Fully Unintegrated Parton Correlation Functions and Factorization

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Discussion of:

- **Collins, TCR, Stasto, PRD77:085009,2008**
- **TCR, PRD78:074018,2008**

DESY MC discussion meeting – May, 11,2009

Factorization

- QCD: Fundamental microscopic theory of strong interactions.

- Asymptotic Freedom: Over short distances, strong coupling is small:

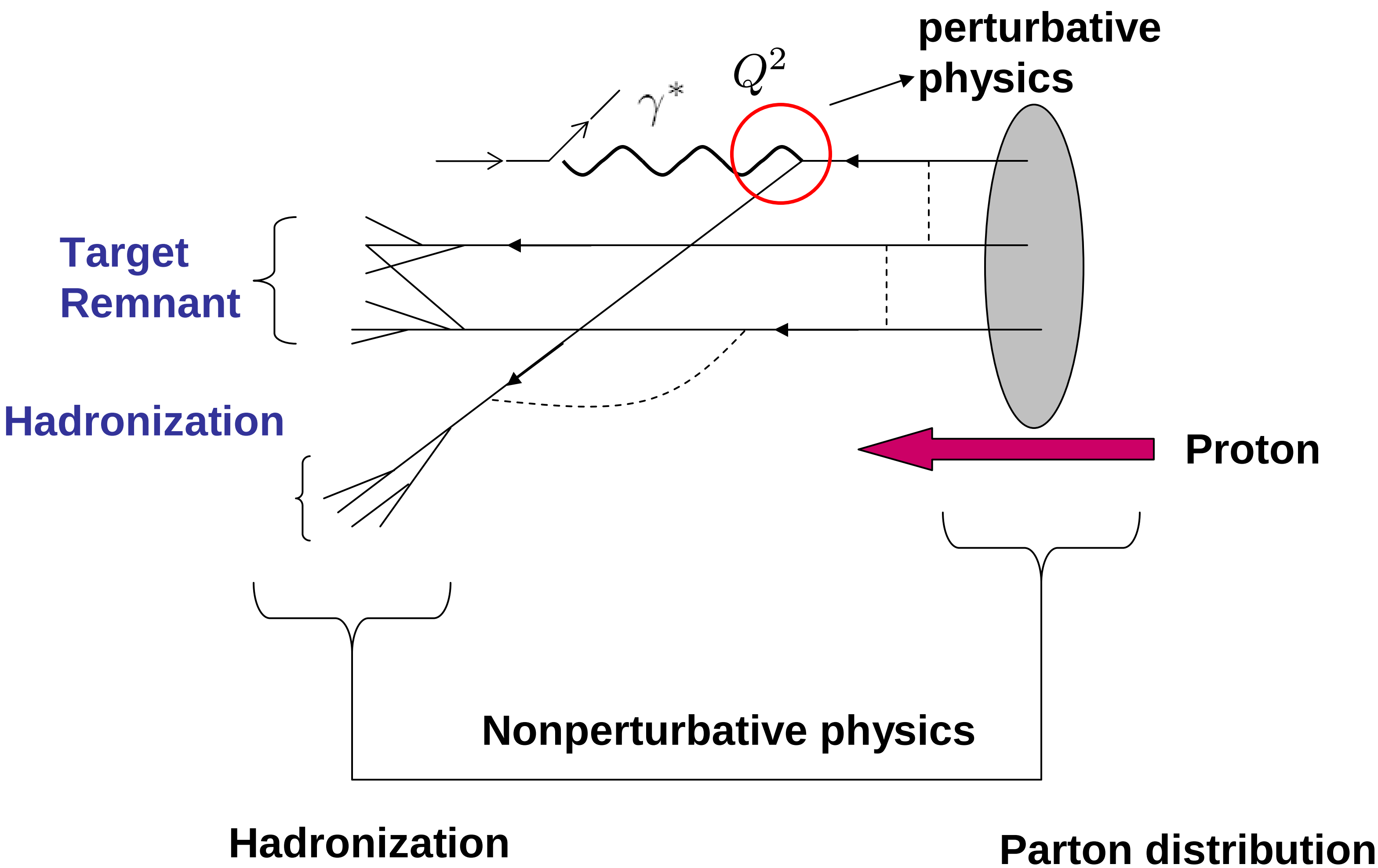
$$d \sim 1/Q << 1/\Lambda_{QCD} \Longrightarrow \alpha_s(Q^2) << 1$$

- Any collision with hadrons involves both long and short distance scales.

- Must be possible to systematically separate any QFT calculation into large scale and short scale pieces.

Classic Case: DIS

Conventional Parton Model Intuition



Factorization

- Statement of a factorization theorem.

$$\sigma = C \otimes f + \mathcal{O}\left(\left(\Lambda/Q\right)^a |\sigma|\right) \qquad a > 0$$

Hard scattering coefficient

Universal parton distribution function

Power suppressed corrections

(A parton correlation function)

In This Talk:

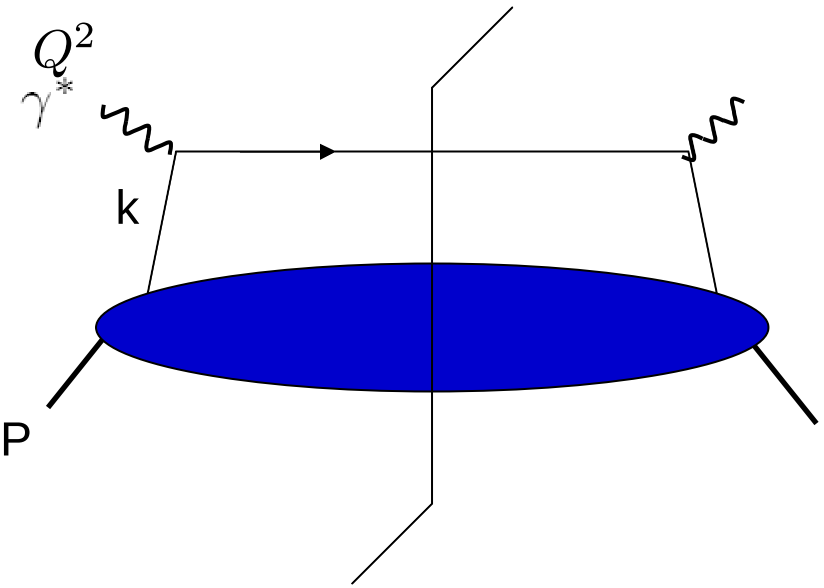
- Very careful review of standard parton model kinematics in DIS – back to basics.
- Argue for a more general treatment of factorization.
- Overview of a factorization formalism that maintains exact overall kinematics.
- Main complications and unsolved problems.
- Relation to parton showers, resummation, etc… ?

Types of PDFs

DIS – Naïve Handbag Diagram

$$k = (k^+, k^-, \mathbf{k}_\perp)$$

$\sim Q$ $\quad \sim \Lambda^2/Q$ $\quad \sim \Lambda$



- What should be used for k in hard scattering?
  - Can categorize level of approximation by the type of PDF that is used.

Types of PDFs

- Integrated PDFs:
  - Standard x-dependent PDFs - k_T and k^2 are integrated over in the definition.
  - Well-known operator definitions of classic factorization theorems.

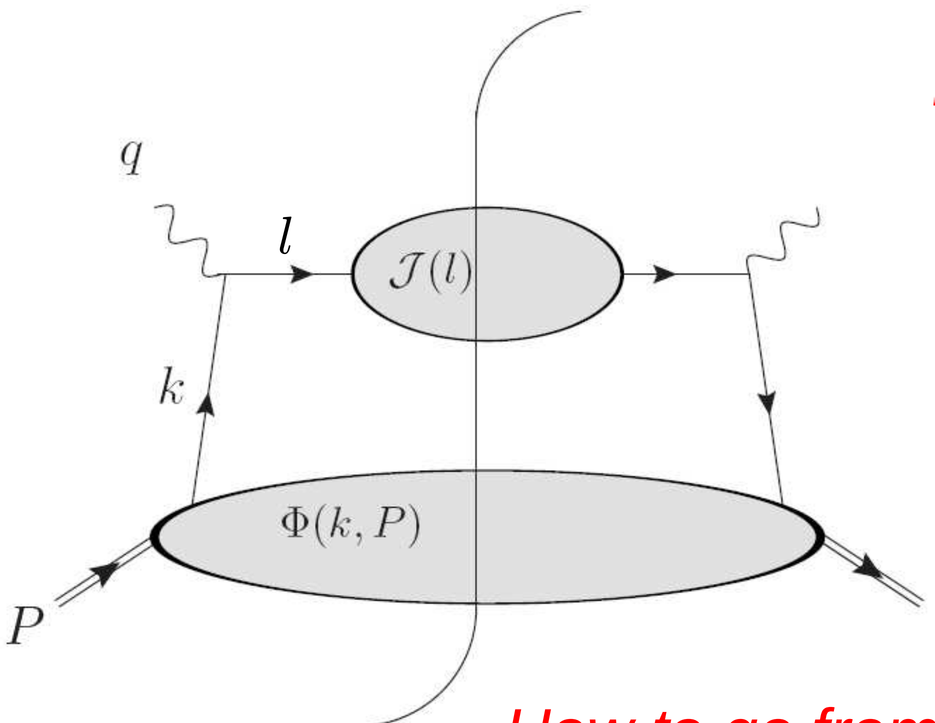
- Unintegrated PDFs:
  - Depend on k_T, *but still integrated over invariant energy*.
  - Some open problems with operator definitions, and problems with factorization in hadron-hadron collisions.

- Parton Correlation Functions (Including *Fully Unintegrated* PDFs):
  - Differential in all components of four-momentum.
  - Refers to *fully* unintegrated PDFs as well as jet-factors, and soft factors.

Unapproximated LO graph

(Additional soft/collinear gluons to be considered later.)

$$W^{\mu\nu}(q,P) = \sum_j \frac{e_j^2}{4\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma^\mu \mathcal{J}(k+q)\gamma^\nu \Phi(k,P)]$$



large component — small component

$$P = \left(P^+, \frac{M^2}{2P^+}, \mathbf{0}_T \right)$$

$$q = \left(-xP^+, \frac{Q^2}{2xP^+}, \mathbf{0}_T \right)$$

How to go from this to the parton model?

Reproducing the basic parton model:

- **Utilize parton model kinematics:**
  - $k = (k^+, k^-, \mathbf{k}_T)$
  - $l = k + q = \left(k^+ - xP^+, \frac{Q^2}{2xP^+} + k^-, \mathbf{k}_T\right)$
  - $\text{small } l^2 \text{ means } k^+ \approx xP^+$
- **Inside target bubble write:** $k \rightarrow (xP^+, k^-, \mathbf{k}_T)$
- **Inside the jet bubble write:** $l \rightarrow \left(l^+, \frac{Q^2}{2xP^+}, \mathbf{0}_T\right)$
- **Use parton model values <u>in hard vertex</u>:**

$$k \rightarrow \hat{k} = (xP^+, 0, \mathbf{0}_T)$$
$$l \rightarrow \hat{l} = \left(0, \frac{Q^2}{2xP^+}, \mathbf{0}_T\right)$$

- **Keep only large Dirac matrices.**

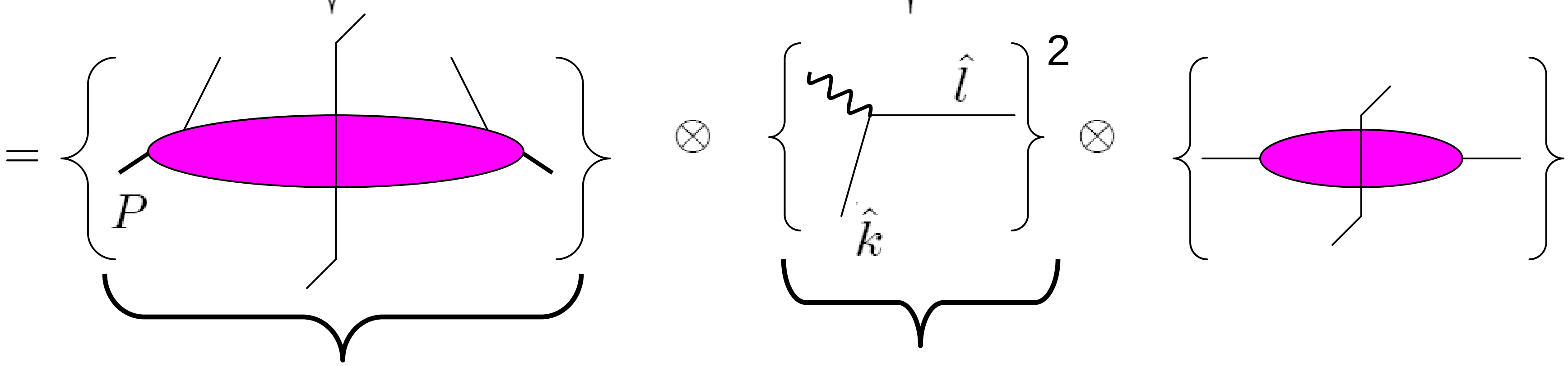
Reproducing the basic parton model:

Set to one by unitarity:

- **Hadronic Tensor Becomes:**

$$W^{\mu\nu}(q,P) \simeq$$

$$\frac{e_j^2}{4\pi}\left\{\int \frac{dk^- d^2\mathbf{k}_T}{(2\pi)^4}\Phi_j^+(x_{Bj}P^+,k^-,\mathbf{k}_T)\right\}\mathrm{Tr}\left[\gamma^\mu\gamma^+\gamma^\nu\gamma^-\right]\left\{\int dl^+ \mathcal{J}_j^-(l^+,q^-,\mathbf{0}_T)\right\}$$



Parton Distribution???

LO partonic structure functions.

- UV divergent – requires renormalization.

Note shift in final state kinematics !!

- Wilson lines needed for gauge invariance.

The Standard PDF

Operator definition:

(Reproduces integral form up to c.t.)

$$f_j(x_{\rm Bj},\mu) \;=\; \int \frac{dw^-}{4\pi} e^{-ix_{\rm Bj}p^+w^-} \langle p|\bar\psi(0,w^-,\mathbf{0}_T) V_w^\dagger(u_{\rm J})\gamma^+ V_0(u_{\rm J})\psi(0)|p\rangle_R$$

$$u_{\rm J} = (0,1,\mathbf{0}_T)$$

Light-like Wilson lines for gauge invariance:

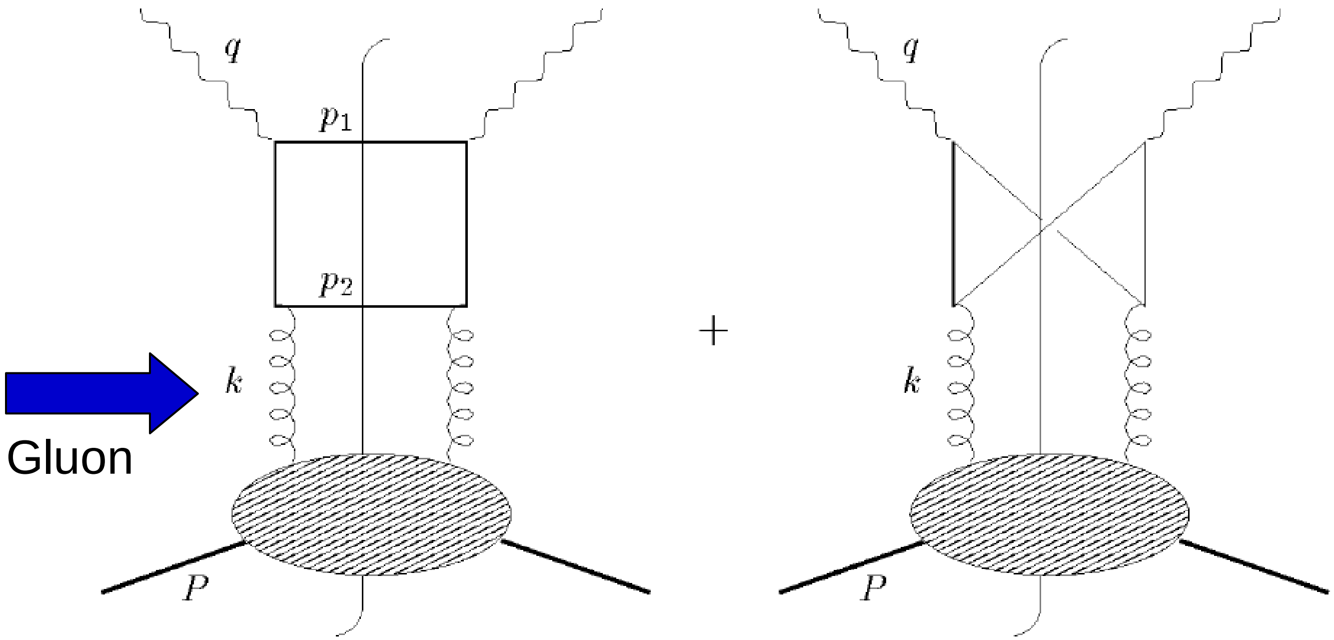
$$V_w(n) = P\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(w+\lambda n)\right)$$

$$V_w^\dagger(u_{\rm J})V_0(u_{\rm J}) = P\exp\left(-ig\int_0^{w^-} d\lambda\, u_{\rm J}\cdot A(\lambda u_{\rm J})\right)$$

How good is standard approx?

Example: $c\bar{c}$ photoproduction

Target Parton = Gluon

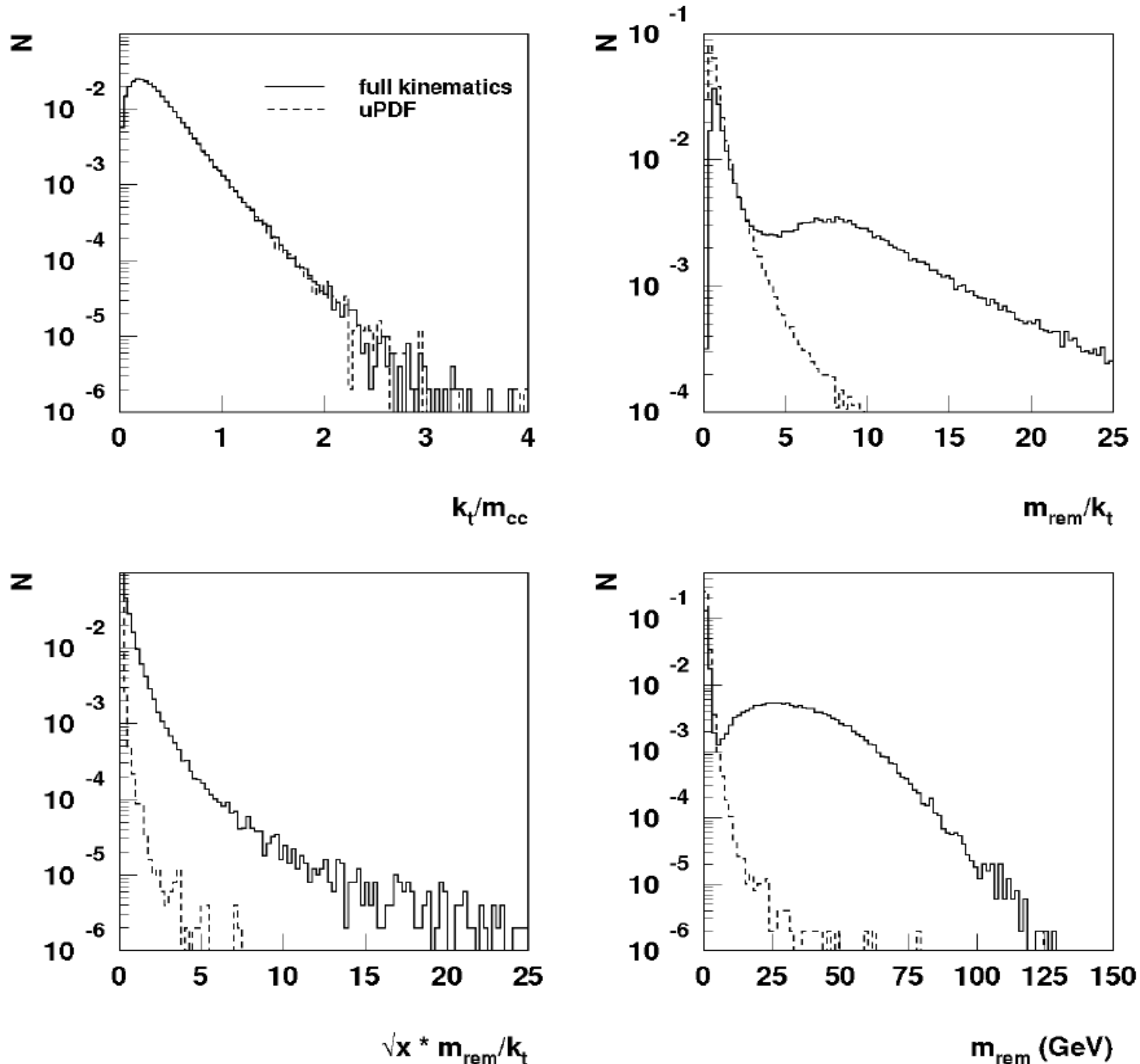


- Try:
  - Keeping k_T dependence, but approximating minus component. (TMD PDFs)
  - Exact kinematics. (Fully unintegrated PDFs)

Errors in final state kinematics

$c\bar{c}$
Pair-production

$$k^2 = -\frac{k_T^2 + x m_{ren}^2}{1-x}$$



(From Collins and Jung, 2005)

Standard Kinematical Approximations

- Re-assignment of final state kinematics – mismatch between true kinematics and parton model approximation.
- Standard factorization works if we look at observables integrated over final state phase space.
- Does not necessarily work point-by-point in phase space.
- In some cases, *exact* kinematics are needed.
- What about factorization?
  - **Need a factorization formula that is accurate point-by-point in initial/final state momentum.**

Proposal:

- Set up new factorization with subtraction formalism using exact over-all kinematics of initial and final states.

  Basic Approach Formalized in: Collins Phys.Rev.D65:094016,2002, Collins, Zu, JHEP 03 (2005) 059

- Maintain explicit factors for final states.

- Non-perturbative factors should depend on all components of four-momentum. (Fully unintegrated parton correlation functions.)

  (PDFs, jet factors, soft factors)

- New approach to factorization is needed.

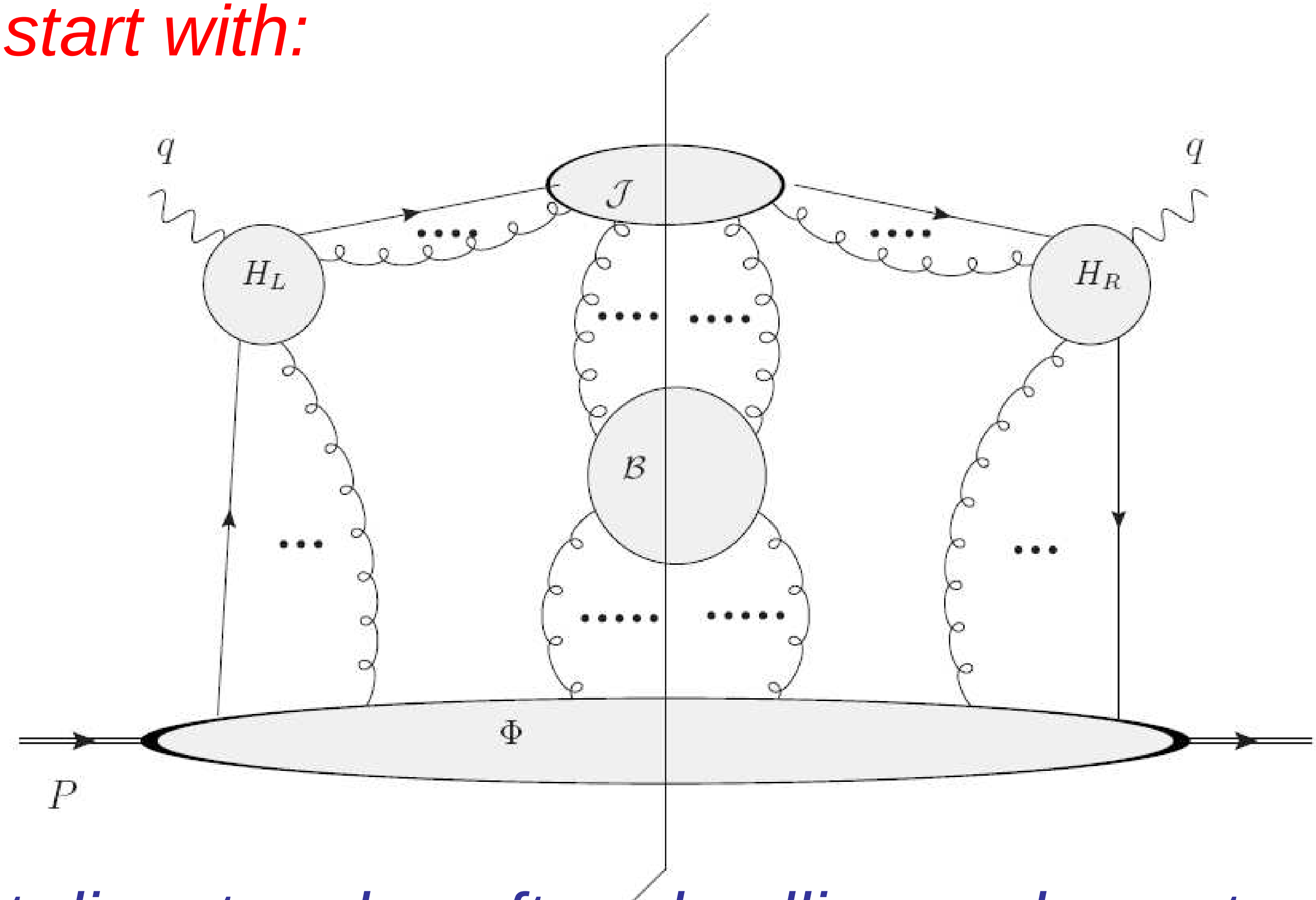
What is needed for factorization?

- General factorization formula with power suppressed corrections point-by-point in final state phase space.
- Well-defined operator definitions for PCFs. Needed for universality.
- Hard scattering matrix calculated with on-shell Feynman graphs.
- Higher orders rely on subtractive formalism – *detailed treatment of lowest order hard scattering needed as a first step.*

General Graphical Structure

(From Collins, TCR, Stasto, PRD77:085009,2008)

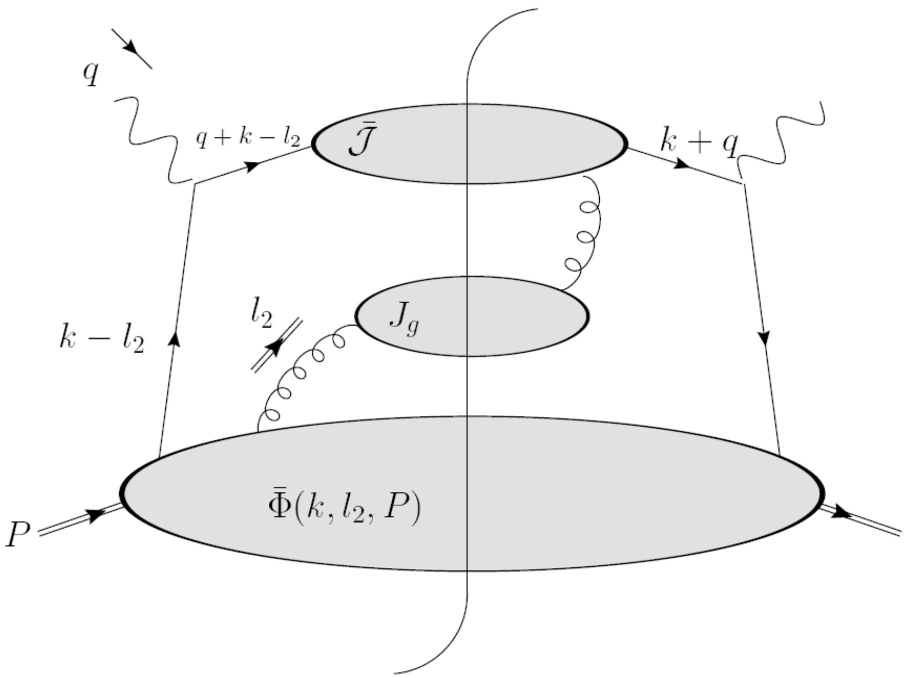
Should start with:

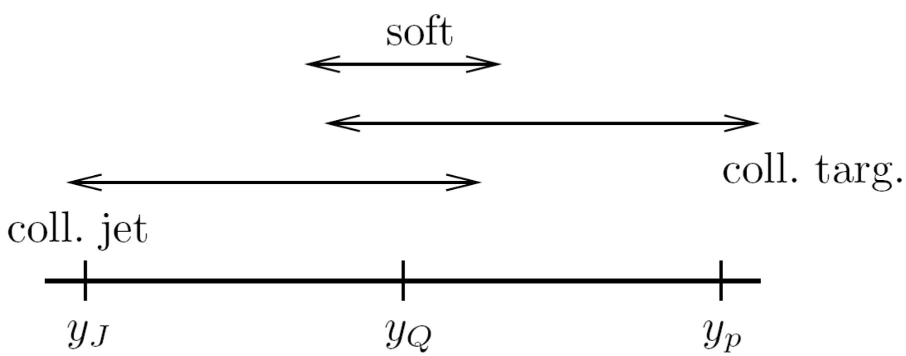


Must disentangle soft and collinear gluons to get topological factorization…

Graphical example:

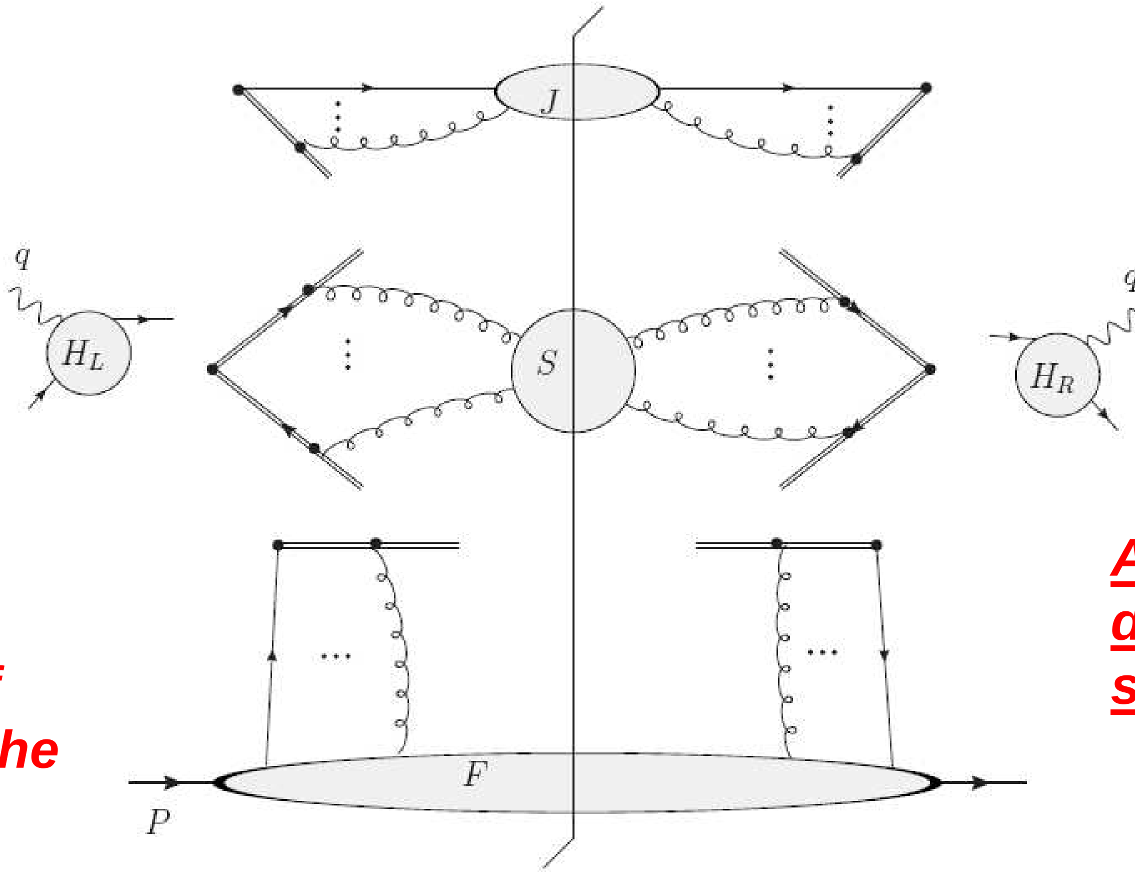
Single extra gluon





Range of gluon rapidity

Topological Factorization:



Also need double counting subtractions.

A formula of this type is the goal.

$$\sigma = C \otimes F \otimes J \otimes S + \mathcal{O}((\Lambda/Q)^a|\sigma|), \qquad a > 0$$

Full Factorization

$$P_{\mu\nu}W^{\mu\nu} = \int \frac{d^4k_{\mathrm{T}}}{(2\pi)^4}\frac{d^4k_{\mathrm{J}}}{(2\pi)^4}\frac{d^4k_S}{(2\pi)^4}(2\pi)^4\delta^{(4)}(q+P-k_{\mathrm{T}}-k_{\mathrm{J}}-k_S)\times$$
$$\times\, |H(Q,\mu)|^2\ S(k_S, y_{\mathrm{T}}, y_{\mathrm{J}}, \mu)\ \underbrace{F_{\mathrm{mod}}(k_{\mathrm{T}}, y_p, y_{\mathrm{T}}, y_s, \mu)}\ \underbrace{J_{\mathrm{mod}}(k_{\mathrm{J}}, y_{\mathrm{J}}, y_s, \mu)}$$

Fourier Transform

$$\tilde{F}_{\mathrm{mod}}(w, y_p, y_{\mathrm{T}}, y_s, \mu) = \frac{\langle p|\bar{\psi}(w)V_w^{\dagger}(n_s)I_{n_s;w,0}\frac{\gamma^+}{2}V_0(n_s)\psi(0)|p\rangle_R}{\langle 0|I^{\dagger}_{n_{\mathrm{T}};w,0}V_w(n_{\mathrm{T}})V_w^{\dagger}(n_s)I_{n_s;w,0}V_0(n_s)V_0^{\dagger}(n_{\mathrm{T}})|0\rangle_R}$$

$$\tilde{J}_{\mathrm{mod}}(w, y_{\mathrm{J}}, y_s, \mu) = \frac{\langle 0|\bar{\psi}(w)V_w^{\dagger}(-n_s)I_{-n_s;w,0}\gamma^- V_0(-n_s)\psi(0)|0\rangle_R}{\langle 0|I^{\dagger}_{-n_s;w,0}V_w(-n_s)V_w^{\dagger}(n_{\mathrm{J}})I_{n_{\mathrm{J}};w,0}V_0(n_{\mathrm{J}})V_0^{\dagger}(-n_s)|0\rangle_R}$$

Summary so far

- Looking at details of final states requires precise treatment of kinematics.
- Full factorization for scalar theory.
- Detailed treatment of factorization for the case of a single outgoing jet.
- Derived formula using candidate operator definitions for the PCFs.
- Account for arbitrary number of gluon exchanges (in the Abelian case).

Fully Unintegrated Approach

Advantages

- **Generality.**
- **Needed for complicated events and for details of final states.**
- **Hard scattering is an ordinary function – not generalized function (e.g. delta-functions/plus-distribution).**
- **Starting point to address complications with unintegrated PDFs.**

Disadvantages

- **Very complicated – much theoretical work is still needed.**
- **PCFs depend on too many parameters. Difficult to use for practical applications.**
- **Complications with ward identities for non-Abelian gauge theory**

Next-to-leading order

- With the detailed treatment of LO, can obtain NLO expression directly via subtraction formalism – we now know exactly what to subtract.
- Double counting subtractions should give factorized expression with power suppressed corrections point-by-point in phase.
- Explicit mapping from exact to approximate momentum variables is needed. It should be chosen consistent with point-by-point factorization.
- For now get NLO hard scattering q-qbar production graphs.

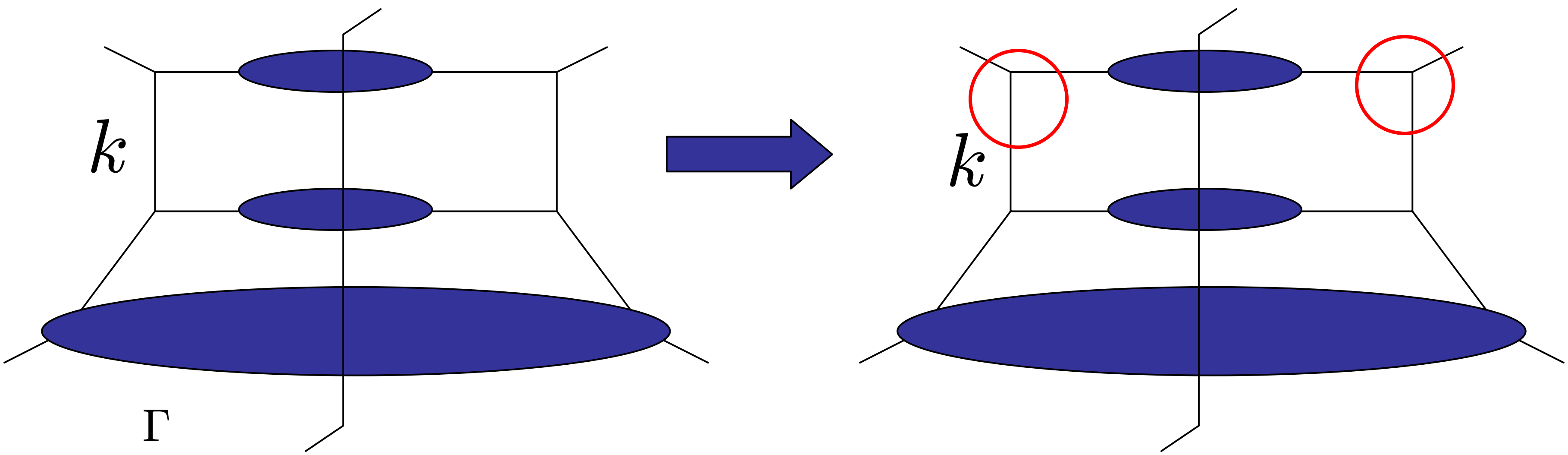
(TCR, PRD78:074018,2008)

Higher Order Hard Scattering

(Overview of subtraction approach)

- Explicit implementation of subtractive formalism:

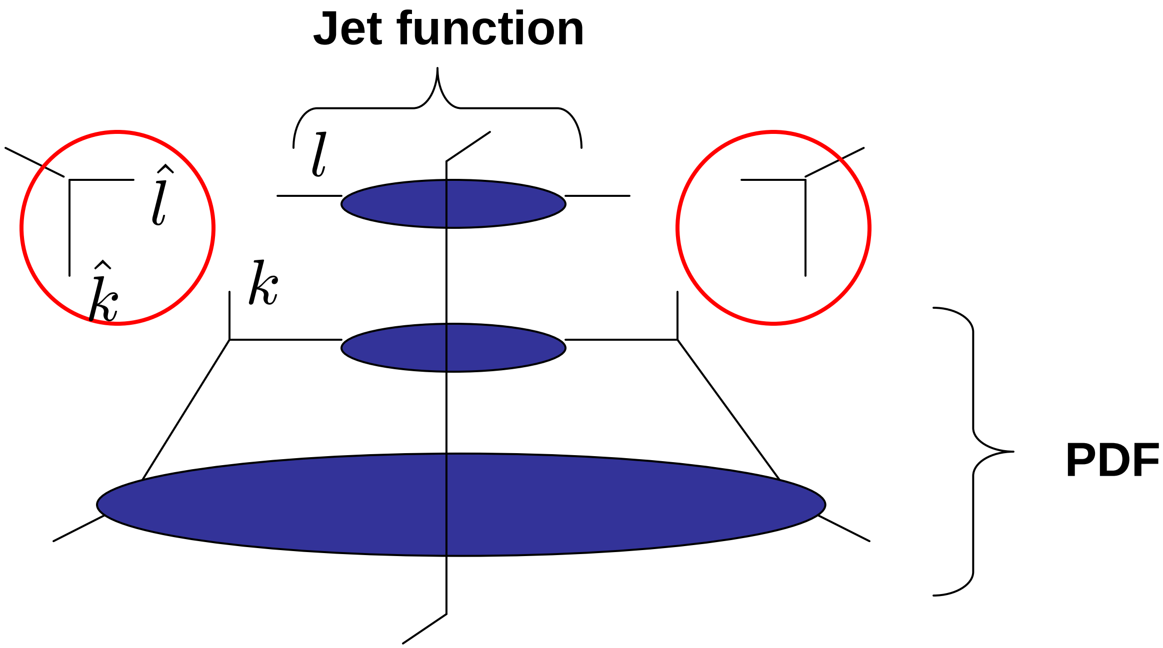
  $\phi^3_{(6)}$ - Scalar theory



$$\Gamma = T_{LO}\Gamma + \mathcal{O}((\Lambda/Q)\Gamma) + \mathcal{O}(\alpha_s(Q^2))$$

Higher Order Hard Scattering

- Mapping of exact to approximate momentum variables:

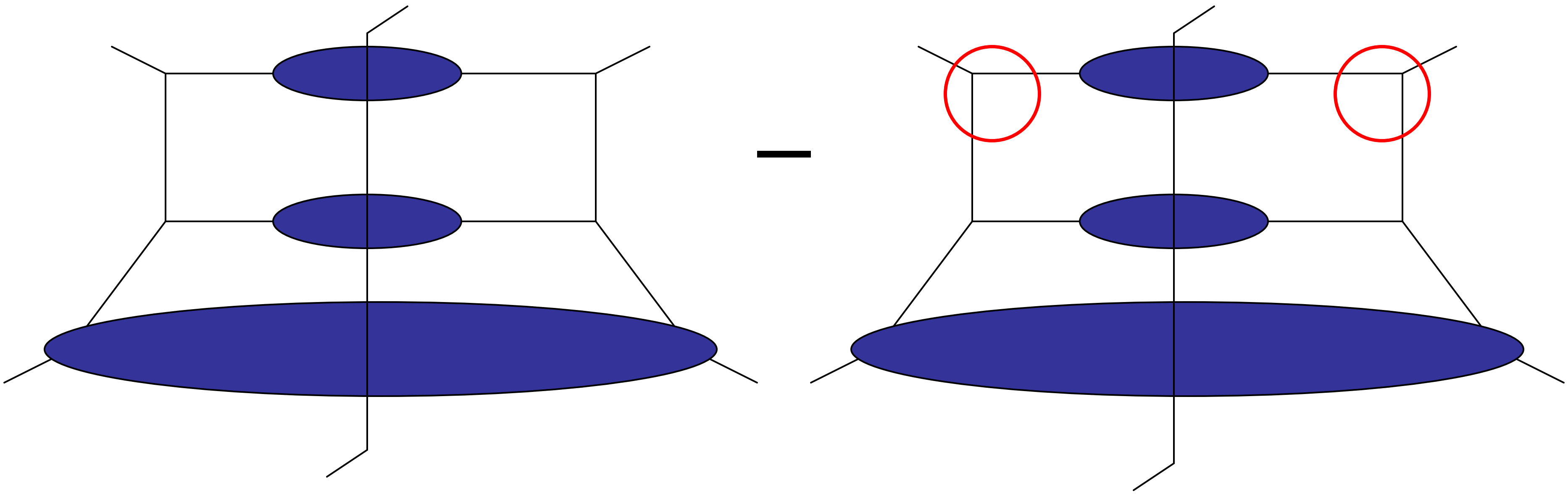


$$\hat{k} = (x_B P^+, 0, \mathbf{0}); \qquad \hat{l} = (0, q^-, \mathbf{0})$$

$$\sigma \sim C \otimes F(k, P) \otimes J_1(l)$$

Higher Order Hard Scattering

- Error in LO calculation:

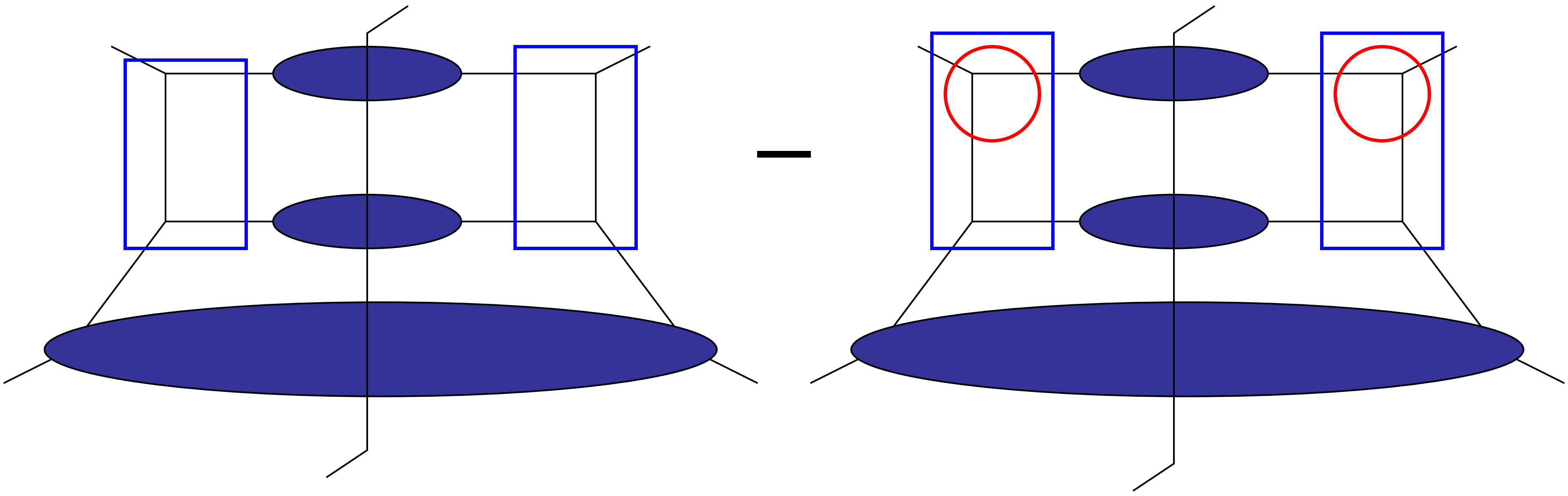


$$\Gamma = T_{LO}\Gamma + \underbrace{(\Gamma - T_{LO}\Gamma)}$$

Non-vanishing for wide angles.

NLO Hard Scattering

- Apply approximation appropriate for wide-angle regime:

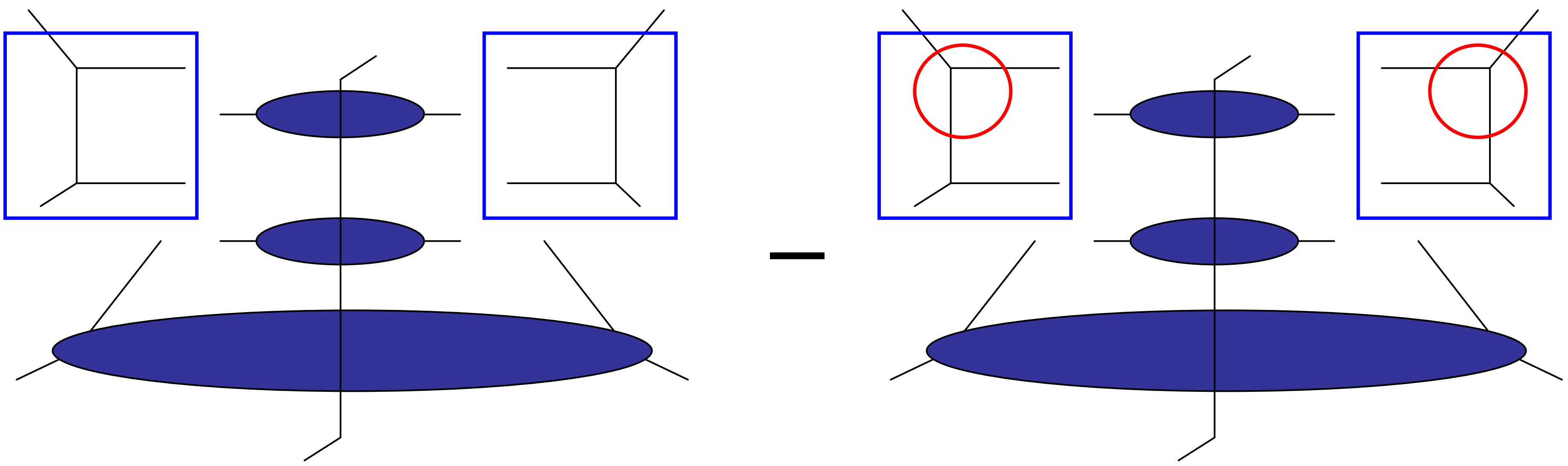


$$\Gamma = T_{LO}\Gamma + \underbrace{T_{NLO}(\Gamma - T_{LO}\Gamma)} + \mathcal{O}((\Lambda/Q)\Gamma) + \mathcal{O}(\alpha_s(Q^2)^2)$$

Calculate error term using approximation appropriate for wide angles.

NLO Hard Scattering

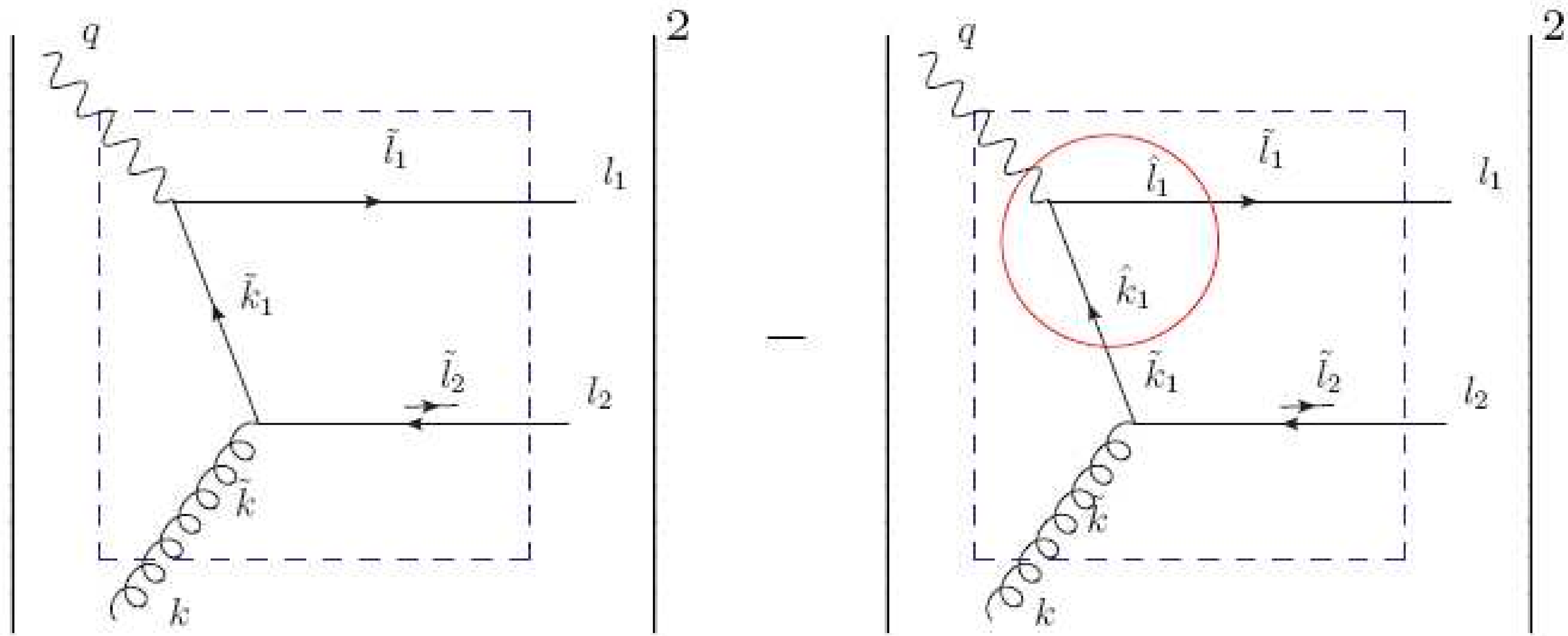
- Apply NLO approximations:



$$\sigma \sim C_{NLO} \otimes F(k, P) \otimes J_1(l) \otimes J_2(l)$$

NLO Hard Scattering

- Structure of fully unintegrated hard scattering coefficient: layered approximations.



NLO Hard Scattering

- Explicit expressions have been obtained for including crossed diagrams and antiquark target.

- Mapping from exact to approximate variables in wide angle region:

$$\tilde{k} = \left(-q^+ + \frac{\tilde{l}_{1,t}^2}{2\tilde{l}_1^-} + \frac{\tilde{l}_{2,t}^2}{2\tilde{l}_2^-}, 0, \mathbf{0}_t \right)$$

$$\tilde{l}_1 = \left(\frac{\tilde{l}_{t,1}^2}{2\tilde{l}_1^-}, \tilde{l}_1^-, \tilde{\mathbf{l}}_{1,t} \right)$$

$$\tilde{l}_2 = \left(\frac{\tilde{l}_{t,2}^2}{2\tilde{l}_2^-}, \tilde{l}_2^-, \tilde{\mathbf{l}}_{2,t} \right)$$

$$\tilde{\mathbf{l}}_{1,t} = \mathbf{l}_{1,t} - \mathbf{k}_t/2$$

$$\tilde{l}_1^- = l_1^- - k^-/2$$

$$\tilde{\mathbf{l}}_{2,t} = \mathbf{l}_{2,t} - \mathbf{k}_t/2$$

$$\tilde{l}_2^- = l_2^- - k^-/2$$

Open problems / future work

- Evolution equations?
  - Relation to CSS formalism?
  - Parton Showering

- Recovery of other approaches in appropriate limits? (e.g. BFKL, CCFM, etc…)

- Extension to other NLO processes.

- Non-Abelian Ward identities?

- Hadron-Hadron collisions?