Asymptotic and rms Kicks due to HOMs in the 3.9 GHz Cavity

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Workshop: Higher order Mode Measurements in Superconducting Accelerating Cavities

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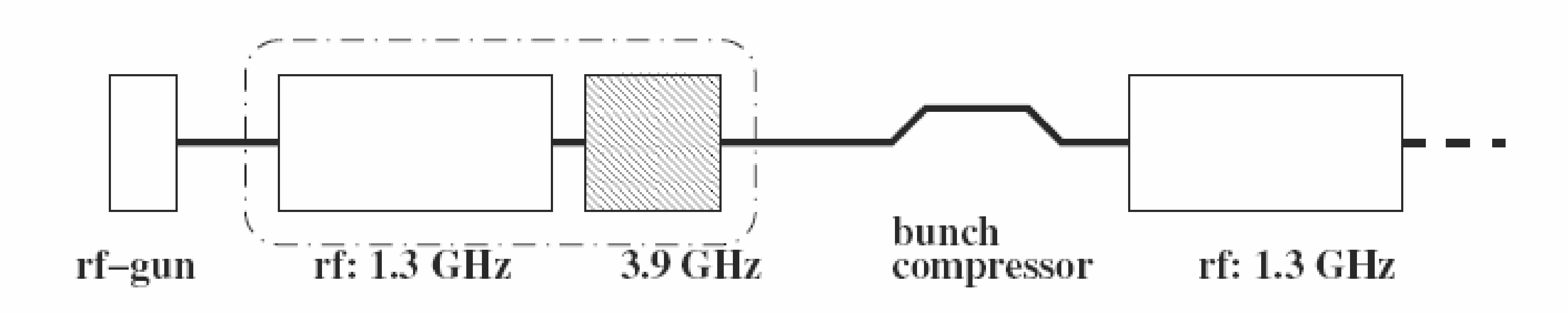
Introduction

Third harmonic (3.9 GHz) cavity will be used to compensate nonlinear distortions of the longitudinal phase space due to cosine-like curvature of the cavity voltage of the 1.3 GHz TESLA cavities.



Number of cacities	4
Number of cells / cavity	9
active length	~ 35 cm
Acc. Gradient	~ 15 MV/m

Beam:
130 MeV at 3.9 GHz Cav.
$\gamma\varepsilon$ = 1 mm mrad
Spot size ~ 0.2 mm
Beam divergence:
~ 20 μrad

Long Range Wakefields

Multipole long range wakefield (m=1 is dipole mode)
Sum over all modes:

$$W_{\parallel}^{(m)}(s) \;=\; -\sum_n \omega_n \left(\frac{R^{(m)}}{Q}\right)_n \cos(\omega_n\, s/c)\;\exp(-1/\tau_n\; s/c)$$

$$W_{\perp}^{(m)}(s) \;=\; c\sum_n \left(\frac{R^{(m)}}{Q}\right)_n \sin(\omega_n\, s/c)\;\exp(-1/\tau_n\; s/c).$$

R/Q Ohm/m^2

$$\frac{R^{(m)}}{Q} \;=\; \frac{1}{r^{2m}}\frac{2\,k^{(m)}(r)}{\omega}$$

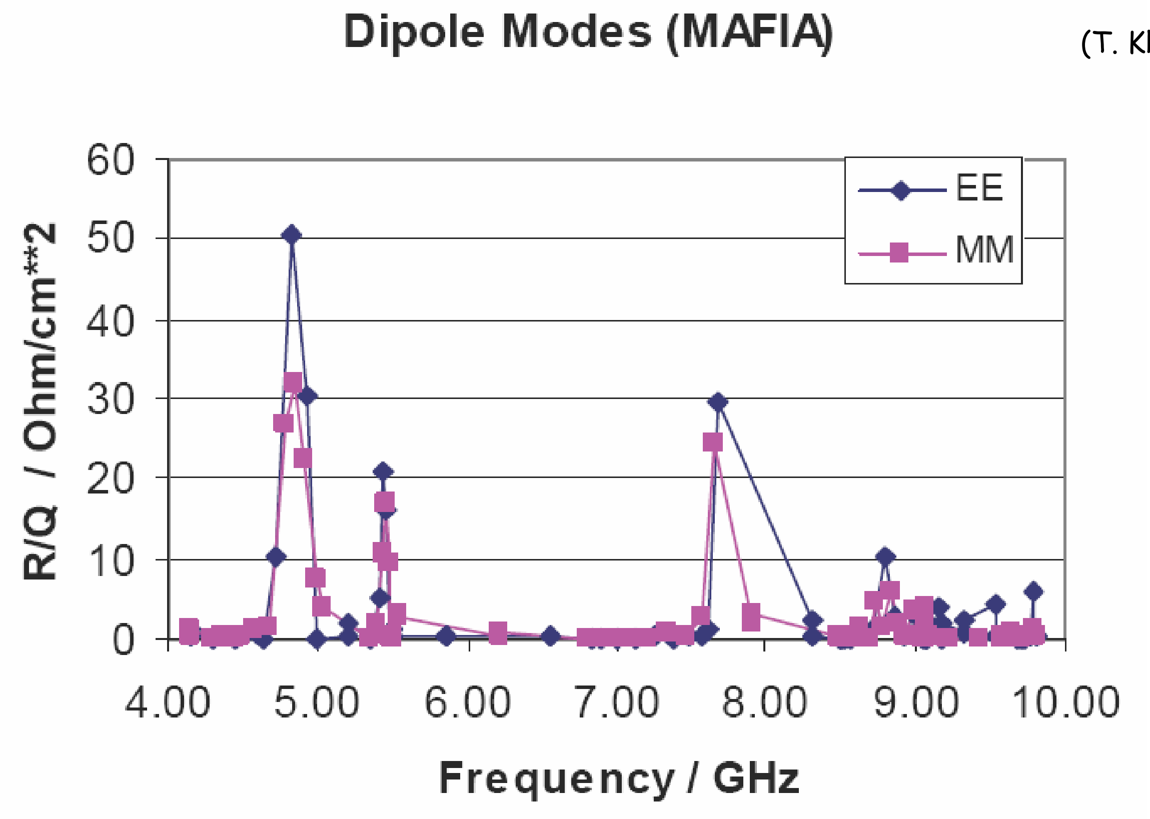
$$k^{(m)}(r) \;=\; \frac{\left|V_L^{(m)}(r)\right|^2}{4\,U^{(m)}}$$

Q-value

$$\tau_n \approx \frac{2\,(Q_{ext})_n}{\omega_n}$$

HOMs of the 3rd harmonic cavity

(T. Khabibouline, N. Solyak, R.W. **TESLA-FEL 2003-01**)



Kick (amplitude) due to one mode:

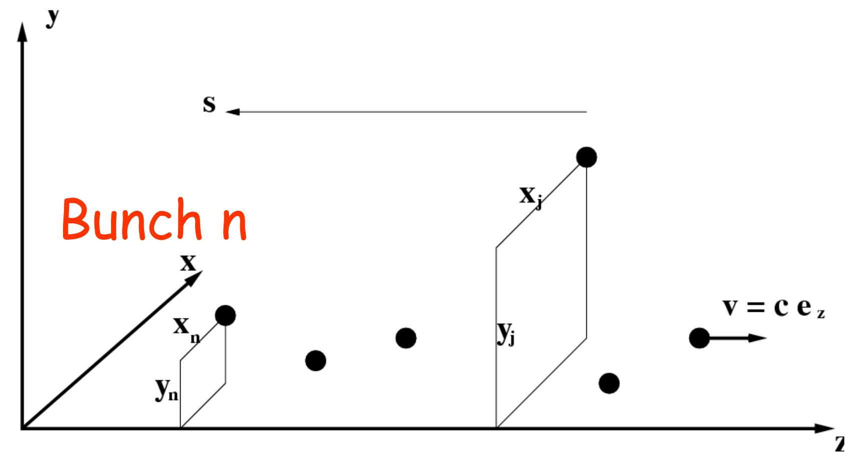
$$\hat{\theta}_n = \frac{e\,q}{E_n} c \frac{R^{(1)}}{Q} x_0$$

Q = 1 nC
E_n= 130 MeV
x_0 = 1 mm

f / GHz	$R^{(1)}/Q$ / Ω/cm^2	G_1 / Ω	$k^{(1)}$ / $\mathrm{V/(pC\,cm^2)}$	$\hat{\theta}$ / μrad
4.8	40	275	0.603	0.96
5.4	20	430	0.339	0.48
7.6	30	470	0.716	0.72

Kick ~ 0.5 … 1 μrad (one cavity)

Kick due to one HOM



Kick on bunch n in a bunch train of N bunches

Frequency of dipole mode:
$\omega_1 = 2\,\pi\, f_1$

f_{fu} = 3.9 GHz
N_{fb} = free buckets

$$\boldsymbol{\theta}_n = \hat{\theta}_n \sum_{j<n} \left(\frac{x_j}{x_0}\, \boldsymbol{e_x} + \frac{y_j}{x_0}\, \boldsymbol{e_y}\right) \sin(\omega_1\,(s_n - s_j)/c)\;\; \exp(-1/\tau_1\;(s_n - s_j)/c)$$

Constant bunch offset and constant bunch to bunch spacing: $\Delta t = \frac{\Delta s}{c} = n_{fb}\,\frac{1}{f_{fu}}$

$$\theta_n \;=\; \hat{\theta}\;\sum_{j=1}^{n-1} \sin(\delta\,(n-j))\;\exp(-d\,(n-j))$$

$$\delta = \omega_1\,\Delta t = 2\pi\frac{f_1}{f_{fu}}\,n_{fb} \qquad d = \frac{\omega_1}{2\,Q_1}\,\Delta t = 2\pi\frac{f_1}{f_{fu}}\,n_{fb}\,\frac{1}{2\,Q_1} \qquad D = i\,\delta - d$$

Kick due to one HOM (cont.)

Kick on bunch n in a bunch train of N bunches with constant bunch offset

$$\theta_n = \hat{\theta} \sum_{j=1}^{n-1} \sin(\delta\,(n-j))\,\exp(-d\,(n-j))$$

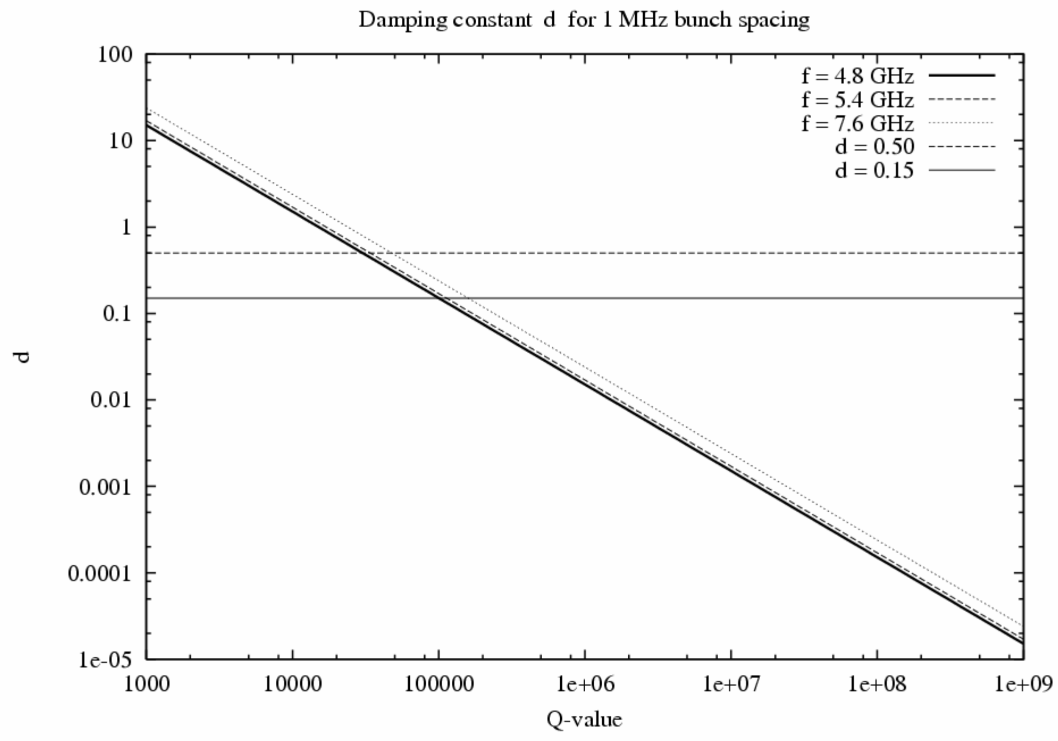
$$\delta = \omega_1\,\Delta t = 2\pi \frac{f_1}{f_{fu}}\, n_{fb}$$

"Random phases"

$$\Delta\delta = 2\pi \frac{1}{f_{fu}}\, n_{fb}\,\Delta f_1 = 0.36^\circ \frac{\Delta f_1}{\text{kHz}}$$

Damping constant d versus Q

d = 0.15 <-> Q ~ 10^5



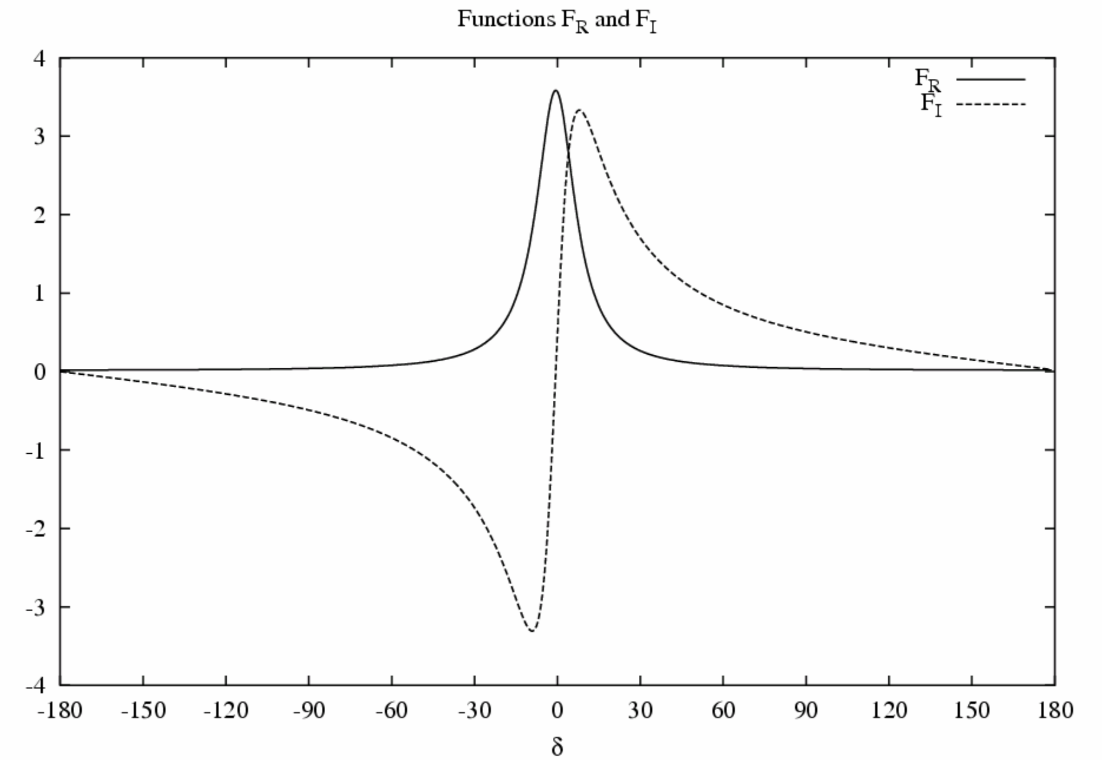
Asymptotic Kick

$$\theta_n = \hat{\theta}\ \mathrm{Im}\,(S_n) \qquad\qquad D = i\,\delta - d$$

$$S_n = \sum_{j=1}^{n-1} \exp\left((n-j)\,D\right)$$

$$S_n = \frac{1-\exp((n-1)\,D)}{\exp(-D)-1} \longrightarrow \frac{1}{\exp(-D)-1}, \quad \text{for } n \to \infty$$

$$\begin{aligned} F_R(\delta,d) &= \frac{1}{2} + \lim_{n\to\infty} \mathrm{Re}\,(S_n) \\ &= \frac{1-\mathrm{e}^{-2\,d}}{2\,(1-2\,\mathrm{e}^{-d}\cos(\delta)+\mathrm{e}^{-2\,d})} \\ F_I(\delta,d) &= \lim_{n\to\infty} \mathrm{Im}\,(S_n) \\ &= \frac{\mathrm{e}^{-d}\sin(\delta)}{1-2\,\mathrm{e}^{-d}\cos(\delta)+\mathrm{e}^{-2\,d}}. \end{aligned}$$



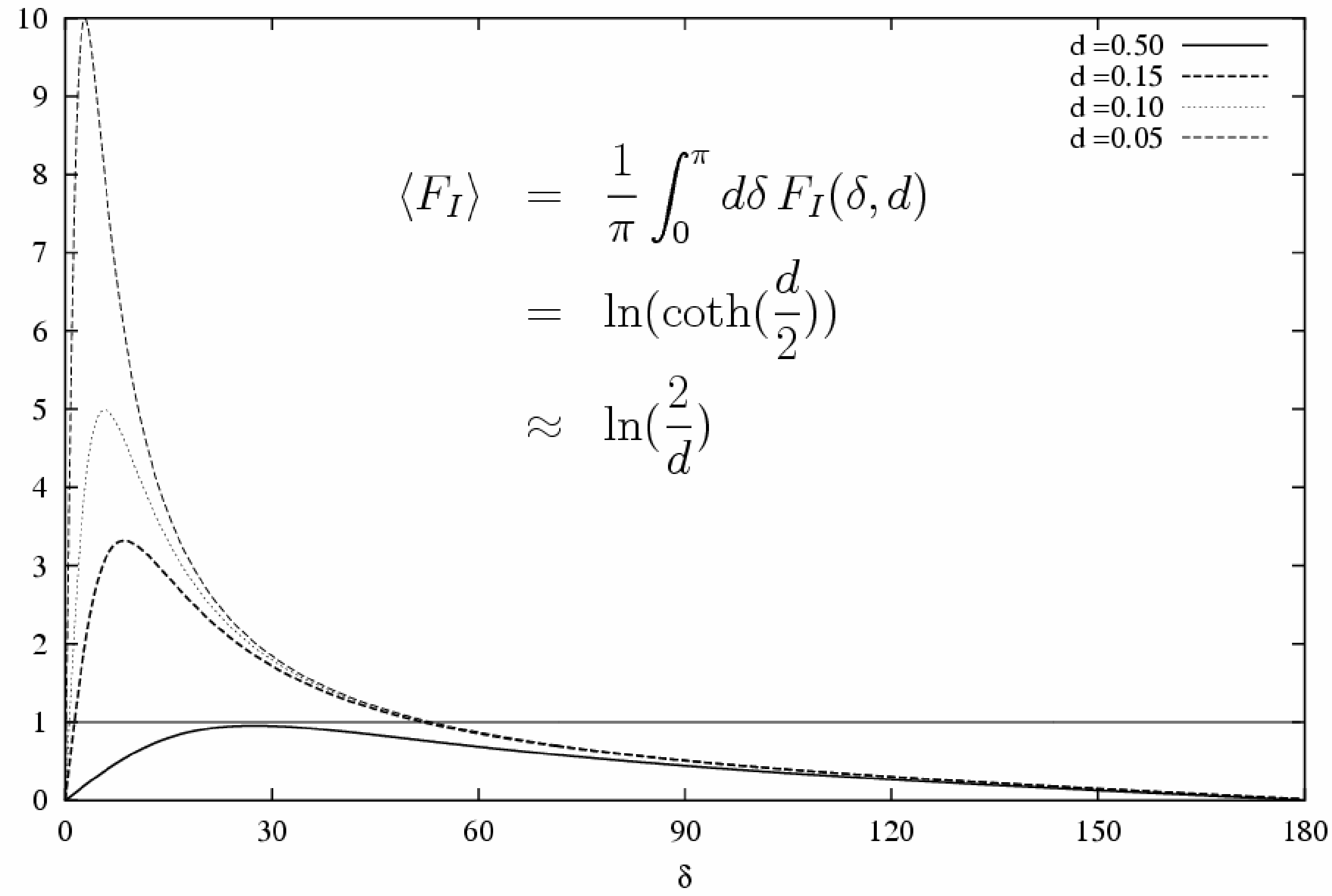
Asymptotic Kick (cont.)

$$\theta_n = \hat{\theta}\ \left(F_I(\delta, d) - \mathrm{Im}\,(R_n) \right)$$

$$D = i\,\delta - d$$

$$R_n = \frac{\exp((n-1)\,D)}{\exp(-D) - 1}$$

Function F_I



$$\langle F_I \rangle = \frac{1}{\pi}\int_0^{\pi} d\delta\, F_I(\delta, d) = \ln(\coth(\frac{d}{2})) \approx \ln(\frac{2}{d})$$

Probability

$F_I > 1$ ~29 %

$F_I > 10$ ~ 3 %,

but

$\max(F_I) = 50000$

for $d = 10^{-5}$

RMS kick due to one HOM

$$\theta_n = \hat{\theta} \; (F_I(\delta, d) - \mathrm{Im}\,(R_n) \,)$$

Average over the bunch train !

$$\langle \theta \rangle / \hat{\theta} := F_I - \frac{1}{N} \sum_{n=1}^{N} \mathrm{Im}(R_n)$$

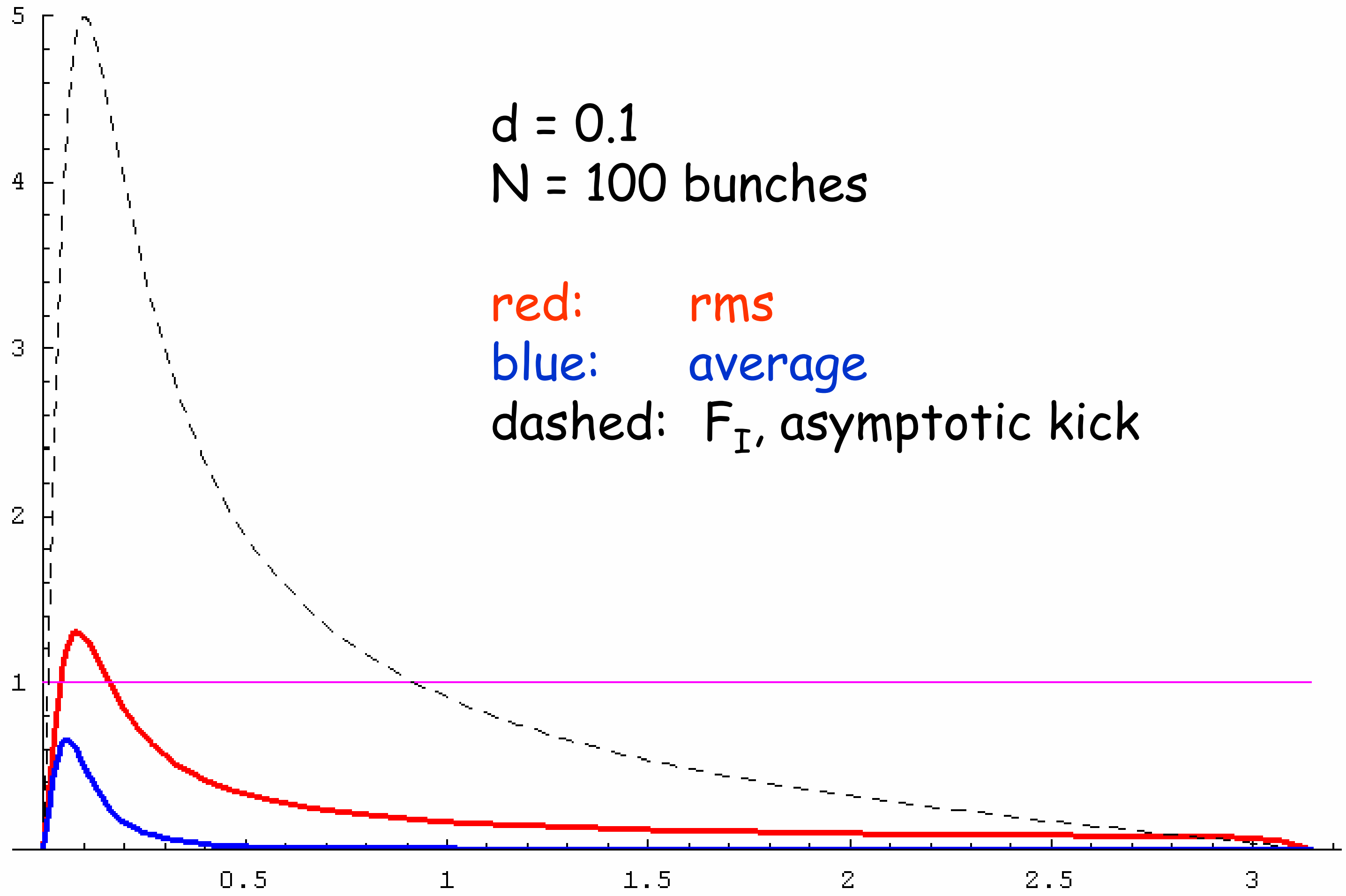
$$rms(\theta) / \hat{\theta} := \sqrt{\frac{1}{N} \sum_{n=1}^{N} (\mathrm{Im}(R_n))^2 - \left(\frac{1}{N} \sum_{n=1}^{N} \mathrm{Im}(R_n) \right)^2}$$

The Rms over the bunch train does not depend on the asymptotic Kick function F_I

RMS kick due to one HOM (cont.)

$$rms(\theta)/\hat{\theta} := \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\mathrm{Im}(R_n))^2 - \left(\frac{1}{N}\sum_{n=1}^{N}\mathrm{Im}(R_n)\right)^2}$$

$$\langle\theta\rangle/\hat{\theta} := F_I - \frac{1}{N}\sum_{n=1}^{N}\mathrm{Im}(R_n)$$

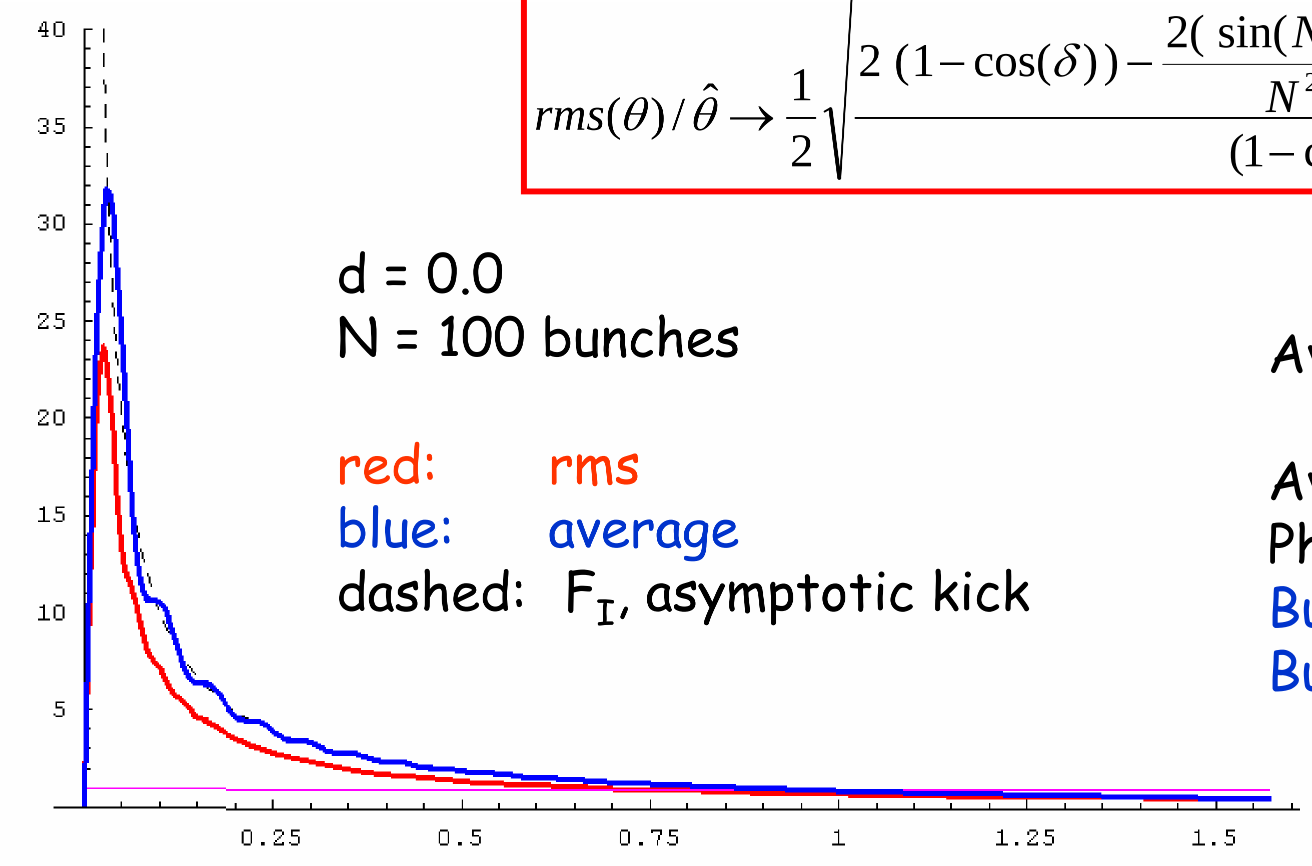


No damping: RMS and average

$$F_I \rightarrow \frac{1}{2}\cot\left(\tfrac{\delta}{2}\right)$$

$$\langle \theta \rangle / \hat{\theta} \rightarrow \frac{1}{2(\cos(\delta)-1)}\left(\frac{\sin(N\delta)}{N} - \sin(\delta)\right)$$

$$rms(\theta)/\hat{\theta} \rightarrow \frac{1}{2}\sqrt{\frac{2\,(1-\cos(\delta)) - \frac{2(\,\sin(N\delta)\,)^2}{N^2} + \frac{\sin(2N\delta)\,(\tan(\delta))^2}{N}}{(1-\cos(\delta))^2}}$$



d = 0.0
N = 100 bunches

red: rms
blue: average
dashed: F_I, asymptotic kick

Averaging the average

Average over the Phase δ:
Bunch Average = 1.55
Bunch RMS = 1.25

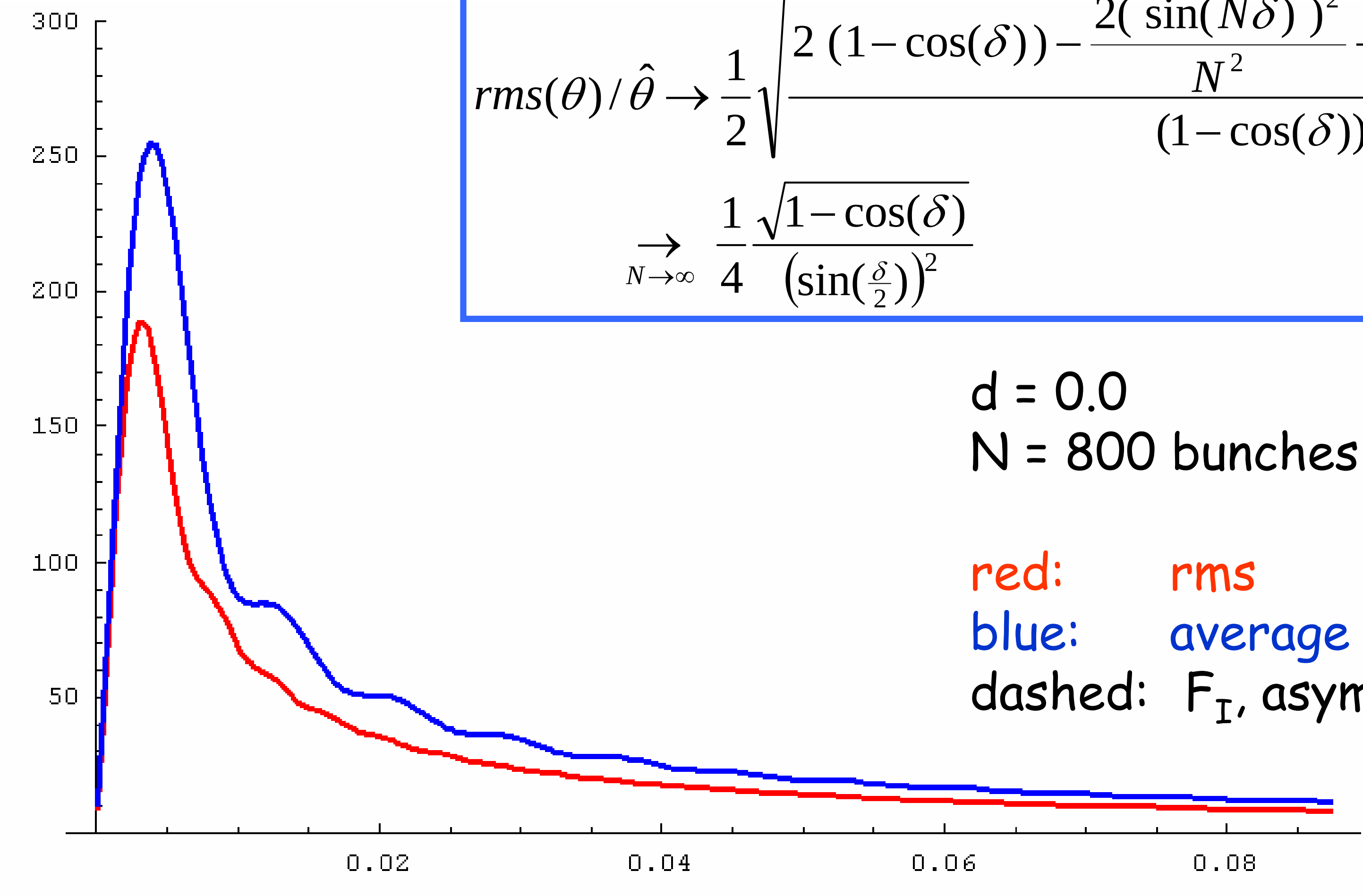
No damping: RMS and average (cont.)

$$F_I \to \frac{1}{2}\cot(\tfrac{\delta}{2})$$

$$\langle\theta\rangle/\hat{\theta} \to \frac{1}{2(\cos(\delta)-1)}\left(\frac{\sin(N\delta)}{N}-\sin(\delta)\right) \underset{N\to\infty}{\longrightarrow} \frac{1}{2}\cot(\tfrac{\delta}{2})$$

$$rms(\theta)/\hat{\theta} \to \frac{1}{2}\sqrt{\frac{2\,(1-\cos(\delta))-\frac{2(\,\sin(N\delta)\,)^2}{N^2}+\frac{\sin(2N\delta)\,(\tan(\delta))^2}{N}}{(1-\cos(\delta))^2}}$$

$$\underset{N\to\infty}{\longrightarrow} \frac{1}{4}\frac{\sqrt{1-\cos(\delta)}}{\left(\sin(\tfrac{\delta}{2})\right)^2}$$



d = 0.0
N = 800 bunches

red: rms
blue: average
dashed: F_I, asymptotic kick

10 degree

Conclusions

- The kicks due to HOMs in the 3.9 GHz cavity have been calculated for a constant offset of all bunches

- Analytic formulas have been obtain for different cases
  1. Asymptotic kick
  2. Average and rms kick
  3. Average and rms kick with no damping
  4. Average and rms kick with no damping and many bunches $N \to \infty$

- Even an small dumping constant seems to be acceptable, if only short bunch trains are used (say < 100 bunches) Further investigations are required
- The operation with long bunch trains require HOM dampers

- A possible solution if one hits a HOM resonce:

  a small change in the bunch-to-bunch spacing (one 1.3 GHz bucket), say 3903 instead 3900 free 3.9 GHz buckets gives a large change of phase δ)