AKK update: AKK08 fragmentation functions

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Outline

1. Single hadron inclusive production and pQCD
   - Introduction
   - Measurements of single hadron inclusive production
   - Outline of factorization
2. FFs at large x
   - Cross section in the fixed order approach
   - Global fits
   - Predictions using FFs from e^+e^-
   - Theoretical improvements
   - Latest global fit: AKK08 (Nucl. Phys. B)
3. Improving the small x region
   - Soft gluon logarithms in splitting functions
   - Unified approach
   - Fits to data

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$\mathrm{XS}(? \rightarrow i + X)$: partonic final state, pQCD at high E_S
$\rightarrow$ series in QCD coupling $a_s(E_S) \sim \frac{1}{\ln E_S/\Lambda_{\rm QCD}}$

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Basic approach

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e^+e^-

$pp(\bar{p})$

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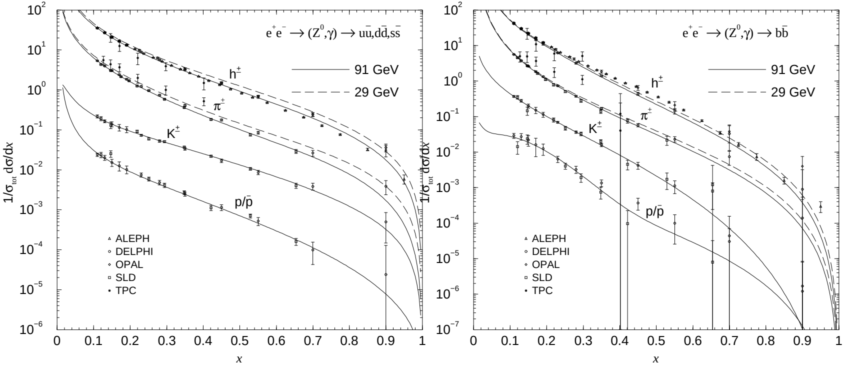
- h^+, h^- separately

Quality of fits



From Albino-Kniehl-Kramer (AKK) fit of 2005
(no unidentified particle data, maybe contaminated with e, μ, ...)

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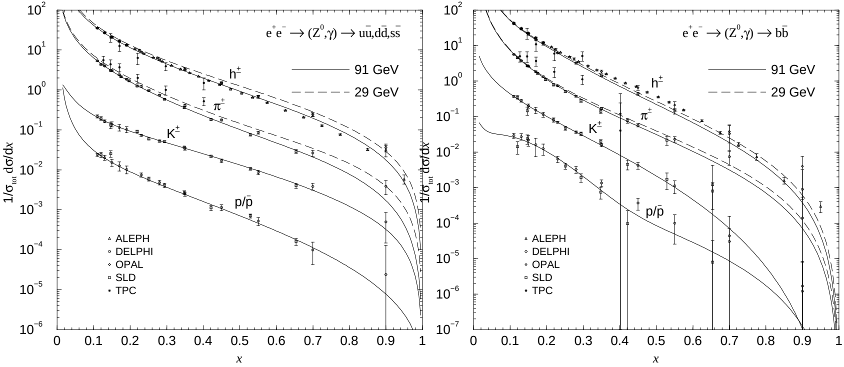

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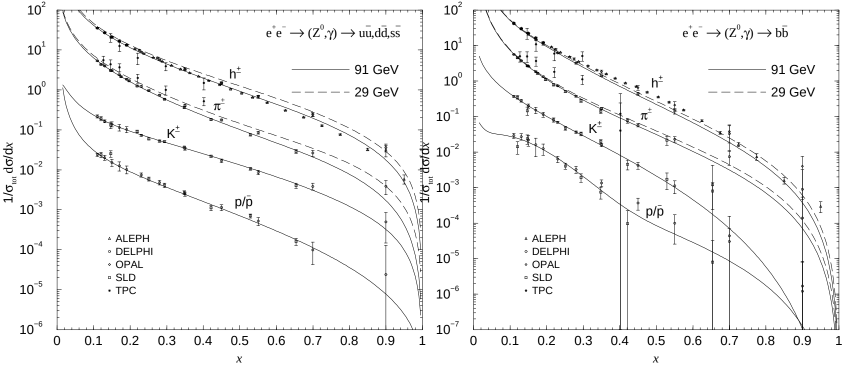

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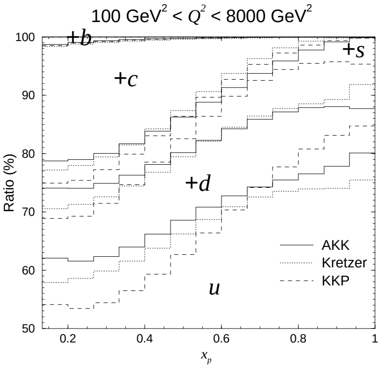
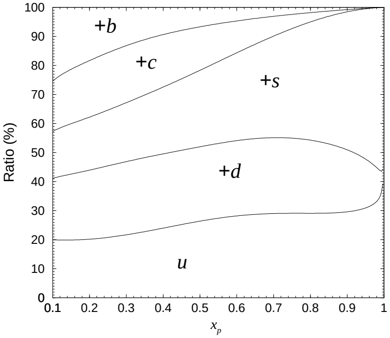
More recently: HKNS, DSS, AKK08

$\gamma^* p \to h + X$ data from HERA

Ratios of quark fragmentation contributions in ? $\to h + X$





ep: High Q @ H1

e^+e^-: $\sqrt{s} = 91$ GeV

(This is for unidentified hadrons $\simeq$ charged pions)

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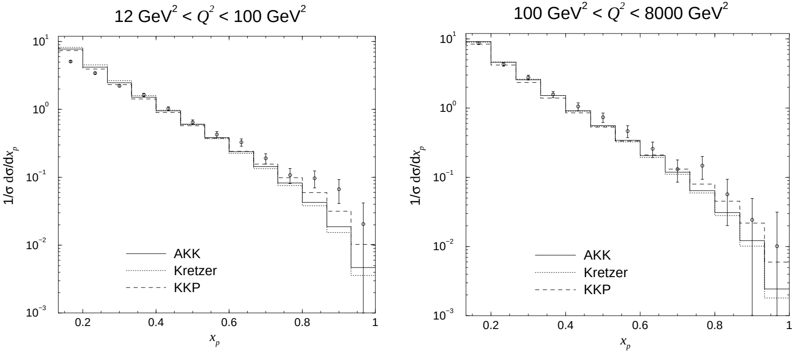
Albino et al. 2007



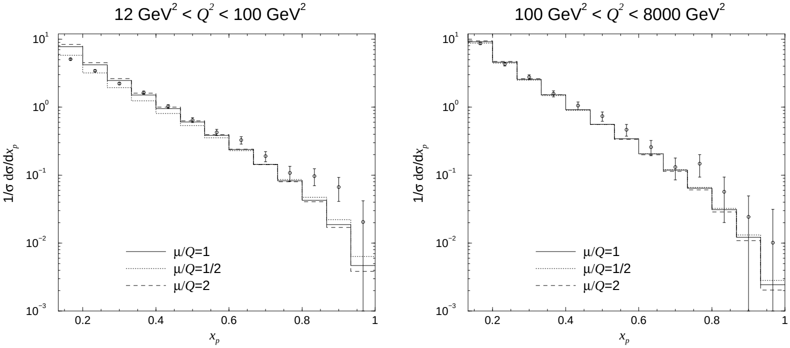
If good constraints on all pion FFs → agreement at large Q,

then disagreement at small Q from theoretical errors

(e.g. large/small x resummation, detected hadron mass, etc.)

$\gamma^* p \to h + X$ data from HERA

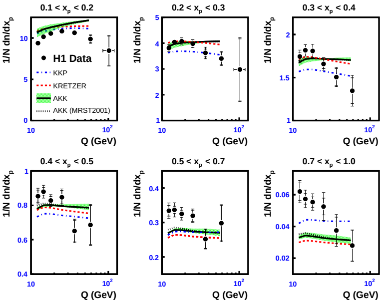
Perturbative errors from scale variation

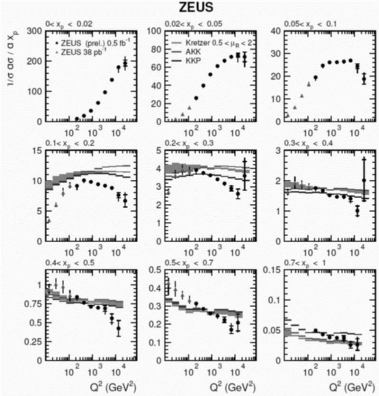


Largest at small Q and small/large x $\rightarrow$ resummation may help

$\gamma^* p \to h + X$ data from HERA

New improved data from HERA





New improved FFs needed

Detected hadron mass effects (Albino et al., 2006-2008)

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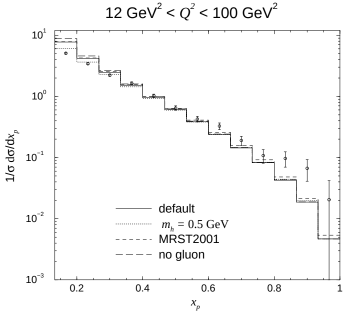

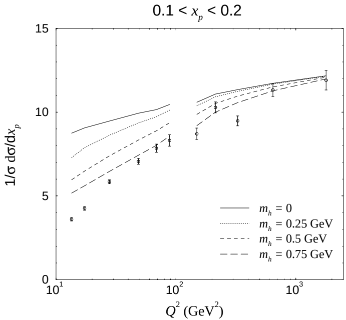

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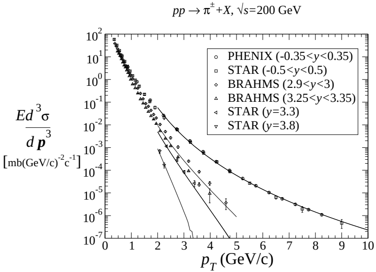
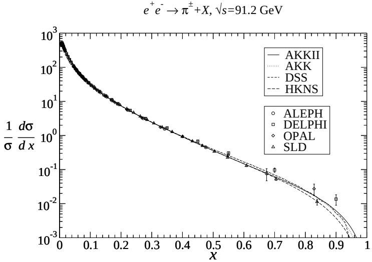
- Albino et al. (2008): resummation in DGLAP evolution
  $\longrightarrow$ modifies theoretical errors

Detected hadron mass effects

"Naive" FO calculation is sufficient





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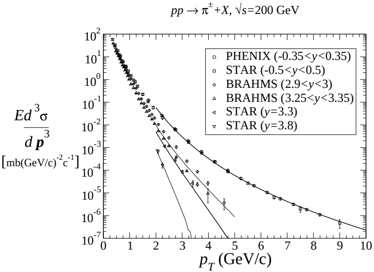

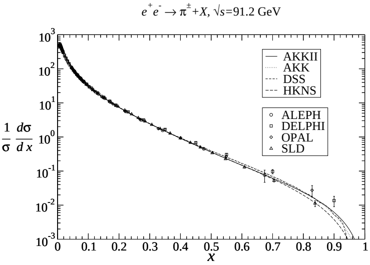

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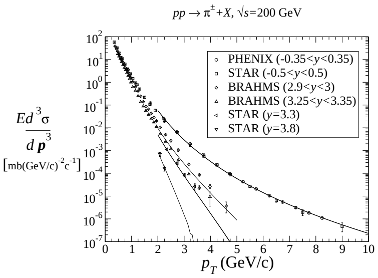

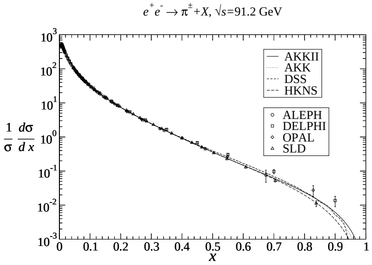

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(Second caveat: Complex decay patterns)

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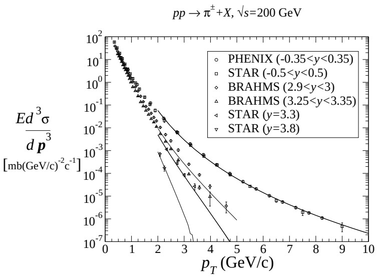

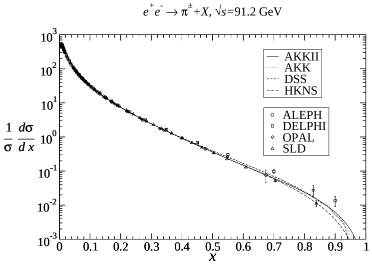

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Fitting m_h subtracts unaccounted-for small x, E_S effects

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AKK08: Fit m_h to e^+e^- data

Particle	Fitted mass (MeV)	True mass (MeV)
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- $\pi^\pm$: overshoot $\longrightarrow$ e.g. $\rho(770) \to \pi^+ + \pi^-$

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$\Lambda/\overline{\Lambda}$	1127.0	1115.7

- $\pi^{\pm}$: overshoot $\longrightarrow$ e.g. $\rho(770) \rightarrow \pi^+ + \pi^-$
- Ks: undershoot $\longrightarrow$ complicated decays ($K^* \rightarrow \pi + K, \ldots$)
  undershoot for $K_S^0 \simeq$ for $K^{\pm} \rightarrow$ SU(2) isospin

Detected hadron mass effects

AKK08: Fit m_h to e^+e^- data

Particle	Fitted mass (MeV)	True mass (MeV)
$\pi^\pm$	154.6	139.6
$K^\pm$	337.0	493.7
$p/\bar{p}$	948.8	938.3
K_S^0	343.0	497.6
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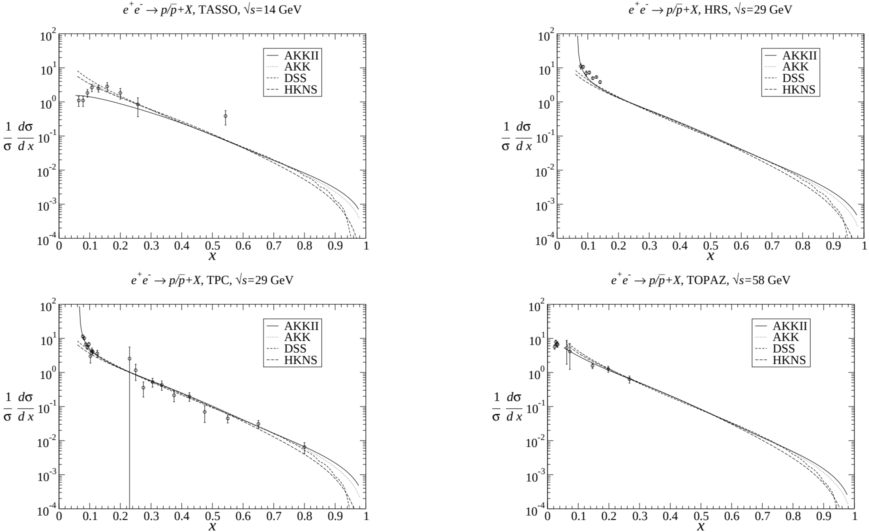
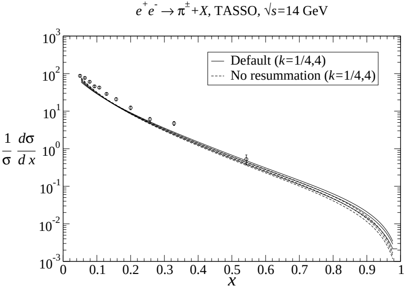
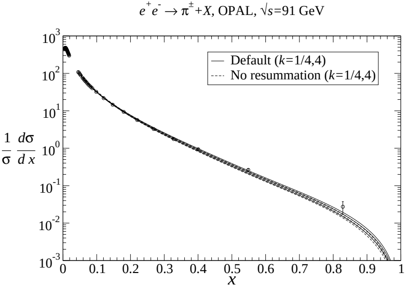
- baryons ($p/\overline{p}$, $\Lambda/\overline{\Lambda}$): $\simeq +1\%$ $\longrightarrow$ resonances slightly heavier
  good environment to study partonic fragmentation

Detected hadron mass effects



Large x resummation





AKK08:

H	χ^2 Main fit	χ^2 Unres. fit
$\pi^{\pm}$	518.7	519.0
$K^{\pm}$	416.6	439.4
$p/\bar{p}$	525.2	538.0
K_S^0	317.2	318.7
$\Lambda/\bar{\Lambda}$	273.1	325.7

Systematic errors

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No systematic effects: $P(\{f_i^e\},\{f_i^t\}) \propto \exp\left[-\frac{1}{2}\chi^2 = \sum_i \left(\frac{f_i^t - f_i^e}{\sigma_i}\right)^2\right]$

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Unknown systematic effects may give $|\lambda_K| \gg 1$

Systematic errors

$\pi^{\pm}$ production data

Expect $|\lambda_K| \simeq 1$

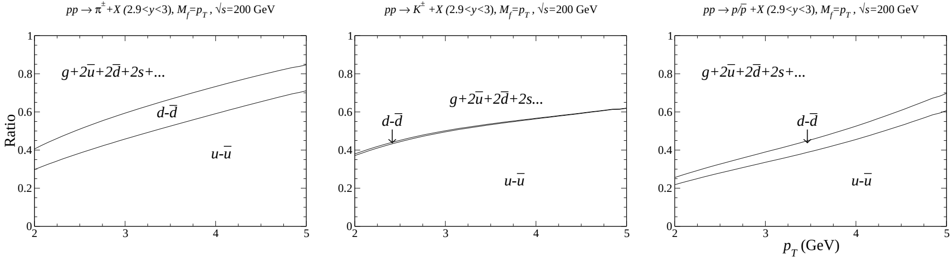
AKK08:

Collaboration	$\sqrt{s}$ (GeV)	# data	Norm. (%)	χ^2_{DF}	λ_K
TASSO	12	5	20	0.50	0.21
TASSO	14	10	8.5	0.92	-1.26
TASSO	22	1	6.3	0.01	-0.08
TASSO	30	4	20	0.57	0.69
TASSO	34	10	6	1.07	0.62
TASSO	44	7	6	1.99	0.66
ALEPH	91.2	22	3	0.61	-0.55
BRAHMS, $y \in [2.9, 3]$	200	8	11,7,8(13),	0.96	-1.76, -1.12, -1.22, -0.32, -0.13
$y \in [3.25, 3.35]$		7	2,1(3)	2.68	-2.01, -1.28, -1.80, -0.37, -0.32
PHENIX (π^0), $\lvert\eta\rvert < 0.35$	200	13	9.7	0.54	-0.48
STAR (π^0), $\eta = 3.3$	200	4	16	0.70	-0.70
STAR (π^0), $\eta = 3.8$	200	2	16	0.57	-0.31
STAR, $\lvert y\rvert < 0.5$	200	10	11.7	0.49	-0.34

$pp \to h + X$ data from RHIC

Ratios of proton's valence/sea quark fragmentation contributions

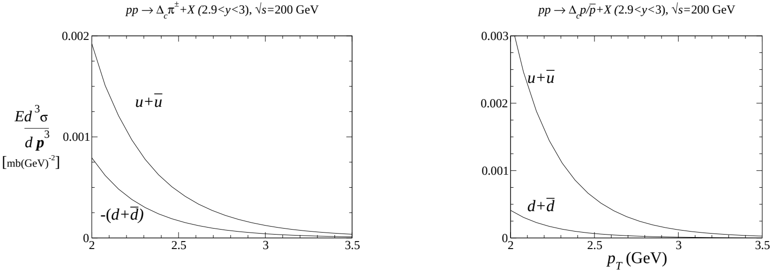
(can be given scheme / scale independent definition)



$pp \to h + X$ data from RHIC

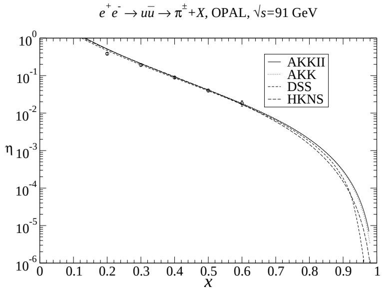
Proton's valence quark fragmentation contributions

to $d\sigma^{h^+} - d\sigma^{h^-}$



Comparison of FF sets

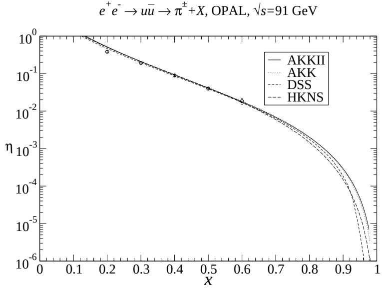
$u \to \pi$ favoured $\therefore$ well constrained by e^+e^-

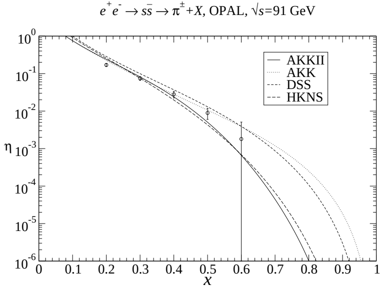


Comparison of FF sets

$u \longrightarrow \pi$ favoured $\therefore$ well constrained by e^+e^-

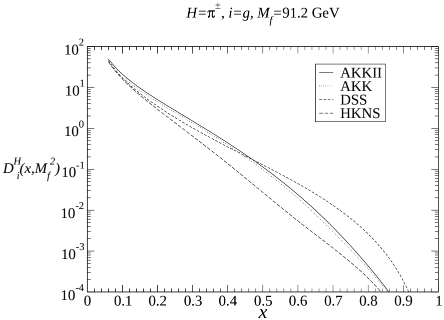
$s \longrightarrow \pi$ unfavoured





Comparison of FF sets

$g \longrightarrow \pi$ constrained mainly by pp



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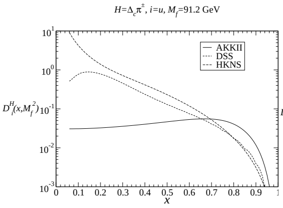
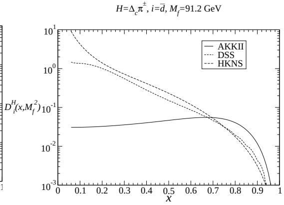
$$D_{u,d}^{h^+} - D_{u,d}^{h^-}$$





HKNS: no $d\sigma^{h^+} - d\sigma^{h^-}$ data, but $\sim$ FF assumptions as DSS

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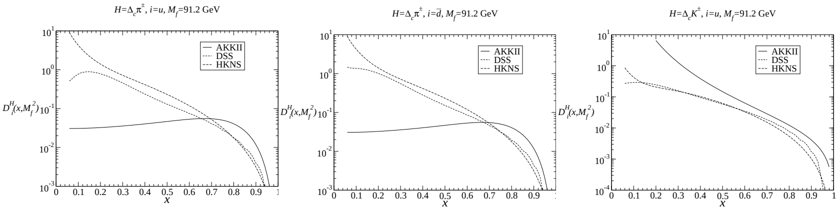

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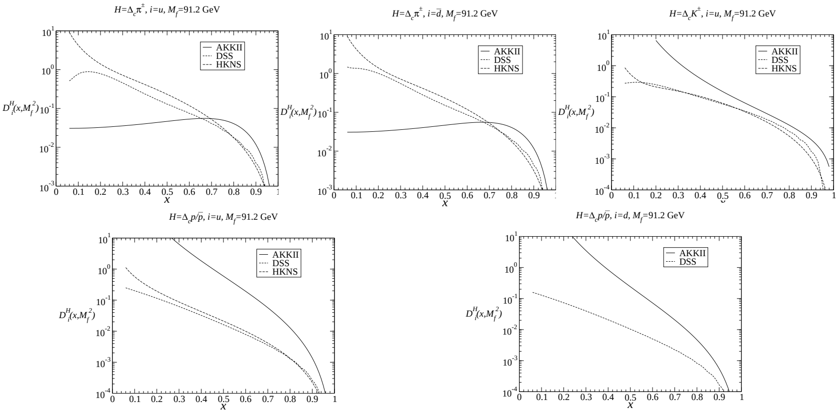
$$D^{h^+}_{u,d} - D^{h^-}_{u,d}$$



BRAHMS $\longrightarrow$ higher FF (HKNS: negative)

HKNS: no $d\sigma^{h^+} - d\sigma^{h^-}$ data, but $\sim$ FF assumptions as DSS

Outline

1. Single hadron inclusive production and pQCD
   - Introduction
   - Measurements of single hadron inclusive production
   - Outline of factorization
2. FFs at large x
   - Cross section in the fixed order approach
   - Global fits
   - Predictions using FFs from e^+e^-
   - Theoretical improvements
   - Latest global fit: AKK08 (Nucl. Phys. B)
3. Improving the small x region
   - Soft gluon logarithms in splitting functions
   - Unified approach
   - Fits to data

Small x: FFs vs. PDFs

Small z: $\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}_{\rm LO\ term} = a_s P^{(0)} = a_s \begin{pmatrix} 0 & \frac{1}{z} \\ 0 & \frac{1}{z} \end{pmatrix}$

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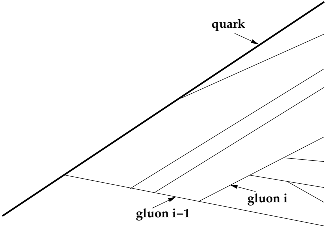
Outline of DLA: Behaviour at small momentum fractions



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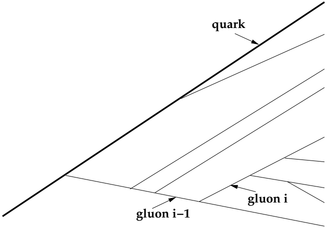

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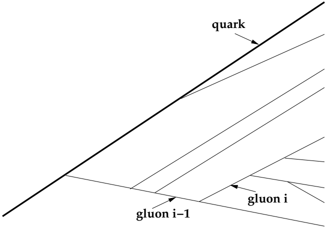

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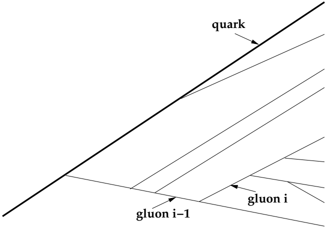

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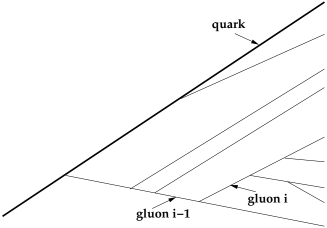

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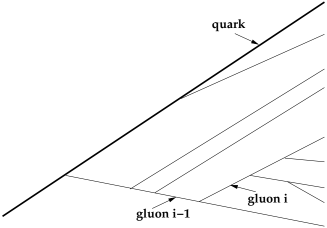

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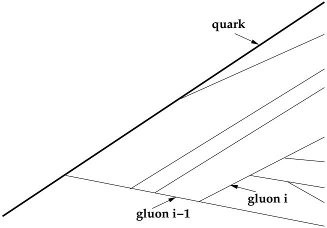

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$\to$ DLA equation, contains all DLs

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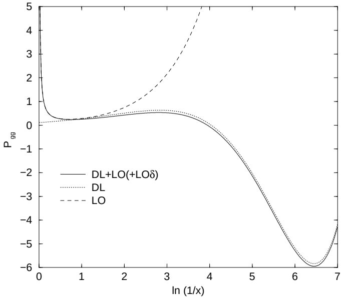

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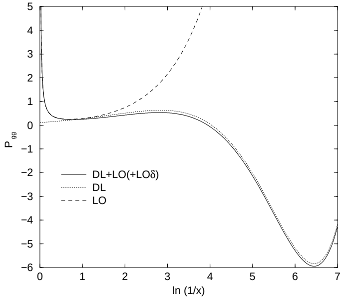

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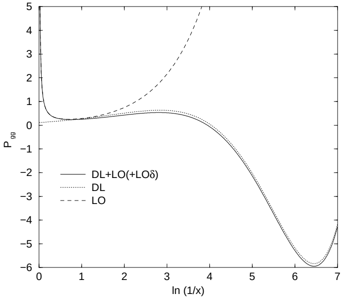

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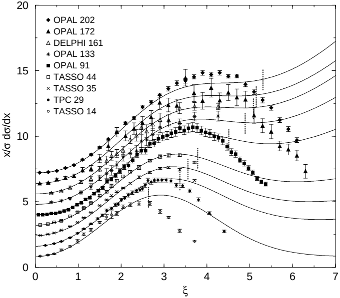
Results

- $P = a_s P^{(0)}$ (old)



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$$\Lambda_{\rm QCD} = 400 \text{ MeV}$$

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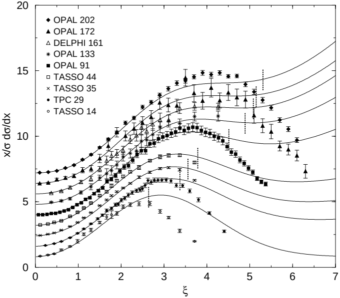

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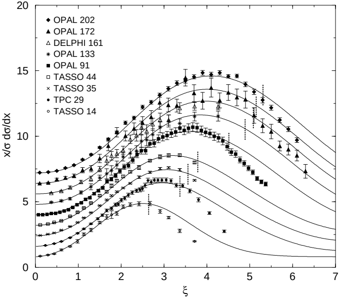

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