Event shapes and jet rates in electron-positron annihilation

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Introduction: **Constants of nature**

I.: **Measurement and extraction of α_s**

II: **Precision calculations**

III: **Numerical results**

Phys. Rev. Lett. 101, (2008), 162001, arxiv:0807.3241;

arxiv:0904.1145, arxiv:0904.1077

Fundamental constants

- The speed of light $c = 299792458\,\text{m s}^{-1}$
- Planck's constant $\hbar = 1.054571628(53) \cdot 10^{-34}\,\text{J s}$
- Constants associated to fundamental forces:
  - Newton's constant $G_N = 6.67428(67) \cdot 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\text{s}^{-2}$
  - Electric charge $e = 1.602176487(40) \cdot 10^{-19}\,\text{C}$
  - Fermi's constant $G_F = 1.16637(1) \cdot 10^{-5}\,\text{GeV}^{-2}(\hbar c)^3$
  - Strong coupling constant $\alpha_s(m_Z) = 0.1176(20)$
- Masses of elementary particles, mixing angles, etc.

In theoretical physics it is common to set $c = \hbar = 1$.

The four fundamental forces

We like dimensionless quantities:

Gravitation:

$$G_N m_p^2 \quad = \quad 5.9 \cdot 10^{-39}$$

Electric force:

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} = 0.007297$$

Weak force:

$$G_F m_W^2 = 0.0754$$

Strong force:

$$\alpha_s = \frac{g_s^2}{4\pi} = 0.118$$

The fine-structure constant

Measurement at $Q^2 = 0$ (Thomson limit of Compton scattering, quantum Hall effect, anomalous magnetic moment of the electron):

$$\alpha = \frac{1}{137}$$

Measurement at $Q^2 = m_Z^2$ (LEP, HERA):

$$\alpha = \frac{1}{128}$$

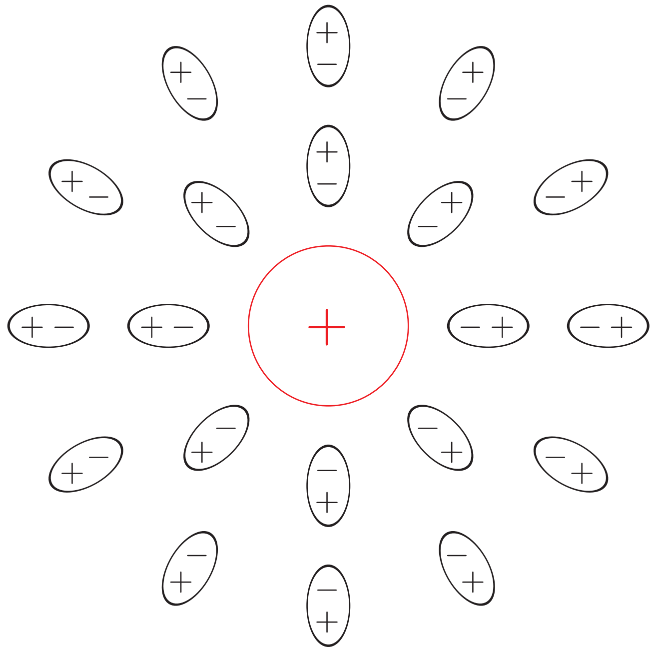
The value is larger at higher scales !

Screening

Qualitative understanding:

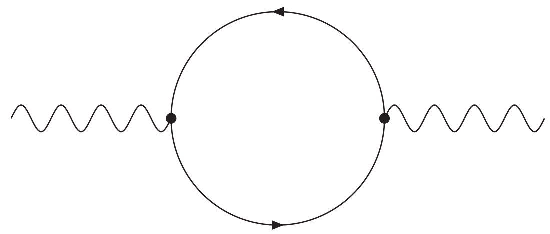
Imagine a bare charge, which is screened by polarization charges.

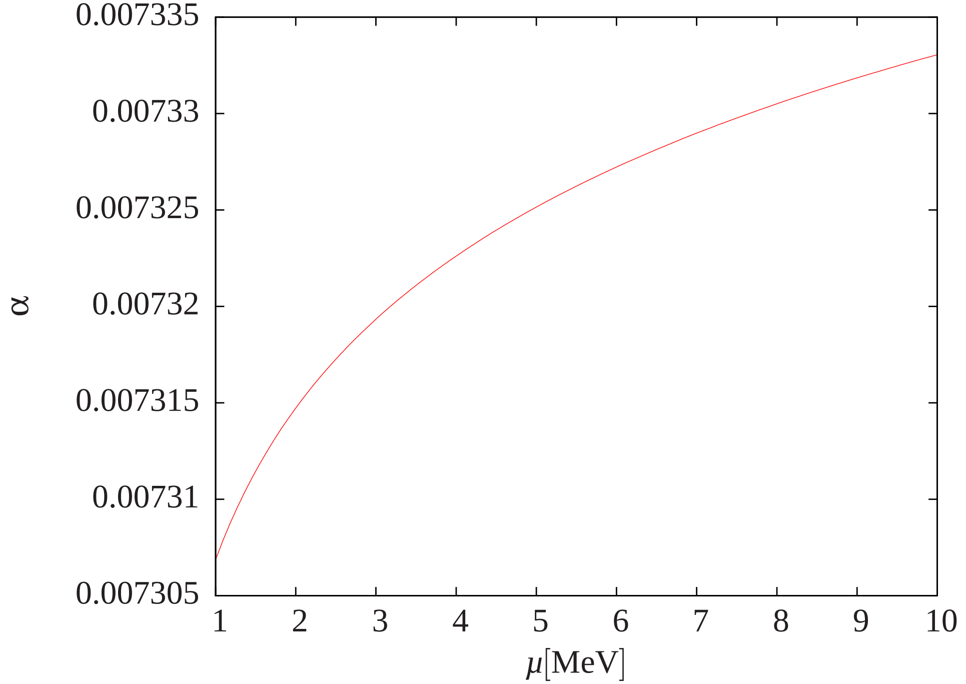
At higher energies we probe shorter distances and screening effects are reduced.



Scale-variation of the electro-magnetic coupling

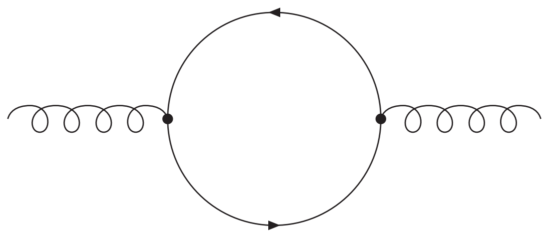
$$\mu^2 \frac{d}{d\mu^2}\left(\frac{\alpha}{4\pi}\right) = \frac{4}{3}\left(\frac{\alpha}{4\pi}\right)^2 + O(\alpha^3)$$



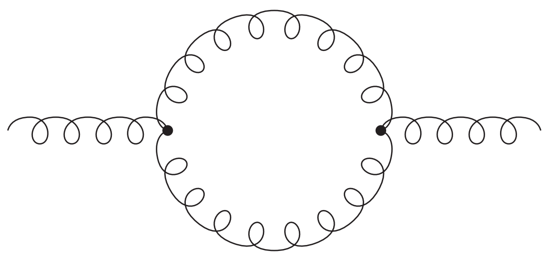


Scale-variation of the strong coupling

$$\mu^2 \frac{d}{d\mu^2}\left(\frac{\alpha_S}{4\pi}\right) = \left(\frac{2}{3}N_f - 11\right)\left(\frac{\alpha_S}{4\pi}\right)^2 + O(\alpha_s^3)$$



The slope is negative !



As before:
Fermion loops give a positive contribution: $2/3N_f$

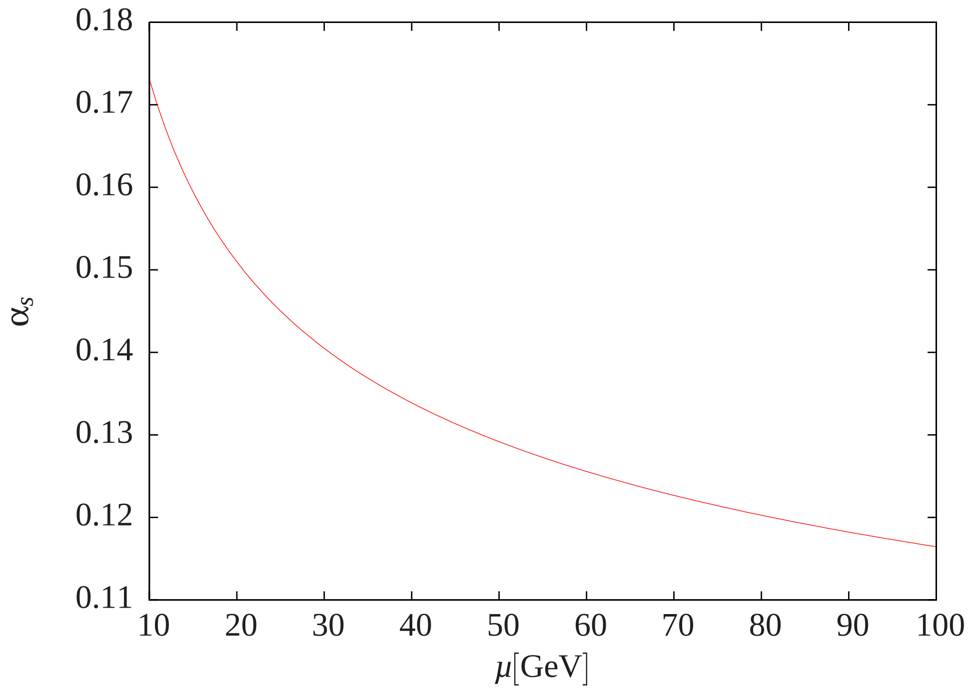
But now:
Boson loops give a negative contribution: -11

For $N_f \leq 16$ the sum is negative !

Gross Wilczek, '73, Politzer, '73

Asymptotic freedom

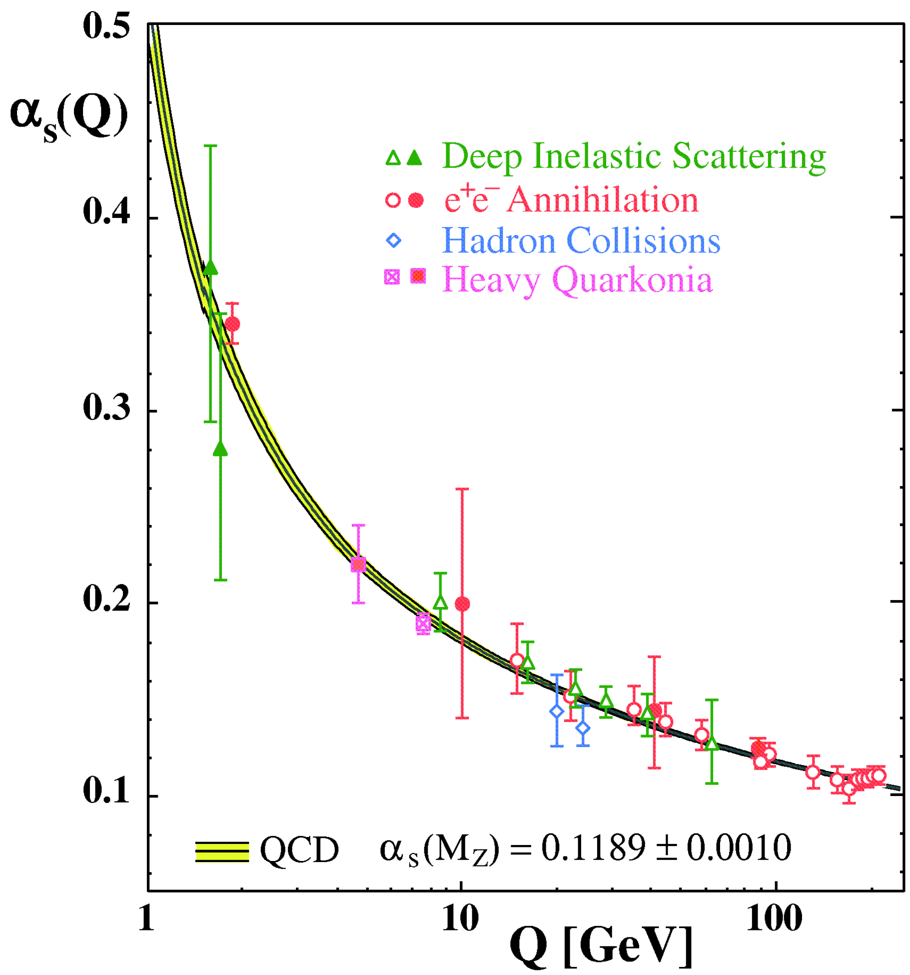
Gross, Politzer and Wilczek: Nobelprize 2004 for the discovery of asymptotic freedom in the theory of the strong interaction.



Measurement of the strong coupling

α_s can be measured in a variety of processes:

- Deep inelastic scattering,
- τ-decays,
- heavy quarkonium,
- electron-positron annihilation,
- hadron collisions, ...



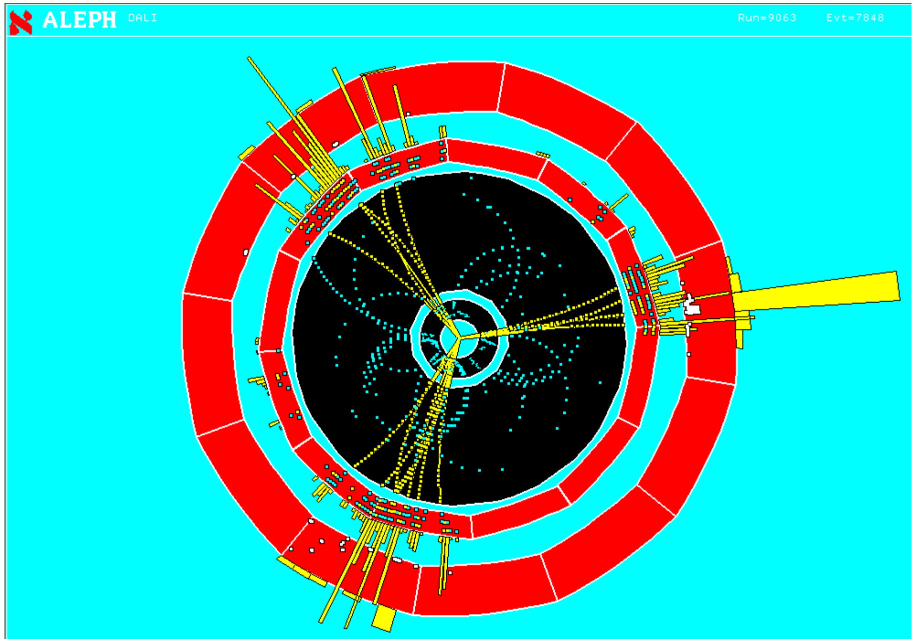
(S. Bethke, ’06.)

The strong coupling from electron-positron annihilation

One possibility: Extract α_s from three-jet events in electron-positron annihilation.

Jets: A bunch of particles moving in the same direction

A three-jet event from the Aleph experiment at LEP:



Jet algorithms

Ingredients:

- a resolution variable y_{ij} where a smaller y_{ij} means that particles i and j are "closer":

$$y_{ij}^{DURHAM} = \frac{2(1-\cos\theta_{ij})}{Q^2} \min(E_i^2, E_j^2)$$

- a combination procedure which combines two four-momenta into one:

$$p^\mu_{(ij)} = p_i^\mu + p_j^\mu.$$

- a cut-off y_{cut} which provides a stopping point for the algorithm.

Event shapes

What experimentalists measure: Event shapes

Example: Thrust

$$T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|}$$

For two particles back-to-back one has

$$T = 1$$

For many particles, isotropically distributed we have

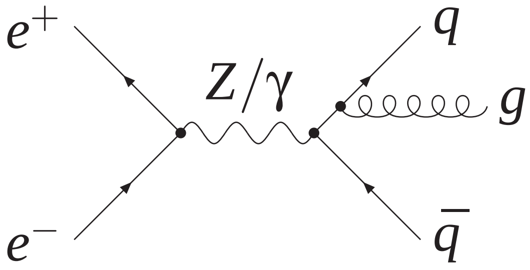
$$T = \frac{1}{2}$$

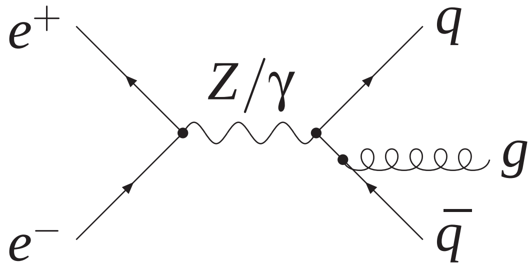
Perturbation theory

Due to the smallness of the coupling constants α and α_s, we may compute an observable at high energies reliable in perturbation theory,

$$\langle O \rangle = \frac{\alpha_s}{2\pi} \langle O \rangle_{LO} + \left(\frac{\alpha_s}{2\pi}\right)^2 \langle O \rangle_{NLO} + \left(\frac{\alpha_s}{2\pi}\right)^3 \langle O \rangle_{NNLO} + \dots$$

Feynman diagrams contributing to the leading order:





Leading order proportional to α_s !

The need for precision

Objectives for LHC: Extract fundamental quantities like α_s to high precision.

Theoretical predictions are calculated as a power expansion in the coupling. Higher precision is reached by including the next higher term in the perturbative expansion.

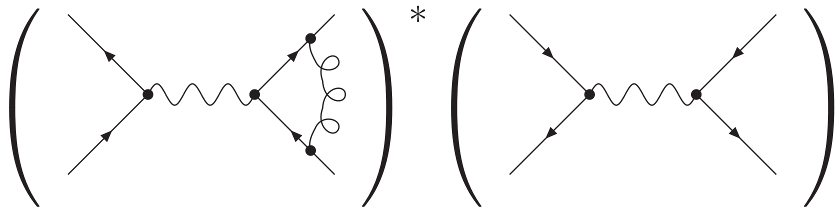
State of the art:

- Third or fourth order calculations for a few selected quantities (R-ratio, QCD β-function, anomalous magnetic moment of the muon).
- NNLO calculations for a few selected $2 \rightarrow 2$ and $2 \rightarrow 3$ processes.
- NLO calculations for $2 \rightarrow n$ ($n = 2, 3, 4$) processes.
- LO calculations for $2 \rightarrow n$ ($n = 2, ..., 8$) processes.

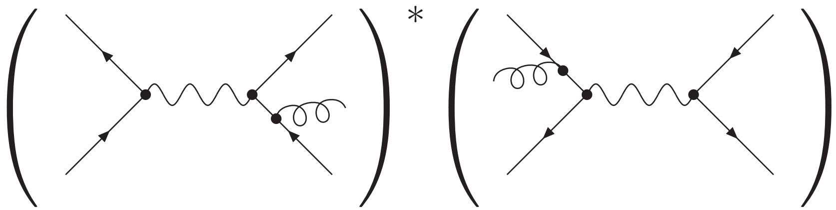
Higher orders in perturbation theory

Higher order contribution to the two-jet cross-section $\sigma \sim |\mathcal{A}|^2$:

Virtual corrections:

2 Re  $\sim \quad g_s^2$

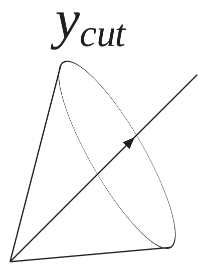
Real emission:

 $\sim \quad g_s^2$

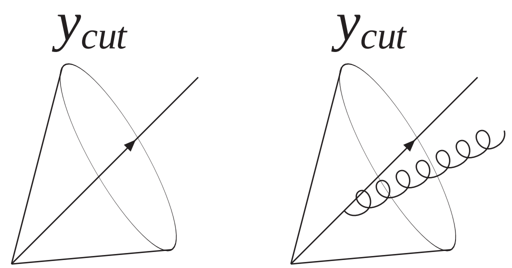
Modeling of jets:

In a perturbative calculation jets are modeled by only a few partons. This improves with the order to which the calculation is done.

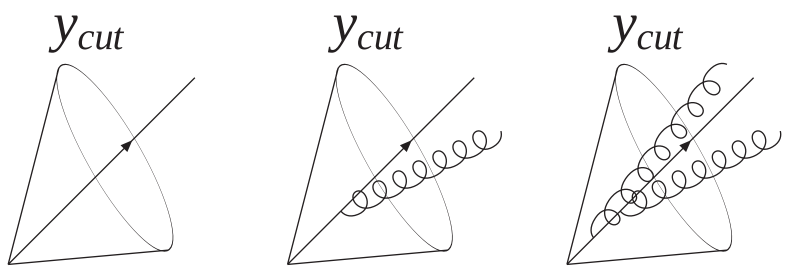
At leading order:



At next-to-leading order:



At next-to-next-to-leading order:



Calculation of observables

Perturbative expansion of the amplitude (LO, NLO, NNLO):

$$|\mathcal{A}_n|^2 = \underbrace{\mathcal{A}_n^{(0)\,*}\mathcal{A}_n^{(0)}}_{\text{Born}} + \underbrace{\left(\mathcal{A}_n^{(0)\,*}\mathcal{A}_n^{(1)} + \mathcal{A}_n^{(1)\,*}\mathcal{A}_n^{(0)}\right)}_{\text{virtual}} + \underbrace{\left(\mathcal{A}_n^{(0)\,*}\mathcal{A}_n^{(2)} + \mathcal{A}_n^{(2)\,*}\mathcal{A}_n^{(0)} + \mathcal{A}_n^{(1)\,*}\mathcal{A}_n^{(1)}\right)}_{\text{two-loop and loop-loop}},$$

$$|\mathcal{A}_{n+1}|^2 = \underbrace{\mathcal{A}_{n+1}^{(0)\;*}\mathcal{A}_{n+1}^{(0)}}_{\text{real}} + \underbrace{\left(\mathcal{A}_{n+1}^{(0)\;*}\mathcal{A}_{n+1}^{(1)} + \mathcal{A}_{n+1}^{(1)\;*}\mathcal{A}_{n+1}^{(0)}\right)}_{\text{loop+unresolved}},$$

$$|\mathcal{A}_{n+2}|^2 = \underbrace{\mathcal{A}_{n+2}^{(0)\;*}\mathcal{A}_{n+2}^{(0)}}_{\text{double unresolved}}\;.$$

$\mathcal{A}_n^{(l)}$: amplitude with n external particles and l loops.

Challenges

What are the bottle-necks ?

- Length: Perturbative calculations lead to expressions with a huge number of terms.
- Integrals: At one-loop and beyond, the occuring integrals cannot be simply looked up in an integral table.
- Divergences: At NLO and beyond, infrared divergences occur in intermediate stages, if massless particles are involved.
- Numerics: Stable and efficient numerical methods are required for the Monte Carlo integration.

Computer algebra

Computer-intensive symbolic calculations in particle physics can be characterized by:

- Need for basic operations like addition, multiplication, sorting ...
- Specialized code usually written by the user
- No need for a system which knows "more" than the user!

CAS on the market:

- Commercial: Mathematica, Maple, Reduce, ...
- Non-commercial: FORM (Vermaseren); GiNaC, ... (Bauer, Frink, Kreckel, Vollinga, ...)

The amplitudes for $e^+e^- \longrightarrow 3$ jets at NNLO

A NNLO calculation of $e^+e^- \longrightarrow 3$ jets requires the following amplitudes:

- Born amplitudes for $e^+e^- \longrightarrow 5$ jets:

  F. Berends, W. Giele and H. Kuijf, 1989;

  K. Hagiwara and D. Zeppenfeld, 1989.

- One-loop amplitudes for $e^+e^- \longrightarrow 4$ jets:

  Z. Bern, L. Dixon, D.A. Kosower and S.W., 1996;

  J. Campbell, N. Glover and D. Miller, 1996.

- Two-loop amplitudes for $e^+e^- \longrightarrow 3$ jets:

  L. Garland, T. Gehrmann, N. Glover, A. Koukoutsakis and E. Remiddi, 2002;

  S. Moch, P. Uwer and S.W., 2002.

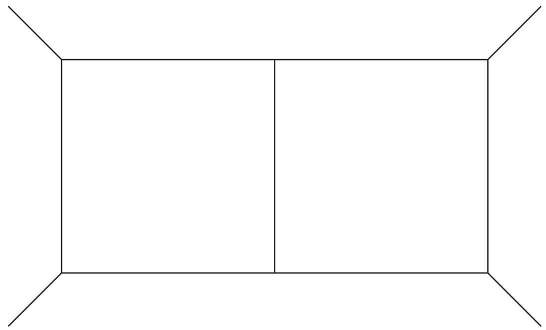
The calculation of two-loop integrals

- Techniques to calculate two-loop integrals
  - Mellin-Barnes transformation, Smirnov '99, Tausk '99.
  - Differential equations, Gehrmann, Remiddi '00.
  - Nested sums, Moch, Uwer, S.W. '01.
  - Sector decomposition (numerical), Binoth, Heinrich, '00.

- Methods to reduce the work-load:
  - Integration-by-parts, Chetyrkin, Kataev, Tkachov '81.
  - Reduction algorithms, Tarasov '96, Laporta '01.
  - Cut technique Bern, Dixon, Kosower, '00

The double-box integral

Two-loop amplitudes for $2 \longrightarrow 2$ processes involve the double-box integral:



- First calculated by Smirnov (planar) and Tausk (non-planar) in 1999.
- Calculation based on Mellin-Barnes representation.
- Result expressed in harmonic polylogarithms.

$$H_{m_1,\ldots,m_k}(x) = \sum_{i_1>i_2>\ldots>i_k>0} \frac{x^{i_1}}{i_1^{m_1} i_2^{m_2} \ldots i_k^{m_k}}, \qquad x = \frac{s}{t}.$$

Multiple polylogarithms

- Definition:

$$\mathrm{Li}_{m_1,...,m_k}(x_1,\dots,x_k) \quad = \quad \sum_{i_1>i_2>\ldots>i_k>0} \frac{x_1^{i_1}}{i_1^{m_1}} \frac{x_2^{i_2}}{i_2^{m_2}} \cdots \frac{x_k^{i_k}}{i_k^{m_k}}.$$

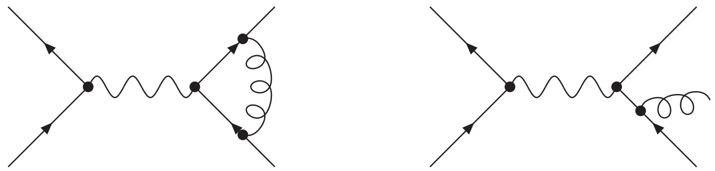
(Goncharov; Borwein, Bradley, Broadhurst and Lisonek)

- Special subsets: Harmonic polylogs, Nielsen polylogs, classical polylogs (Remiddi and Vermaseren, Gehrmann and Remiddi).
- Have also an integral representation.
- Obey two Hopf algebras (Moch, Uwer, S.W.).
- Can be evaluated numerically for all complex values of the arguments (Gehrmann and Remiddi, Vollinga and S.W.).

Infrared divergences and the Kinoshita-Lee-Nauenberg theorem

In addition to ultraviolet divergences, loop integrals can have infrared divergences.

For each IR divergence there is a corresponding divergence with the opposite sign in the real emission amplitude, when particles becomes soft or collinear (e.g. unresolved).



The Kinoshita-Lee-Nauenberg theorem: Any observable, summed over all states degenerate according to some resolution criteria, will be finite.

The subtraction method

The NLO cross section is rewritten as

$$\sigma^{NLO} = \int_{n+1} d\sigma^R + \int_n d\sigma^V = \int_{n+1} \left(d\sigma^R - d\sigma^A\right) + \int_n \left(d\sigma^V + \int_1 d\sigma^A\right)$$

The approximation $d\sigma^A$ has to fulfill the following requirements:

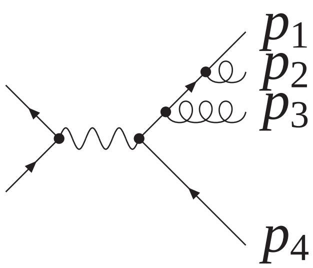
- $d\sigma^A$ must be a proper approximation of $d\sigma^R$ such as to have the same pointwise singular behaviour as $d\sigma^R$ itself.
  Thus, $d\sigma^A$ acts as a local counterterm for $d\sigma^R$.

- Analytic integrability over the one-parton subspace leading to soft and collinear divergences.

Catani, Seymour, ’96

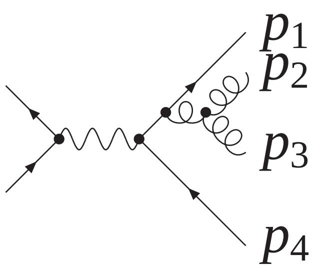
An example involving double unresolved configurations

The leading-colour contributions to $e^+e^- \longrightarrow qgg\bar{q}$.



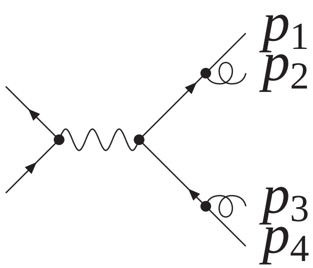
Double unresolved configurations:

- Two pairs of separately collinear particles
- Three particles collinear
- Two particles collinear and a third soft particle
- Two soft particles
- Coplanar degeneracy



Single unresolved configurations:

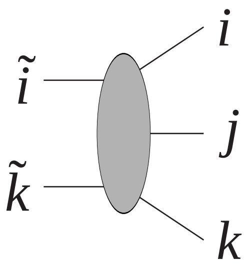
- Two collinear particles
- One soft particle



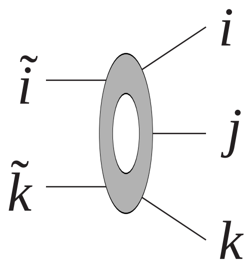
The subtraction method at NNLO

- Singular behaviour
  - Factorization of tree amplitudes in double unresolved limits, Berends, Giele, Cambell, Glover, Catani, Grazzini, Del Duca, Frizzo, Maltoni, Kosower '99
  - Factorization of one-loop amplitudes in single unresolved limits, Bern, Del Duca, Kilgore, Schmidt, Kosower, Uwer, Catani, Grazzini, '99
- Extension of the subtraction method to NNLO Kosower; S.W.; Kilgore; Gehrmann-De Ridder, Gehrmann, Glover, Heinrich; Frixione, Grazzini; Somogyi, Trócsányi and Del Duca;
- Cancellation based on sector decomposition Anastasiou, Melnikov, Petriello; Heinrich;
- Applications:
  - $pp \longrightarrow W$, Anastasiou, Dixon, Melnikov, Petriello '03,
  - $pp \longrightarrow H$, Anastasiou, Dixon, Melnikov, Petriello '05, Catani, Grazzini '08
  - $e^+e^- \longrightarrow$ 2 jets, Anastasiou, Melnikov, Petriello '04, S.W. '06
  - $e^+e^- \longrightarrow$ 3 jets, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07, S.W. '08

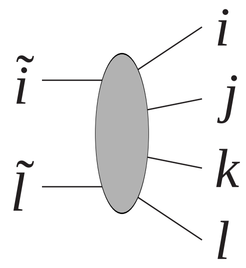
Antenna subtraction terms at NNLO



NLO



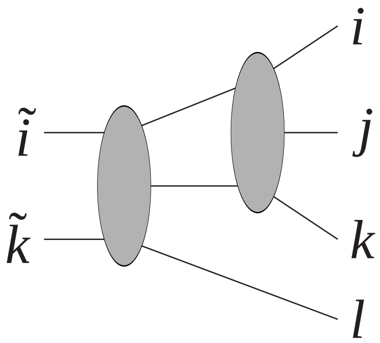
one-loop unresolved



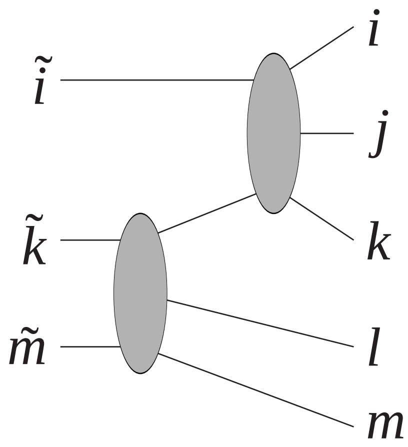
double unresolved

Gehrmann-De Ridder, Gehrmann, Glover, '05

At NNLO also iterated structures:



colour connected



almost colour connected

$e^+e^- \longrightarrow 3$ jets at NNLO

Fully differential Monte-Carlo programs for 3-jet observables at NNLO:

- EERAD3

  Gehrmann-De Ridder, Gehrmann, Glover, Heinrich,

  Phys.Rev.Lett.99:132002,2007,

  Phys.Rev.Lett.100:172001,2008

- MERCUTIO2

  S.W.,

  Phys.Rev.Lett.101:162001,2008

Soft gluons

4 partons:

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi \ln\left(\frac{(1+c_j)(1-c_2)}{2(1-c_2c_j-s_2s_j\cos\phi)}\right) =$$

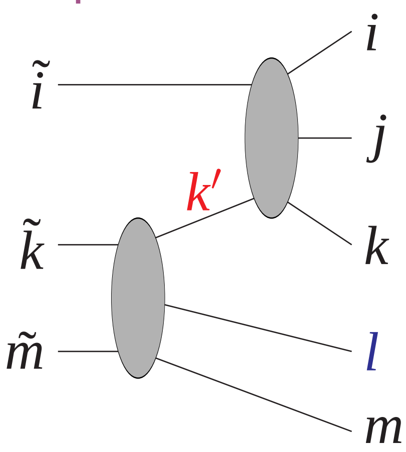
$$= \ln\left(\frac{1-c_2c_j+(c_j-c_2)}{1-c_2c_j+|c_j-c_2|}\right).$$

Non-zero for $c_j < c_2$!

The explicit poles in the four-parton configuration have to cancel: $d\alpha^{soft}$ is needed.

The five-parton contribution has to be independent of the slicing parameter: $-d\alpha^{soft}$ is needed.

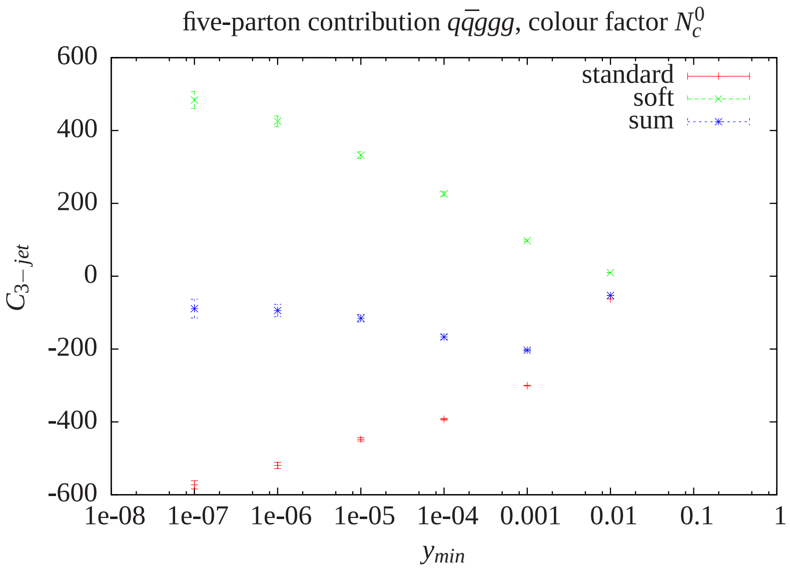
5 partons:



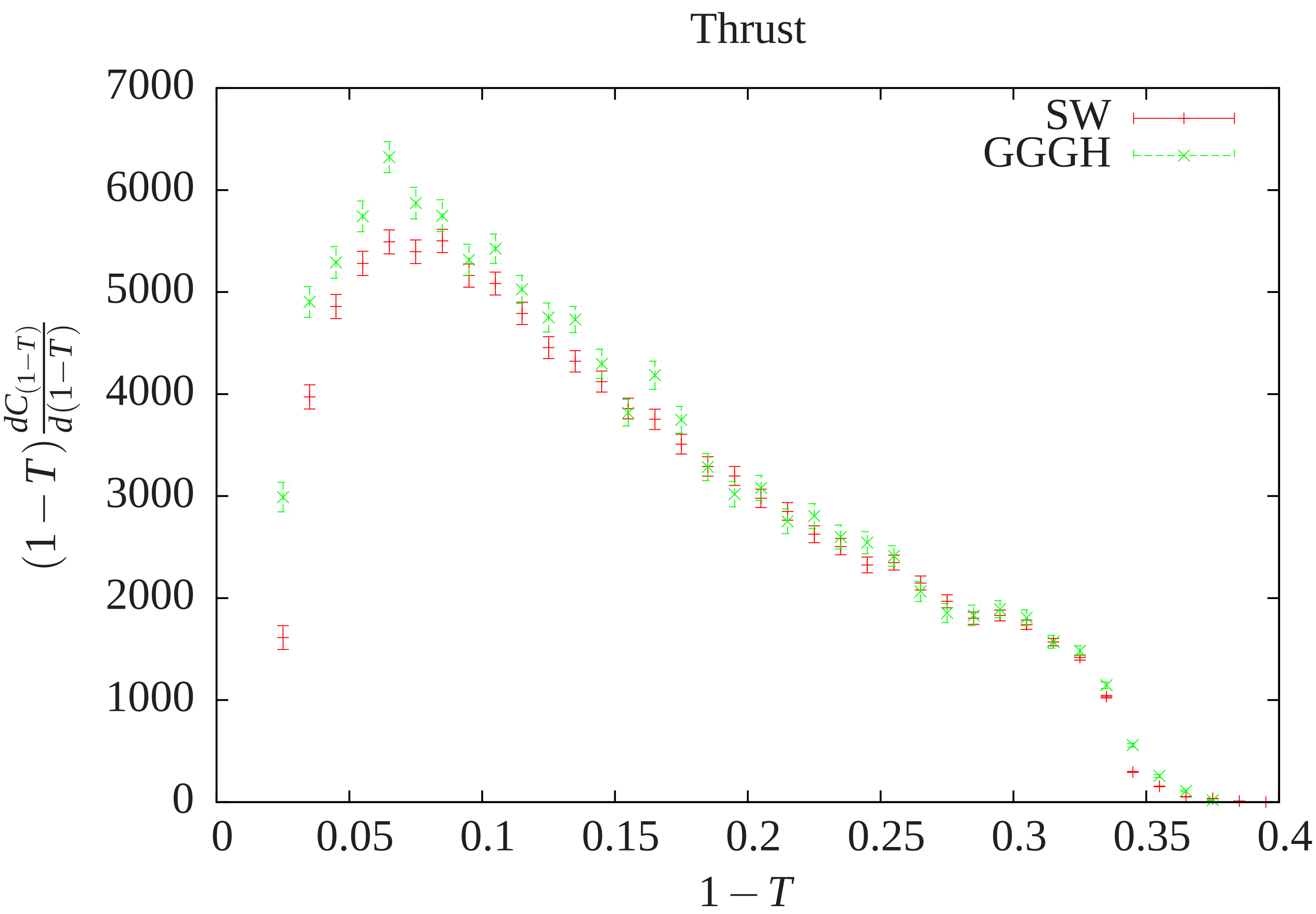
Gluon l soft:

Eikonal factor

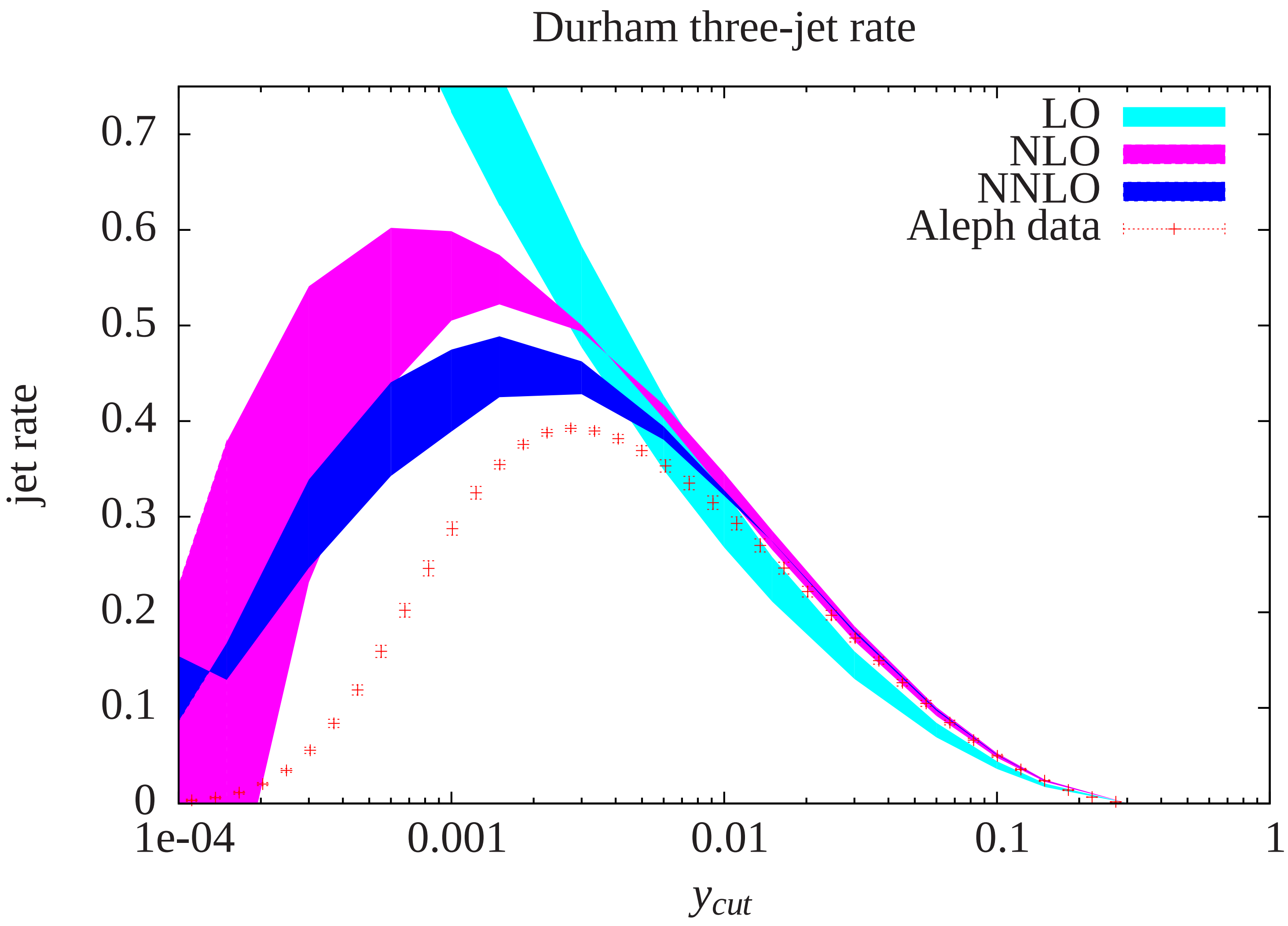
$\mathrm{Eik}(k', l, m)$



Comparison with EERAD3

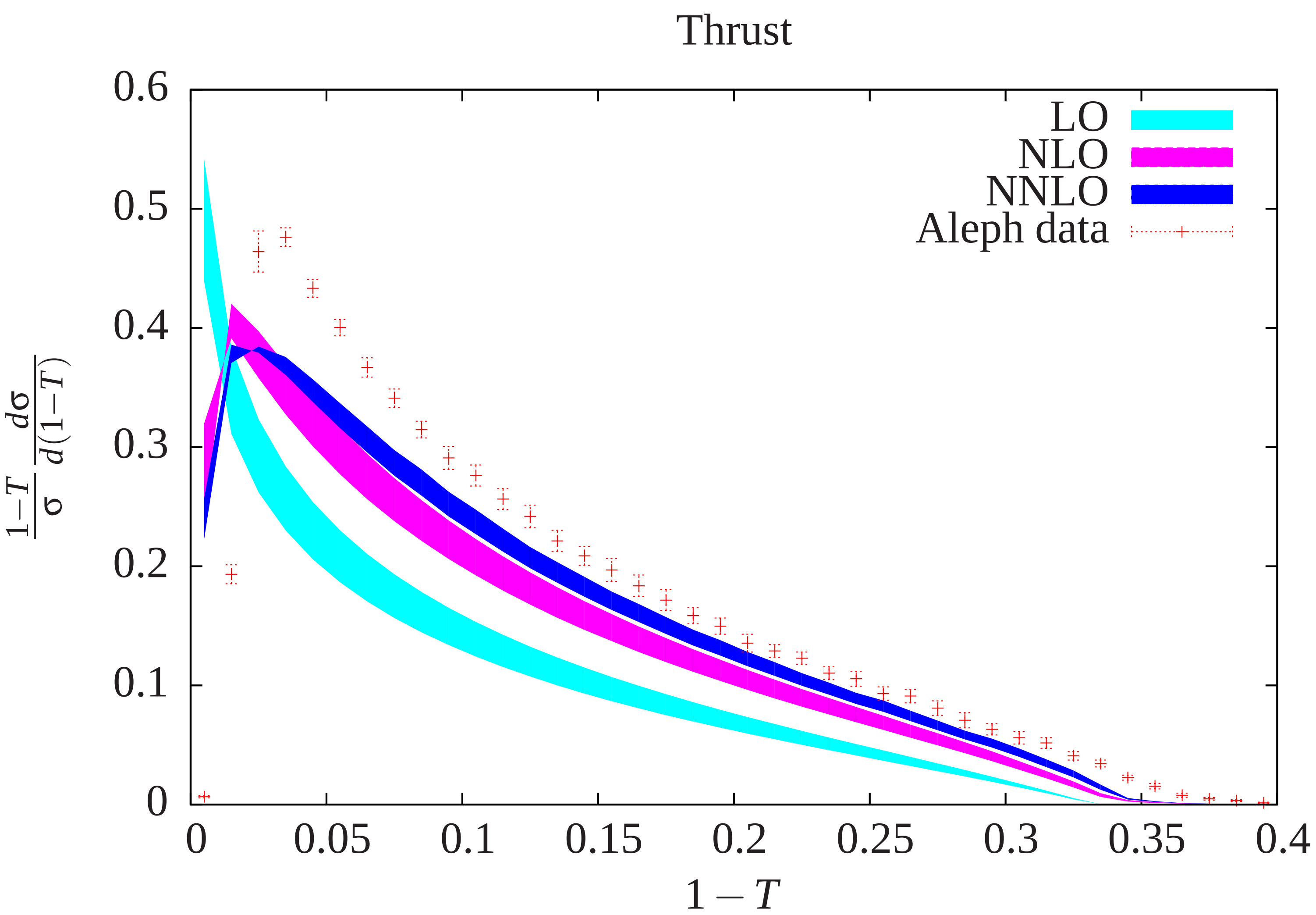


Results for the three-jet rate in electron-positron annihilation



S.W., Phys.Rev.Lett.101:162001,2008

Results for the thrust distribution



S.W., arxiv:0904.1077

Further refinements

Soft-gluon resummation: Perturbative expansion is of the form

$$1 + c_0 \alpha_s + c_1 \alpha_s \ln y_{cut} + c_2 \alpha_s \ln^2 y_{cut} + O(\alpha^2)$$

In the region where $\alpha_s \ln^2 y_{cut} \approx 1$ resum the large logarithms.

Catani, Trentadue, Turnock, Webber, '93; Becher, Schwartz, '08

Power corrections: From the operator product expansion we expect power corrections of the form

$$\frac{\lambda}{Q} + O\left(\frac{1}{Q^2}\right)$$

Dokshitzer, Webber, '97; Davison, Webber, '08

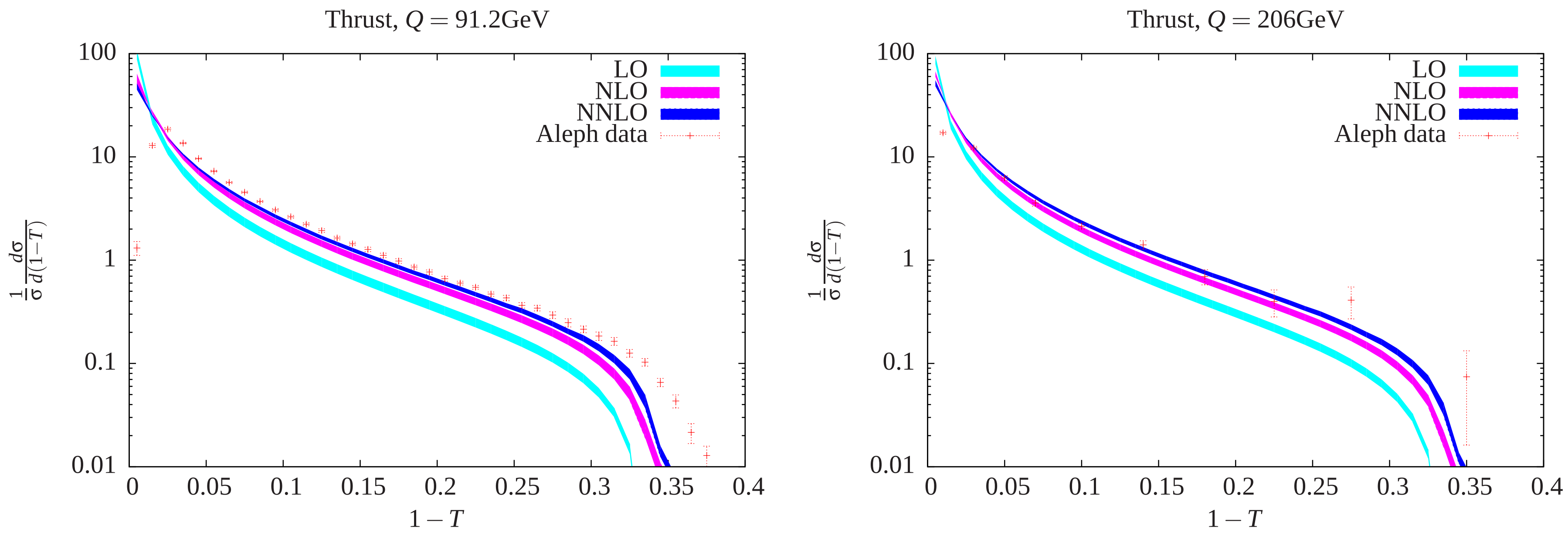
Current world average:

$$\alpha_s(m_Z) \quad = \quad 0.1176(20)$$

Particle Data Group, '08

Results for the thrust distribution

Changing the centre-of-mass energy:



Summary

- α_s is one of the fundamental parameters of nature
- Error on α_s dominated by theory
- NNLO calculations reduce the theoretical uncertainty

  Re-analysis of JADE data, ...
- Calculational techniques developed for $e^+e^- \longrightarrow$ 3 jets can be applied to other processes