Subtractions for NLO and NNLO QCD calculations

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HEP student seminar
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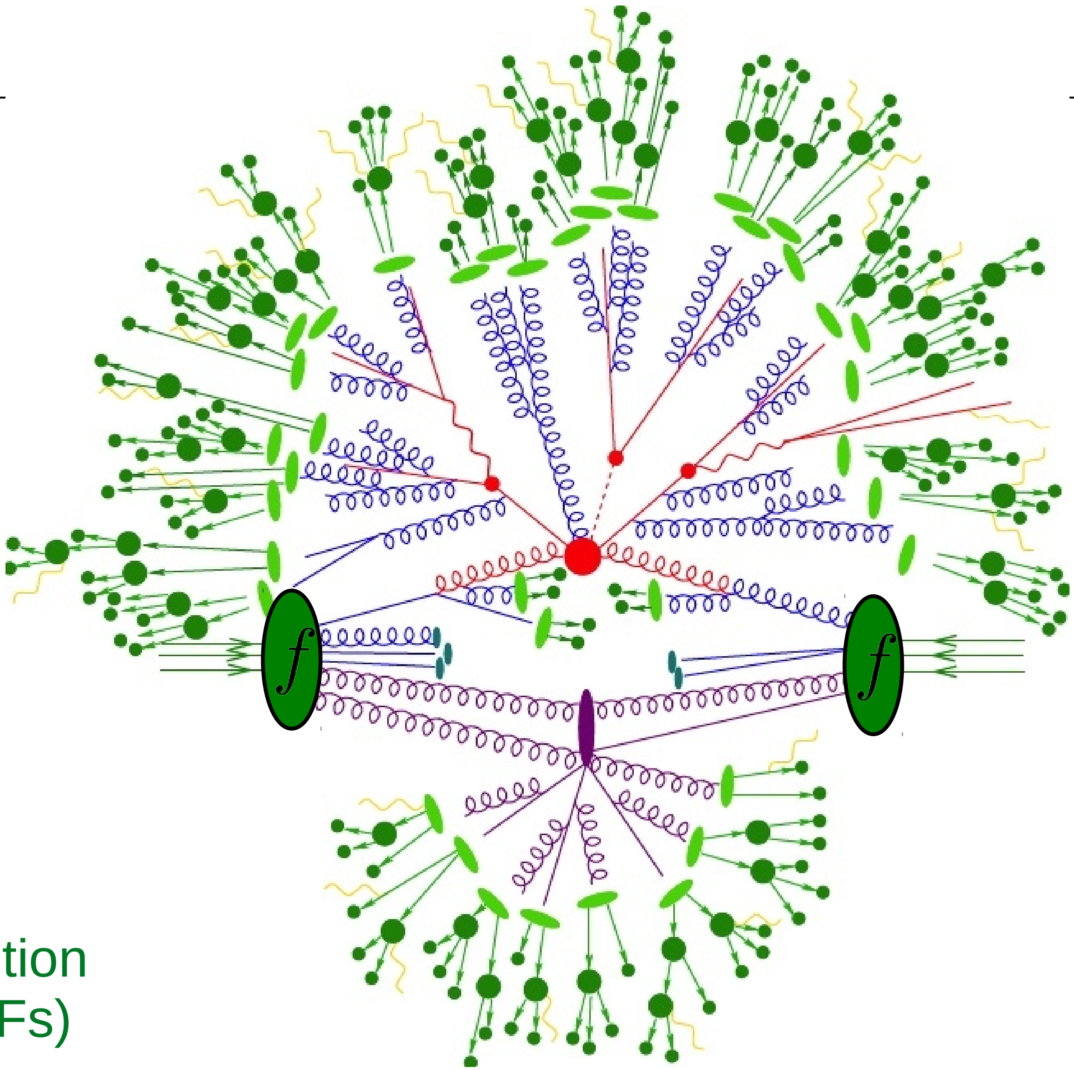
Collider Physics



Multiple scales

- Hard collision
- Proton structure (non-perturbative long distance physics)
- Collinear & soft radiation

All LHC calculations rely on the factorization

Parton distribution functions (PDFs)

$$\sigma = \sum \int dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2)\, \hat{\sigma}(\alpha_S(\mu_R), \mu_R, \mu_F)$$

I want to focus on the partonic cross section.
(Hard scattering, jet algorithm, parton shower, hadronisation model)

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Partonic cross section

For the partonic cross section we can use perturbative QCD

$$\hat{\sigma} \sim \alpha_S^n \left(\sigma_{\mathrm{LO}} + \alpha_S \, \sigma_{\mathrm{NLO}} + \alpha_S^2 \, \sigma_{\mathrm{NNLO}} + \ldots \right)$$

Theoretical Uncertainty:

- Series is truncated and $\alpha_S(M_Z) \sim 0.12$ is not that small
- Estimate of higher orders through scale variation (renormalization/factorization scale)
- Systematically improvable by adding higher orders
- Improvable by resumming large logs (not this talk)

NLO QCD is currently state of the art

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Motivation for NNLO

- Precise predictions needed for
  - Strong coupling constant ($e^+e^- \to 3\,\mathrm{jets}$)
  - PDF (dijet cross section for gluon PDF)
  - For various processes:
    - NLO correction large (e.g. Higgs at hadron colliders)
    - Uncertainty bands large at NLO
    - New channels can appear at NNLO
    - NLO effectively LO

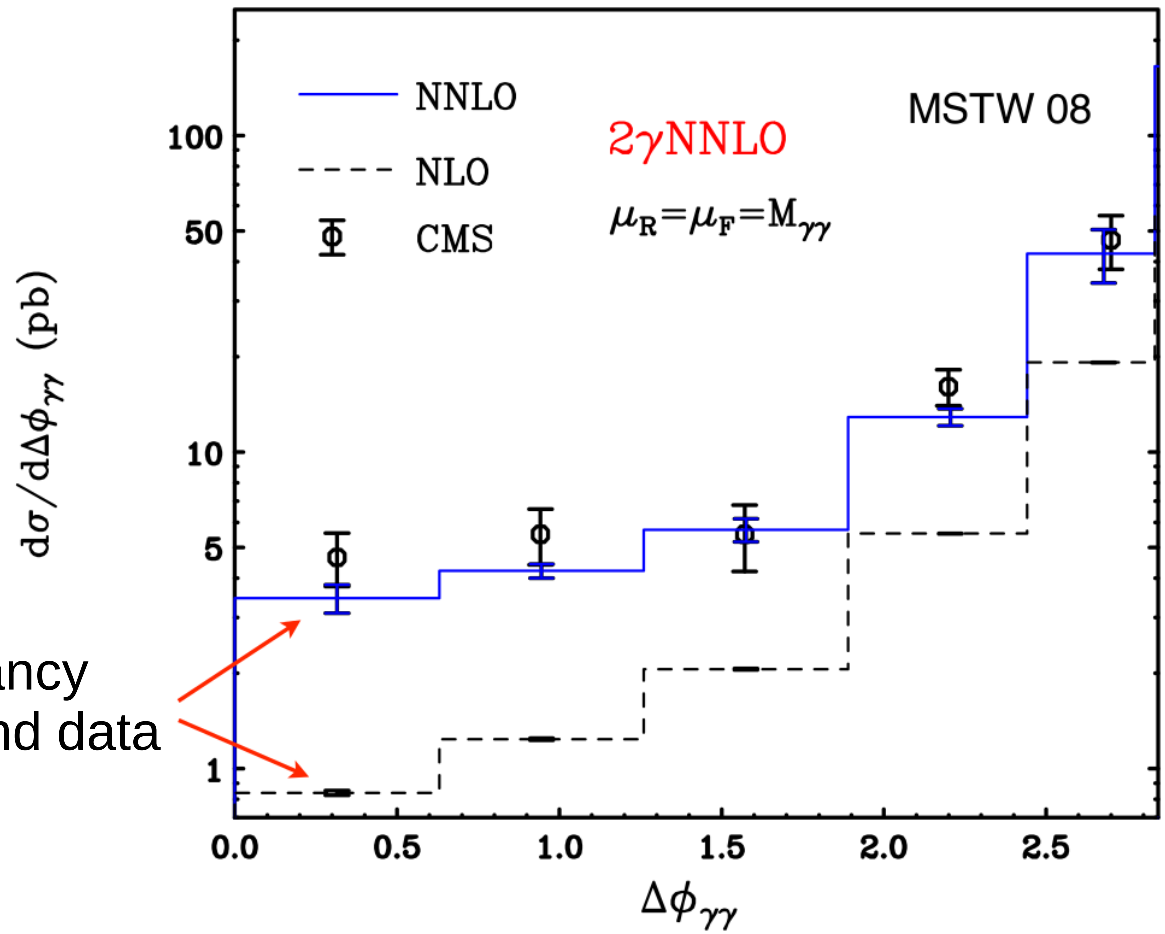
      e.g. jet structures like energy distribution in jet cone
- LHC run II: Experimental precision challenges theory
  - Z production lepton pair p_T spectrum error < 1%
  - A lot of beyond NLO @ QCD (not yet beyond SM)

→ NNLO should become standard

Motivation for NNLO

- NLO effectively LO e.g. opening angle in diphoton
  - At LO back to back, distribution starts at NLO
  - New channel opens at NNLO

large discrepancy between NLO and data

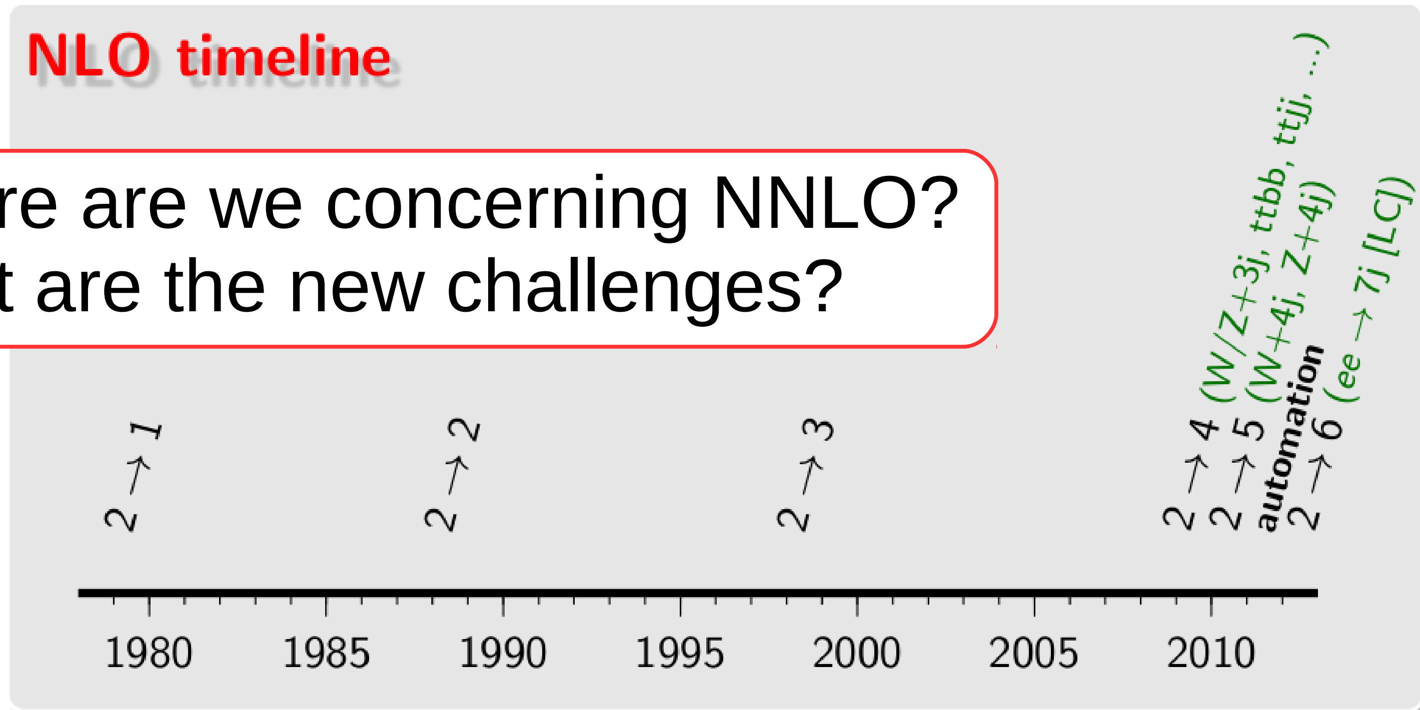


Catani et al. 1110.2375

NLO Revolution

- NLO calculations have progressed significantly around 2010 (+automation)

from Gavin Salam's talk 2012



Where are we concerning NNLO?
What are the new challenges?

2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al] [unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al] [unitarity]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al] [unitarity]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al] [unitarity]
2011: first automation [MadNLO: Hirschi et al] [unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al] [unitarity]
2011: first automation [GoSam: Cullen et al] [feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour] [numerical loops]

NLO Revolution

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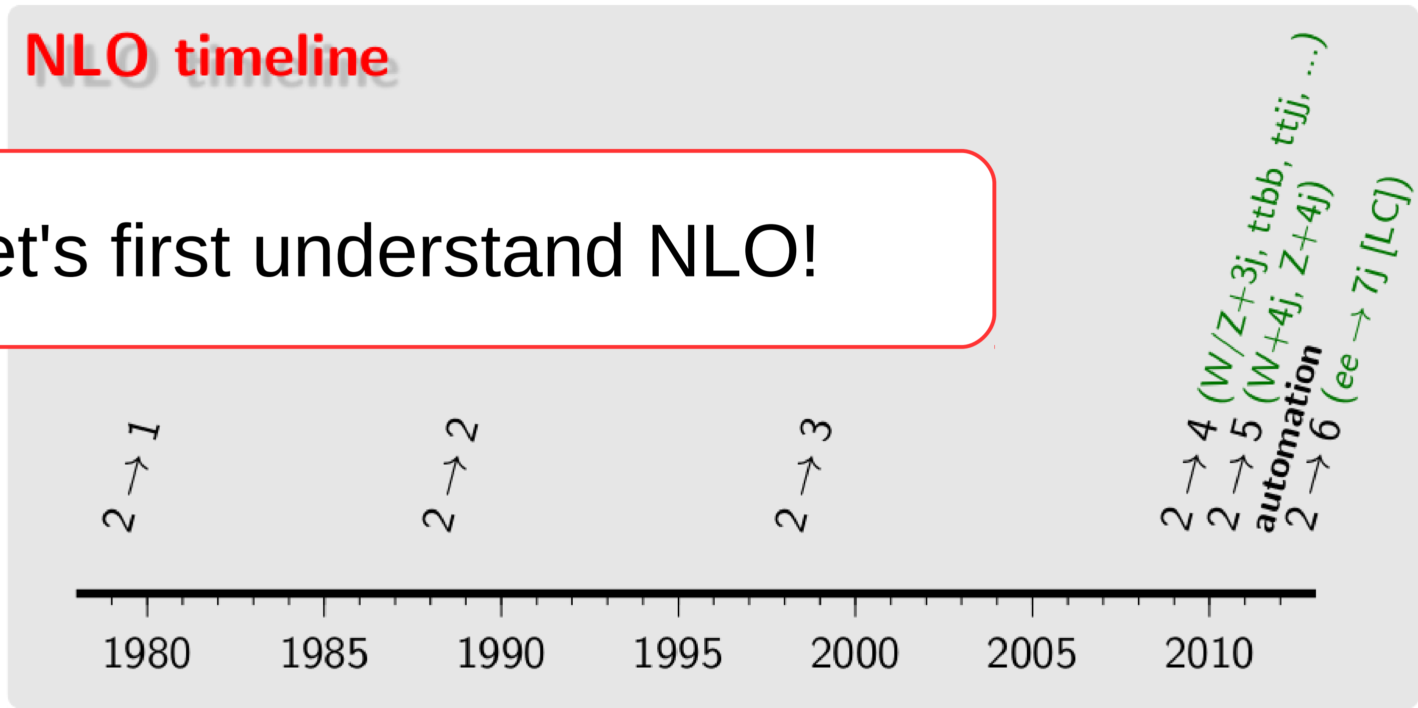
Let's first understand NLO!



2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al] [unitarity]
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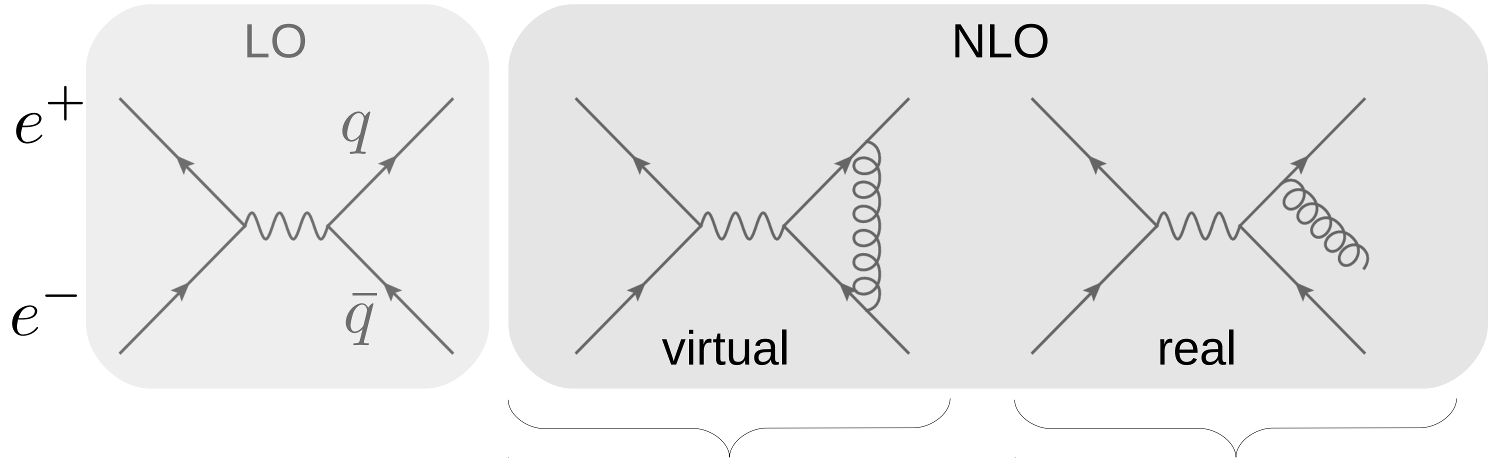
QCD at higher orders

At higher order IR divergences appear. Simple example $e^+e^- \to \text{jets}$



$$\sigma_0 3Q_q^2 \frac{C_F \alpha_s}{2\pi}\left[-\frac{2}{\epsilon^2}+\ldots\right] \qquad \sim \int \frac{dE}{E}\frac{d\theta}{1-\cos\theta}$$

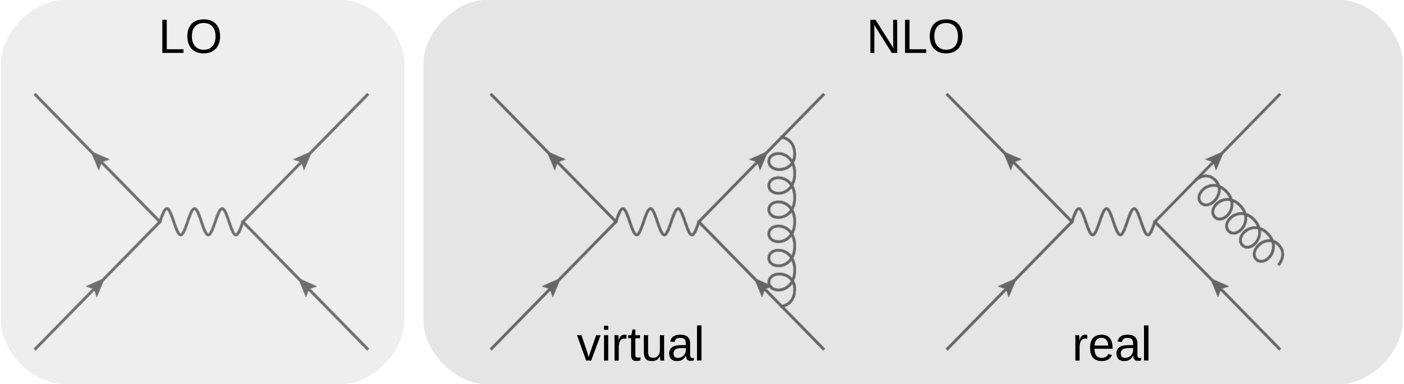
Explicit divergences from loop integral

Soft & Collinear divergence from unresolved radiation. Implicit divergences in phase space.

Sum is finite for IR safe observable… but in the presence of phase space singularities 4 dim. Monte Carlo integration not possible.

QCD @ NLO

IR divergences in $e^+e^- \to \text{jets}$



Generic N-jet cross section:

$$\sigma_{\text{LO}} = \int d\Phi_N B_N(\Phi_N) X_N(\Phi_N)$$

N-parton Born phase space

N-parton matrix element (squared)

Born level cuts (jet definition, rapidity and lepton cuts,...)

$$\sigma_{\text{NLO}} = \int d\Phi_N V_N(\Phi_N) X_N(\Phi_N) + \int d\Phi_{N+1} R_{N+1}(\Phi_{N+1}) X_N(\Phi_{N+1})$$

Explicit IR divergences

Implicit IR divergences

Implicit divergence in real are problematic since we would like to use numerical integration methods → subtractions

NLO Subtraction

$$(X_N(\Phi_{N+1} \to \Phi_N) \longrightarrow X_N(\Phi_N))$$

Add and subtract a term:

$$\sigma_{\mathrm{NLO}} = \int d\Phi_N \, V_N(\Phi_N) X_N(\Phi_N) + \int d\Phi_{N+1} \, R_{N+1}(\Phi_{N+1}) X_N(\Phi_{N+1})$$
$$+ \int d\Phi_{N+1} S_{N+1}(\Phi_{N+1}) X_N(\Phi_N) - \int d\Phi_{N+1} S_{N+1}(\Phi_{N+1}) X_N(\Phi_N)$$

reshuffle:

1/ε cancellation

$$\sigma_{\mathrm{NLO}} = \int d\Phi_N \left(V_N(\Phi_N) + \int d\Phi_1 S_{N+1} \right) X_N(\Phi_N)$$
$$+ \int d\Phi_{N+1} \left(R_{N+1}(\Phi_{N+1}) X_N(\Phi_{N+1}) - S_{N+1}(\Phi_{N+1}) X_N(\Phi_N) \right)$$

Numerically integrable in d=4

NLO Subtraction

$$\sigma_{\rm NLO} = \overbrace{\int d\Phi_N \left(V_N(\Phi_N) + \int d\Phi_1 S_{N+1} \right) X_N(\Phi_N)}^{1/\varepsilon\text{ cancellation}}$$

$$+ \int d\Phi_{N+1} \underbrace{\left(R_{N+1}(\Phi_{N+1}) X_N(\Phi_{N+1}) - S_{N+1}(\Phi_{N+1}) X_N(\Phi_N) \right)}_{\text{Numerically integrable in d=4}}$$

Subtraction term

- reproduces all IR singular limits of real emission (@NLO: one soft + one collinear divergence)
- Need to be able to integrate the subtractions analytically in d=4-2ε.

This is a LOCAL subtraction scheme (match IR point-by-point in phase space)

NLO Subtraction

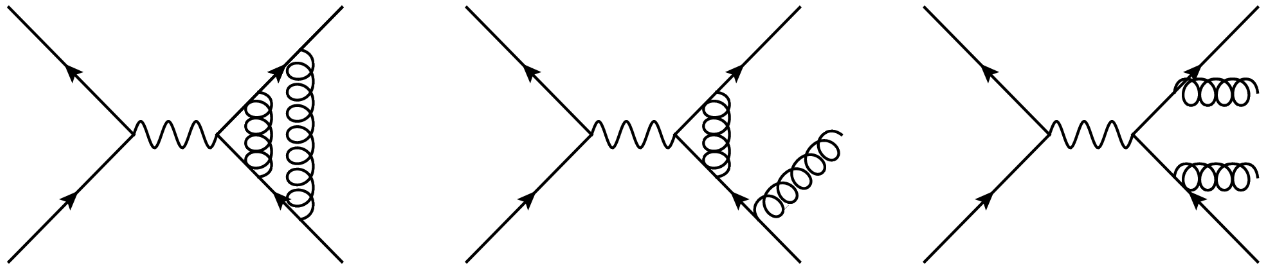
$$\sigma_{\mathrm{NLO}} = \overbrace{\int d\Phi_N \left(V_N(\Phi_N) + \int d\Phi_1 S_{N+1} \right) X_N(\Phi_N)}^{1/\varepsilon \text{ cancellation}}$$

$$+ \int d\Phi_{N+1} \underbrace{\left(R_{N+1}(\Phi_{N+1}) X_N(\Phi_{N+1}) - S_{N+1}(\Phi_{N+1}) X_N(\Phi_N) \right)}_{\text{Numerically integrable in d=4}}$$

@NLO: Has been solved generically (observable independent)

- Unique procedure $S_{N+1} \sim B_N(\Phi_N) \underbrace{CS(\Phi_1)}_{\text{universal factor}}$
- Dipole subtraction
  [Catani, Seymour 1996]
- FKS subtraction [Frixione, Kunszt, Signer 1996]
  – Used in aMC@NLO

QCD @ NNLO



Virtual-Virtual   Real-Virtual   Real-Real

$$\sigma_{\mathrm{NNLO}} = \int_{d\Phi_N} d\sigma^{\mathrm{VV}}_{\mathrm{NNLO}} + \int_{d\Phi_{N+1}} d\sigma^{\mathrm{VR}}_{\mathrm{NNLO}} + \int_{\Phi_{N+2}} d\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}$$

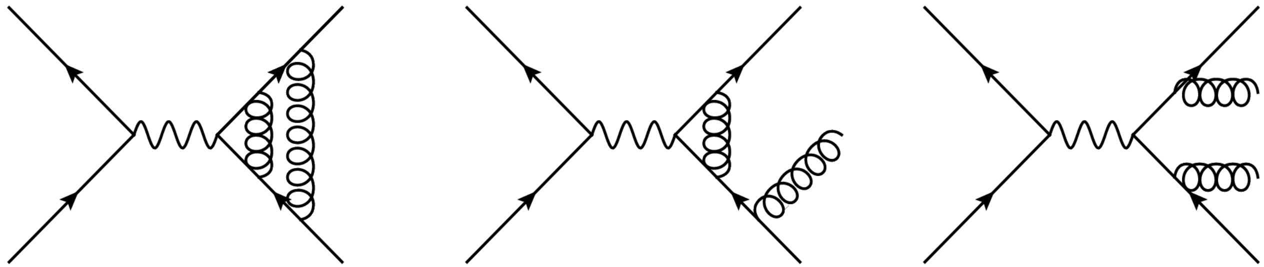
Explicit poles from loop.

Two loop amplitudes are far from automation and only a limited set is known.

Explicit IR poles from loop and implicit soft and collinear divergences from the emission

Double unresolved radiation

QCD @ NNLO



Virtual-Virtual Real-Virtual Real-Real

$$\sigma_{\mathrm{NNLO}} = \int_{d\Phi_N} d\sigma^{\mathrm{VV}}_{\mathrm{NNLO}} + \int_{d\Phi_{N+1}} d\sigma^{\mathrm{VR}}_{\mathrm{NNLO}} + \int_{\Phi_{N+2}} d\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}$$

For a local subtraction method proceed as in NLO

- Major difficulty: singular limits of the real-real radiation

  overlapping and nested singularities

- Built up from scratch, difficult to recycle known NLO results

Need systematic technique to handle singularities
(no general NNLO IR cancellation scheme yet)

Despite challenges various schemes on the market

Local Subtractions at NNLO

Every scheme has advantages and disadvantages

- Sector decomposition [Czakon, Mitov, Binoth, Heinrich 2000] [Anastasiou, Melnikov, Petriello 2004]
  - Higgs, W, Z (2005, 2006)
- Antenna Subtraction [Gehrmann, Glover et al; Weinzierl, 2005]
- STRIPPER, Residue Subtraction [Czakon 2010] [Boughezal, Melnikov, Petriello 2011]
- CoLorFulNNLO [Del Duca, Somogyi, Trocsanyi 2005]
- Projection to Born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi 2015]

	Analytic	FS color	IS color	Azimuthal	Approach
Antenna	☺	☺	☺	x	Subtraction
STRIPPER	x	☺	☺	☺	Subtraction
Colorful	☺	☺	x	☺	Subtraction
P2B	☺	☺	☺	-	Subtraction
q_T	☺	x	☺	-	Global (slicing)
N-Jettiness	☺	☺	☺	-	Global (slicing)

Global Subtractions

Examples of global subtraction schemes (slicing)

- q_T subtraction [Catani, Grazzini 2007]
  - Only for color-single final state
  - Successfully applied to essentially all diboson processes
- N-jettiness [Stewart, Tackmann, Waalewijn 2010; Gaunt, Stahlhofen, Tackmann, Walsh 2015] [Boughezal, Focke, Liu, Petriello 2015]
  - More generic than q_T, catches all IR behaviors also for jet processes (universal).

Advantages (to local subtraction schemes)

- Possible to recycle existing NLO and resummation tools
  - Singular limit well studied (resummation in limits where large logs spoil convergence) → use known factorization theorems
  - Systematically organizes calculation (poles)

Responsible for significant recent progress (Dibosons, Boson+Jet)

Global Subtractions

- Phase Space slicing:

  Split into two regions

$$\sigma(X) \equiv \int d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \underbrace{\int^{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}}_{\sigma(\mathcal{T}_{\rm cut})} + \underbrace{\int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}}$$

Born-level cuts and measurements

- Singular NNLO piece
- No additional radiation resolved
- VV & double unresolved limit of RV & RR

- NLO for Born +1 jet
- free of singularities
- Resolved & single unresolved of RV & RR
- Numerically integrable

- Resolution parameter (here $\mathcal{T}_N$)
  - Physical IR safe variable and analytically integrable for $\mathcal{T}_N < \mathcal{T}_{\rm cut}$
  - must separate double unresolved limit from NLO singular limits.

Global Subtractions

Subtraction such that analytic integration is possible:

$$\sigma(X) \equiv \int d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \sigma(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

$$= \sigma^{\rm sub}(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \underbrace{\left(\sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})\right)}_{\Delta\sigma(\mathcal{T}_{\rm cut})}$$

- $\sigma^{\rm sub}(\mathcal{T}_{\rm cut})$
  - Must reproduce singular limit

    $$\sigma^{\rm sub}(\mathcal{T}_{\rm cut}) = \sigma^{\rm sing}(\mathcal{T}_{\rm cut})[1 + \mathcal{O}(\mathcal{T}_{\rm cut})] \qquad \Delta\sigma(\mathcal{T}_{\rm cut}) \xrightarrow{\mathcal{T}_{\rm cut} \to 0} 0$$

  - The cut must be sufficiently small to be able to neglect $\Delta\sigma(\mathcal{T}_{\rm cut})$
    - However, N+1-jet NLO quickly becomes much less stable (computational cost increases substantially)
  - Must be explicitly calculable
    - Just take singular limit of cross section for small $\mathcal{T}_N$

Global Subtractions

Subtraction such that analytic integration is possible:

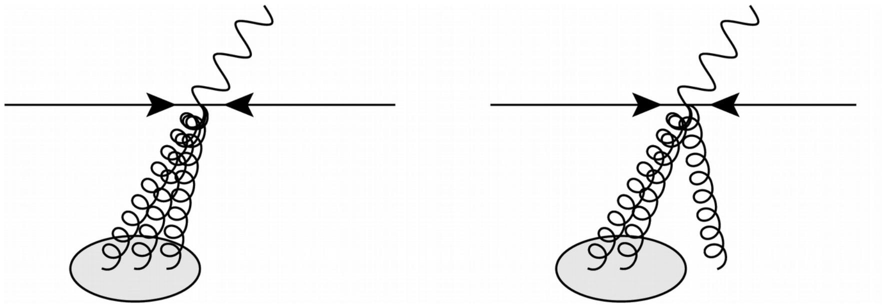
$$\sigma(X) \equiv \int d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \sigma(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

$$= \underbrace{\sigma^{\rm sub}(\mathcal{T}_{\rm cut})}_{\sigma^{\rm sing}(\mathcal{T}_{\rm cut})} + \int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \Delta\sigma(\mathcal{T}_{\rm cut})$$

- Recycle existing NLO and resummation tools
  - Singular limit well studies (resummation in limits where large logs spoil convergence)
  - Use known factorization theorems
  - Neglected contribution is of $\mathcal{O}(\mathcal{T}_{\rm cut})$
    - However suppression weakens with higher order due to logarithmic enhancement
    - Improvement by adding power corrections (this talk, later)

Global Subtractions

- q_T subtraction: [Catani, Grazzini 2007]
  - Resolution variable: boson transverse momentum q_T
  - Successfully applied to essentially all diboson processes
  - Only for color-single final state:
    - Does not resolve double unresolved from NLO singular limit



Same q_T

- N-Jettiness [Stewart, Tackmann, Waalewijn 2010; Gaunt, Stahlhofen, Tackmann, Walsh 2015] [Boughezal, Focke, Liu, Petriello 2015]
  - More generic variable than q_T, catches all IR behaviors also for jet processes (universal).
  - All ingredients for NNLO of most V+1 jet processes available.

N-Jettiness Definition

- Event shape defined for N-jet observables

N+2 massless jet axes (two beams for hadron colliders)

Final-state partons

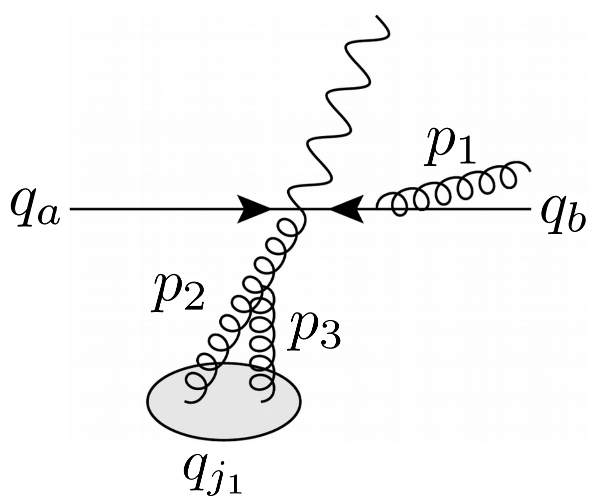
$$\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2 q_i \cdot p_k}{Q_i} \right\}$$

Normalization factor
e.g. $Q_i = Q$ invariant mass
$Q_i = p_i E_i$ geometric measure

- Construct q_i by using IR safe jet algorithm

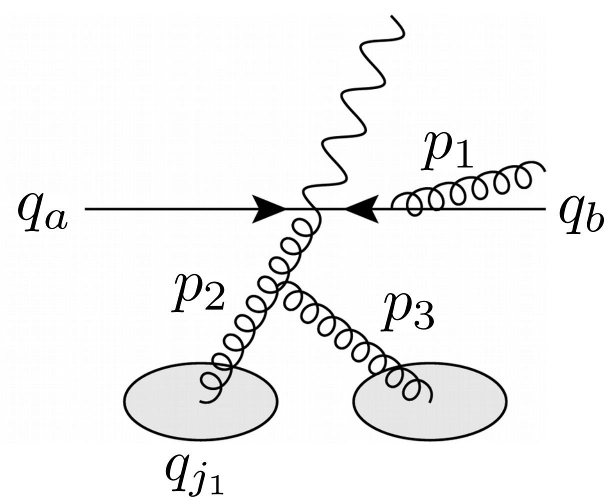
$\mathcal{T}_N \to 0$

Radiation soft or collinear



$\mathcal{T}_N > 0$

+1 additional jet (radiation is resolved)



N-jettiness Subtractions

$$\sigma = \sigma^{\rm sing}(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N} + \mathcal{O}(\mathcal{T}_{\rm cut})$$

- Need the singular piece! Use factorization formula for N-jettiness from SCET! [Stewart, Tackmann, Waalewijn 2010]

Jet functions

$$\sigma^{\rm sing}(\mathcal{T}_N < \mathcal{T}_N^{\rm cut}) = \int H \otimes B \otimes B \otimes \prod_n^N J_n \otimes S$$

Hard Wilson coefficient
2-loop virtual
(color space matrices)

Beam functions
collinear ISR

Soft function
describes soft radiation

- Each function IR finite (operator definition in SCET)
- To obtain the fixed order singular piece just expand and collect terms
  - For NNLO each ingredient at two loops needed
  - Last years: essentially all ingredients for 2→2 at LHC available!

Factorization Formula for N-jettiness

$$\sigma^{\rm sing}(\mathcal{T}_N < \mathcal{T}_N^{\rm cut}) = \int H \otimes B \otimes B \otimes \prod_n^N J_n \otimes S$$

Ingredients at fixed order NNLO

- Hard function W/H+jet: [Gehrmann, Tancredi; + Jaquier, Glover, Koukoutsakis 2011]
  - Only process dependent piece
    - Some known, many not
- Beam functions NLO: [Stewart, Tackmann, Waalewijn 2009, 2010] NNLO: [Gaunt, Stahlhofen, Tackmann 2014]
  - Universal for any N (depends only on parton flavor)
  - Requires matching onto PDFs
- Jet functions NNLO: [Becher, Neubert 2006; Becher, Bell 2010]
- Soft function (generally color space matrix)
  - 2 partons: [Kelley, Schartz, Schabinger, Zhu 2011; Monni, Gehrmann, Luisoni 2011] [Hornig, Lee, Walsh, Zuberi 2011; Kang, Labun, Lee 2015]
  - 3 partons: pp → V+1 jet [Boughezal, Liu, Petriello 2015]
  - Unknown for general N