Phase space effects on dipole showers and the DGLAP equation

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Is the Dipole Cascade Inconsistent with DGLAP Evolution?

DGLAP: Renormalization group equation. A single scale.

Jet evolution: Scale = virtuality $\sim p_\perp^2$. Energy $\approx \infty$

Dipole cascade: Two scales: Dipole mass and virtuality $\sim p_\perp^2$

In e^+e^--annihilation high $p_\perp$ gluons have a very limited range in rapidity

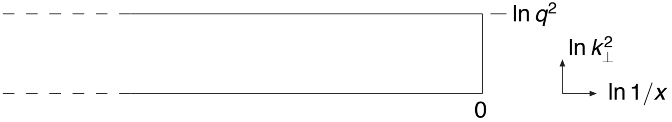
$\Rightarrow$ Phase space limitations are important

DGLAP evolution in timelike cascades

Renormalization group equation describes the modification when a single scale parameter is varied.

Example: Gluon emission in a jet with virtuality M but infinite energy (i.e. energy $>> M$)

In e^+e^--annihilation this corresponds to events where there is no jet with transverse momentum $q_t > M$.



$z_g \equiv$ the fraction of the positive lightcone momentum P_+.

The DGLAP evolution equation reads (in LL approximation):

$$D(Q,x) = \delta(1-x) + \frac{\bar{\alpha}}{2}\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\int_0^1\frac{dz}{z}[D(k,x/(1-z)) - D(k,x)] \tag{1}$$

The k^2-integral has a cutoff Q_0^2

The z-integral extends down to 0.

Moments

The contributions to the moments $D_N(Q) = \int dx\, x^N D(Q,x)$ for 1 and 2 loops:

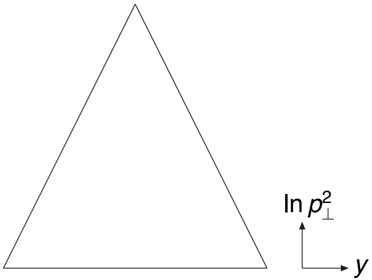
1 loop: $\bar{\alpha} \ln N \ln(Q/Q_0)$

2 loops: $\frac{1}{2}(\bar{\alpha} \ln N \ln(Q/Q_0))^2$

The sum exponentiates

e^+e^--annihilation

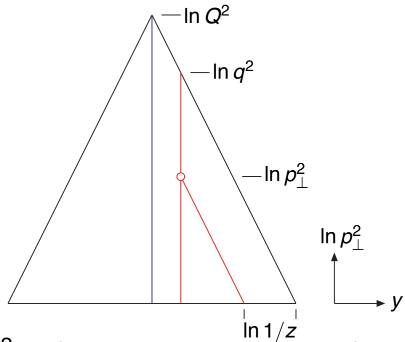
With no restriction on $p_\perp$ ("unbiased" events) phase space boundaries become important



Large $p_\perp \Rightarrow$ limited rapidity range

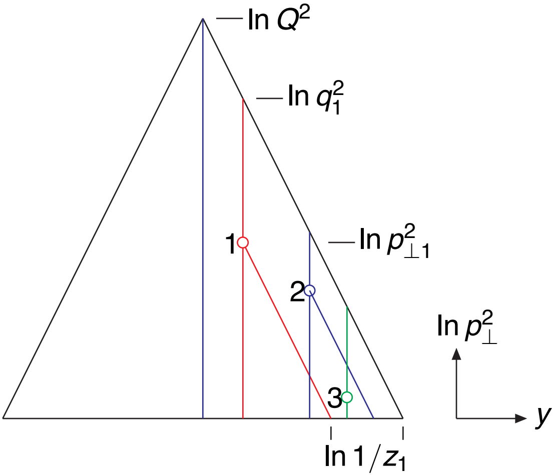
Angular ordered cascade à la HERWIG

(Marchesini-Webber, NPB238, 1, 1984)



q^2 defines the angle (rapidity). Fixed angle $\Rightarrow$ the z-integral gets a lower limit Q_0/q.

Cascade



Integral equation:

$$D(q,x) = \delta(1-x) + \frac{\bar{\alpha}}{2}\int_{Q_0^2}^{q^2}\frac{dk^2}{k^2}\int_{Q_0/k}^{1}\frac{dz}{z}[D(k,x/(1-z)) - D(k,x)]. \tag{2}$$

Note that q or k is not the transverse momentum of an emitted gluon, but the maximum $p_\perp$ possible for that rapidity (angle).

Lower limit for z-integral: Q_0/k

1 loop contribution to the Nth moment (if $N < Q/Q_0$):

$$\bar{\alpha}\int_{1/N}^{1}\frac{dz}{z}\int_{Q_0/z}^{q}\frac{dk}{k} = \bar{\alpha}\int_{1/N}^{1}\frac{dz}{z}\ln(Qz/Q_0)$$
$$= \bar{\alpha}\left(\ln N \ln(Q/Q_0) - \frac{1}{2}\ln^2 N\right) \quad (3)$$

2 loop contribution:

$$\bar{\alpha}^2\int_{Q_0}^{Q}\frac{dk_1}{k_1}\int_{Q_0}^{k_1}\frac{dk_2}{k_2}\int_{Q_0/k_1}^{1}\frac{dz_1}{z_1}\int_{Q_0/k_2}^{1}\frac{dz_2}{z_2}$$
$$= \frac{1}{2}\bar{\alpha}^2\left(\int_{1/N}\frac{dz}{z}\int_{Q_0/z}^{Q}\frac{dk}{k}\right)^2 = \frac{1}{2}(1\ \text{loop})^2 \quad (4)$$

k_t-ordered dipole cascade à la ARIADNE

(G.G.-U.Pettersson NPB306, 746, 1988)

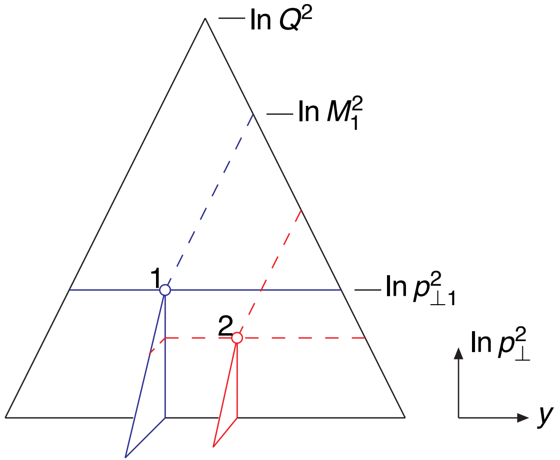
Emissions factorize only for strongly ordered cascades

The event structure is dominated by hardest emissions

Aim: Find best factorizing approximation when the emissions are not strongly ordered

In the k_t-ordered dipole cascade implemented in ARIADNE a dipole emission depends on two scales

Dipole mass and dipole virtuality


$\ln Q^2$
$\ln M_1^2$
1
$\ln p_{\perp 1}^2$
2
$\ln p_\perp^2$
y

The quark recoil is given by

$$D(s, k_{t,max}, x) = \delta(1-x)\, S +$$
$$+ \sum_n \prod_{i=1}^n \left\{ \frac{\bar{\alpha}}{2} \int \frac{dk_{t,i}^2}{k_{t,i}^2} \theta(k_{t,i-1} - k_{t,i}) \int_{k_{t,i-1}^2/(k_{t,i}^2 z_{i-1})}^1 \frac{dz_i}{z_i} S_i \right\} \times$$
$$\times \delta(x - \prod_i (1-z_i)) \theta(k_{t,n} - Q_0)\, S_{n+1} \quad (5)$$

$$S_i = \exp\left\{ -\frac{\bar{\alpha}}{4} \ln(k_{t,i-1}^2/k_{t,i}^2) \ln(k_{t,i-1}^2/(k_{t,i}^2 z_{i-1}^2)) \right\},$$
$$\text{with} \;\; k_{t,0} \equiv k_{t,max}, \;\; k_{t,n+1} \equiv Q_0, \;\; z_0 \equiv k_t^2/s. \quad (6)$$

2 scales $\Rightarrow$ No simple renormalization group equation

No simple exponentiation

1 loop contribution

Single gluon: Full phase space allowed.

$$Q_0 < k_t < Q, \;\; |y| < \ln(Q/k_t) \\ \text{or } k_t^2/Q^2 < z < 1 \tag{7}$$

Change order of integration ⇒

$$\frac{\bar{\alpha}}{2}\int_{1/N}^{1}\frac{dz}{z}\int_{Q_0^2}^{Q^2 z}\frac{dk_t^2}{k_t^2} = \frac{\bar{\alpha}}{2}\int_{1/N}^{1}\frac{dz}{z}\ln(Q^2 z/Q_0^2) \\ = \bar{\alpha}\left(\ln N \ln(Q/Q_0) - \frac{1}{4}\ln^2 N\right). \tag{8}$$

2 loop contribution:

Add contribution from the “forward” dipole $M^2_{\text{forward}} = k^2_{t1}/z_1$

Limits: $k^2_{t1}/Q^2 < z_1 < 1, \;\; k_{t2} < k_{t1}, \;\; k^2_{t2} z_1/k^2_{t1} < z_2 < 1$

Adding contributions from $z_2 > z_1$ and $z_2 < z_1$ gives:

$$\frac{\bar{\alpha}^2}{2}\left(\ln^2(Q/Q_0)\ln^2 N - \frac{2}{3}\ln(Q/Q_0)\ln^3 N + \frac{5}{48}\ln^4 N\right)$$
$$= \frac{\bar{\alpha}^2}{2}\left(\ln N \ln(Q/Q_0) - \frac{1}{4}\ln^2 N\right)\left(\ln N \ln(Q/Q_0) - \frac{5}{12}\ln^2 N\right) \quad (9)$$

Conclusions

In timelike cascades as e^+e^--ann. phase space boundaries are important. Limits emission of soft gluons

More important for quark recoil are non-singular terms in the splitting function and non-factorizing contributions from hard gluon emissions

(See B. Andersson - GG - C. Sjögren, NPB380, 391, 1992)

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