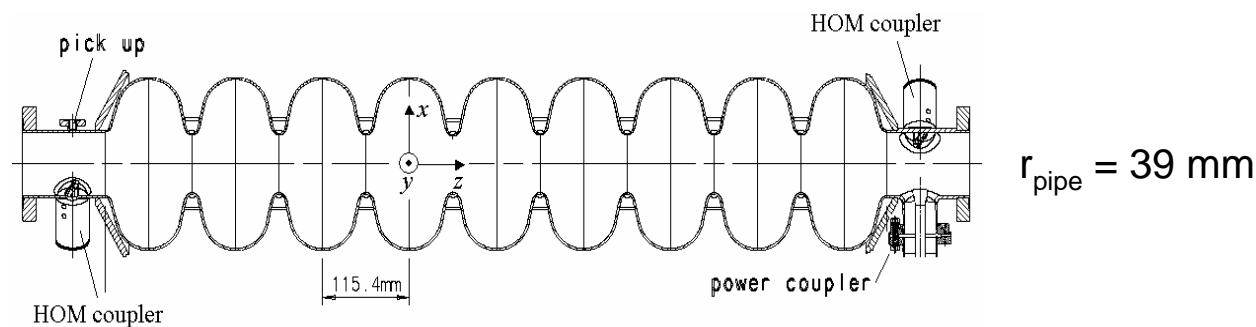
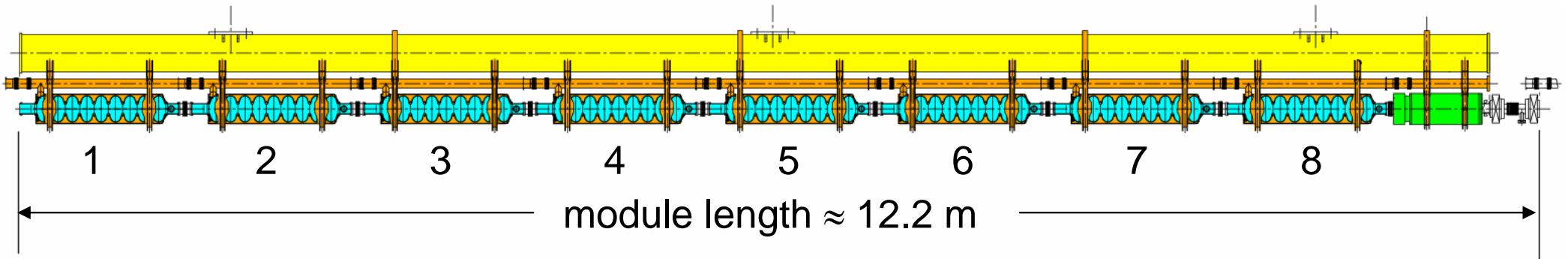


# (Higher Order) Modes in Cavity Structures



TESLA cavity with main coupler  
and two HOM couplers



# cutoff frequencies & cavity bands

from R.Wanzenberg, TESLA 2001-33

$r_{\text{pipe}} = 39 \text{ mm}$ , cutoff frequencies:

monopole modes	dipole modes	quadrupole modes
$f_c/\text{GHz}$	$f_c/\text{GHz}$	$f_c/\text{GHz}$
<b>2.942 (TM)</b>	<b>2.253 (TE)</b>	<b>3.737 (TE)</b>
4.688 (TE)	4.688 (TM)	6.283 (TM)
6.753 (TM)	6.523 (TE)	8.204 (TE)
8.583 (TE)	8.583 (TM)	10.298 (TM)
...	...	...

TESLA cavity, bands:

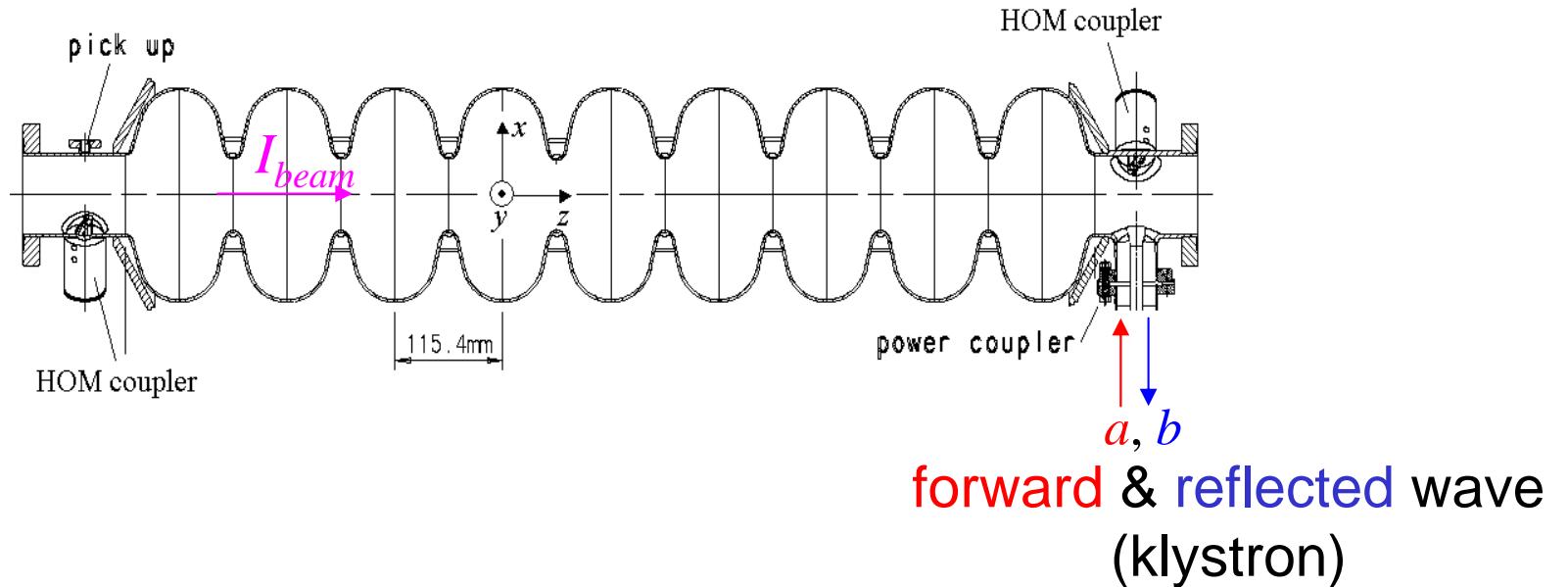
TM monopole modes $f/\text{GHz}$	dipole modes $f/\text{GHz}$	quadrupole modes $f/\text{GHz}$
<u><b>1.275 ... 1.30</b></u>	1.63 ... 1.80	2.30 ... 2.32
2.38 ... 2.45	<u><b>1.84 ... 1.89</b></u>	2.47 ... 2.49
2.67 ... 2.76	2.48 ... 2.58	3.21 ... 3.46
3.39 ... 3.67	2.82 ... 3.01	3.56 ... 3.62
...	3.08 ... 3.12	<u><b>3.66 ... 3.69</b></u>
	...	3.85 ... 3.87

$f < f_c$ : calculate cavities

$f > f_c$ : coupled cavities → calculate modules (or less? or more?)



## below $f_c$ : accelerating mode



steady state (after filling):

$$a = I_{beam} \sqrt{\frac{2Qk_{\parallel}}{\omega}} + V_{acc} \sqrt{\frac{\omega}{2Qk_{\parallel}}} \left( \frac{1}{2} - j \frac{\delta\omega Q}{\omega} \right)$$

$$b = -I_{beam} \sqrt{\frac{2Qk_{\parallel}}{\omega}} + V_{acc} \sqrt{\frac{\omega}{2Qk_{\parallel}}} \left( \frac{1}{2} + j \frac{\delta\omega Q}{\omega} \right)$$

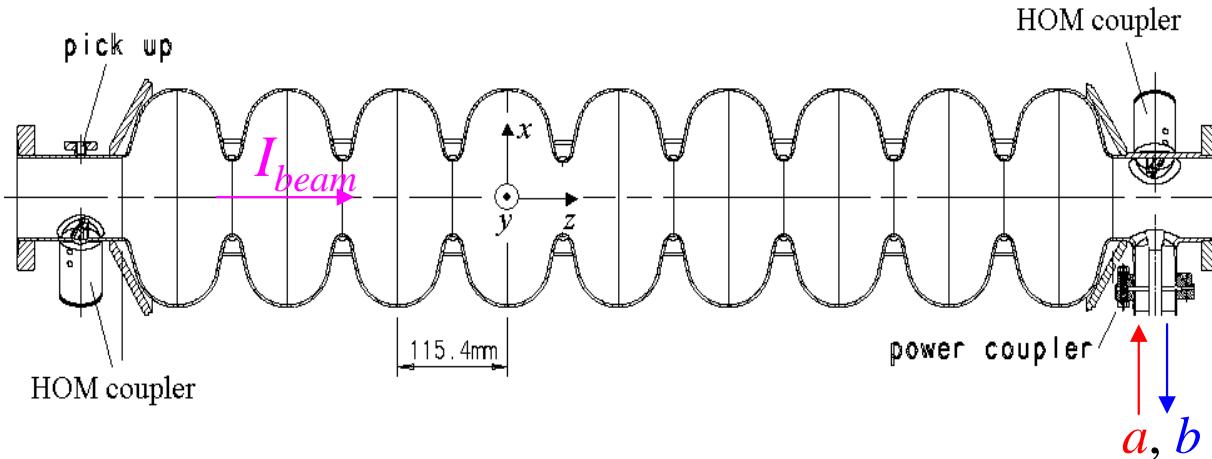
$$(a, b) \leftrightarrow (I_{beam}, V_{acc})$$

with:

- $Q$  = external quality
- $k_{\parallel}$  = longitudinal loss parameter  
=  $|V_{acc}|^2 / 4W_{tot}$
- $\omega$  = frequency
- $\delta\omega$  = detuning



## below $f_c$ : accelerating mode external quality & coupler kick



$$\mathbf{E} = a\mathbf{E}^{(a)} + b\mathbf{E}^{(b)}$$

$$\mathbf{B} = a\mathbf{B}^{(a)} + b\mathbf{B}^{(b)}$$

$$\mathbf{V} = \int (\mathbf{E} + c\mathbf{u}_z \times \mathbf{B}) \exp\left(j \frac{\omega}{c} z\right) dz = a\mathbf{V}^{(a)} + b\mathbf{V}^{(b)}$$

← coupler kick depends on forward and backward wave!

special case:  $b = 0$  (matched operation)

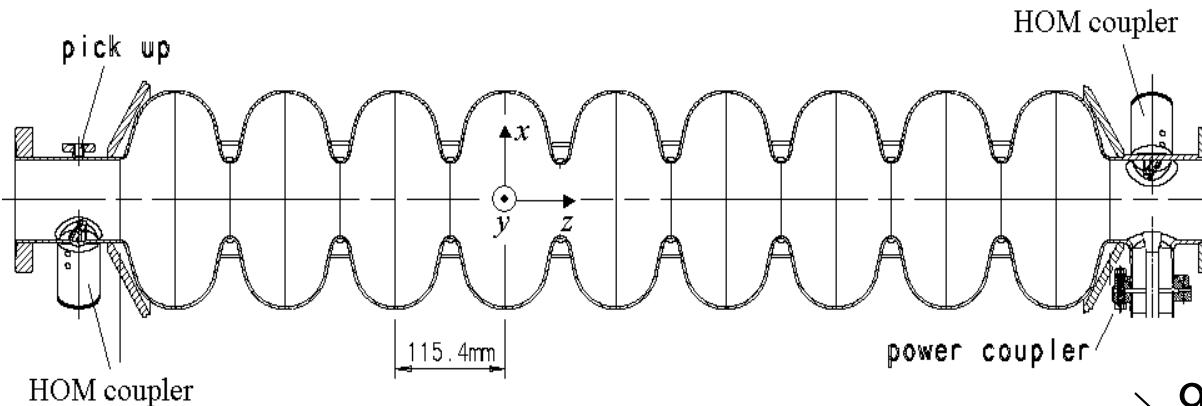
$$V_{acc} = \mathbf{V} \cdot \mathbf{u}_z \text{ longitudinal voltage}$$

$$V_t = \mathbf{V} \cdot \mathbf{u}_t \text{ transverse voltage with } \mathbf{u}_t = \mathbf{u}_{x/y}$$

$$\text{normalized kick: } k_{t,n} = V_t / V_{acc}$$



# below $f_c$ : accelerating mode simplifications



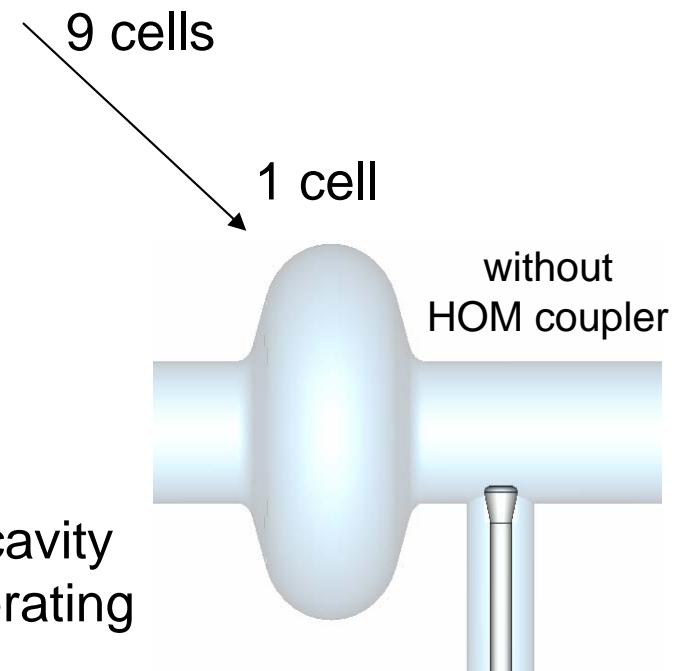
## coupler field:

establish energy flow to (or from cavity) !!!  
either with beam or by wall losses  
or by  $dW/dt \neq 0$

## scale field:

energy in 9 cell cavity  $\approx 9 \times$  energy in 1 cell cavity  
for the same input power or the same accelerating field

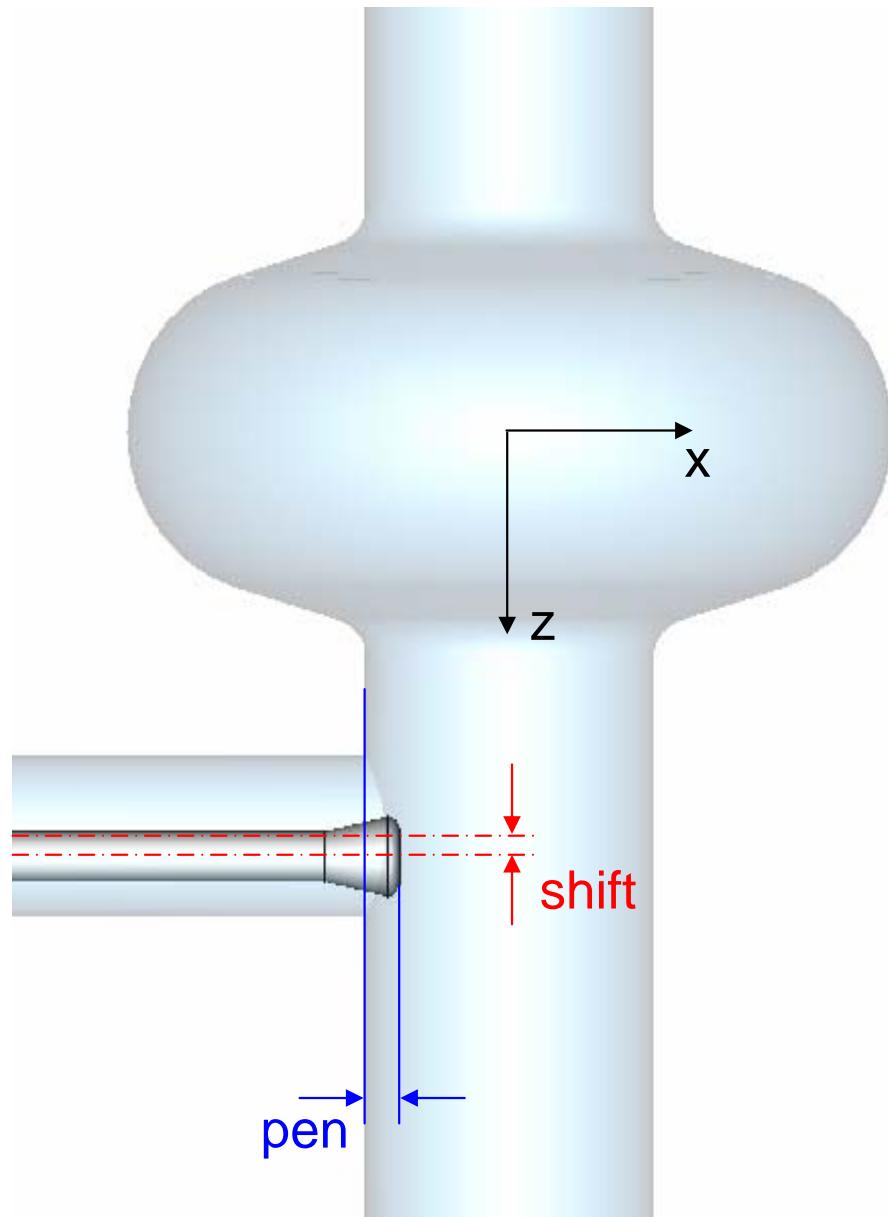
$$\rightarrow Q_{\text{ext},9\text{cell}} \approx 9 \times Q_{\text{ext},1\text{cell}}$$



complete documentation (with all tricks):  
[http://adweb.desy.de/~mpymax/mafia/HOM\\_Coupler/index.html](http://adweb.desy.de/~mpymax/mafia/HOM_Coupler/index.html)



# below $f_c$ : accelerating mode external quality & coupler kick



penetration depth & displacement  
of inner conductor

scaled to TESLA cavity:

$$\frac{Q_{ext}}{10^6} \cdot \frac{10^6 \cdot V_x}{V_z}$$

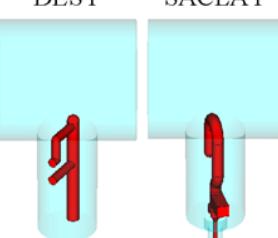
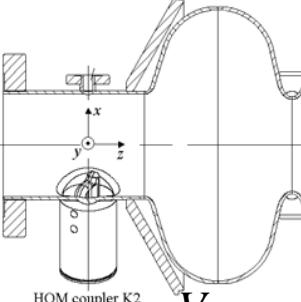
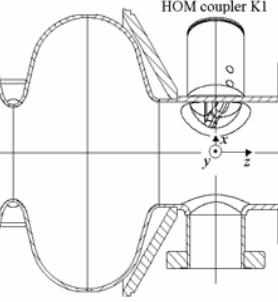
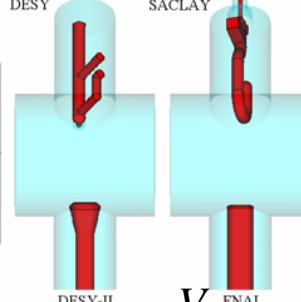
MWS-discretization: 30lines@2GHz

pen/mm \ shift/mm	-5	0	5
4.5	3.347 19.9+j35.9	4.490	
6	2.466 47.6+j40.9	3.384 30.6+j54.3	
7.5	1.781 84.5+j50.0	2.4482 58.7+j65.0	3.987 37.4+j68.1
9	1.272 130.3+j56.9	1.940 93.4+j83.3	3.464 65.1+j88.9
10.5	0.9662	1.663	2.583 100.9+j86.5
12		1.351	2.099 141.1+j65.0



# below $f_c$ : accelerating mode

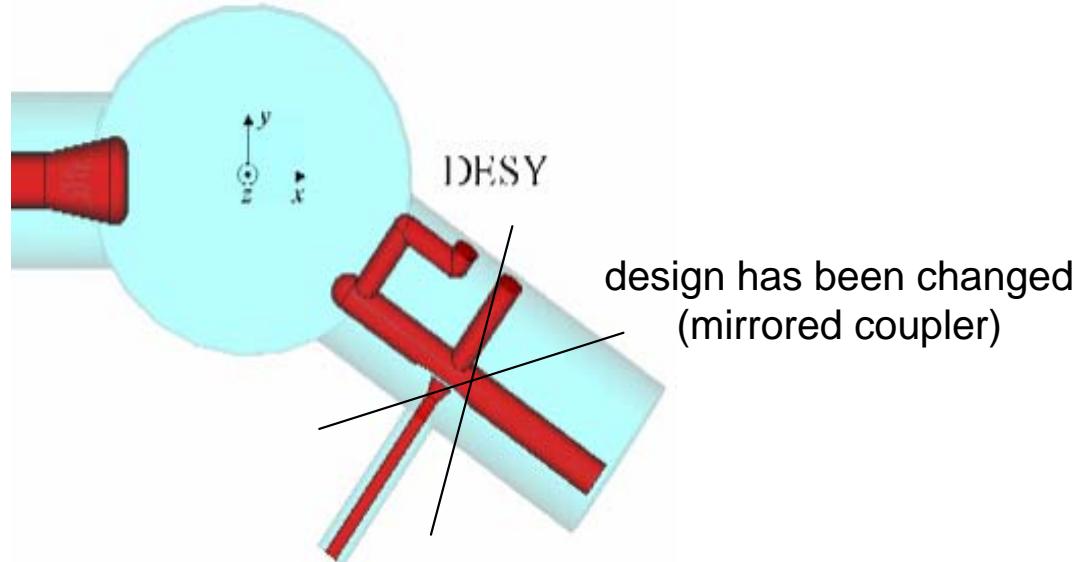
## kick by main coupler & HOM couplers:

	desy	saclay
upstream	<p style="text-align: center;">DESY      SACLAY</p>  $\frac{V_x}{V_{\parallel}} = (-53.0 + j14.3) \cdot 10^{-6}$ $\frac{V_y}{V_{\parallel}} = (-33.4 + j2.8) \cdot 10^{-6}$	 $\frac{V_x}{V_{\parallel}} = (-20.1 + j24.5) \cdot 10^{-6}$ $\frac{V_y}{V_{\parallel}} = (-26.2 + j12.5) \cdot 10^{-6}$
downstream	<p style="text-align: center;"><math>Q_{ext} \approx 2.5 \cdot 10^6</math></p>  $\frac{V_x}{V_{\parallel}} = (2 + j54) \cdot 10^{-6}$ $\frac{V_y}{V_{\parallel}} = (32 + j6) \cdot 10^{-6}$	 $\frac{V_x}{V_{\parallel}} = (34 + j45) \cdot 10^{-6}$ $\frac{V_y}{V_{\parallel}} = (28 + j15) \cdot 10^{-6}$

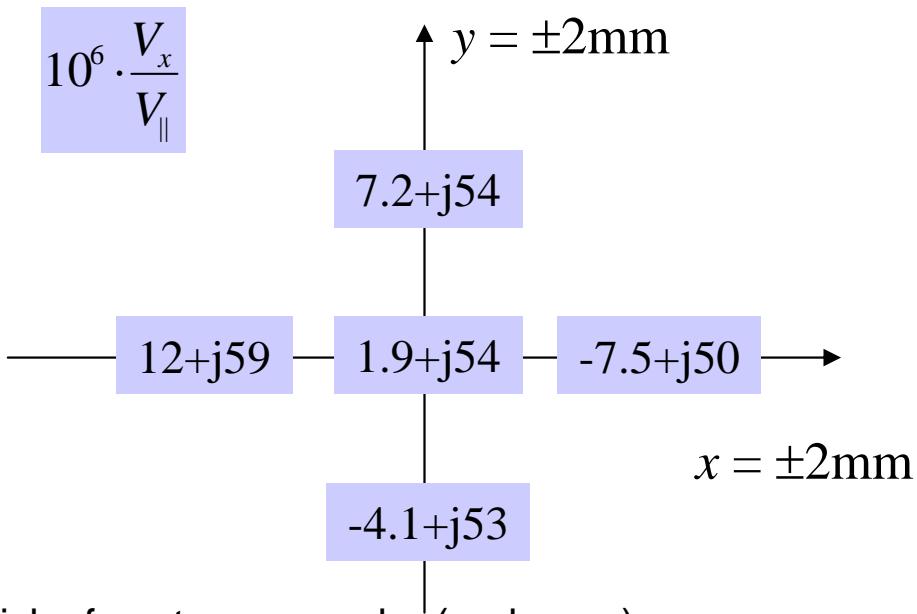
phase reference is center of cell 1



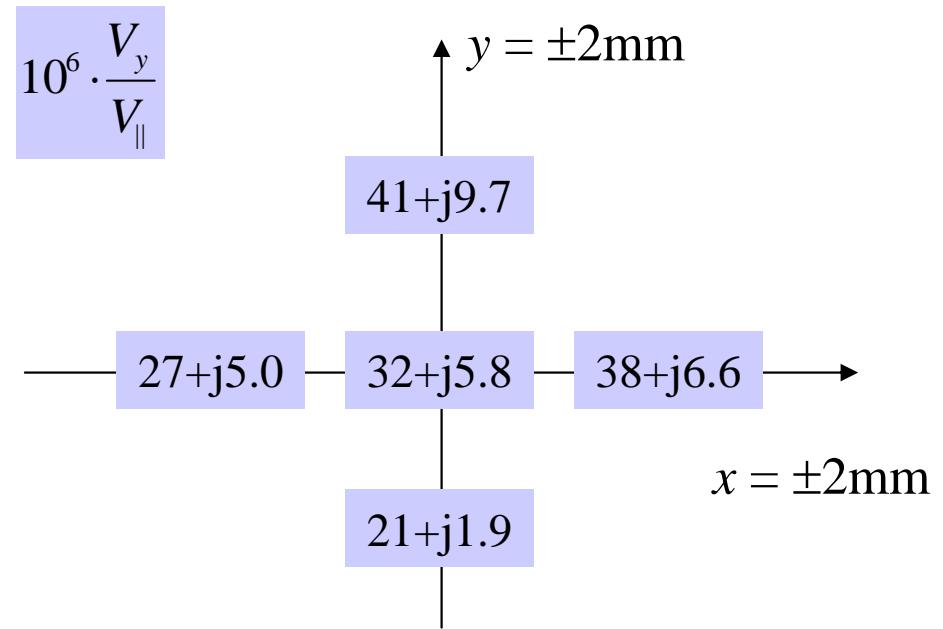
# below $f_c$ : accelerating mode kick by main & HOM couplers



horizontal kick



vertical kick



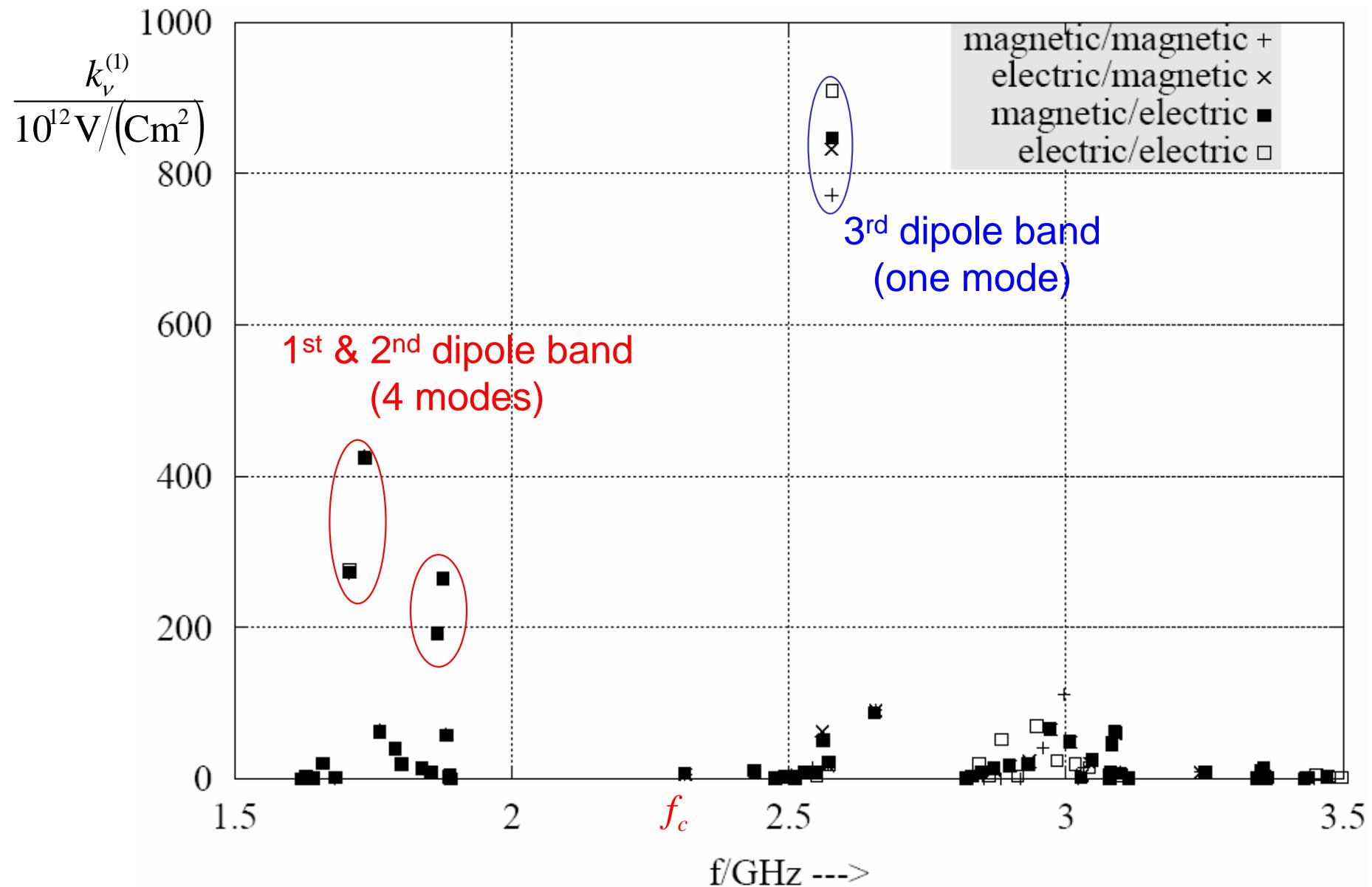
kick of upstream coupler (and more):

[http://www.desy.de/~dohlus/2004/2004.09.holgers\\_seminar/asym&kick\\_sep2004.pdf](http://www.desy.de/~dohlus/2004/2004.09.holgers_seminar/asym&kick_sep2004.pdf)

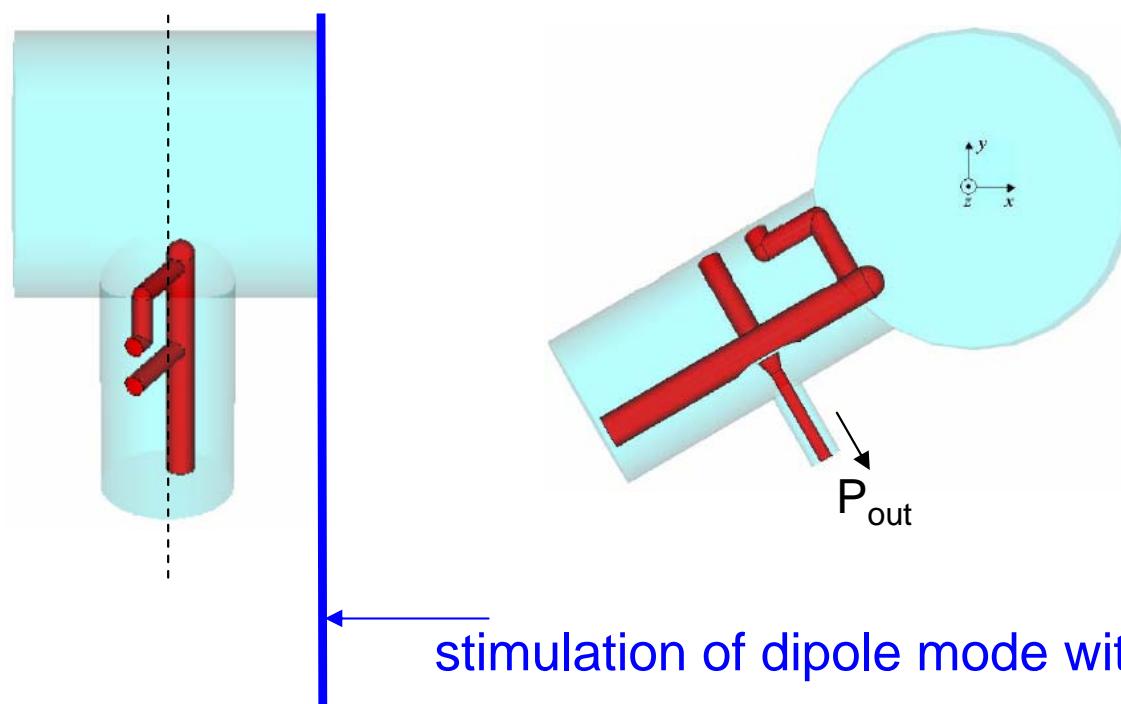
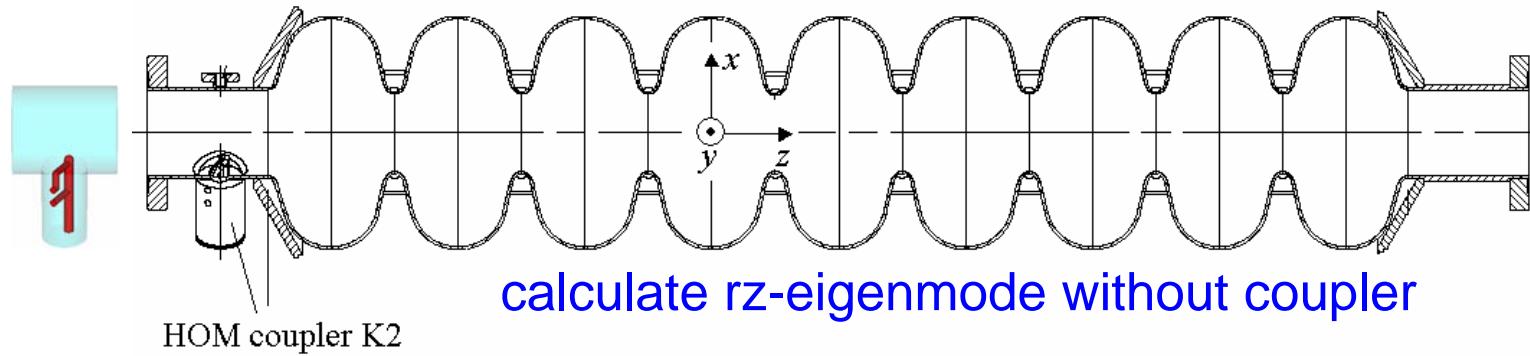


## below $f_c$ : dipole modes

frequencies & loss parameters from rz eigenmode calculation



**below  $f_c$ : dipole modes  
pseudo field matching**

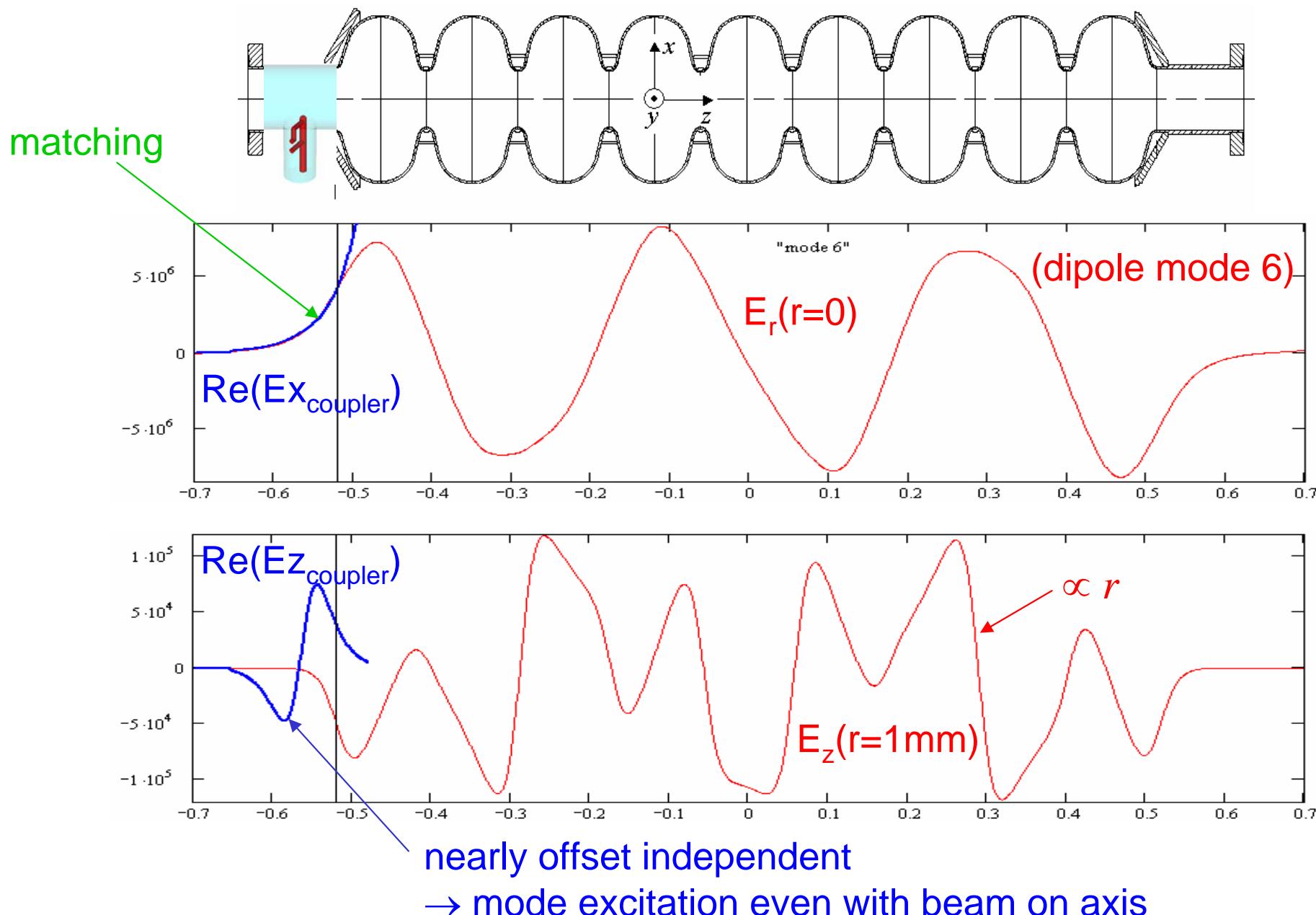


stimulation of dipole mode with horizontal polarization

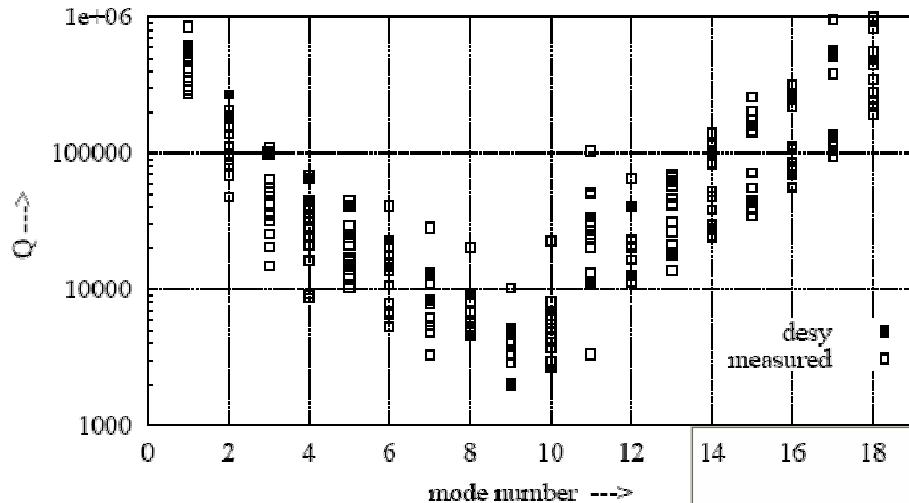


# below $f_c$ : dipole modes

pseudo field matching; estimate Qext & field asymmetry



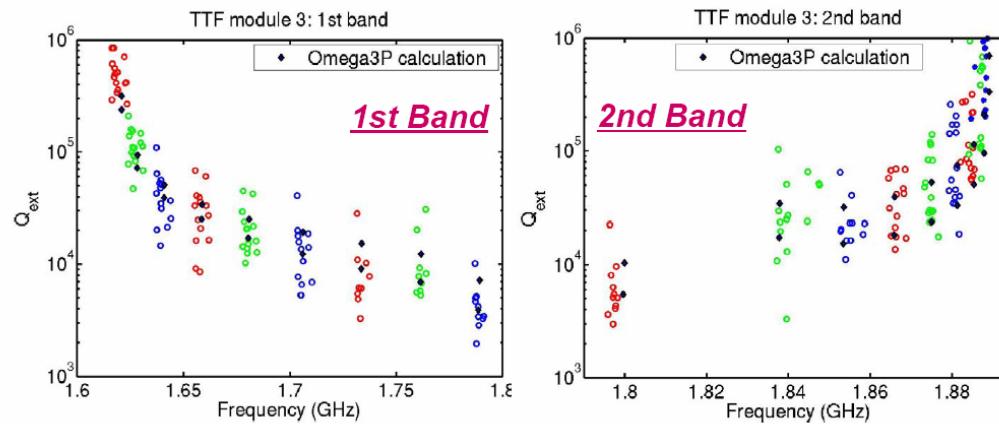
# below $f_c$ : dipole modes estimated Q<sub>ext</sub>



even with “flagship” supercomputers:

## TDR Cavity – HOM Damping (Omega3P)

Comparing measurements (color) with Omega3P (black) complex eigenmode solutions ( $Q_{\text{ext}} = f_{\text{real}} / 2f_{\text{imag}}$ ) shows data scatter around ideal cavity results due to shape deformations



Accelerating Cavity Design for the International Linear Collider

A. Kabel, ...

ICAP 2006, Chamonix Mont-Blanc, October 2-6, 2006

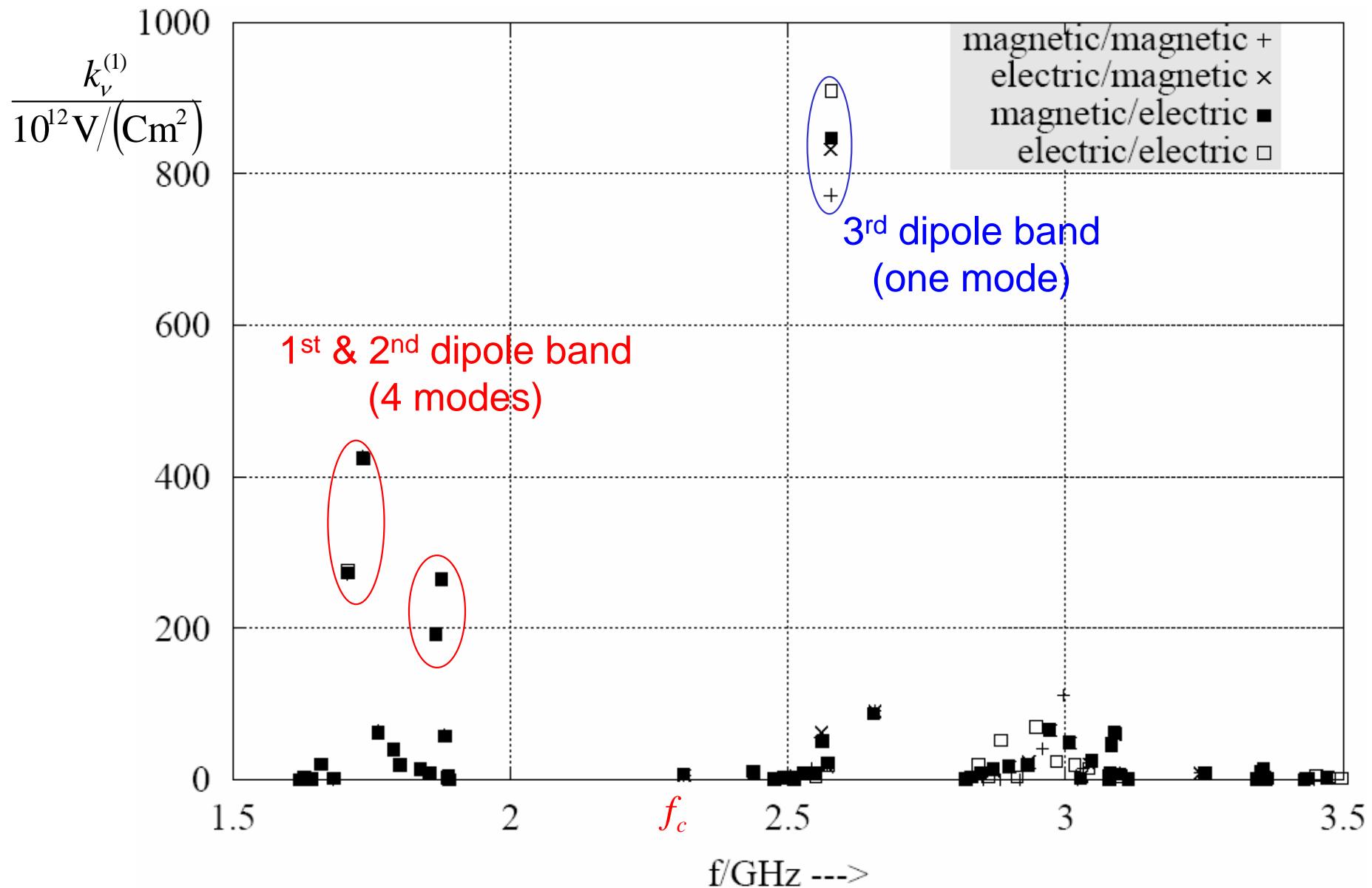


0.53 M quadratic elements, 3.5 M DOFs, 512 CPU with 300 GB on Seaborg, Second Arnoldi with MUMPs, 1 hour per dipole band

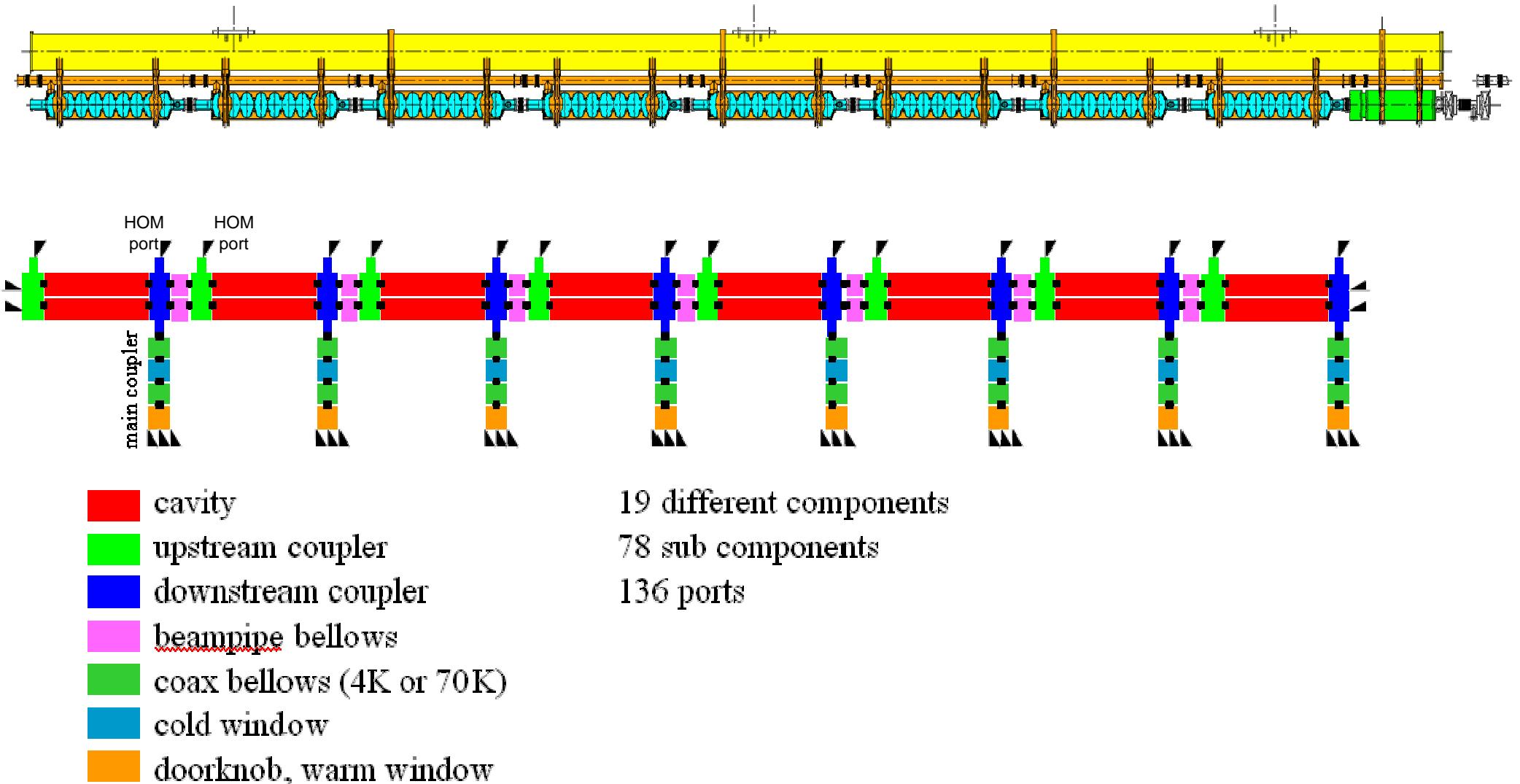


## above $f_c$ : 3<sup>rd</sup> dipole band

frequencies & loss parameters from rz eigenmode calculation

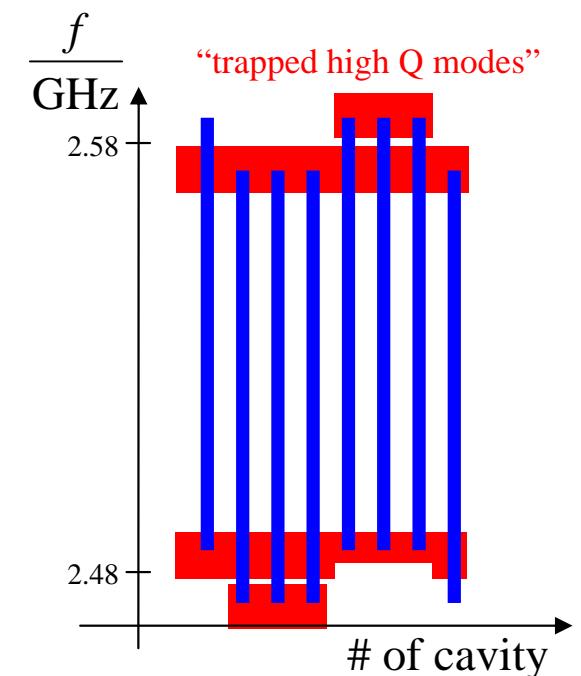
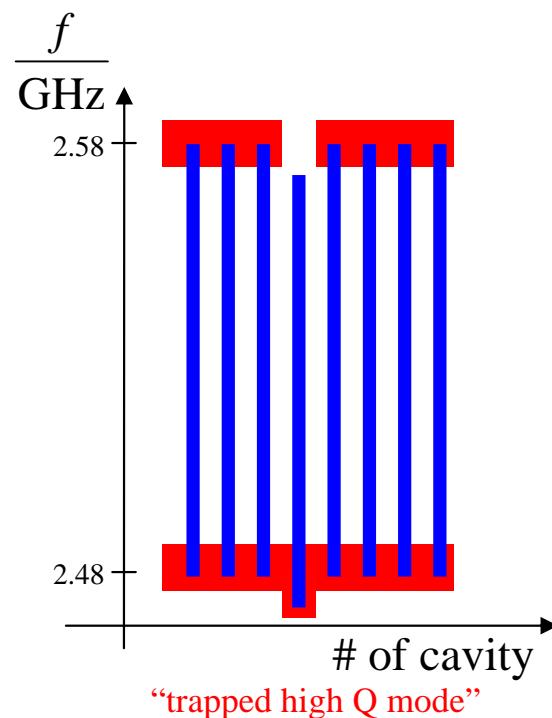
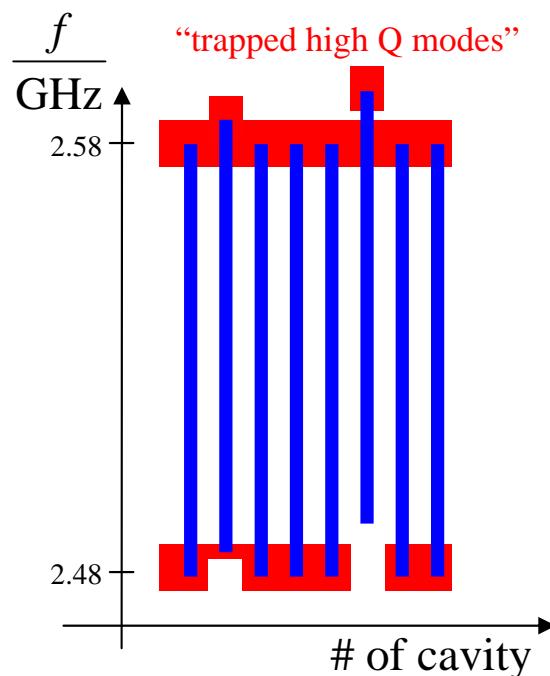


# above $f_c$ : 3<sup>rd</sup> dipole band coupled scattering matrix calculation



distributed modes (many cavities)  
and localized modes

eg. 2.5906 GHz mode  
cavity 7 / module 3



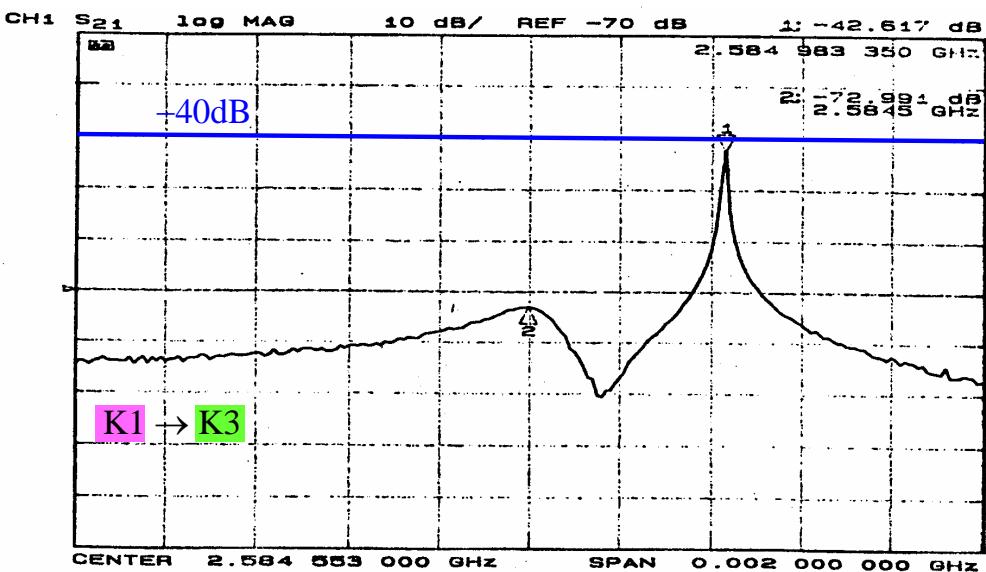
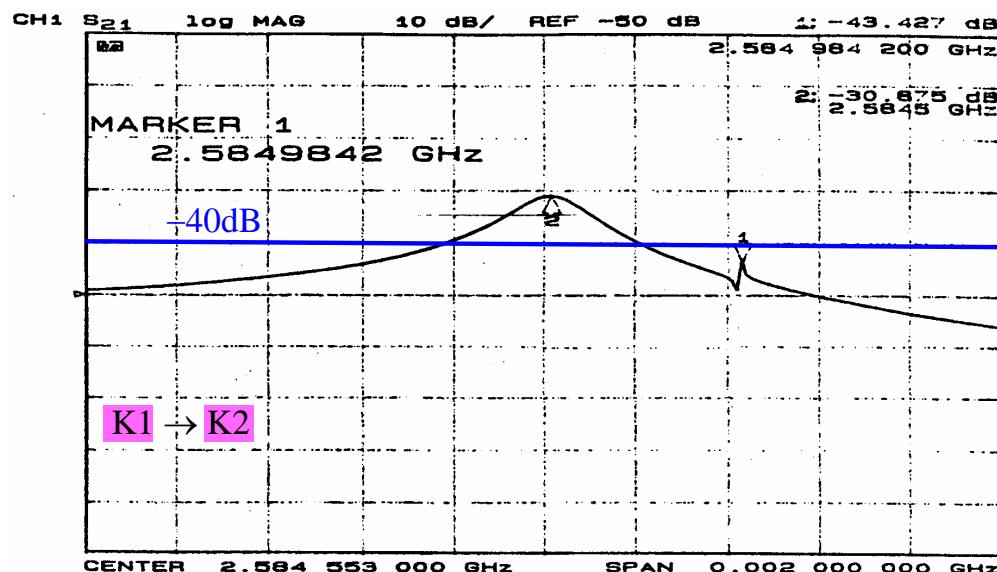
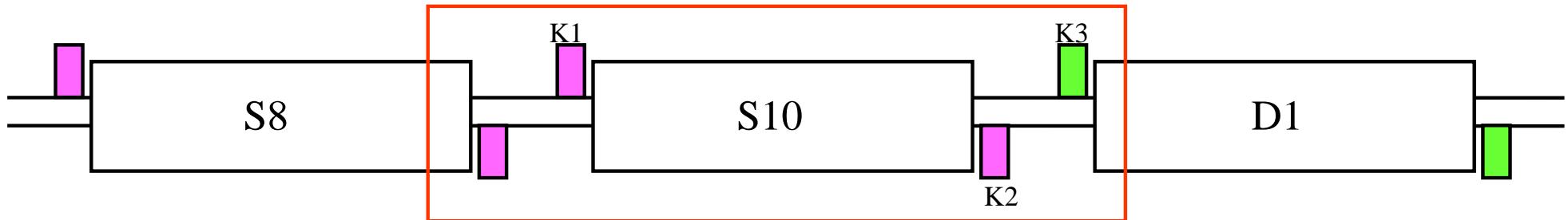
inhomogeneous modules  
(e.g. module 1; cavity S8/S10/D1)

SACLAY coupler - vertical

DESY coupler - horizontal

wrong polarization of coupler fields  
cancellation of coupling fields

**S10:  $\Delta f \approx 10$  MHz  $\Rightarrow$  trapped mode**

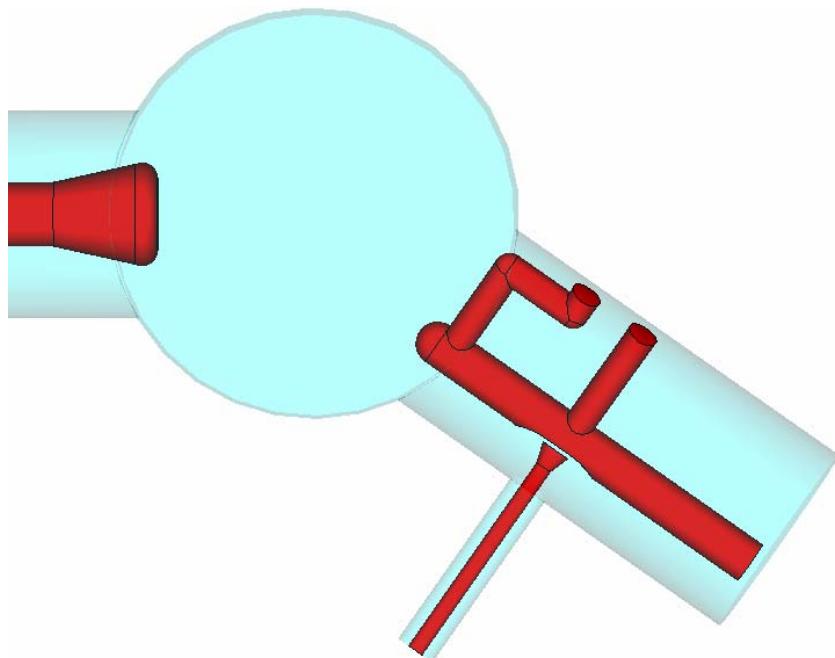


from TESLA meeting Saclay Apr. 2002

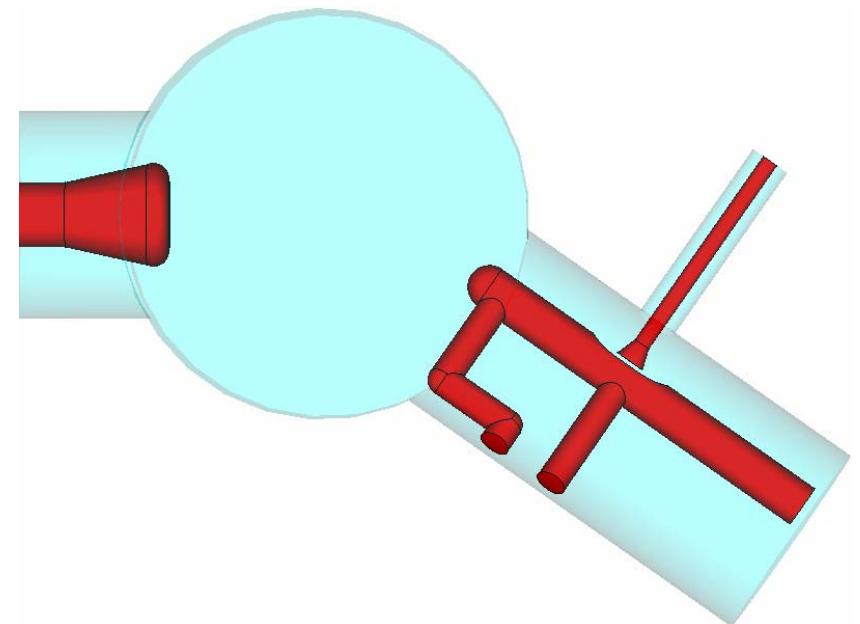


**above  $f_c$ : 3<sup>rd</sup> dipole band  
modified downstream coupler**

old HOM coupler  
orientation



new HOM coupler  
orientation



## questions / problems

**what determines the polarization of dipole modes ?**

random geometry errors  $\leftrightarrow$  asymmetries by design

**what types of polarizations ?**

linear / elliptical / circular

is there an orthogonality (of modes that are related  
to the same rz-mode)

**spread of mode properties** (same mode, different cavities)

e.g. dipole bands 1 & 2:

$\Delta Q \sim 1$  order of magnitude

$\Delta f \sim 5$  MHz

**modes above fc**

what modes have to be considered?

how to treat them? (how many cavities or modules are involved?)

**trapped modes**

