The exclusive NLO non-singlet kernel

A prototype of the NLO Parton Shower for the Initial-State

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More talks are here http://jadach.web.cern.ch/

Outline

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Mission statement

- "NLO Parton Shower Monte Carlo" is regarded as highly desirable for the QCD calculations at LHC, while completely unfeasible in practice.
- Presented work demonstrates proof of existence of NLO PS MC for the initial state QCD (non-singlet evolution)
- It involves re-calculation of the NLO DGLAP evolution kernels in the exclusive (unintegrated) form, following Furmanski-Petronzio method (1982).
- Prototype MC with new exclusive kernels performs exactly NLO DGLAP evolution on its own (no external PDFs!) following closely the "collinear factorization theorems" of QCD and the "Matrix Element $\times$ Phase Space" approach.
- This opens new avenues for a new class of better practical QCD calculations in MC form for the coming two decades of the LHC experiments.

Outline

Main problems on the way to NLO QCD exclusive MC evolution

Main obstacles on the way to NLO exclusive MC evolution:

- **A** The internal phase space of NLO DGLAP kernel is unlimited ($k^T \to 0$), while in the parton shower MC it is limited due to ordering and/or initial condition.
- **B** Switching freely from one to another evolution time variable in the MC should be at hand: k^T, angle and virtuality (k^-). Soft limit (Colour Coherence) will dictate the final choice.
- **C** Issues related to quark-pair and gluon-pair production: running coupling constant, UV renorm. scale versus IR factorization scale, extra phase space absent at LO, Colour Coherence in the soft limit and more.

Problems **A** and **B** solved, **C** under study (seems feasible).
See next slides for some details...

(A) Limited internal phase space of NLO kernel due to ordering in MC While in orthodox DGLAP it is not limited! See below:

$$D^{nlo}_{DGLAP}(z) = \int_0^1 \frac{d\alpha_2}{\alpha_2}\frac{d\alpha_1}{\alpha_1}\delta_{z=1-\alpha_1-\alpha_2}\int_0^{2\pi} d\varphi_1 \int_0^Q \frac{dk_1^T}{Q}\,\rho\left(\dots\frac{k_1^T}{Q}\dots\right)$$

where ρ comes from Feynman diags. transformed by factorization.
In the iterative solution of the evolution equation in the MC we have

$$\dots\int_{k^T_{i-1}}^{k^T_{i+1}} \frac{dk_i^T}{k^T_{i+1}}\,\rho\left(\dots\frac{k_i^T}{k^T_{i+1}}\dots\right)\dots = \dots\int_{y_{\min}}^1 dy_i\;\rho(\dots y_i\dots)$$

To implement exactly NLO DGLAP we must put $y_{\min}\to 0$ in the MC.
Solution found by going back to the very foundations of the factorization plus some “Monte Carlo wizardry” – already implemented and tested.

Backup solution: evaluation analytically $\Delta D^{nlo}(z,y_{\min}) = \int_0^{y_{\min}} dy\;\rho(\dots y\dots)$ and adding it to virtual corrections in NLO kernel. Not tested.

(B) Switching freely from one to another evolution time variable in the MC Which choice is the best?

- In the NLO kernel evaluation μ_F of $\overline{MS}$ dimens. regulariz. we mapped into:
  - (a) virtuality $S(k_1, k_2) = \sqrt{-(P - k_1 - k_2)^2}$,
  - (b) eikonal minus variable $S = \max(k_1^-, k_2^-)$
  - (c) transverse momentum $S = \max(|\mathbf{k}_1|, |\mathbf{k}_2|)$
  - (d) rapidity $S = \max(\frac{k_1^-}{k_1^+}, \frac{k_2^-}{k_2^+})$

  where $k_i = (k^0, \mathbf{k}, k^3)$ are momenta of real emitted partons and $k^\pm = k^0 \pm k^3$.
- Real parton phase space is closed up from the above $S(k_1, k_2) < Q$.
- Resulting NLO DGLAP kernel is the same for all choices (a)-(d)
- Option (c) is best aligned with $\overline{MS}$ (and extensions towards BFKL).
- Soft limit clearly drags us into option (d).
- In the NLO MC parton shower context (a) and (b) are disfavored, (c) is useful for testing and...
- the angular variable (d) is perhaps the best, as it fits well the soft limit and does not lead to “dead zones”, when combining two emission chains in the future MC for Drell-Yan and DIS.

(C) Issues related to quark-pair and gluon-pair production
Running coupling constant, colour coherence, etc.

- ... still under study, but
- characteristic cancellations between various diagrams due to colour coherence are reproduced and understood
  M. Slawińska and A. Kusina, IFJ-PAN-IV-2009-2, arXiv: ...
- ...more work needed.... but no major problems expected.

Our definition of exclusive (fully unintegrated) PDF for the Monte Carlo

$$D(x,Q) = \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} \frac{d^3 k_i}{2k_i^0}\; \tilde{\rho}^{(n)}(P; k_1, k_2 \ldots, k_n)\; \Theta_{S(k_1,k_2\ldots,k_n)<Q}\; \delta_{1-x=\sum \alpha_i},$$

$\alpha_i = \frac{k_i^+}{2P^0}$, $P = (P^0, 0, 0, P^0)$ is initial parton momentum, $k_i, i = 1, .., n$ momenta of n emitted real partons. For S we take maximum rapidity $S = \max(\frac{\mathbf{k}_1^2}{\alpha_1^2}, \frac{\mathbf{k}_2^2}{\alpha_2^2}, \ldots \frac{\mathbf{k}_n^2}{\alpha_n^2})$ or maximum transverse momentum $S = \max(|\mathbf{k}_1|, |\mathbf{k}_2|, \ldots, |\mathbf{k}_n|)$.

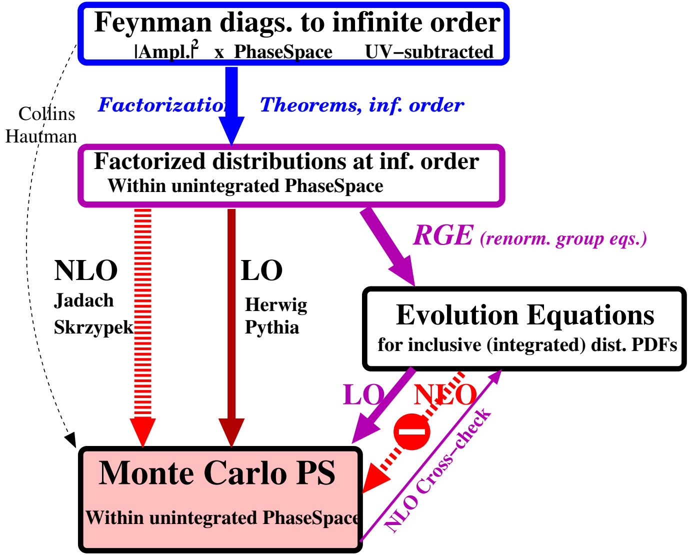
The raw distribution $\rho^{(n)}$ originates from UV subtracted Feynman diagrams – it includes certain dummy IR regulators.

The actual $\tilde{\rho}^{(n)}$ in MC is the result of projection of $\rho^{(n)}$ down to LO+NLO level using methodology of the "factorization theorems", SUCH THAT the inclusive PDF $D(x,Q)$ obeys EXACTLY evolution equation

$$\frac{\partial}{d \ln Q} D(x,Q) = \int dzdu\; \mathcal{P}^{LO+NLO}(z)\; D(u,Q)\; \delta_{x=zu}$$

of the NLO DGLAP with the standard DGLAP inclusive kernels $\mathcal{P}^{LO+NLO}(z)$.

Projection $\boxed{\rho^{(n)} \to \tilde{\rho}^{(n)}}$ is of course the main issue.


Feynman diags. to infinite order
|Ampl.|2 x PhaseSpace UV−subtracted
Factorization Theorems, inf. order
Collins
Hautman
Factorized distributions at inf. order
Within unintegrated PhaseSpace
RGE (renorm. group eqs.)
NLO
Jadach
Skrzypek
LO
Herwig
Pythia
Evolution Equations
for inclusive (integrated) dist. PDFs
LO
NLO
NLO Cross−check
Monte Carlo PS
Within unintegrated PhaseSpace

Summarizing our aims:

Our new NLO Parton Shower Monte Carlo for QCD Initial State Radiation should be:

- Based firmly on Feynman Diagrams (ME) and LIPS
- Based rigorously on the collinear factorization (EGMPR-CFP, CSS)
- Implementing exactly NLO $\overline{MS}$ DGLAP evolution **!**
- Implementing fully exclusive PDFs.
- NLO evolution done by MC itself using EXClusive NLO kernels.

Potential gains

While retaining exact NLO DGLAP evolution, excellent starting point for extensions:

- Possible extension towards CCFM, BFKL (low x limit)
- Correct soft limit and built-in colour coherence
- More realistic description of the quark thresholds
- The use of exact amplitudes for multigluon emission, the analog of Coherent Exclusive Exponentiation in QED (Jadach,Was,Ward)
- Better connection between hard process ME and the shower parts, as compared with MC@NLO and the likes
- In particular no negative weigh events, no ambiguity of defining last emission before hard process, etc.
- Providing better tool for exploiting HERA DATA for LHC (fitting F_2 directly with MC)
- And more!!!

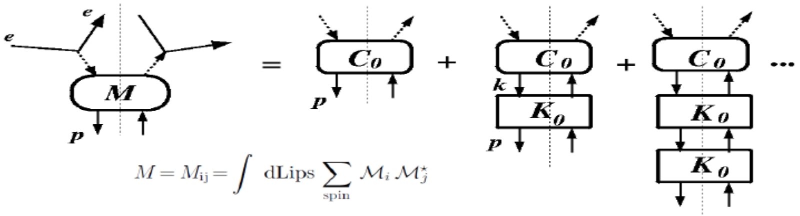
Outline

Re-Calculating NLO DGLAP kernels

- NLO DGLAP QCD kernels were calculated directly from the Feynman diags by Furmanski and Petronzio (+Curci in some papers) 1979-82.
- They did it using dimensional regularization ($\overline{MS}$), and axial gauge.
- They exploited/extended “QCD factorization theorem” of EGMPR (Ellis, Georgi, Machacek, Politzer and Ross, PLB78, 1978).
- I shall summarize briefly FP/EGMPR scheme and focusing on the C_F^2 part of double gluonstrahlung in non-singlet kernel
- “Numerical MC model” of the exclusive version of the NLO kernel (part of it) will be constructed
  – old integrated NLO kernel results of FP reproduced.
- Visualizing/analyzing singularity structure in the internal phase space of NLO kernel, etc.

EGMPR scheme of collinear factorization (1978)

"Raw" factorization of the IR collinear singularities



$$M = M_{ij} = \int \mathrm{dLips} \sum_{\mathrm{spin}} \mathcal{M}_i \mathcal{M}_j^*$$

- Cut vertex M: spin sums and Lips integrations over all lines cut across
- C_0 and K_0 and are 2-particle irreducible (2PI)
- C_0 is IR finite, while K_0 encapsulates all IR collinear singularities
- Use of the axial gauge essential for the proof
- Formal proof given in EGMPR NP B152 (1979) 285
- Notation next slide

$$M = C_0(1 + K_0 + K_0^2 + \cdots) = C_0 \frac{1}{1 - K_0} \equiv C_0 \Gamma_0$$

EGMPR scheme of collinear factorization (1978)

Factorization of EGMPR improved by Furmanski and Petronzio (80):

$$F = C_0 \cdot \frac{1}{1-K_0} = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right),$$

$$= \left\{ C_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0} \right\} \otimes \left\{ \frac{1}{1-\left(\mathbf{P}K_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}\right)} \right\}_{\otimes},$$

$$\Gamma\left(\alpha, \frac{1}{\epsilon}\right) \equiv \left(\frac{1}{1-K}\right)_{\otimes} = 1 + K + K \otimes K + K \otimes K \otimes K + \ldots,$$

$$K = \mathbf{P}K_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}, \quad C = C_0 \cdot \frac{1}{1-(1-\mathbf{P})\cdot K_0}.$$

Ladder part Γ corresponds to MC parton shower and C is the hard process part. For projection operator **P** see next slide...

EGMPR scheme of collinear factorization, cont.

Projection operator

$$\mathbf{P} = P_{spin}\, P_{kin}\, PP$$

consists of

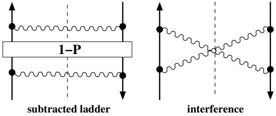
- the kinematic (on-shell) proj. operator P_{kin},
- spin proj. operator P_{spin}
- and the pole part PP extracting $\frac{1}{\epsilon_{IR}^{k}}$ part.

Multiplication symbol $\cdot$ means full phase space integration $d^n k$ while convolution $\otimes$ only the integration over the 1-dim. lightcone variable.

NLO Kernel extraction up second order

We shall work out C_F^2 part of double bremsstrahlung:

$$K_0 \cdot [(1-\mathbf{P}) \cdot K_0] + K_0 =$$



Which is second order term in the expansion:

$$\Gamma = \frac{1}{1-K} = 1 + \mathbf{P}K_0 + \mathbf{P}K_0 \cdot [(1-\mathbf{P}) \cdot K_0] + [\mathbf{P}K_0] \otimes [\mathbf{P}K_0] + \cdots ,$$

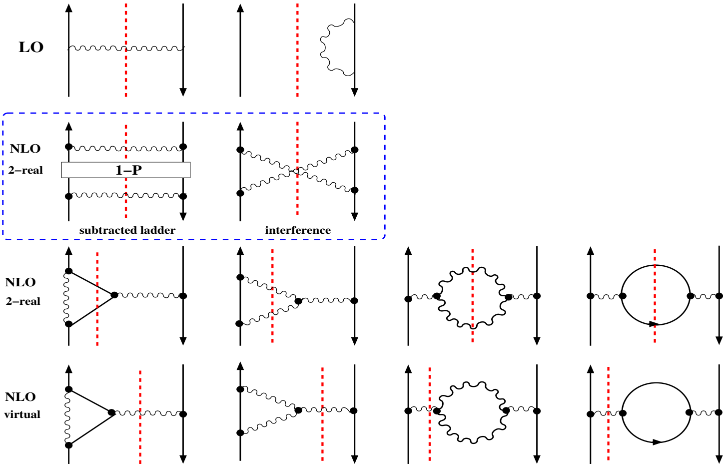
$$C = C_0 \frac{1}{1-(1-\mathbf{P})K_0} = C_0 + C_0 \cdot (1-\mathbf{P}) \cdot K_0 + C_0 \cdot (1-\mathbf{P}) \cdot K_0 \cdot [(1-\mathbf{P}) \cdot K_0)] + \dots$$

Original CFP Pole-part PP_1 method:

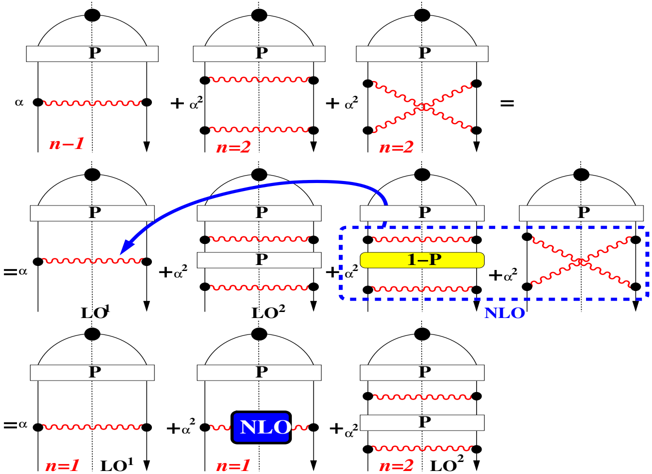
$$\frac{\alpha_R}{\pi} P(\alpha_R, x) = \frac{\alpha_R}{\pi} P^{(0)}(x) + \left(\frac{\alpha_R}{\pi}\right)^2 P^{(1)}(x) = \alpha_R \frac{\partial}{\partial \alpha_R} Res_1 \, \Gamma(\alpha_R, x)$$

$$= Res_1 \left\{ \mathbf{P}K_0 \right\} + 2 Res_1 \left\{ \mathbf{P}K_0 \cdot [(1-\mathbf{P}) \cdot K_0] \right\}.$$

Diags. contributing to non-singlet kernels up to NLO

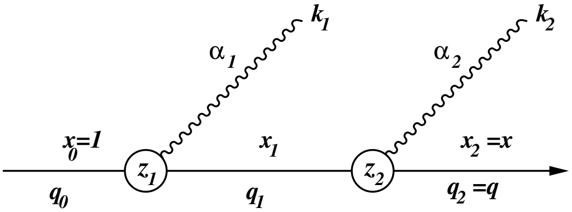


QCD collinear factorization transforms $n = 2 \rightarrow 1$!



Before we get back $n = 2$ for NLO in the MC we have to learn how does it look like!

Kinematics of the double emission



Kinematics of the double/multiple real emission

$$x_i = \frac{\zeta \cdot q_i}{\zeta \cdot q_0}, \quad \alpha_i = \frac{\zeta \cdot k_i}{\zeta \cdot q_0}, \quad q_i = q_0 - \sum_{i=1}^{n} k_i,$$

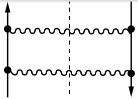
$$q_0^2 = 0, \quad q_0 = (E, 0, 0, E),$$

$$\zeta = (1, 0, 0, -1), \quad \zeta^2 = 0,$$

$$k_i^2 = 0, \quad k_i = (k_i^0, \mathbf{k}_i, k_i^3).$$

Ladder (uncrossed) diagram, C_F^2 part

Start from expression for the ladder diagram



$$F(\bar{C}_0 K_0 K_0) = \mathcal{N} C_F \int \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \delta_{1-x=\alpha_1+\alpha_2} \int d^{2+2\epsilon}\mathbf{k}_1 d^{2+2\epsilon}\mathbf{k}_2 \; \mu^{-4\epsilon} \; \Theta(S(k_1,k_2) < Q)$$
$$\times \left\{ \frac{T_1(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} + \frac{T_2(\alpha_1,\alpha_2,\epsilon)}{\alpha_2^2} \frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} + \frac{T_3(\alpha_1,\alpha_2)}{\alpha_2} \frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\mathbf{k}_1^2} \right\} \frac{1}{q^4(k_1,k_2)}$$

where γ-trace factors and the second propagator are

$$\begin{aligned}
T_1(\alpha_1,\alpha_2) &= (1+x^2+x_1^2)\alpha_1\alpha_2,\\
T_2(\alpha_1,\alpha_2,\epsilon) &= (1+x_1^2)(x^2+x_1^2)+\epsilon T_2'(\alpha_1,\alpha_2),\\
T_2'(\alpha_1,\alpha_2) &= (1-x_1)^2(x^2+x_1^2)+(x-x_1)^2(1+x_1^2),\\
T_3(\alpha_1,\alpha_2) &= (1+x^2+x_1^2)x_1,\\
-q^2(k_1,k_2) &= \frac{1-\alpha_2}{\alpha_1}\,\mathbf{k}_1^2 + \frac{1-\alpha_1}{\alpha_2}\,\mathbf{k}_2^2 + 2\mathbf{k}_1\cdot\mathbf{k}_2.
\end{aligned}$$

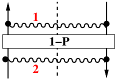
The above integral in $4+2\epsilon$ dimensions is finite for $\epsilon = \epsilon_{IR} > 0$.

For the cap function we choose maximum k^T: $S(k_1,k_2) = \max(k_1^T, k_2^T)$.

Ladder final result, agrees with CFP

k-integration yields relatively simple result in the α-space

$$F(\bar{C}_0 \mathbf{P} K_0 (1-\mathbf{P}) K_0)_{1P} = \left(C_F \frac{\alpha}{\pi}\right)^2 \frac{1}{2\epsilon} \int\limits_{\alpha_1+\alpha_2=1-x} \frac{\alpha_1}{\alpha_1^2+\delta^2} \frac{\alpha_2}{\alpha_2^2+\delta^2}$$
$$\times \left[T_1 \frac{1}{x} + T_2^0 \left\{ \frac{1}{x_1^2} \left(\frac{2\alpha_1\alpha_2}{x} + \ln\frac{(1-\alpha_1)^2}{x} + \ln\frac{\alpha_1}{\alpha_2} \right) - \frac{(1-\alpha_2)}{x x_1} \right\} - T_3 \frac{\alpha_1\alpha_2}{x x_1} \right.$$
$$\left. + \frac{T_2'(\alpha_1,\alpha_1) - 2T_2'''(\alpha_1,\alpha_1)}{x_1^2} \right].$$



and after integrating over the α-variables we get

$$F(\bar{C}_0 \mathbf{P} K_0 (1-\mathbf{P}) K_0)_{1P} = \left(C_F \frac{\alpha}{\pi}\right)^2 \frac{1}{2\epsilon}$$
$$\times \left\{ -4\frac{1+x^2}{1-x}\left[\ln\frac{1}{\delta} + \ln(1-x)\right] + \frac{-1+\frac{5}{2}(1+x^2)}{1-x} \ln^2 x + (2-4x)\ln x + 3(1-x) \right\}.$$

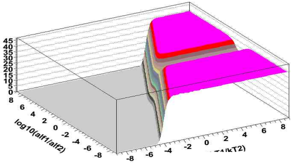
The above reproduces CFP paper Table 1, column 3+4.

OUR MAIN ACTIVITY STARTS HERE, ie. WHERE CFP HAVE FINISHED.

ALL THE ABOVE WAS JUST AN INTRODUCTION.

Outline

How the soft counterterm works? One ladder diagram ⊖ counterterm.



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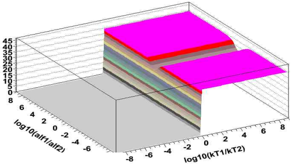

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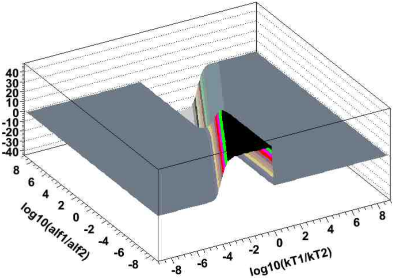
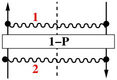
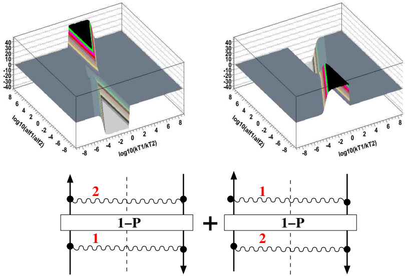




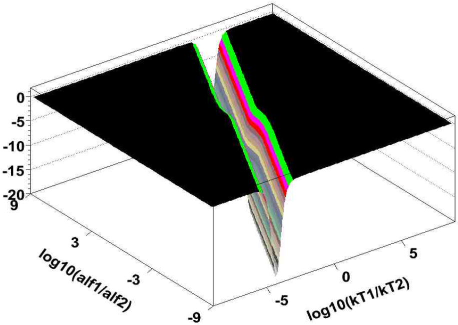
Plot of integrand in sudakovian variables $\ln\frac{\alpha_1}{\alpha_2}$ and $\ln y = 2\ln\frac{k_1^T}{k_2^T}$. Virtuality ordering?

A disaster for the Monte Carlo: Huge double-log cancellations!!!

Adding two ladder diagrams (BE-symmetrization)



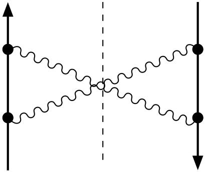
BE–symmtrized subtracted ladder

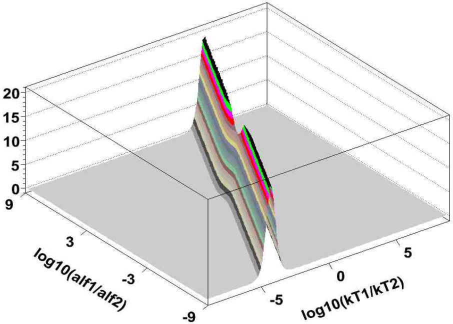


Doubly logarithmic IR divergences (triangles) cancel when adding 2 diagrams. Single logarithmic IR log remains!
We know it from the table of CFP paper that it goes away also.

Who cancells it? The interference (crossed ladder) graph! See next...

Inside LIPS: Crossed-ladder interference graph



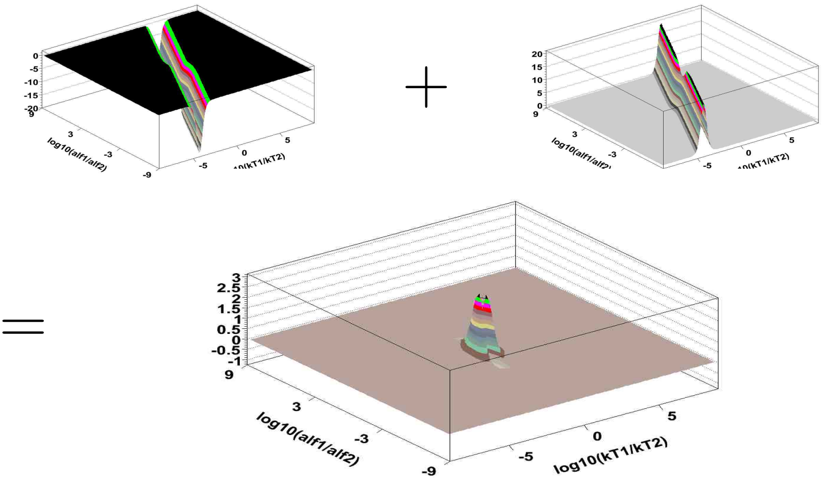


The interference graph looks almost as a mirror image of the previous!

$$F(\bar{C}_0 K_0 K_0) = N \int \frac{d\alpha_1}{\alpha_1} \frac{d\alpha_2}{\alpha_2} \delta_{1-x=\alpha_1+\alpha_2} \int d^{2+2\epsilon}\mathbf{k}_1 d^{2+2\epsilon}\mathbf{k}_2\ \mu^{-4\epsilon}\ \theta_{\max(\mathbf{k}_1^2,\mathbf{k}_2^2)\leq Q^2} \frac{1}{q^4(k_1,k_2)}$$
$$\times \left\{ \frac{2T_1^x(\alpha_1,\alpha_2)}{\alpha_1\alpha_2} - T_{2a}^x(\alpha_1,\alpha_2)\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\alpha_1\mathbf{k}_2^2} - T_{2b}^x(\alpha_1,\alpha_2)\frac{2\mathbf{k}_1\cdot\mathbf{k}_2}{\alpha_2\mathbf{k}_1^2} + T_3^x(\alpha_1,\alpha_2)\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{\mathbf{k}_1^2\mathbf{k}_2^2} \right\}$$

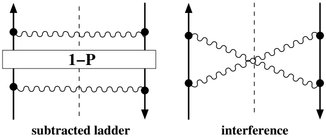
Also treated by FOAM using THE SAME parametrization of LIPS.

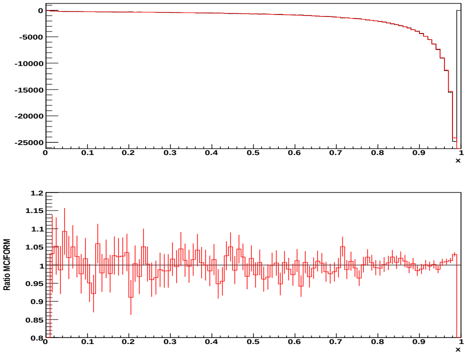
Symmetrized Ladder + Interference. IR goes away!!! Short range correl.



NLO exclusive kernel induces short-range correlation in the sudakov plane!
Factor 50 cancellation! Less strong without averaging over φ.

Reproducing Furmanski-Petronzio analytically and numerically, of course...





Comparing MC integral over exclusive NLO kernel with analytical kernel of CFP:

$$\mathcal{P}^N(z) = \frac{1+3x^2}{16(1-x)} \ln^2(x) + \frac{2-x}{4} \ln(x) + \frac{3}{8}(1-x)$$

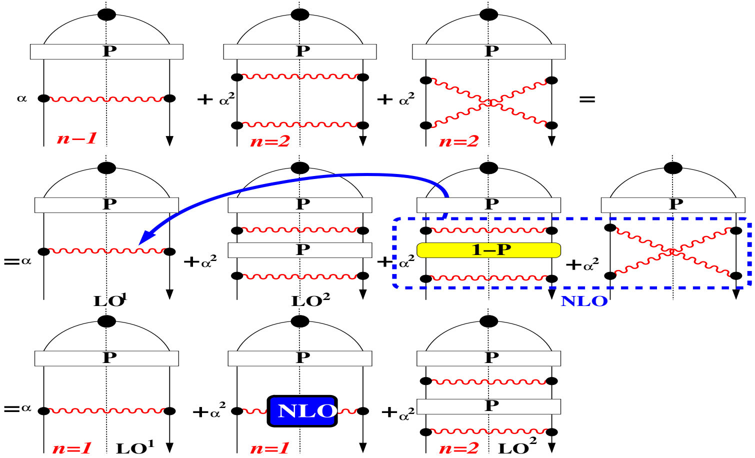
$$= \frac{1}{2!} \int \frac{d^3k_2}{2k_2^0} \int \frac{d^3k_1}{2k_1^0} \; \delta_{1=\max(|\mathbf{k}_1|,|\mathbf{k}_2|)/Q} \; \delta_{1-x=\alpha_1+\alpha_2} \; b_2^N(k_1, k_2)$$

Partial summary, Part II

- Diagram symmetrization removes double-log cancellations between distant regions of the phase space
- Adding interference graph removes soft IR singularity (single log)
- Nonzero when $kT1 \sim kT2$ and $\alpha_1 \sim \alpha_2 \sim 1$;
  Just short range correlation vanishing in the soft limit!
- The same pattern for other evolution time variables (done)
- WHAT NEXT?
- More diagrams and recalculate all DGLAP NLO kernels in the exclusive form.
- Even more urgent: Establish methodology of reinstalling the NLO unitegrated kernel in the LO MC, by means of reweighting LO events. See next Part III.

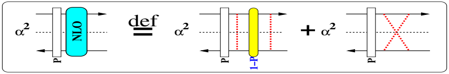
Outline

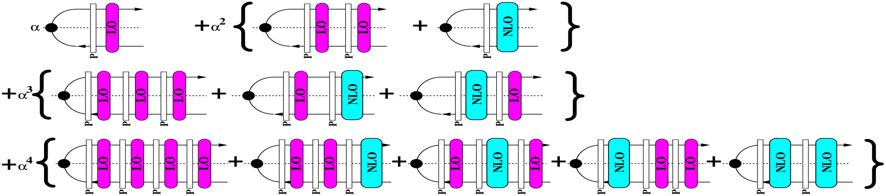
QCD collinear factorization transforms $n = 2 \rightarrow 1$!



We want to get back $n = 2$ for NLO in MC!

General scheme of exclusive NLO insertion for $n = 3, 4, ..\infty$





NLO decomposition in powers of α in factorization theorems. Example $n = 4$:

$$D_4^{L+N}(t,x) = e^{-S} \left(\prod_{i=1}^{4} \int \frac{d^3 k_i}{2k_i^0} \theta_{t_{i+1}>t_i} \right) \delta_{1-x=\sum_{i=1}^{3} \alpha_i} \; \rho_4^{L+N}(k_4, k_3, k_2, k_1),$$

$$\begin{aligned} \rho_4^{L+N}(k_4,k_3,k_2,k_1) &= \rho^L(k_4|x_3)\,\rho^L(k_3|x_2)\,\rho^L(k_2|x_1)\,\rho^L(k_1|x_0) \\ &+ \rho^L(k_4|x_3)\,\rho^L(k_3|x_2)\,b_2^N(k_2,k_1|x_0) + \rho^L(k_4|x_3)\,b_2^N(k_3,k_2|x_1)\,\rho^L(k_1|x_0) \\ &+ b_2^N(k_4,k_3|x_2)\,\rho^L(k_2|x_1)\,\rho^L(k_1|x_0) \\ &+ b_2^N(k_4,k_3|x_2)\,b_2^N(k_2,k_1|x_0) \end{aligned}$$

Outline

Implanting NLO exclusive kernel into parton shower LO MC

THREE-FOLD TESTING FRAMEWORK:

- Markovian MC with standard inclusive LO+NLO DGLAP kernel (gluonstrahlung part)
- Markovian MC with EXCLUSIVE LO+NLO DGLAP kernel. NEW!!!
- Analytical integration leading to NLO DGLAP kernel following Curci-Furmanski-Petronzio (1980).

Backup MC evolution model with INCLUSIVE/INTEGRATED kernels

Distributions of MC with the INCLUSIVE NLO kernel $\boxed{\mathcal{P}^{L+N}(\alpha, z) = \mathcal{P}^L(z) + \alpha\mathcal{P}^N(z)}$:

$$
\begin{aligned}
D^{L+N}(t,x) &= e^{-S}\delta(1-x)\\
&+ e^{-S}(\alpha T)(\mathcal{P}^L + \alpha\mathcal{P}^N)(x)\\
&+ e^{-S}\frac{(\alpha T)^2}{2!}(\mathcal{P}^L + \alpha\mathcal{P}^N)\otimes(\mathcal{P}^L + \alpha\mathcal{P}^N)(x)\\
&+ e^{-S}\frac{(\alpha T)^3}{3!}(\mathcal{P}^L + \alpha\mathcal{P}^N)\otimes(\mathcal{P}^L + \alpha\mathcal{P}^N)\otimes(\mathcal{P}^L + \alpha\mathcal{P}^N)(x)\\
&+ e^{-S}\frac{(\alpha T)^4}{4!}(\mathcal{P}^L + \alpha\mathcal{P}^N)\otimes(\mathcal{P}^L + \alpha\mathcal{P}^N)\otimes(\mathcal{P}^L + \alpha\mathcal{P}^N)\otimes(\mathcal{P}^L + \alpha\mathcal{P}^N)(x) + \dots .
\end{aligned}
$$

"Crazy decomposition" into a products of LO and NLO factors:

$$
\begin{aligned}
D^{L+N}(t,x) &= e^{-S}\delta(1-x)\\
&+ e^{-S}(\alpha T)[\mathcal{P}^L + \alpha\mathcal{P}^N](x)\\
&+ e^{-S}\frac{(\alpha T)^2}{2!}[\mathcal{P}^L\otimes\mathcal{P}^L + \alpha\mathcal{P}^N\otimes\mathcal{P}^L + \alpha\mathcal{P}^L\otimes\mathcal{P}^N + \alpha^2\mathcal{P}^N\otimes\mathcal{P}^N](x)\\
&+ e^{-S}\frac{(\alpha T)^3}{3!}[\mathcal{P}^L\otimes\mathcal{P}^L\otimes\mathcal{P}^L + \alpha\mathcal{P}^L\otimes\mathcal{P}^L\otimes\mathcal{P}^N + \alpha\mathcal{P}^L\otimes\mathcal{P}^N\otimes\mathcal{P}^L + \dots](x)\\
&+ e^{-S}\frac{(\alpha T)^4}{4!}[\mathcal{P}^L\otimes\mathcal{P}^L\otimes\mathcal{P}^L\otimes\mathcal{P}^L + \alpha\mathcal{P}^L\otimes\mathcal{P}^L\otimes\mathcal{P}^L\otimes\mathcal{P}^N + \dots](x) + \dots .
\end{aligned}
$$

Where $T = (t - t_0)$, $\alpha' \to \alpha$ and $S = (t - t_0)\alpha\left(\ln\frac{1}{\delta} + \frac{3}{4}\right)$.

Backup MC evolution model with INCLUSIVE/INTEGRATED kernels

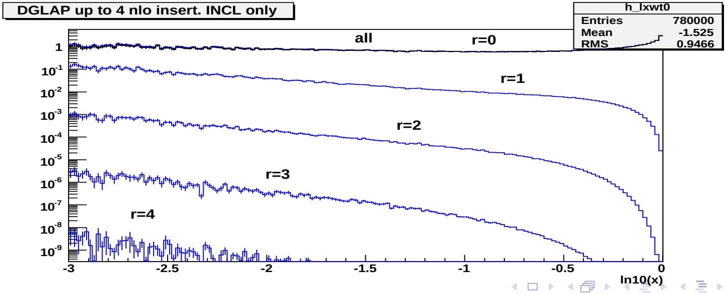
The basic LO MC distribution for n "emissions"

$$D_n^L(z,t) = e^{-S} = e^{-S}\mathcal{P}^{(0)}(z_1)\mathcal{P}^{(0)}(z_2)\dots\mathcal{P}^{(0)}(z_n)\theta_{t_0<t_1<t_2<\dots<t_n<t}$$

is upgraded to NLO level using MC weight:

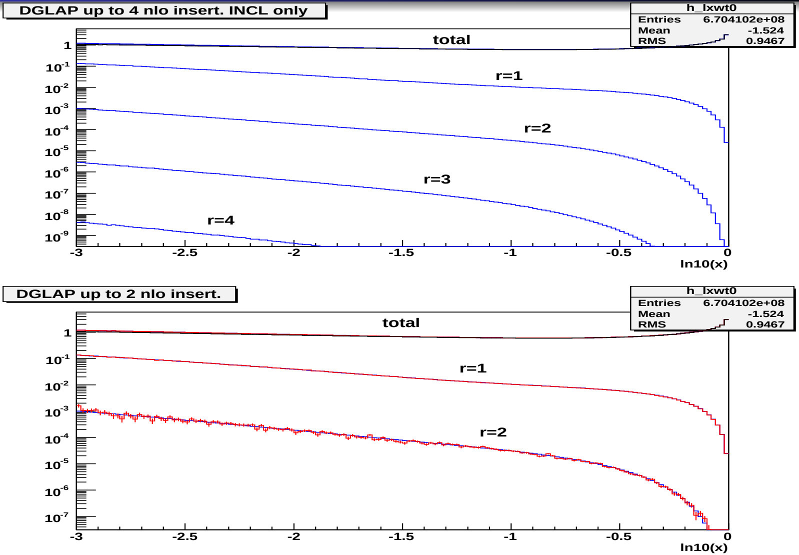
$$w = \frac{\prod_{i=1}^n(\mathcal{P}^{(0)}(z_i)+\alpha\mathcal{P}^{(1)}(z_i))}{\prod_{i=1}^n \mathcal{P}^{(0)}(z_i)} = \sum_{\{d\}} \alpha^{r(d)} \prod_{i=1}^n \frac{\mathcal{P}^{(d_i)}(z_i)}{\mathcal{P}^{(0)}(z_i)},$$

where $\{d\}$ is the set of all 2^n partitions $d=(d_1,d_2,d_3,\dots,d_n)$, $d_i=0,1$.
$r(d)=\sum_i d_i$ is the number of NLO factors in a given term/partition.
MC result below shows slices in $r=0,1,2,3\dots$. We may limit $r(d)\leq 4$.

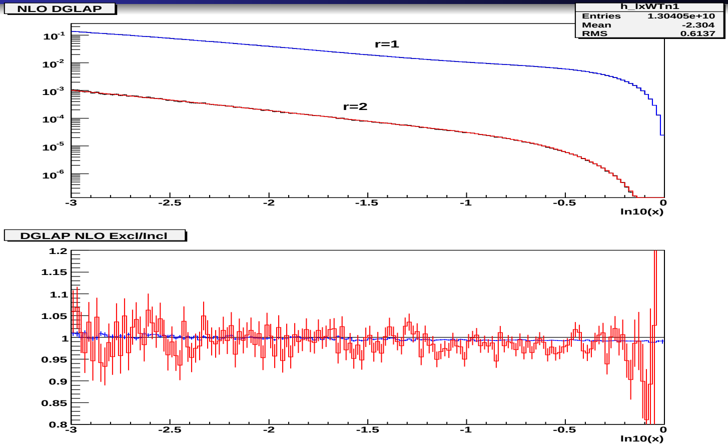


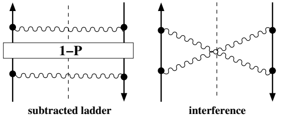
Next slides show the most important new result in this talk!

ExclusiveNLO/InclusiveNLO MC: total and slices in No. of insertions



Exclus./Inclusive NLO: Slices in No. of inserts. **NO ε TERM**



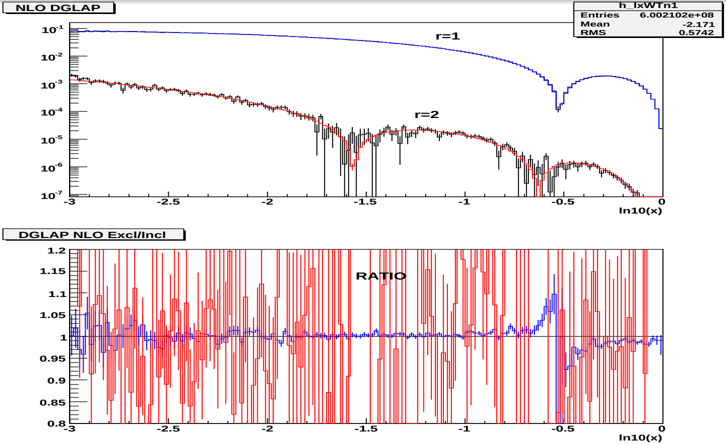


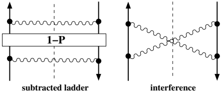
MC with Exclusive NLO kernels compared with standart DGLAP

More on what is in these plots:

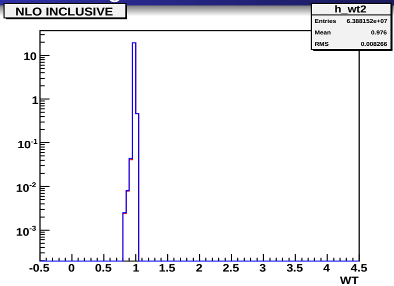
- Both evolutions on top of the same Markovian LO MC. (It can be put easily on top of non-Markovian CMC.)
- MC weights positive, weight distributions very reasonable, see next slides.
- Evolution range from 10GeV to 1TeV
- LO pre-evolution staring from $\delta(1-x)$ at 1GeV to 10GeV provides initial x-distribution for the LO+NLO continuation.
- As before only C_F^2 part of gluonstrahlung.
- Non-ruining α_S.
- Term due to ε part of γ-traces omitted. See next slides.
- NLO virtual corrections omitted.

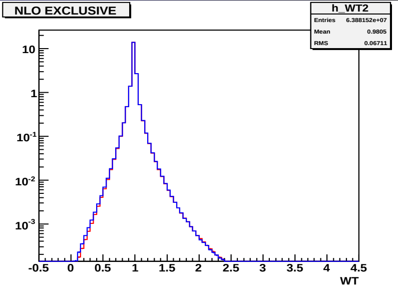
Exclus./Inclusive NLO: Slices in No. of inserts. WITH ε TERM

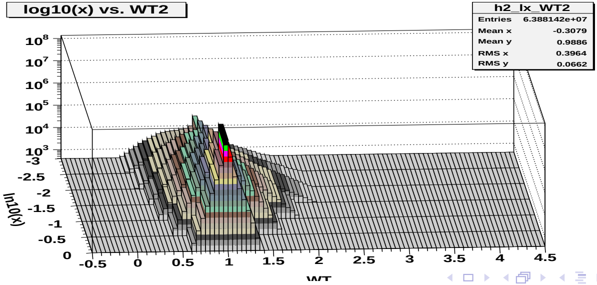




Excellent weight distribution!



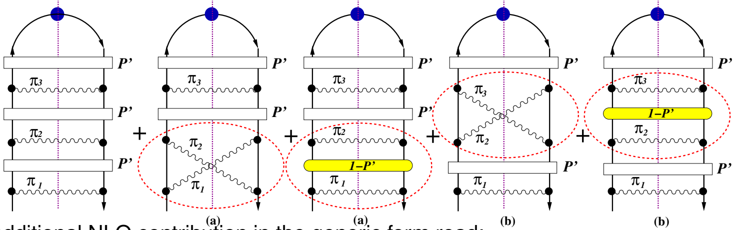




Outline

1st nontrivial case: 3 gluons; 1 NLO insert + 1 LO spectator

Factorization-reorganized diags. contributing to LO+NLO (our subset) for 3 gluons:



The additional NLO contribution in the generic form read:

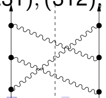
$$D_3^N(t,x) = \frac{1}{3!}\sum_{\{\pi\}} e^{-S} \int \frac{d^3k_{\pi 3}}{2k^0_{\pi 3}} \int \frac{d^3k_{\pi 2}}{2k^0_{\pi 2}} \int \frac{d^3k_{\pi 1}}{2k^0_{\pi 1}}\; \delta_{x_0-x=\alpha_{\pi 1}+\alpha_{\pi 2}+\alpha_{\pi 1}}\; \theta_{e^t>|\mathbf{k}_{\pi_i}|>e^{t_0}}$$

$$\big[\rho^N(k_{\pi 3},k_{\pi 2}|x_1^\pi)\;\rho^L(k_{\pi 1}|x_0)\;\theta_{\max(|\mathbf{k}_{\pi 3}|,|\mathbf{k}_{\pi 2}|)>|\mathbf{k}_{\pi 1}|}$$

$$+\rho^L(k_{\pi 3}|x_2)\;\rho^N(k_{\pi 2},k_{\pi 1}|x_0)\;\theta_{|\mathbf{k}_{\pi 3}|>\max(|\mathbf{k}_{\pi 2}|,|\mathbf{k}_{\pi 1}|)}\big]$$

where the sum is over permutations $\{\pi\}=(123),(213),(132),(231),(312),(321)$:

What is dropped at NLO? The NNLO's like this



To understand better we have to go back to LO, n=3 gluons...
... and re-examine carefully how to get LO from exact matrix element

The $n = 3$ distr. at LO reads (ph.sp. is a simplex in $\mathbf{k}$):

$$D_3^L(t,x) = e^{-S} \int \frac{d^3k_3}{2k_3^0} \int \frac{d^3k_2}{2k_2^0} \int \frac{d^3k_1}{2k_1^0}\; \delta_{1-x=\alpha_1+\alpha_2+\alpha_3}\; \theta_{e^t>|\mathbf{k}_i|>e^{t_0}}$$
$$\rho^L(k_3|x_2)\; \rho^L(k_2|x_1)\; \rho^L(k_1|x_0)\; \theta_{|\mathbf{k}_3|>|\mathbf{k}_2|>|\mathbf{k}_1|},$$

where $\rho^L(k_i|x_{i-1}) \sim \frac{1+z_i^2}{(1-z_i)|\mathbf{k}_i|^2}$.
One small step closer back to Feyn.Diags.$\times$Ph.Sp.:
Ph.Sp. is hyperrectangle $e^t > |\mathbf{k}_i| > e^{t_0}$, explicit BE symmetrization:

$$D_3^L(t,x) = \frac{1}{3!} e^{-S} \int \frac{d^3k_3}{2k_3^0} \int \frac{d^3k_2}{2k_2^0} \int \frac{d^3k_1}{2k_1^0}\; \delta_{1-x=\alpha_1+\alpha_2+\alpha_3}\; \theta_{e^t>|\mathbf{k}_i|>e^{t_0}}$$
$$\rho^L(k_{3*}|x_{2*})\; \rho^L(k_{2*}|x_{1*})\; \rho^L(k_{1*}|x_0).$$

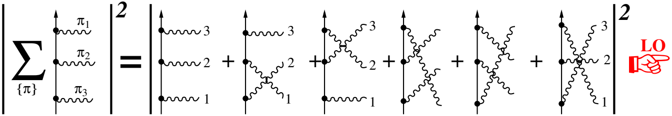
The permutation $\pi_k = (1^*, 2^*, 3^*)$, implicitly depends on all $\mathbf{k}_i$, must obey $|\mathbf{k}_{3*}| > |\mathbf{k}_{2*}| > |\mathbf{k}_{1*}|$. π_k needed to calculate x_i^*.

Trace back to Feyn.Diags.×Ph.Sp. cont.

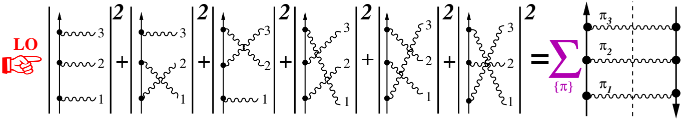
The same with $\sum$ over permutations $\{\pi\} = (123), (213), (132), (231), (312), (321)$:

$$D_3^L(t,x) = e^{-S} \int \frac{d^3k_3}{2k_3^0} \int \frac{d^3k_2}{2k_2^0} \int \frac{d^3k_1}{2k_1^0}\; \delta_{1-x=\alpha_1+\alpha_2+\alpha_3}\; \theta_{e^t>|\mathbf{k}_i|>e^{t_0}}$$
$$\frac{1}{3!}\sum_{\{\pi\}} \rho^L(k_{\pi_3}|x_2^\pi)\; \rho^L(k_{\pi_2}|x_1^\pi)\; \rho^L(k_{\pi_1}|x_0)\; \theta_{|\mathbf{k}_{\pi_3}|>|\mathbf{k}_{\pi_2}|>|\mathbf{k}_{\pi_1}|},$$

where $x_1^\pi = x_0 - \alpha_{\pi_1}$, $x_2^\pi = x_0 - \alpha_{\pi_1} - \alpha_{\pi_2}$. In the axial gauge (Lipatov 1974) each term in the above $\sum_\pi$ is the single diagram squared, in the LO approx.

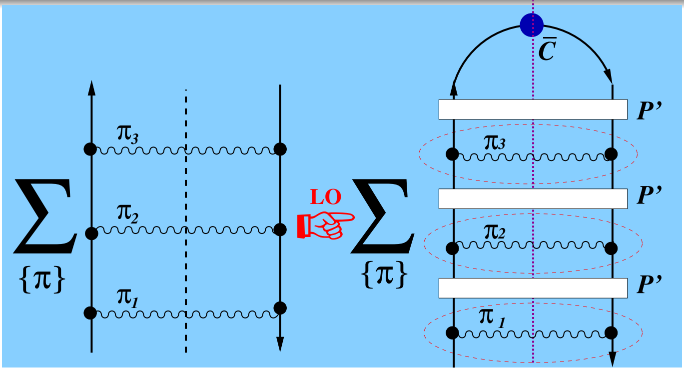


All interferences are at least NLO hence



Three factorials: $\frac{1}{n!}\sum_{\{\pi\}}\int_{\rm PhSp} = \frac{1}{n!}\; n!\; \frac{L^n}{n!}$. (Bose, FeynDiags., PhSp.)

Last step: Projection operator P' of the coll. factorization



- Projection operator $\mathbf{P}'$ is (almost) that of the collinear QCD factorization, but not quite, see other slides at http://jadach.web.cern.ch/
- $\bar{\mathbf{C}}$ closes phase space from the above $\max_i(q_i) < Q$, and closes spin(or) index. It mimics the phase space and ME of the hard process.
- Finally the distribution of the LO Monte Carlo (RGE-like) is obtained: $\rho^L(k_{\pi_3}|x_2^\pi)\ \rho^L(k_{\pi_2}|x_1^\pi)\ \rho^L(k_{\pi_1}|x_0)\ \theta_{|\mathbf{k}_{\pi_3}|>|\mathbf{k}_{\pi_2}|>|\mathbf{k}_{\pi_1}|}$.

1 NLO insert for n=3; 1 LO spectator, in fine detail

Inclusive		Exclusive		
○ ○ ○	$\rightarrow$	○ ○ ○	ρ_3^L	leading order
● ○	$\rightarrow$	● ● ○	ρ_{3a}^N	1nlo + 1lo spector after
○ ●	$\rightarrow$	○ ● ●	ρ_{3b}^N	1nlo + 1lo spector before
	$\rightarrow$	● ○ ●	ρ_{3b}^N	spillover!!!

$$D_3^{L+N}(t,x) = \frac{1}{3!} e^{-S} \int \frac{d^3k_3}{2k_3^0} \int \frac{d^3k_2}{2k_2^0} \int \frac{d^3k_1}{2k_1^0}$$

$$\times\, \delta_{x_0 - x = \alpha_1+\alpha_2+\alpha_3} \prod_i \theta_{e^t > |\mathbf{k}_i| > e^{t_0}}\ \rho_3^{L+N}(k_3,k_2,k_1),$$

$$\rho_3^{L+N}(k_3,k_2,k_1) = \sum_\pi \Big(\rho_3^L(k_{\pi_3},k_{\pi_2},k_{\pi_1}) + \rho_{3a}^N(k_{\pi_3},k_{\pi_2},k_{\pi_1}) + \rho_{3b}^N(k_{\pi_3},k_{\pi_2},k_{\pi_1}) \Big),$$

$$\rho_3^L(k_3,k_2,k_1) = \rho^L(k_3|x_2)\ \rho^L(k_2|x_1)\ \rho^L(k_1|x_0)\ \theta_{|\mathbf{k}_3|>|\mathbf{k}_2|>|\mathbf{k}_1|},$$

$$\rho_{3a}^N(k_3,k_2,k_1) = \rho^L(k_3|x_2)\ b_2^{\theta N}(k_2,k_1|x_0)\ \theta_{|\mathbf{k}_3|>|\mathbf{k}_2|},$$

$$\rho_{3b}^N(k_3,k_2,k_1) = b_2^{\theta N}(k_3,k_2|x_1)\ \rho^L(k_1|x_0)\ \theta_{|\mathbf{k}_3|>|\mathbf{k}_1|}.$$

and for convenience we redefine $b_2^{\theta N}(k_b,k_a|x_0) \equiv 2!\ \theta_{|\mathbf{k}_b|>|\mathbf{k}_a|}\ b_2^N(k_b,k_a|x_0)$.

TECHNICAL PROBLEM:
ρ_{3b}^N admits both "regular" $|\mathbf{k}_3| > |\mathbf{k}_2| > |\mathbf{k}_1|$ and 'spill-over" $|\mathbf{k}_3| > |\mathbf{k}_1| > |\mathbf{k}_2|$ configurations. Can we cope with that?

THE SOLUTION: Exclusive 1 NLO insert for n=3; MC weight

At a given point in the phase space, from the sum over $3! = 6$ permutations in NLO distribution, for ρ^N_{3a} only one $\pi^\bullet = (1,2,3)$ survives
and for ρ^N_{3b} the two $\pi^\bullet = (1,2,3),(2,1,3) = \pi^\bullet_a, \pi^\bullet_b$ contribute.
The resulting MC weight is:

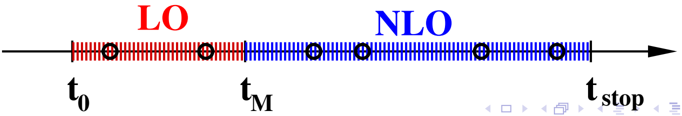
$$w = 1 + w^N_{3a} + w^N_{3b}$$

$$w^N_{3a} = \frac{b_2^{\theta N}(\tilde{k}_2, \tilde{k}_1|x_0)}{\rho^L(\tilde{k}_2|x_1)\ \rho^L(\tilde{k}_1|x_0)}\theta_{\tilde{t}_2 > t_M},$$

$$w^N_{3b} = \frac{b_2^{\theta N}(\tilde{k}_3, \tilde{k}_2|x_1)}{\rho^L(\tilde{k}_3|x_2)\ \rho^L(\tilde{k}_2|x_1)}\theta_{\tilde{t}_3 > t_M} + \frac{b_2^{\theta N}(\tilde{k}_3, \tilde{k}_1|x_1^{\pi^\bullet_b})}{\rho^L(\tilde{k}_3|x_2)\ \rho^L(\tilde{k}_1|x_0)}\ \frac{\rho^L(\tilde{k}_2|x_0)}{\rho^L(\tilde{k}_2|x_1)}\theta_{\tilde{t}_3 > t_M},$$

where $x_1^{\pi^\bullet_b} = x_0 - \alpha_2$.

Finally, the additional cutoff delaying the onset of NLO corrections to $t > t_M$ is also introduced.



Summary and Prospects

- First serious feasibility study of the true NLO exclusive MC parton shower is under construction, well advanced...
- What next? Workplan well defined:
- Short range aim: Complete non-singlet.
- Middle range aim: Complete singlet.
- Speed up the MC weight calculation.
- Better documentation needed on what was done.
- NLO MC for W/Z production for LHC, including SANC electroweak library.
- NLO MC for DIS@HERA and an example of BSM processes at LHC

DGLAP Collinear QCD ISR Evolution and Monte Carlo. The state of art.

1970 1980 1990 2000 2010

LO

Moments OPE: (74) QCD: Georgi+Politzer

Diagramatic: (72) QED: Gribov+Lipatov; (77) Altarelli+Parisi

Monte Carlo: 10 years → (85) Sjostrand; (88) Marchesini,Webber

NLO

Moments OPE: (78) Floratos+Ross+Sachrajda

Diagramatic: (81) Curci+Furmanski+Petronzio

Monte Carlo: 27 years later → (08) Jadach Skrzypek

WE ARE HERE!!!

NNLO

Moments: (03) Moch+Verm.+Vogt

Diagramatic: (03) Moch+Verm.+Vogt

Monte Carlo: (15) ???