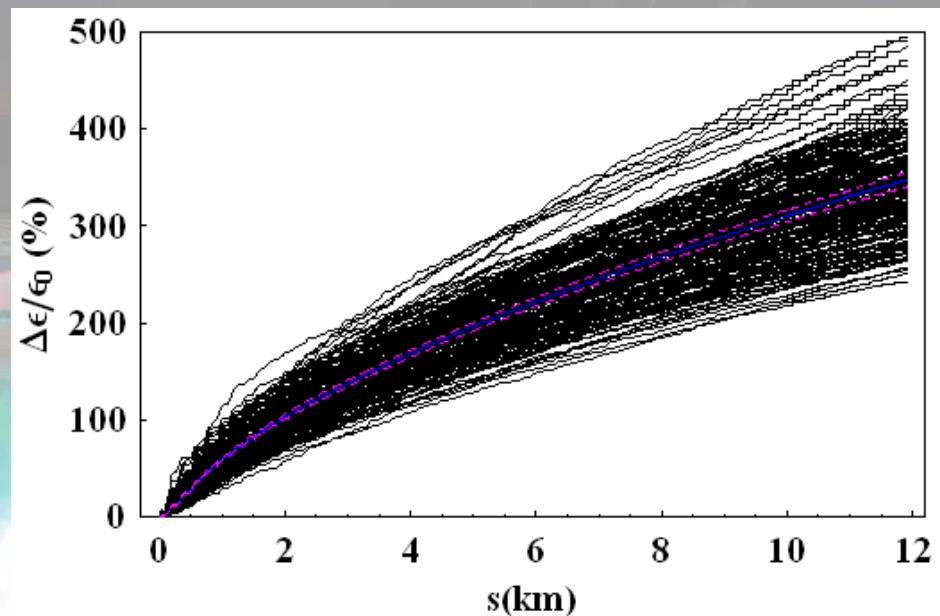
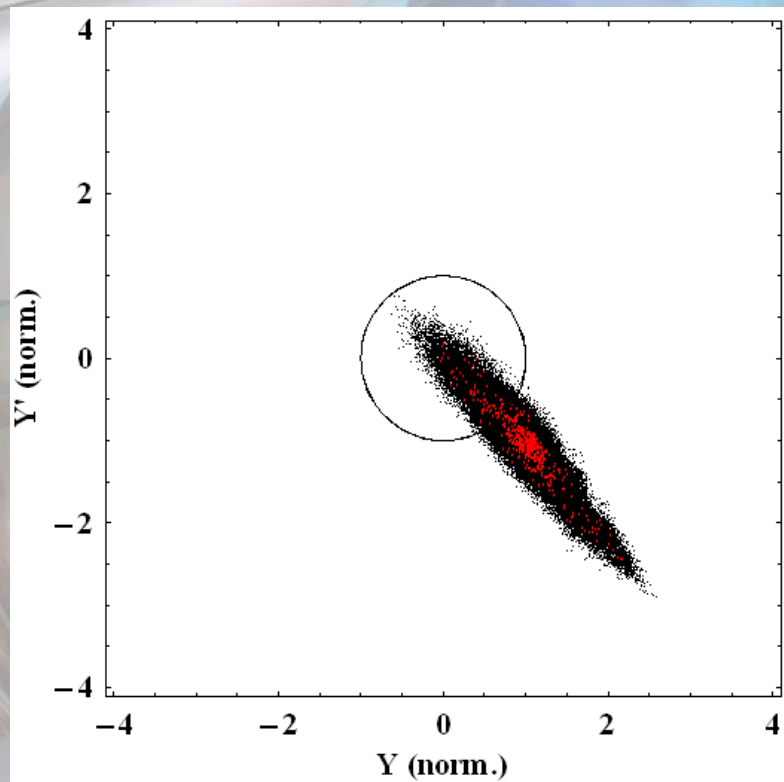
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

Emittance Dilution From Cross-Coupling Wakes $\Rightarrow Y_{\text{offset}} = 0$



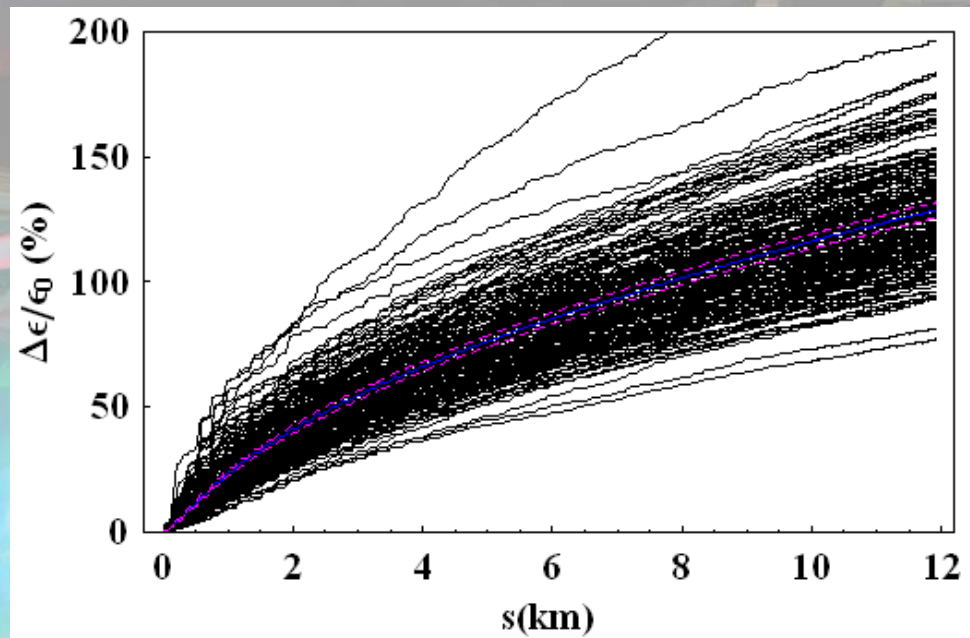
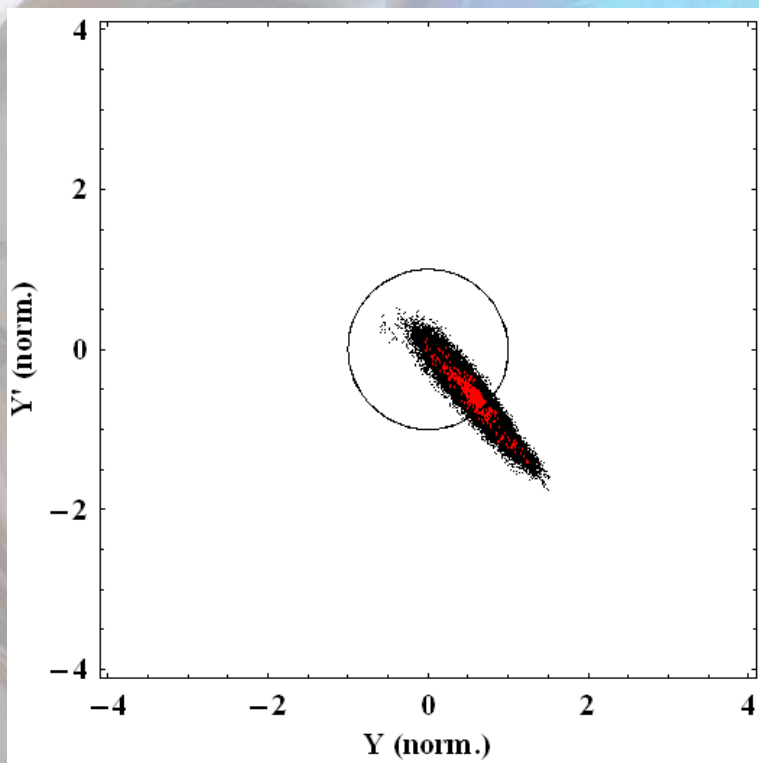
- 500 particles in all simulations.
- Initial horizontal offset ~400 μm
- No vertical offset!
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- *Random azimuthal phase about mean of $\pi/4$*
- Long + short range wakes included in simulations

Fixed Azimuthal Phase Distribution (Yoff=0).



- 500 particles in all simulations and 200 machines shown
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270\mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

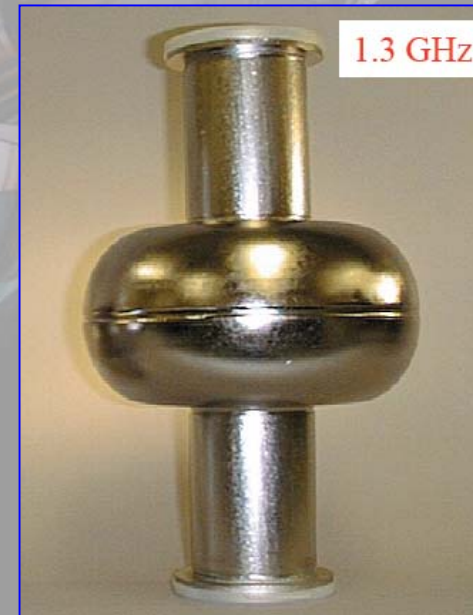
Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$).



- 500 particles in all simulations and 200 machines shown
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Random azimuthal orientation of couplers (0 to 2π about a mean value of $\pi/4$).
- Long range wakes included in simulations

Amelioration of Large Emittance Dilution?

- Increase the damping of the HOM modes? At present the damping is $\sim 10^4 - 10^5$. It may be difficult to reduce the damping even further?
- The dipole frequency degeneracy's are already split – by ~ 400 kHz to 800 kHz. Increasing the splitting (which can be achieved by making markedly asymmetrical cavities) to say 10 MHz (or more) may allow the influence of mode coupling to be minimal over the length of the linac.
- Splitting the tune of the linac. Present design is 60/60 – horizontal/vertical. This may be the most straightforward to implement. \Rightarrow We investigate splitting the tune in this talk.



Re-entrant cavity shape is shown. Taken from Rong-Li Geng SRF 2005

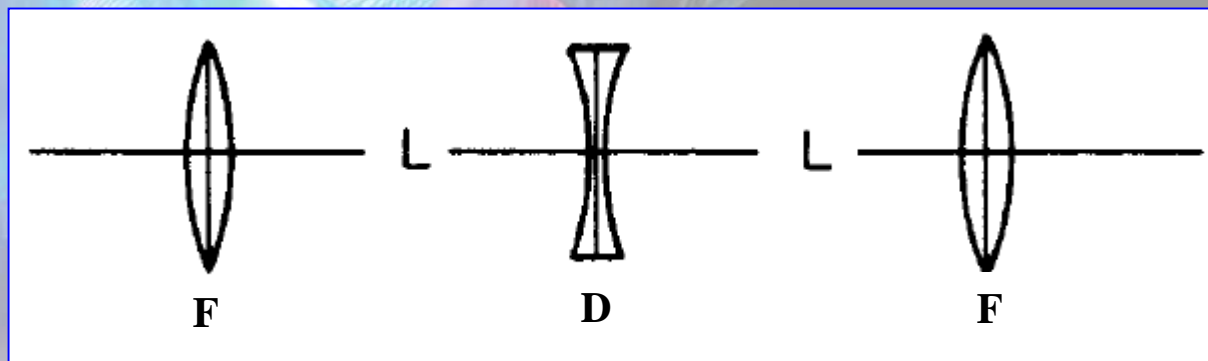
Split Tune of Lattice

- Present lattice has phase advance per cell of 60 degrees per cell.
- Each lattice cell consists of two groups of:
quadupoles + 2 cryo-modules + quadrupole + 2 cryo-modules.
- There are 12 cavities per cryo-module.
- If we split the tune then the coupling should be out of resonance.
- We look at 61/60 through 90/60.

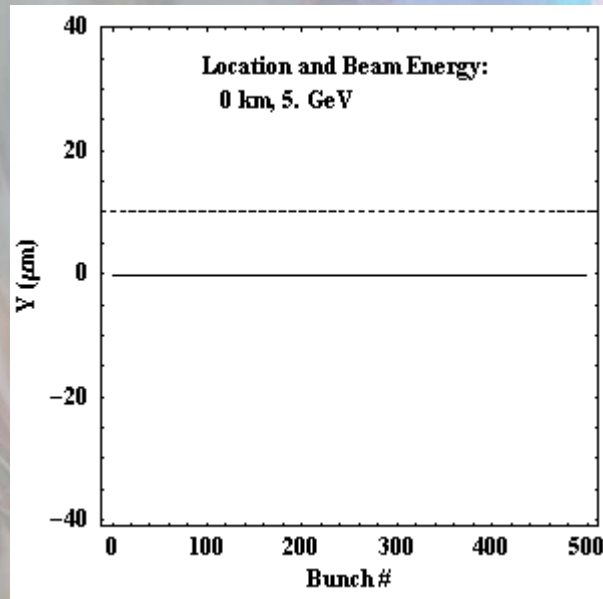
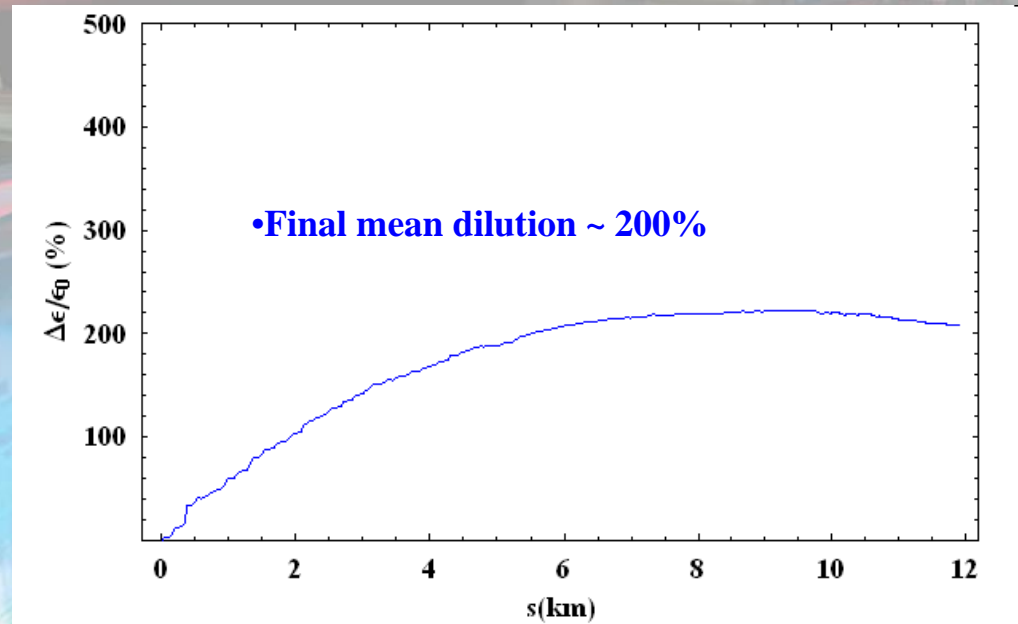
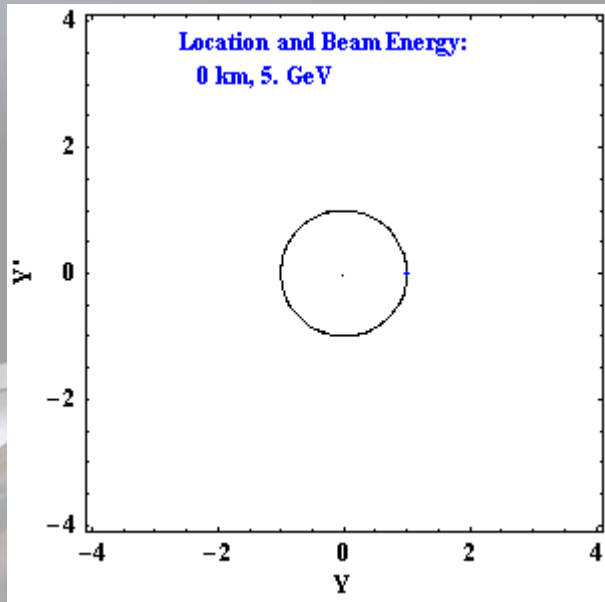
• Single FODO array shown.

• We split the tune by raising the F functions for the focusing quadrupole with respect to the D quadrupole.

• The D quadrupole is then readjusted in order to achieve 60 in the vertical plane.

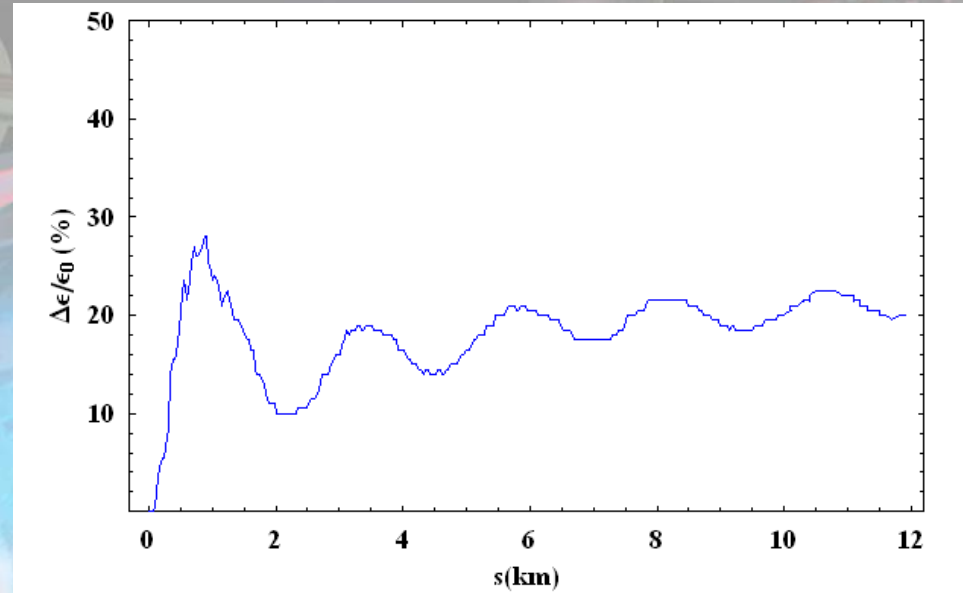
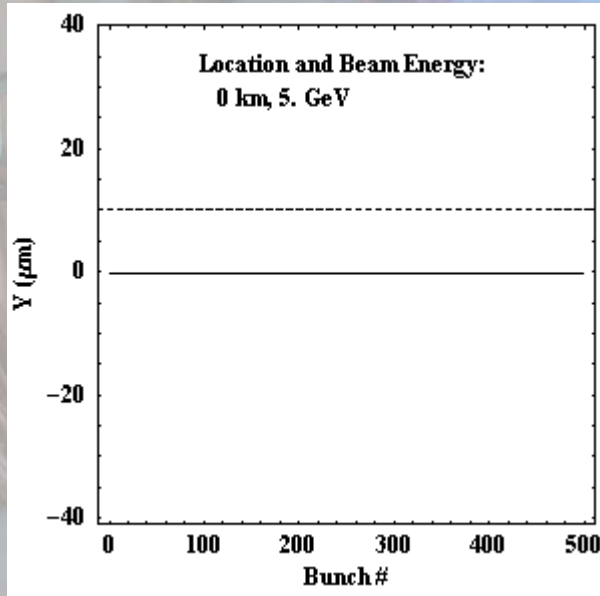
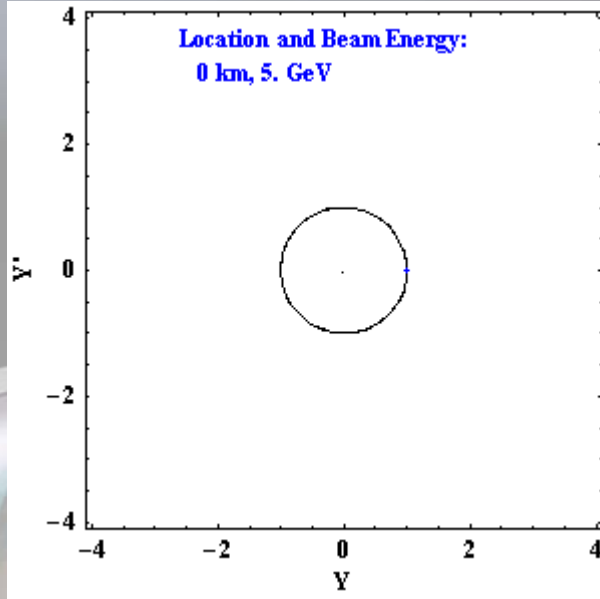


Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 61/60*



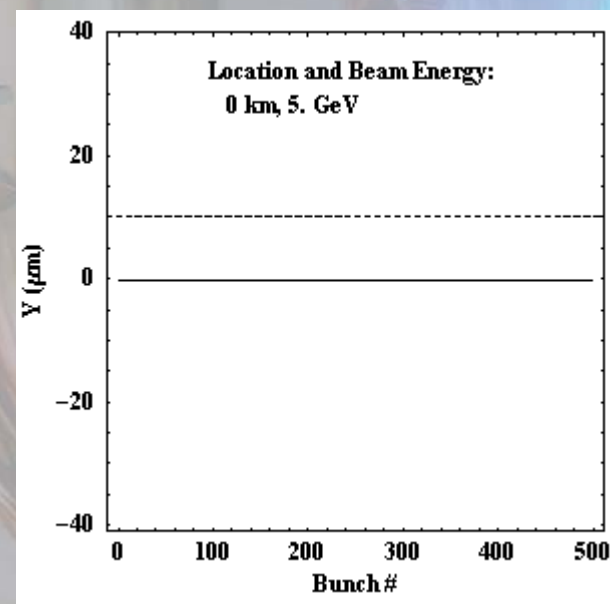
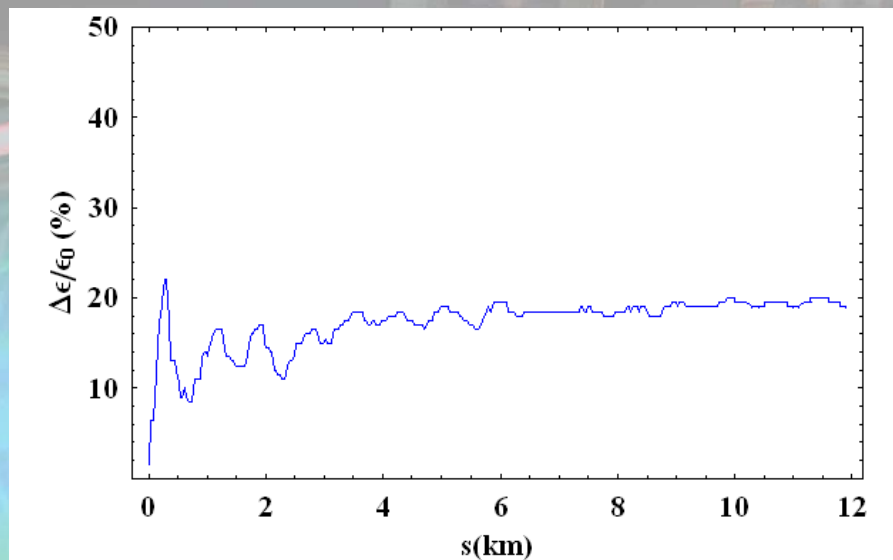
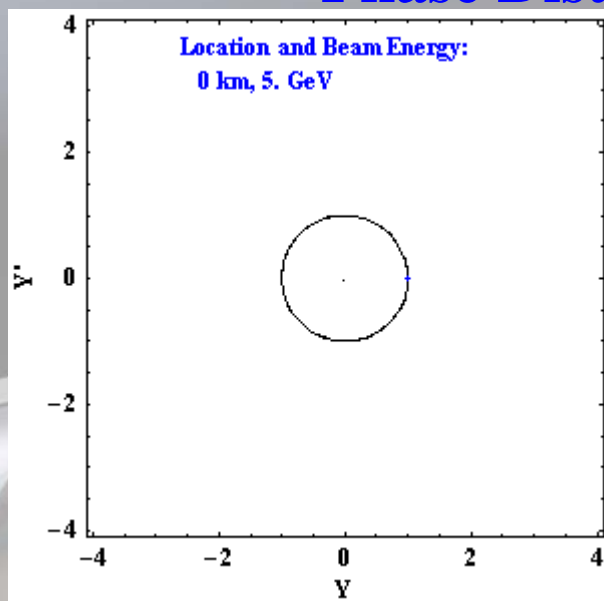
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 70/60*



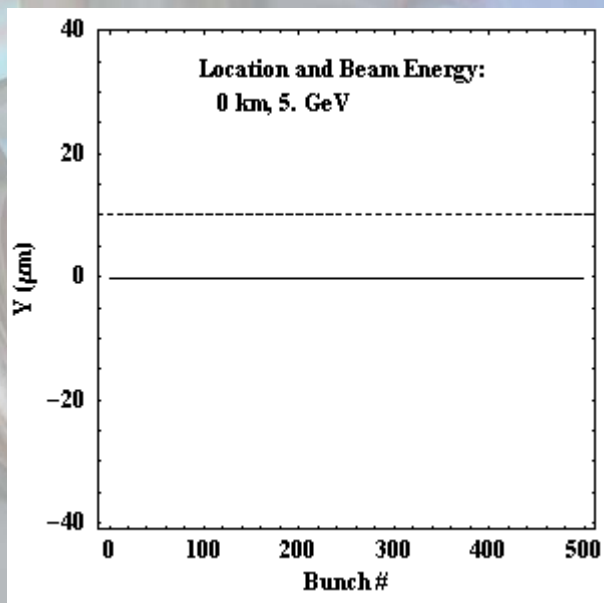
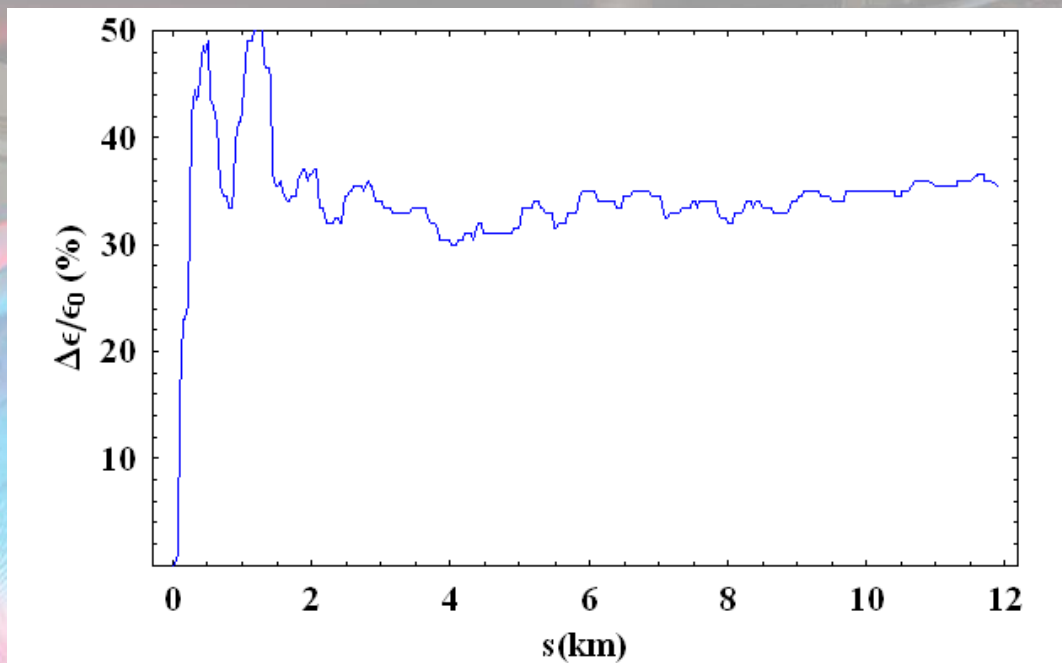
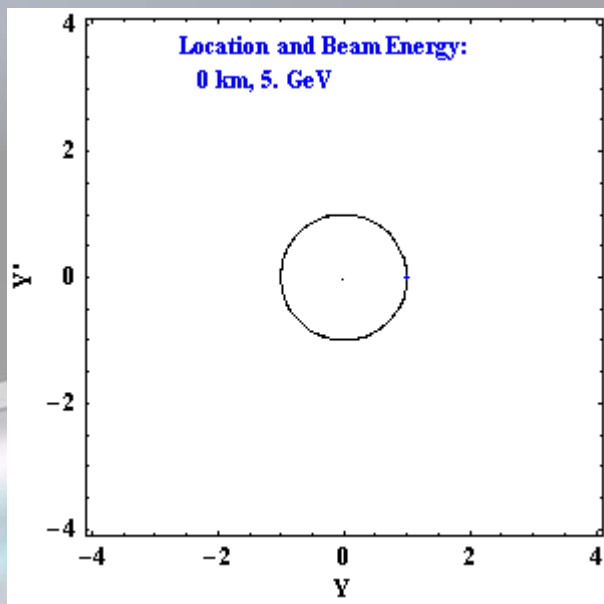
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- *Fixed azimuthal orientation of couplers.*
- **Long range wakes included in simulations**

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$). *Split Tune 90/60*



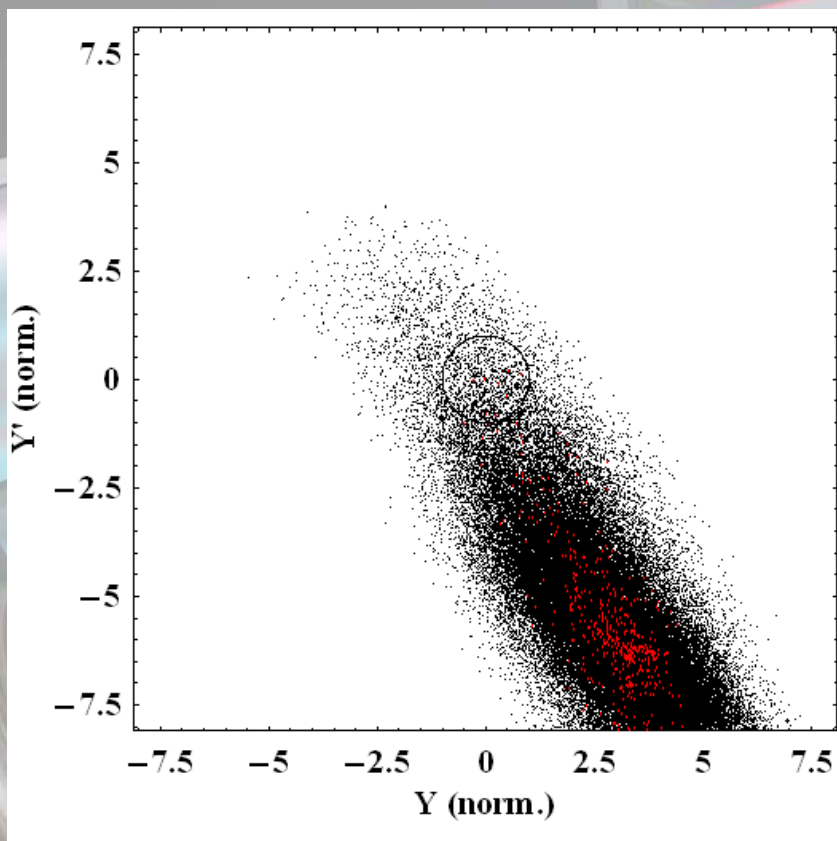
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}$, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- **Long range wakes included in simulations**

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 90/60*

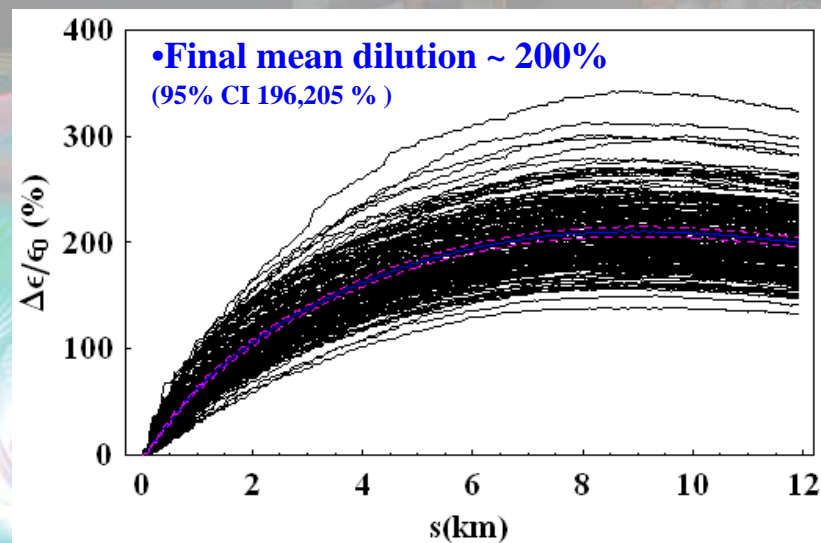


- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}$, 270 μm)
- Limited randomization of azimuthal phase*
–Max 10% of mean.
- Long range wakes included in simulations**

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 61/60*

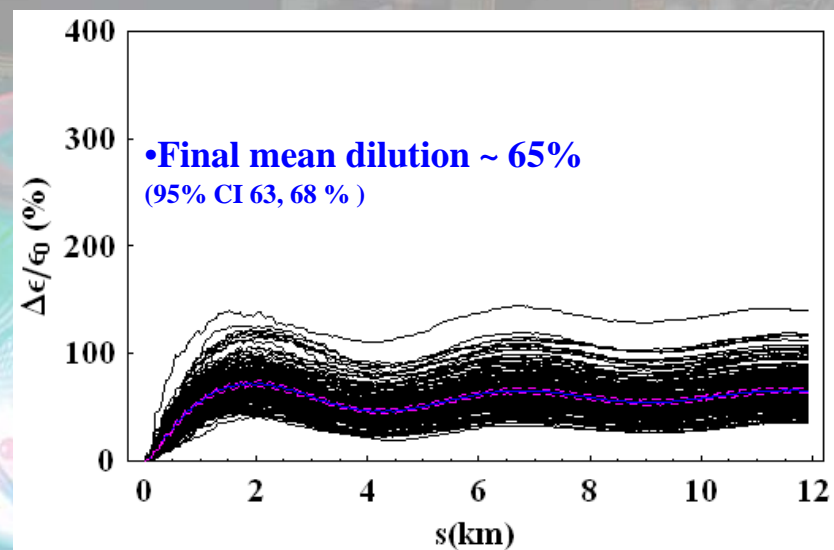
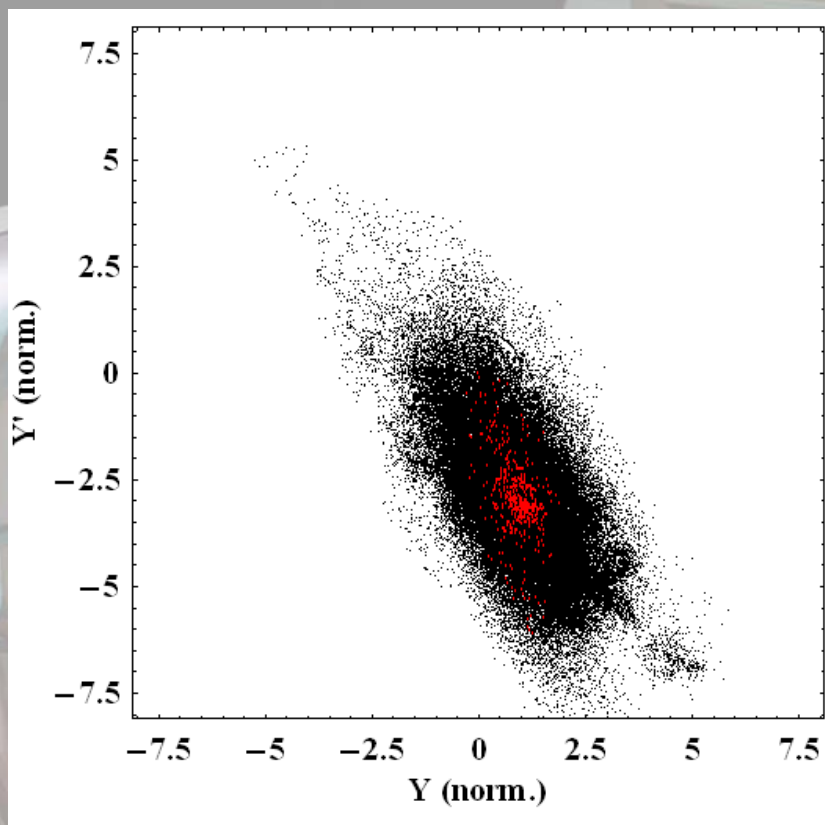


Small circle represents scaling w.r.t. energy at *end* of linac.



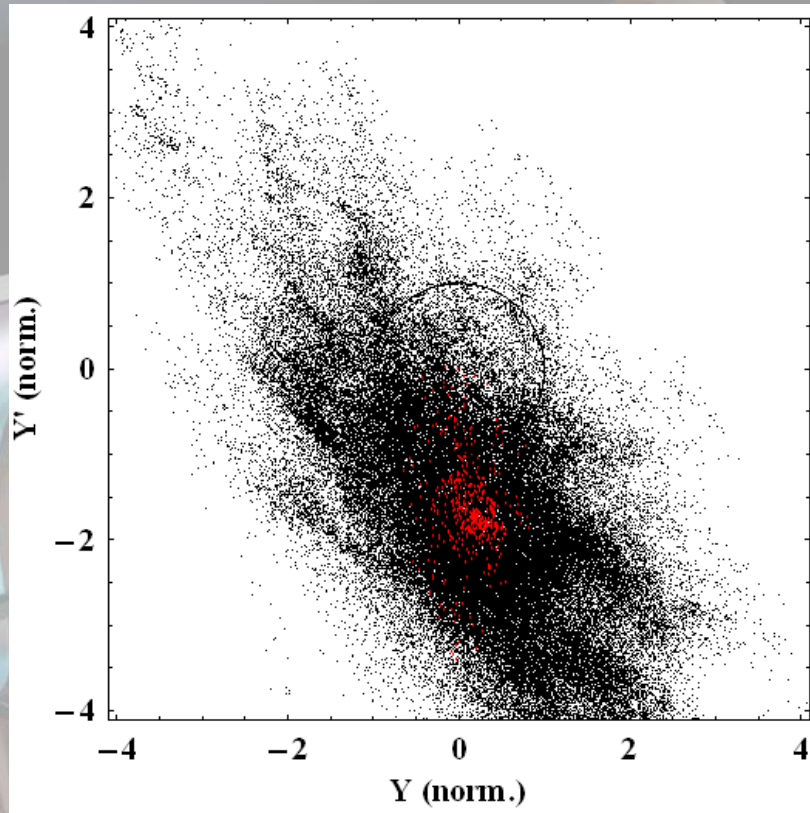
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 65/60*



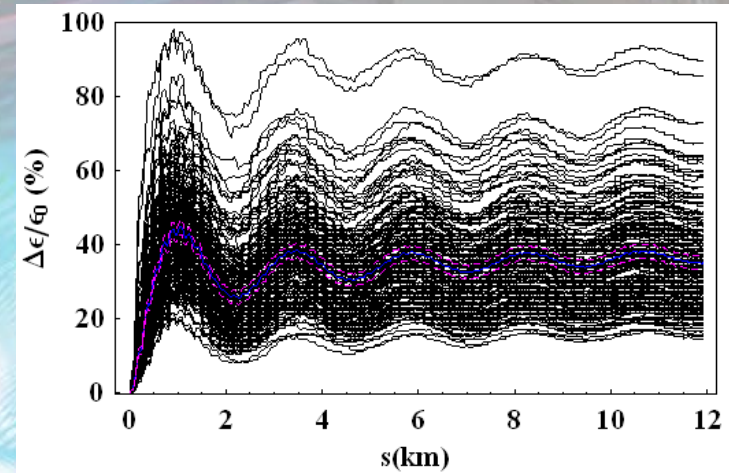
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$). *Split Tune 70/60*



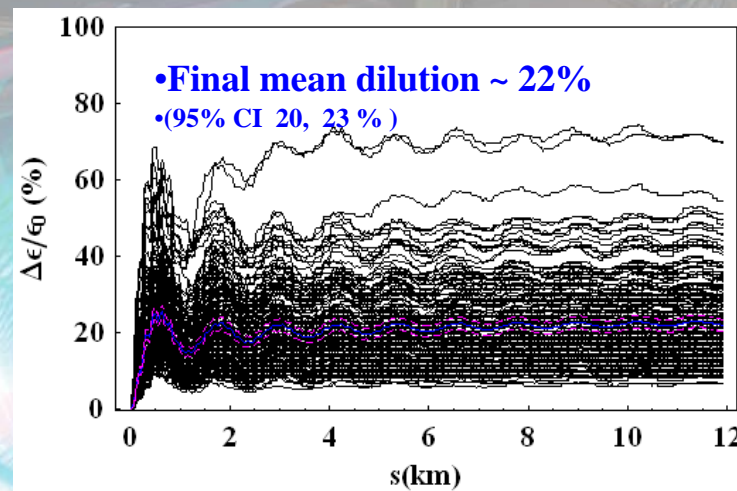
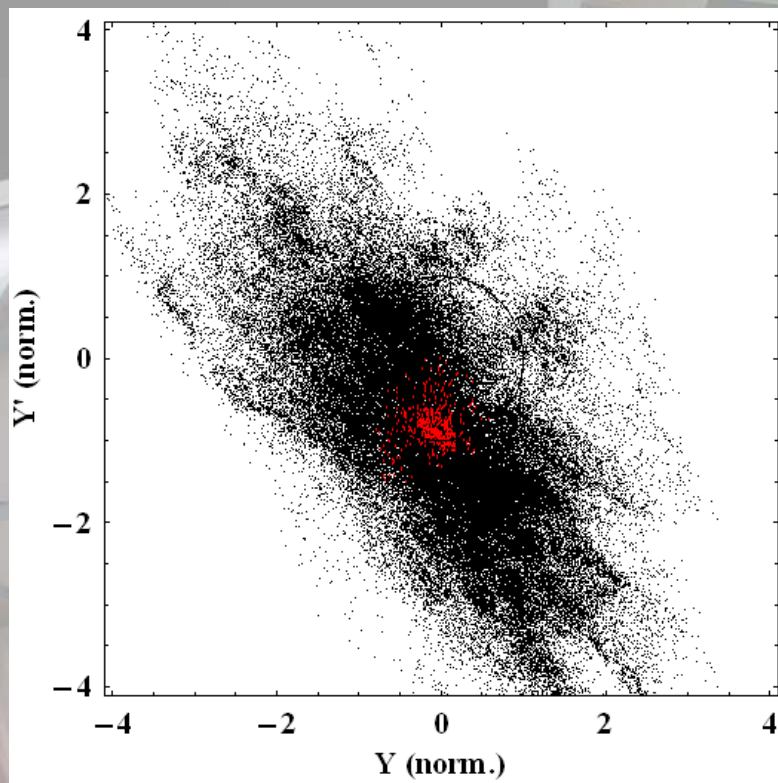
These have been rescaled
relative to the previous ones

•Final mean dilution $\sim 35\%$ (95% CI 33, 37 %)



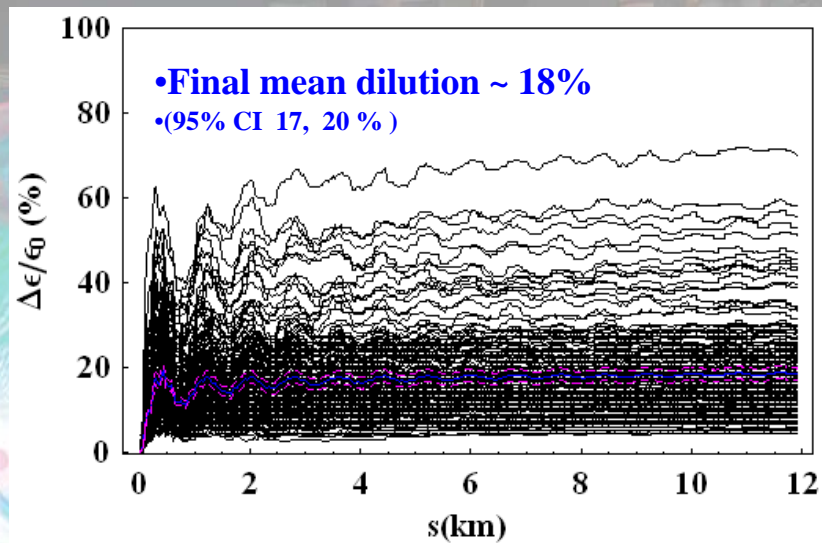
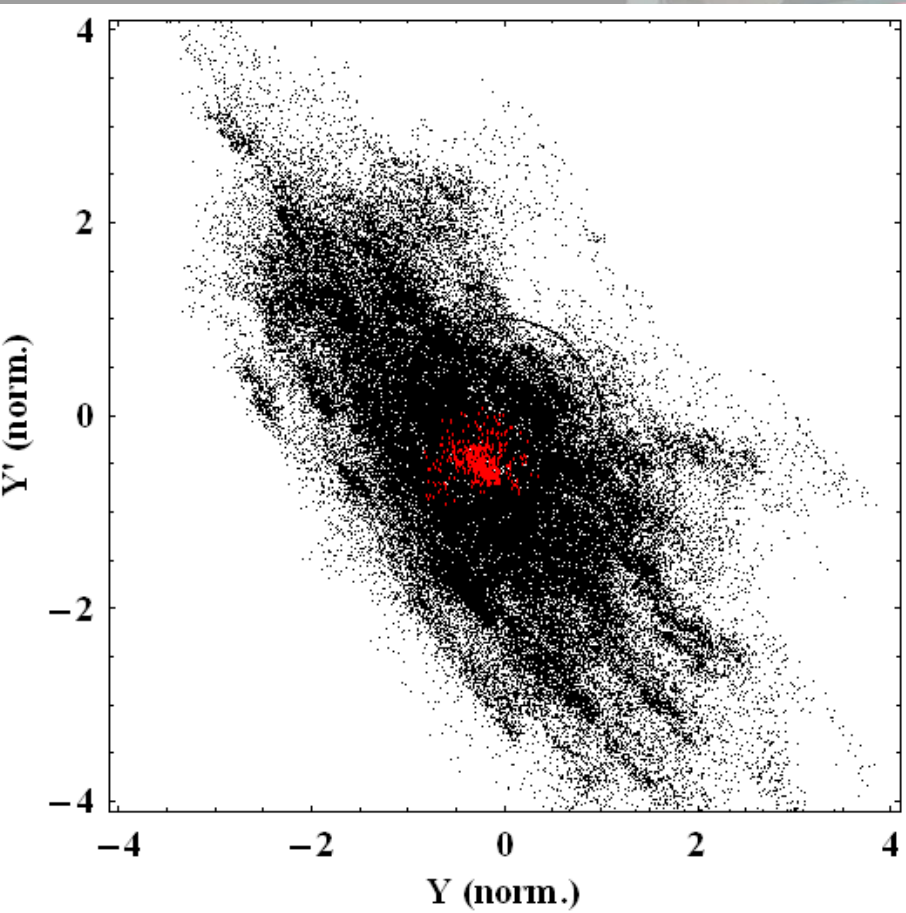
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- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$). *Split Tune 80/60*



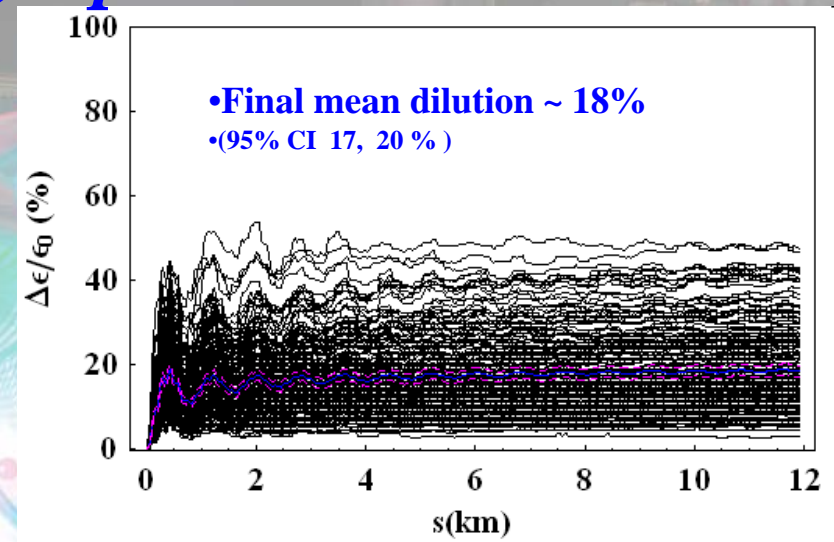
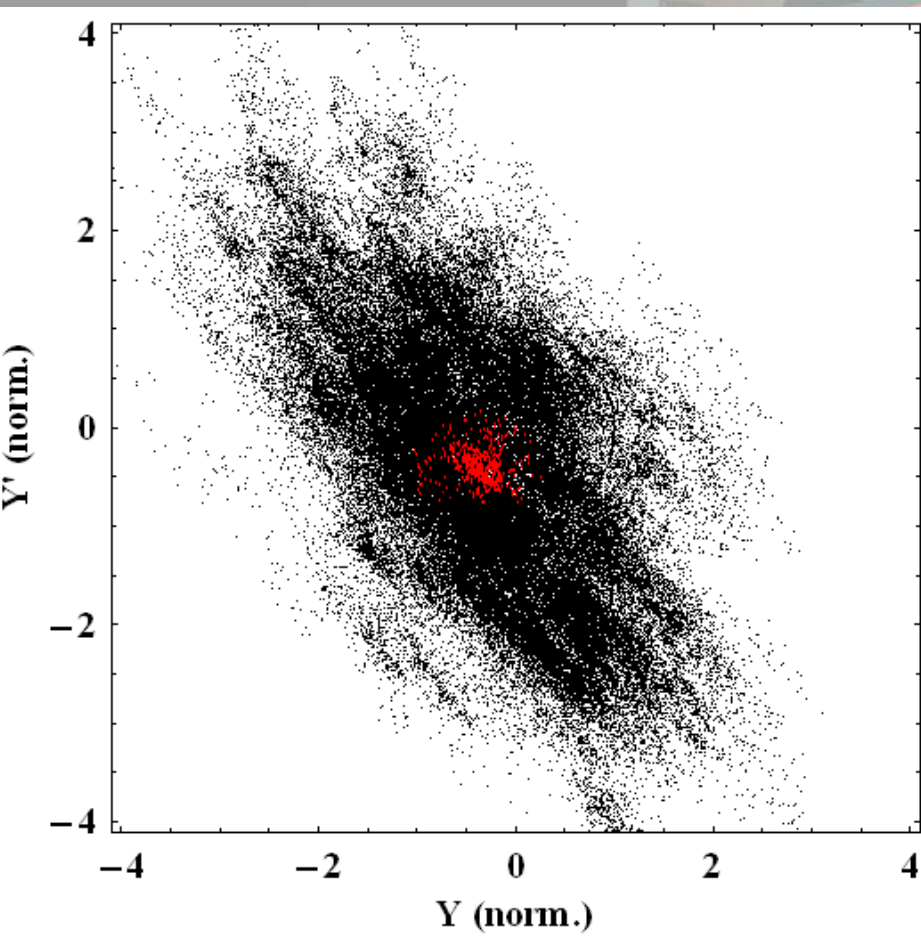
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$). *Split Tune 90/60*



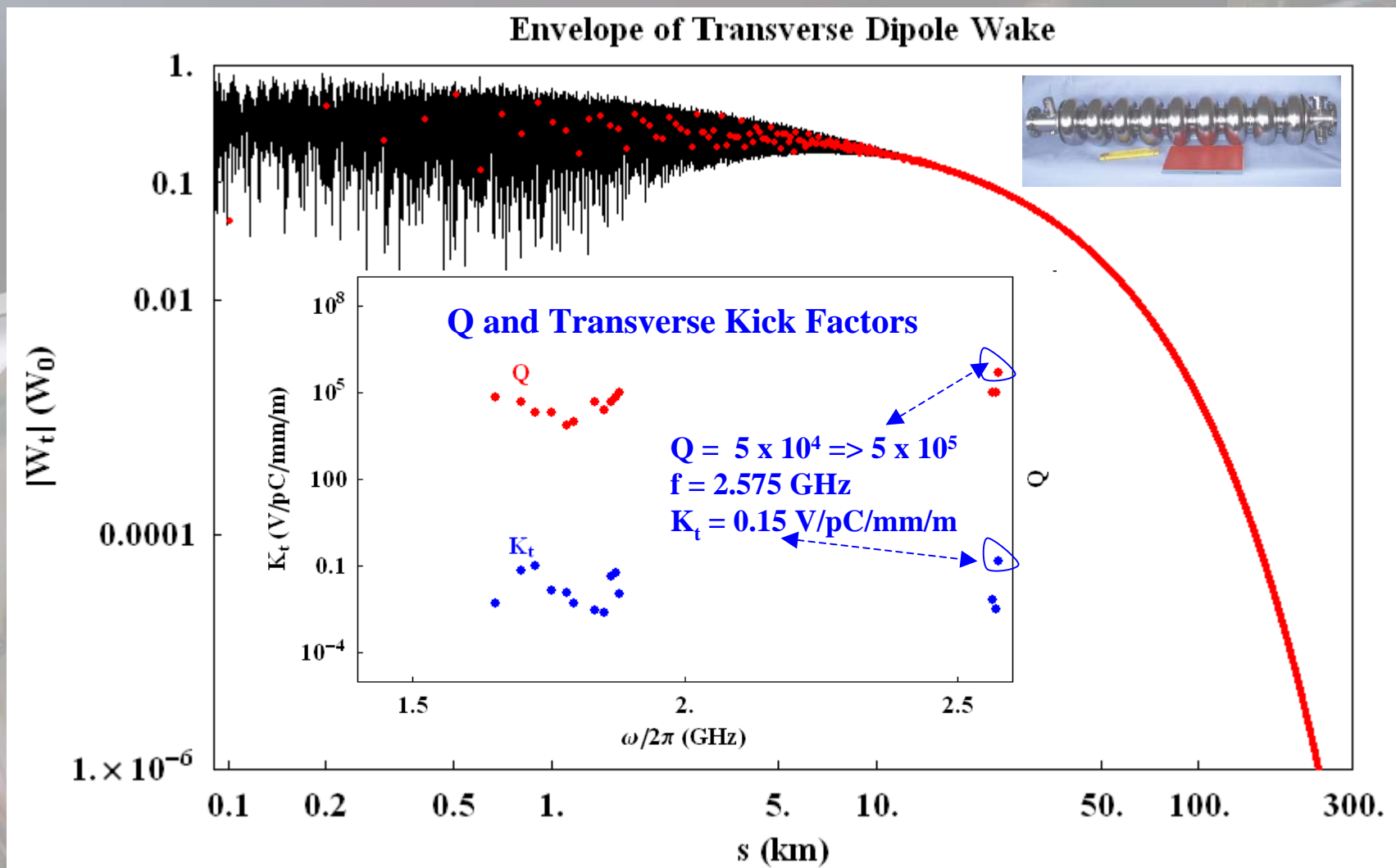
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- *Fixed azimuthal phase.*
- Long range wakes included in simulations

Cross-Coupled Motion, Limited Randomization of Azimuthal Phase Distribution of HOM Couplers ($Y_{\text{off}}=0$). *Split Tune 90/60.*

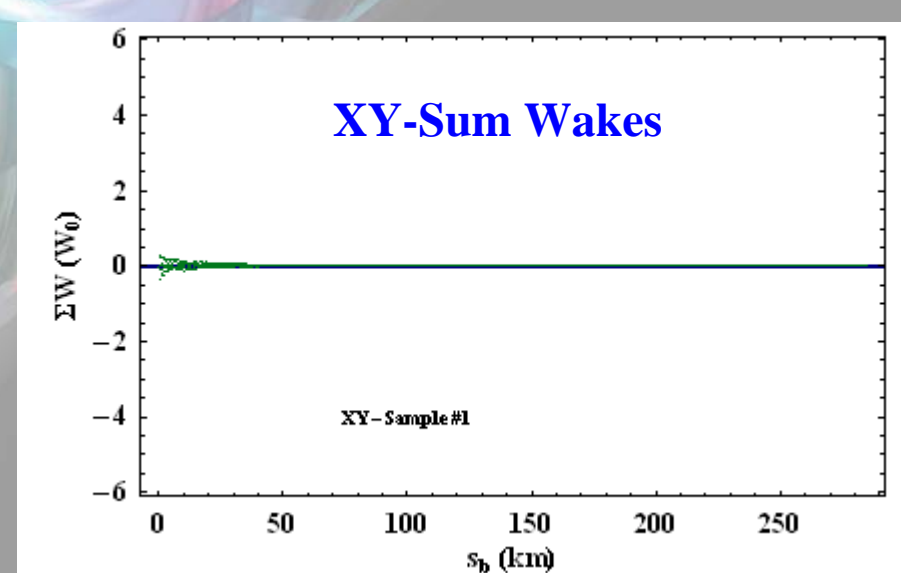
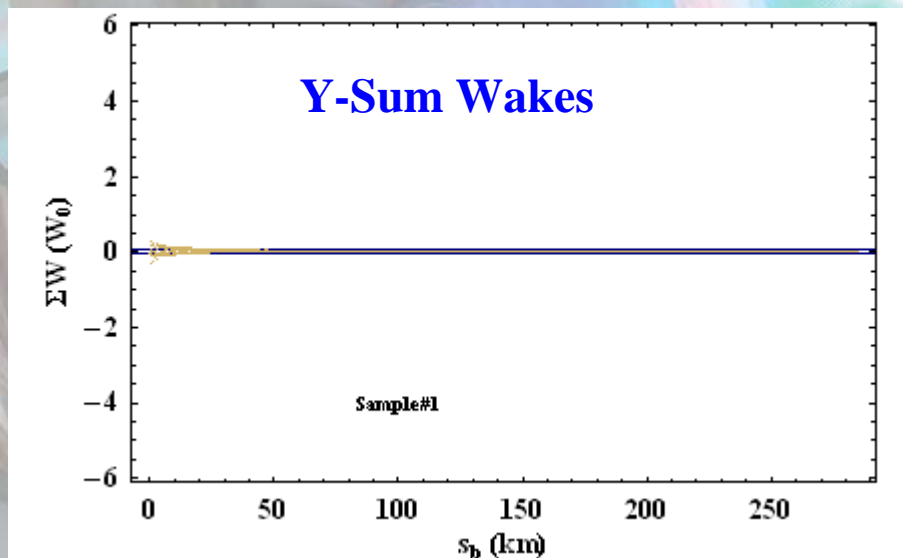
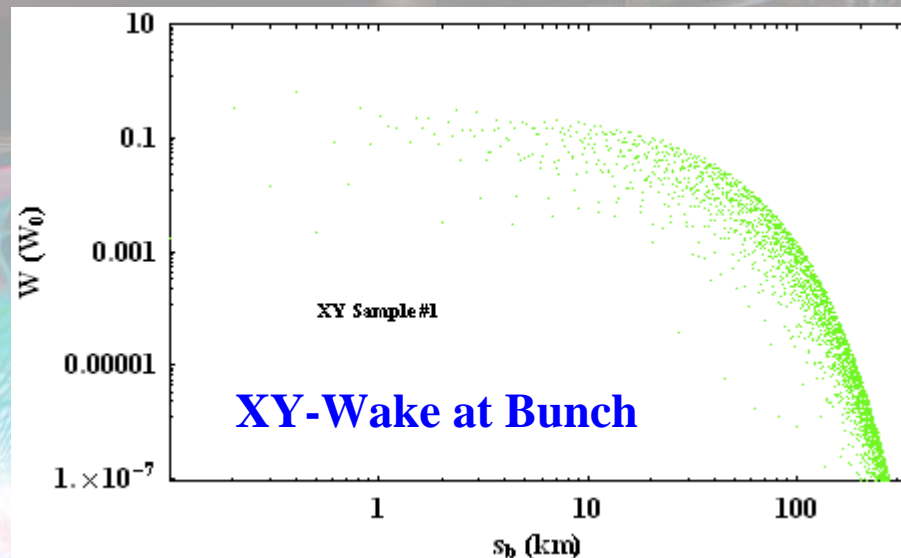
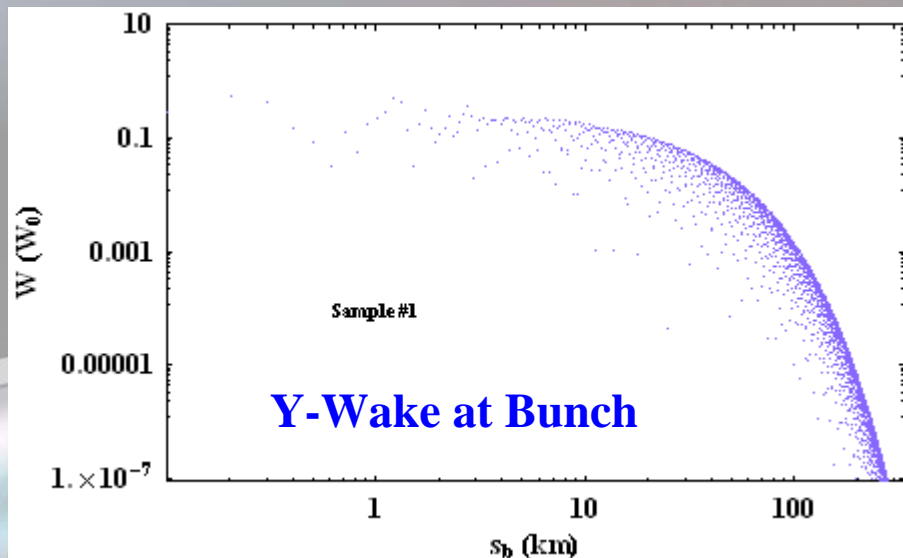


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- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Limited randomization of azimuthal phase*
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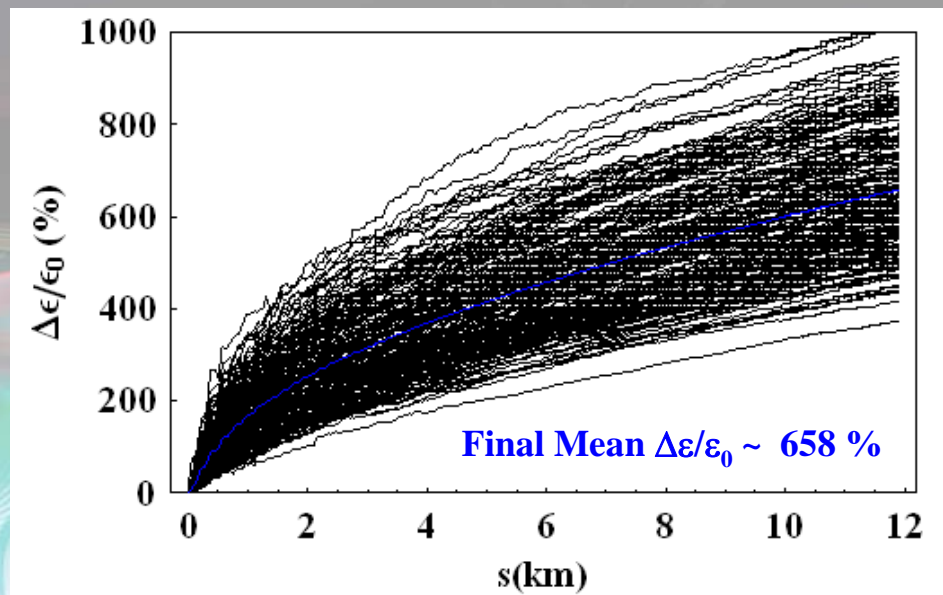
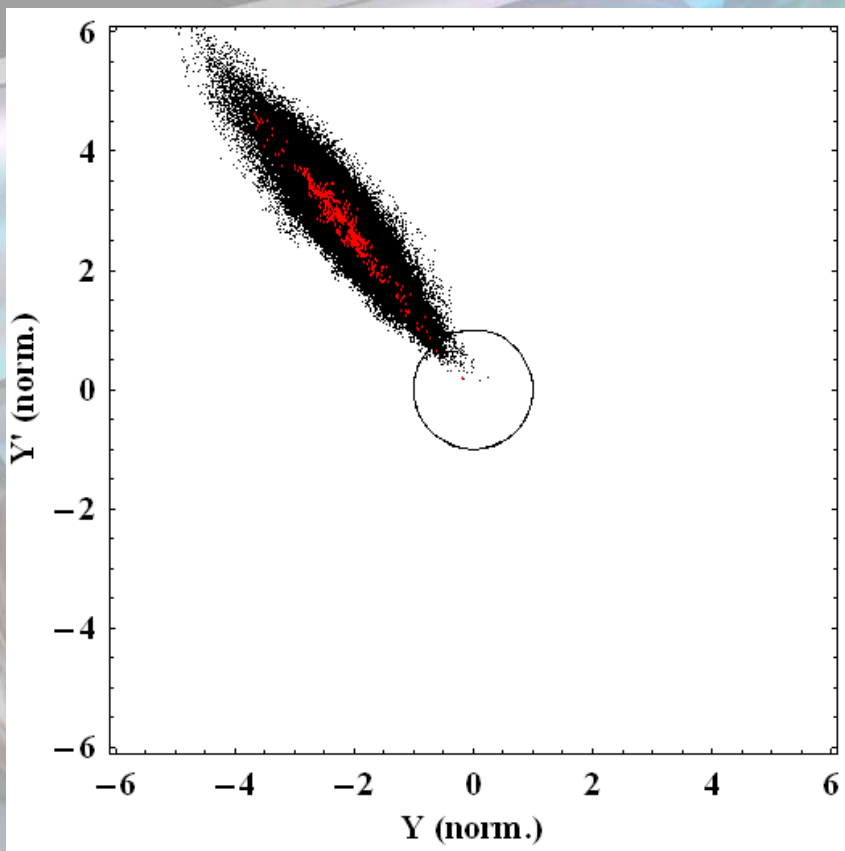
Wakes for Cross-Coupled Motion + Rogue Mode



Wakes Incl. Rogue Mode & Random Errors

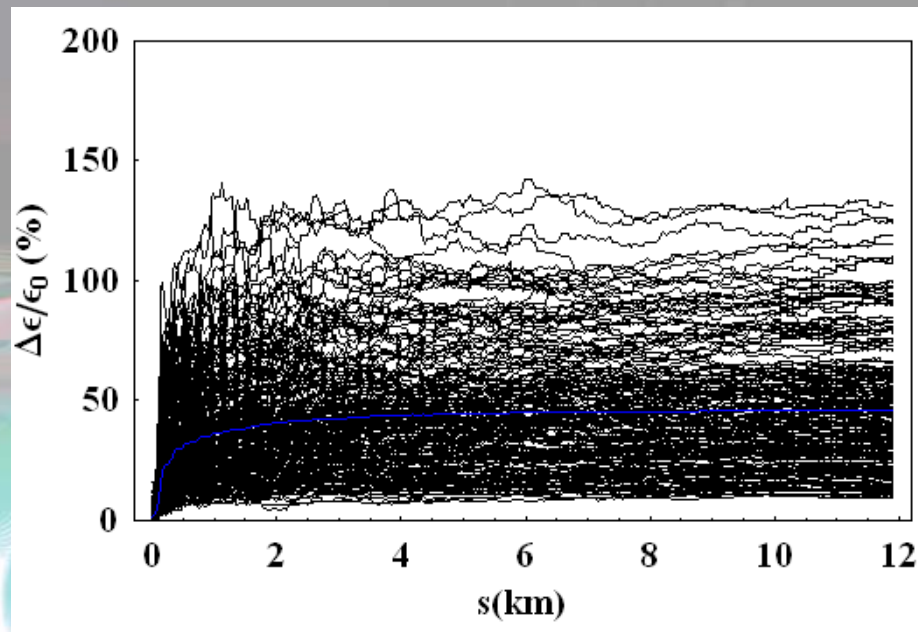
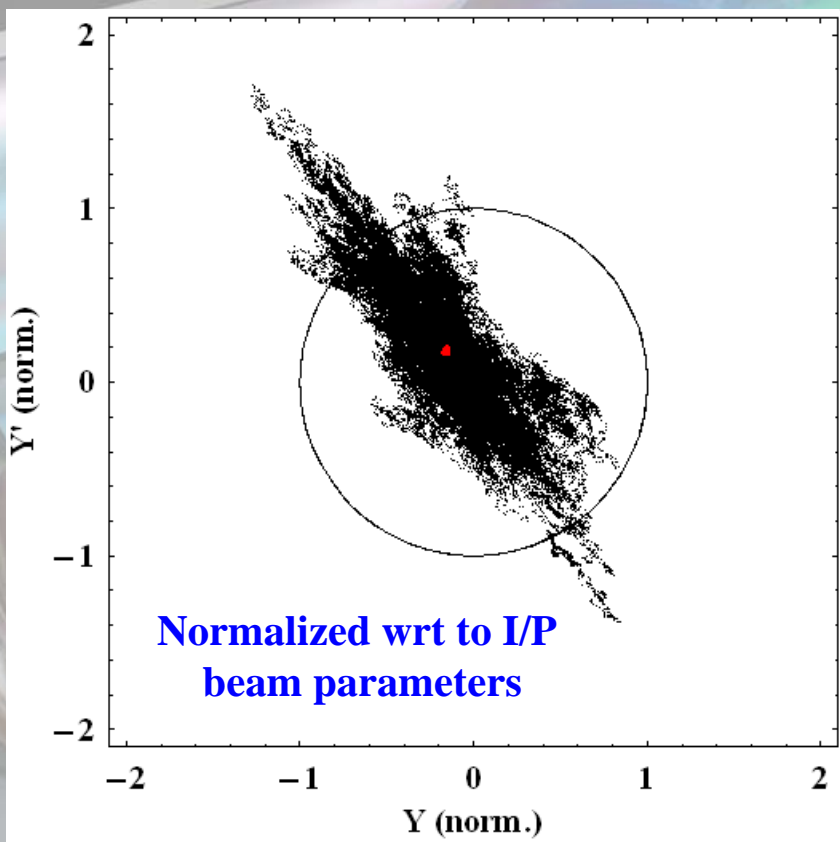


Cross-Coupled Motion At Fixed Azimuthal Phase Distribution + Rogue Mode



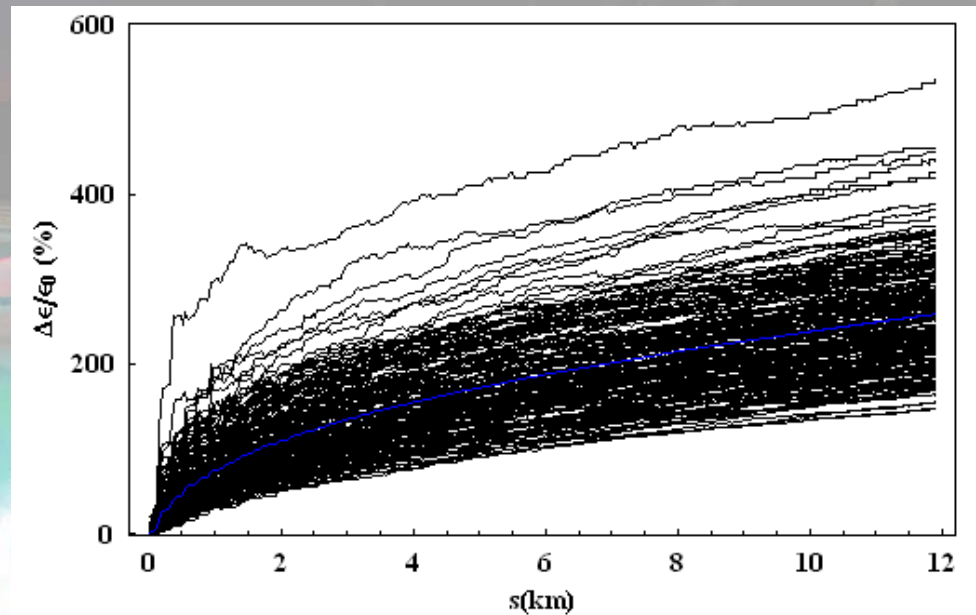
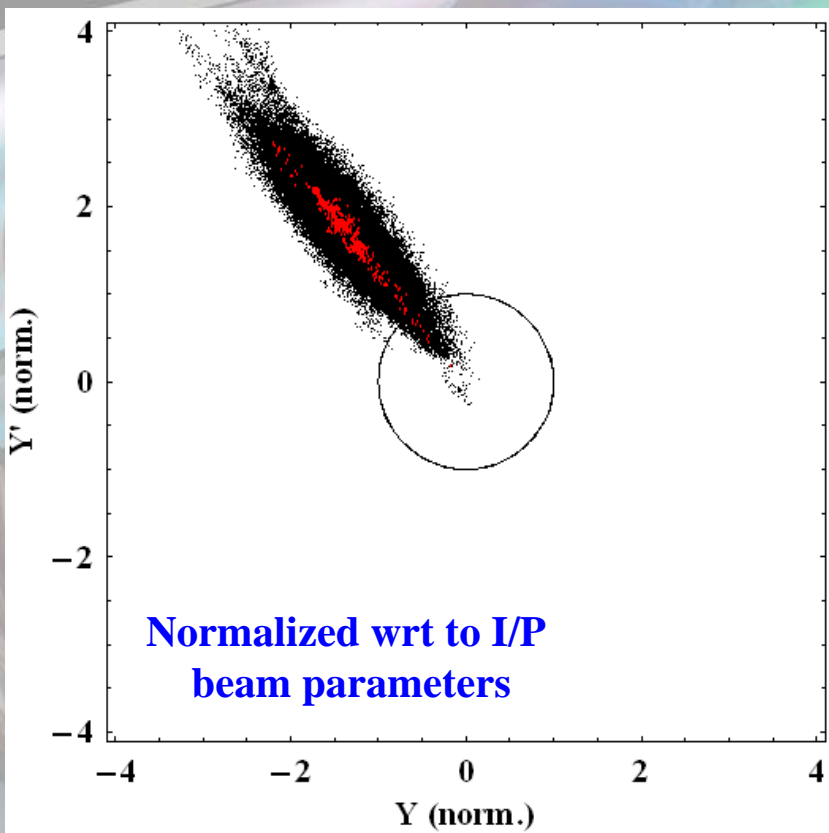
- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset $\sim 10 \mu\text{m}$ and $400 \mu\text{m}$.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Azimuthal phase fixed at $\pi/4$
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included in simulations

Cross-Coupled Motion With Random Azimuthal Phase Distribution + Rogue Mode



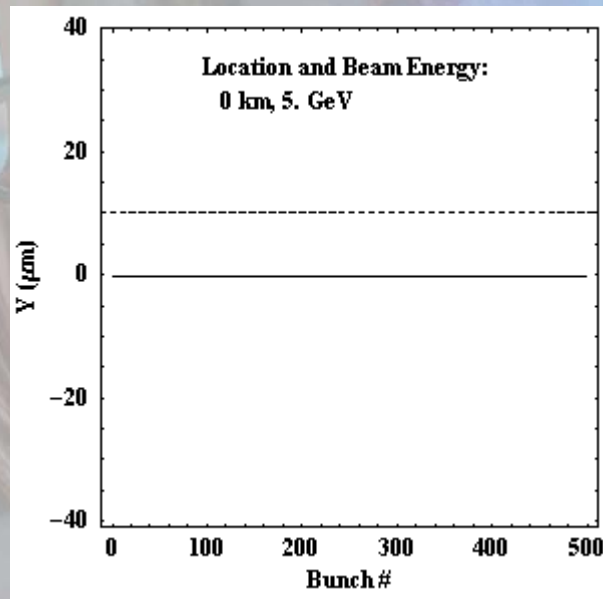
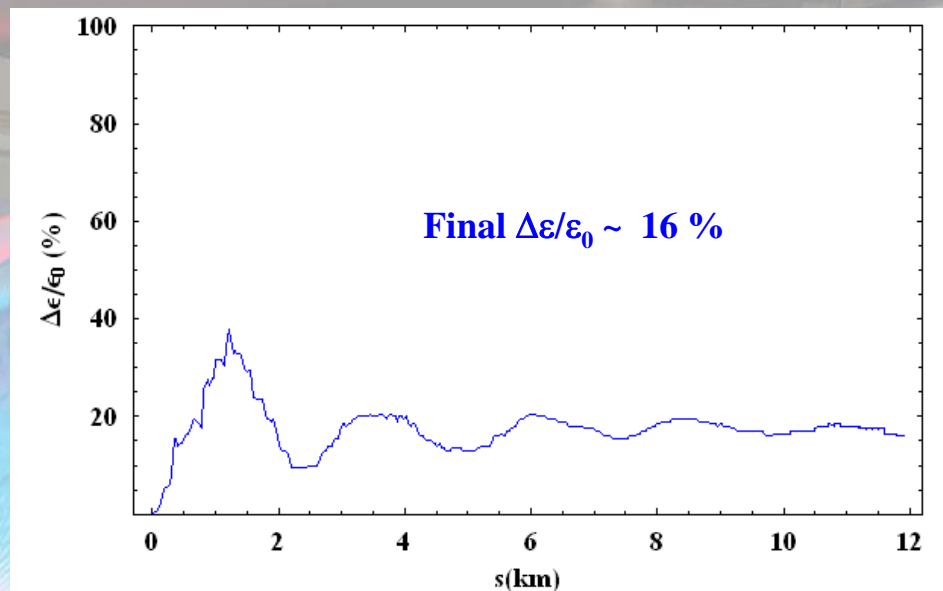
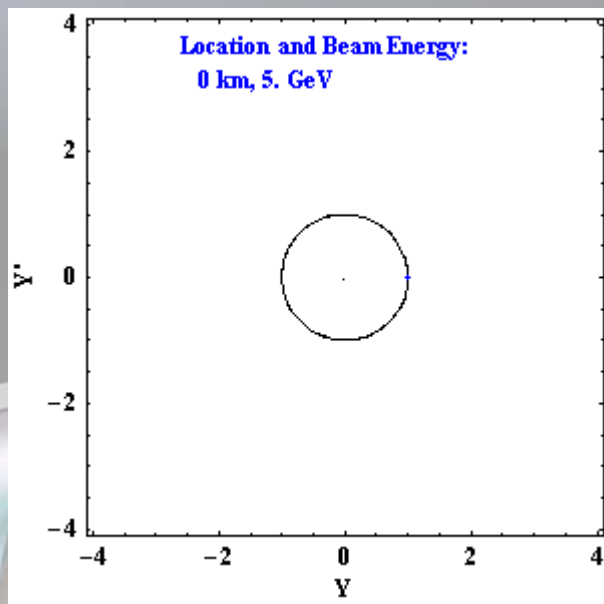
- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset $\sim 10 \mu\text{m}$ and $400 \mu\text{m}$.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- *Random Azimuthal phase (mean $\pi/4$)*
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included in simulations

Cross-Coupled Motion With Limited Random Azimuthal Phase Distribution + Rogue Mode



- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset $\sim 10 \mu\text{m}$ and $400 \mu\text{m}$.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Azimuthal phase randomly distributed – Max 10% of mean.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included in simulations

Split Tune 70/60.

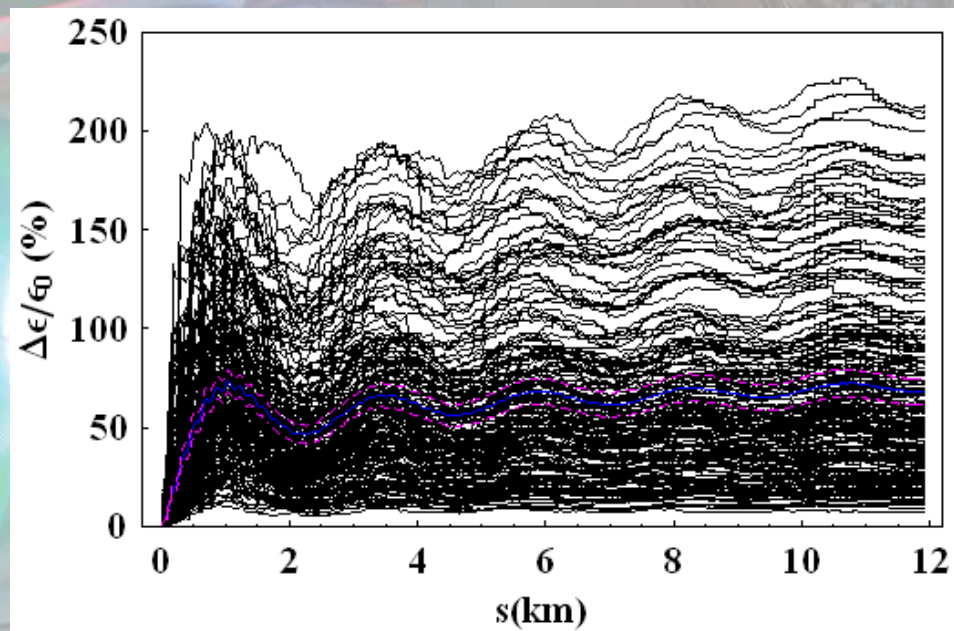
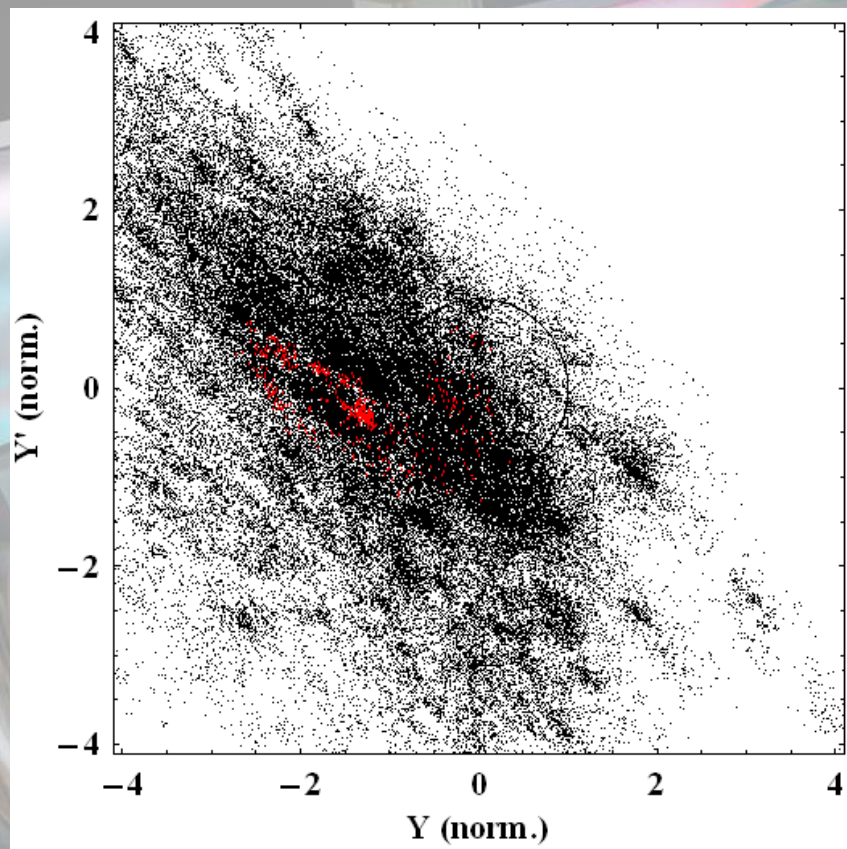


- Only mode-cross coupling gives rise to emittance dilution
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

Cross-Coupled Motion

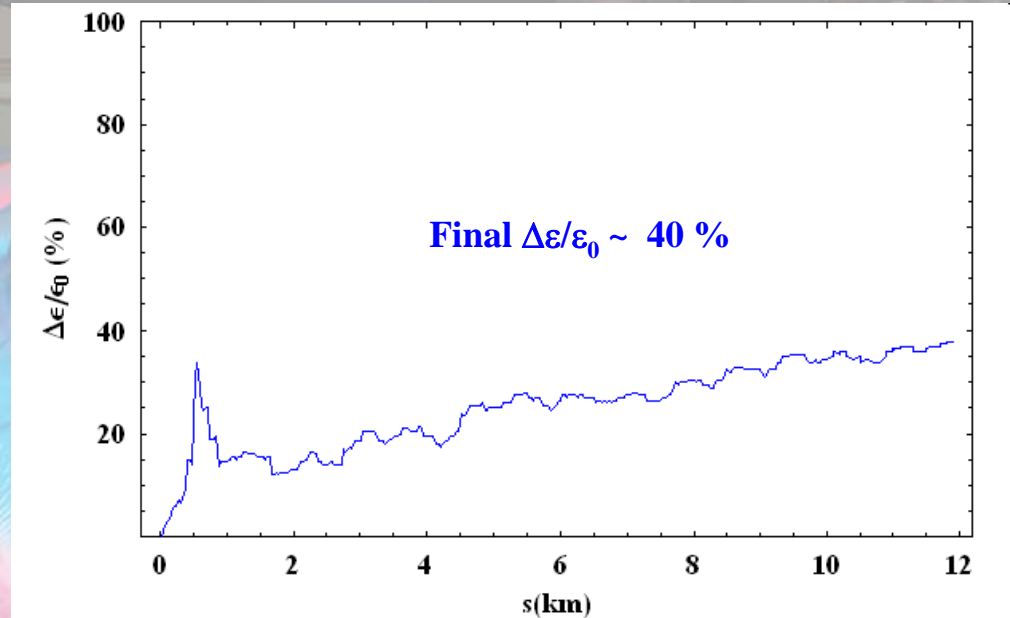
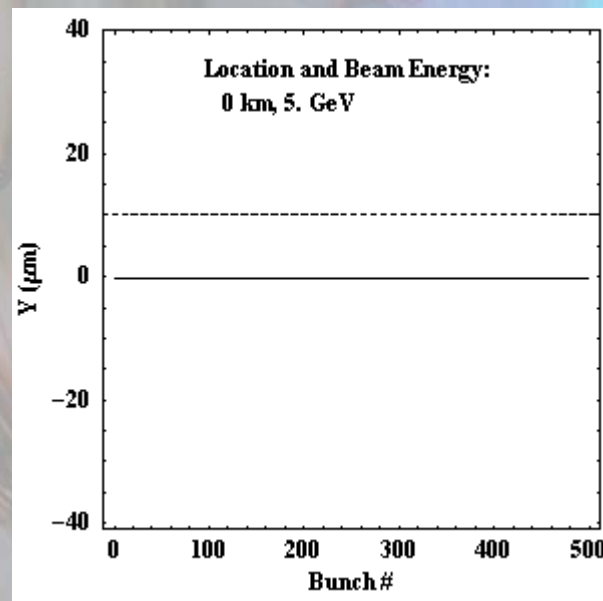
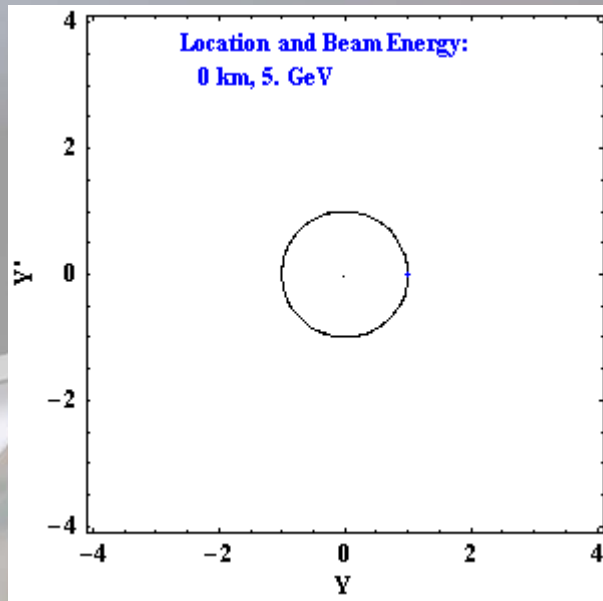
Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode

$X_{\text{off}} \sim \sigma_x$, $Y_{\text{off}}=0$. *Split Tune 70/60.*



- 500 particles in all simulations.
- Initial vertical and horizontal offset =0 and 270 μm .
- c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included

Split Tune 90/60.

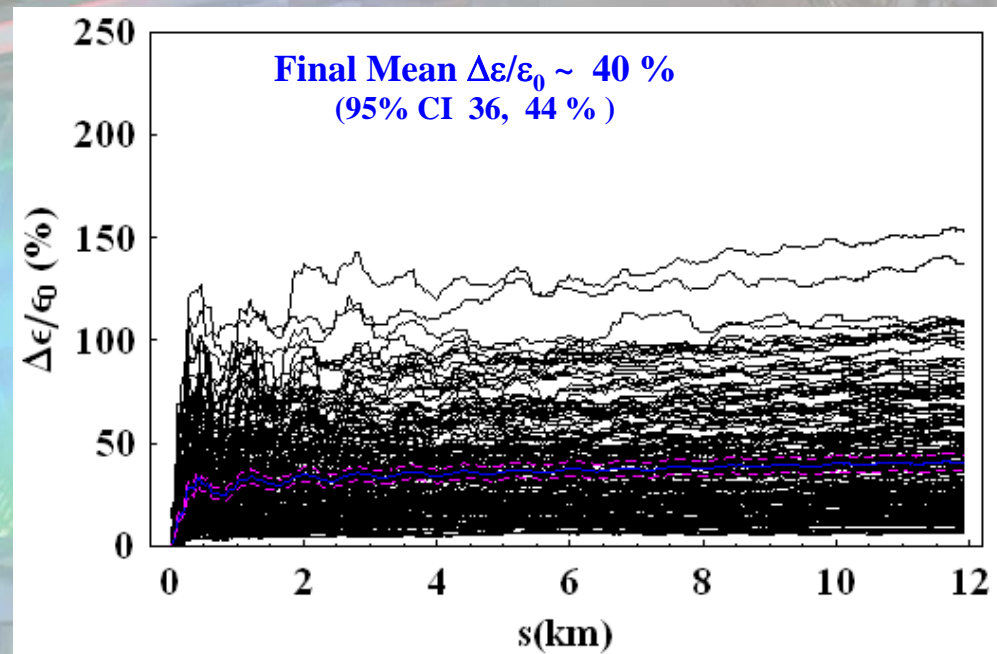
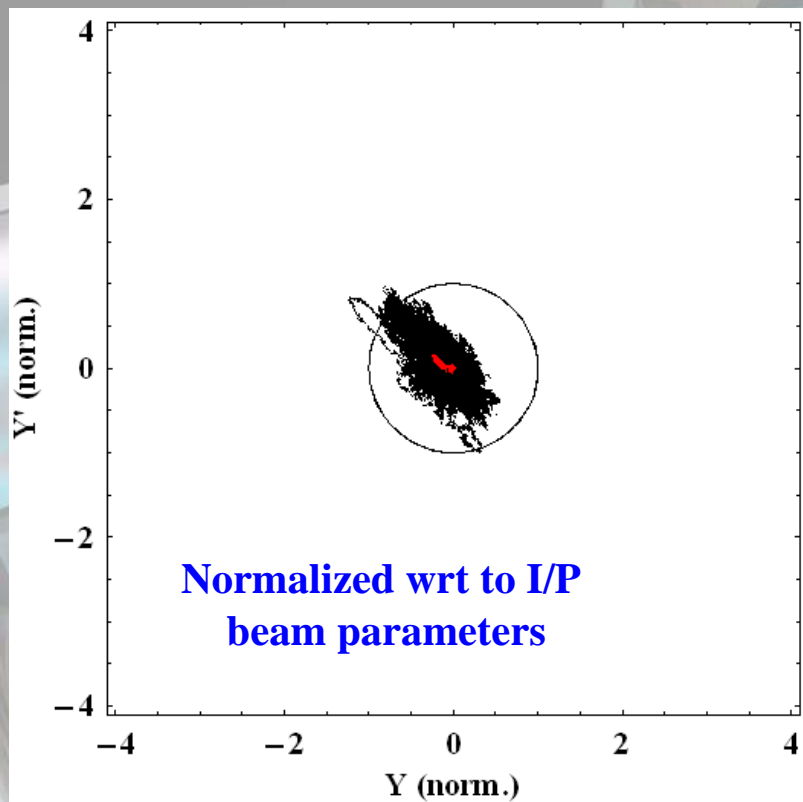


- Only mode-cross coupling gives rise to emittance dilution
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

Cross-Coupled Motion

Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode

$X_{\text{off}} \sim \sigma_x$, $Y_{\text{off}}=0$. *Split Tune 90/60.*



- 500 particles in all simulations.
- Initial vertical and horizontal offset =0 and 270 μm .
- c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included

Mode Coupling Summary

- ❑ Mode-coupling is a significant source of emittance dilution.
- ❑ Emittance dilution can be ameliorated by splitting the tune of the lattice.
- ❑ Further experimental data is needed on the actual splitting of the dipole degeneracies in frequencies that is likely to occur in the fabrication of several thousand ILC cavities (experiments at FLASH/DESY will be helpful in this respect –Molloy, Baboi, Ross, Napoly, et al). In addition, simulations are in progress on the splitting of the tune encountered due to the HOM couplers at SLAC (Z. Li, K. Ko et al).
- Analysis of the influence of higher order bands on emittance dilution in the presence of mode coupling is required.
- The effect of purposely distorting the shapes of the cavities in order to increase the frequency degeneracy splitting requires further study.

Further General HOM Studies

- ❑ Study of typical mode splitting over the 40 cavities => Obtain statistics and feed into the beam dynamics study.
- ❑ Cavity position measurement/prediction focussed on a mode with second largest large kick factor, at ~ 1.7 GHz mode (TE111-6, cf TE111-7)
=> Cavity position ascertained to better than 10 μm .
- ❑ Verify this alignment on additional modes.
- ❑ Absolute cavity to cavity alignment studies have already been made - alignments of X/Y 100/250 μm predicted for ACC4 (similar for ACC5).
- ❑ Proposition: FLASH/TTF measurements of complete band.
=> Analysis of cell to cell misalignment from an a study of the eigenmodes (or double-band circuit analysis).
Motivation –earlier studies on SLAC/KEK X-band structures enabled alignment of cells to be obtained to within tens of microns (modes were localised and hence the cell alignment was mapped from each mode).
For the L-band cavities, in general, the modes are non-local. Thus, an eigenmode analysis of the complete band is needed.