PHYSICS AT THE TERA SCALE
Helmholtz Alliance

QCD vs. MC: Drell-Yan pT distribution

http://www.terascale.de/mc

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Introduction

- What is parton shower?
- Is it theory or just model of the theory?
- Is it able to reproduce know analytic QCD results?
- Do we have freedom to freely choose the free parameters of the shower algorithm?
- If yes, we are happy. If no, what are the constraints, how strict are they, how can we find this constraints, are they general,.....?????
- Marchesini and Dokshitzer have found that the color dipole based showers don't obey the DGLAP evolution. **arXiv:0809.1749** [hep-ph]
- ZN & D. Soper: color dipole model is OK. **arXiv:0901.3587** [hep-ph]
- P. Skands & S. Weinzierl: color dipole model is OK. **arXiv:0903.2150** [hep-ph]

QCD: Drell-Yan process

The NLL expression of the pT distribution was obtained using the renormalization group technique and the result is

$$C = 2e^{-\gamma_E}$$

$$\frac{d\sigma}{d\boldsymbol{p}\,dY} = \int \frac{d\boldsymbol{b}}{(2\pi)^2}\, e^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{b}}$$
$$\times \sum_{a,b} \int_{x_\mathrm{A}}^{1} \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}}\, f_{a/A}\left(\eta_\mathrm{a}, \frac{C^2}{\boldsymbol{b}^2}\right) \int_{x_\mathrm{B}}^{1} \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\, f_{b/B}\left(\eta_\mathrm{b}, \frac{C^2}{\boldsymbol{b}^2}\right)$$
$$\times \exp\left\{-C_\mathrm{F} \int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}^2}{\boldsymbol{k}^2}\, \frac{\alpha_\mathrm{s}(\boldsymbol{k}^2)}{\pi} \left[\log\frac{M^2}{\boldsymbol{k}^2} - \frac{3}{2}\right]\right\}$$
$$\times \sum_{c,d} C_{a,c}\left(\frac{x_\mathrm{A}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}(C^2/\boldsymbol{b}^2)\right) C_{b,d}\left(\frac{x_\mathrm{B}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}(C^2/\boldsymbol{b}^2)\right) H^{(0)}_{c,d}$$

Here

$$C_{q,q'}(z,\alpha_\mathrm{s}) = \delta_{q,q'}\left[\delta(1-z) + \frac{\alpha_\mathrm{s}}{2\pi} \underbrace{\left(C_\mathrm{F}\,(1-z) + \frac{1}{3}\delta(1-z)[\pi^2 - 8]\right)}_{\text{Next-next-leading log}}\right]$$

and

$$C_{q,\mathrm{g}}(z,\alpha_\mathrm{s}) = \mathcal{O}(NNLL)$$

Parton Shower

First we have to define the transverse momentum distribution in a parton shower calculation:

$$
\begin{aligned}
\big(\boldsymbol{p}, Y \big| \rho(t)\big) = \sum_m \frac{1}{m!} \int \big[d\{p,f,s',c',s,c\}_m\big] \big\langle \{s'\}_m \big| \{s\}_m \big\rangle \big\langle \{c'\}_m \big| \{c\}_m \big\rangle \\
\times\, \delta(\boldsymbol{p} - \boldsymbol{D}(p_{\rm A}, p_{\rm B}) \cdot p_{\rm V})\, \delta\!\left(Y - \frac{1}{2}\log\frac{p_{\rm V}\cdot p_{\rm B}}{p_{\rm V}\cdot p_{\rm A}}\right) \\
\times\, \big(\{p,f,s',c',s,c\}_m \big| \rho(t)\big) \ .
\end{aligned}
$$

Its integral gives the total cross section

$$
\int_{-\infty}^{\infty} dY \int d^2\boldsymbol{p}\, \big(\boldsymbol{p}, Y \big| \rho(t)\big) = \big(1 \big| \rho(t)\big)
$$

Putting this measurement function into a shower algorithm

Shower Equation

In the parton shower calculation the energy distribution function is the solution of the the following equations:

$$\frac{d}{dt}\big(\hat{\boldsymbol{p}}, Y\big|\rho(t)\big) = \big(\hat{\boldsymbol{p}}, Y\big|\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)\big|\rho(t)\big)$$

or in another form

$$\begin{aligned}\frac{d}{dt}\big(\hat{\boldsymbol{p}}, \hat{Y}\big|\rho(t)\big) = \sum_m \frac{1}{m!}\int [d\{p, f, s', c', s, c\}_m] \\ \times \big(\hat{\boldsymbol{p}}, \hat{Y}\big|\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)\big|\{p, f, s', c', s, c\}_m\big) \\ \times \big(\{p, f, s', c', s, c\}_m\big|\rho(t)\big) \quad .\end{aligned}$$

Splitting Kernel

And a nice expression for the splitting kernel

$$
\begin{aligned}
&\big(\hat{\boldsymbol{p}}, \hat{Y}\big|\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)\big|\{p,f,s',c',s,c\}_m\big) = \\
&\quad \sum_l \sum_{\zeta_{\rm f}\in\Phi_l(f_l)} \int d\zeta_{\rm p}\ \theta(\zeta_{\rm p}\in\varGamma_l(\{p\}_m,\zeta_{\rm f}))\,\delta\Big(t - T_l(\{\hat{p},\hat{f}\}_{m+1})\Big) \\
&\quad \times \frac{n_{\rm c}(a)n_{\rm c}(b)\,\eta_{\rm a}\eta_{\rm b}}{n_{\rm c}(\hat{a})n_{\rm c}(\hat{b})\,\hat{\eta}_{\rm a}\hat{\eta}_{\rm b}}\ \frac{f_{\hat{a}/A}(\hat{\eta}_{\rm a},\mu_F^2)f_{\hat{b}/B}(\hat{\eta}_{\rm b},\mu_F^2)}{f_{a/A}(\eta_{\rm a},\mu_F^2)f_{b/B}(\eta_{\rm b},\mu_F^2)}\,\big\langle\{s'\}_m\big|\{s\}_m\big\rangle \\
&\quad \times \left[\delta(\hat{\boldsymbol{p}} - \boldsymbol{D}\cdot\hat{p}_{\rm V})\,\delta\!\left(\hat{Y} - \frac{1}{2}\log\frac{\hat{p}_{\rm V}\cdot p_{\rm B}}{\hat{p}_{\rm V}\cdot p_{\rm A}}\right) - \delta(\hat{\boldsymbol{p}} - \boldsymbol{D}\cdot p_{\rm V})\,\delta\!\left(\hat{Y} - \frac{1}{2}\log\frac{p_{\rm V}\cdot p_{\rm B}}{p_{\rm V}\cdot p_{\rm A}}\right)\right] \\
&\quad \times \Big\{\theta(\hat{f}_{m+1}\neq {\rm g})\ C(\hat{f}_l,\hat{f}_{m+1})\,\big\langle\{c'\}_m\big|\{c\}_m\big\rangle\,\overline{w}_{ll}(\{\hat{f},\hat{p}\}_{m+1}) \\
&\qquad + \theta(\hat{f}_{m+1} = {\rm g})\sum_{k\neq l}\big\langle\{c'\}_m\big|t_k(f_k\to\hat{f}_k+\hat{f}_{m+1})\,t_l^\dagger(f_l\to\hat{f}_l+\hat{f}_{m+1})\big|\{c\}_m\big\rangle \\
&\qquad\quad \times\Big[2A_{lk}(\{\hat{p}\}_{m+1})\overline{w}_{lk}(\{\hat{f},\hat{p}\}_{m+1}) - \overline{w}_{ll}(\{\hat{f},\hat{p}\}_{m+1})\Big]\Big\}\ .
\end{aligned}
$$

Evolution Eq.: Collinear Part

It is hard to believe but it simplifies a lot

$$
\begin{aligned}
\frac{d}{dt}\big(\hat{\boldsymbol{p}},\hat{Y}\big|\rho(t)\big)\big|_{\rm coll} = {}& \sum_m \frac{1}{m!}\int \big[d\{p,f,s',c',s,c\}_m\big] \\
&\times \int_0^{1/(1+y)}\frac{dz}{z}\int_0^{2\pi}\frac{d\phi}{2\pi}\,\frac{\alpha_{\rm s}(\mu_{\rm R}^2)}{2\pi}\sum_{\hat{a},a}P_{\hat{a},a}(z,y)\,\frac{f_{\hat{a}/{\rm A}}\left(\frac{\eta_{\rm a}}{z},\mu_{\rm F}^2\right)}{f_{a/{\rm A}}(\eta_{\rm a},\mu_{\rm F}^2)} \\
&\times \bigg[\delta^{(2)}\bigg(\hat{\boldsymbol{p}}-\boldsymbol{p}(\{p\}_m)+\frac{x_{\rm A}}{\eta_{\rm a}}\,\boldsymbol{k}\bigg)\,\delta\Big(\hat{Y}-\hat{Y}(y,k_\perp,\{p\}_m)\Big) \\
&\qquad -\delta^{(2)}(\hat{\boldsymbol{p}}-\boldsymbol{p}(\{p\}_m))\,\delta\Big(\hat{Y}-Y(\{p\}_m)\Big)\bigg] \\
&\times \big(\{p,f,s',c',s,c\}_m\big|\rho(t)\big) \\
&+ (\text{when praton “b” is the emitter})
\end{aligned}
$$

Remember the evolution time is related to the virtuality

$$
t = \log\frac{M^2}{2\hat{p}_l\cdot\hat{p}_{m+1}} = \log\frac{M^2}{yQ^2}
$$

Evolution Eq.: Collinear Part

Defining the partonic cross section by

$$\big(\boldsymbol{p}, Y \big| \rho(t)\big) = \sum_{a,b} \int d\eta_{\rm a} \int d\eta_{\rm b}\ f_{a/A}(\eta_{\rm a}, \mu_{\rm F})\, f_{b/B}(\eta_{\rm b}, \mu_{\rm F})\, H_{a,b}(\boldsymbol{p}, Y; \eta_{\rm a}, \eta_{\rm b}; t)$$

now we have

$$\begin{aligned}\frac{d}{dt}\big(\boldsymbol{p}, Y \big| \rho(t)\big)\big|_{\rm coll} = {} & \sum_{a,b} \int_0^1 d\eta_{\rm a} \int_0^1 d\eta_{\rm b}\ f_{b/B}(\eta_{\rm b}, \mu_{\rm F}) \\ & \times \int_0^{\frac{1}{1+y}} \frac{dz}{z} \int_0^{2\pi} \frac{d\phi}{2\pi}\, \frac{\alpha_{\rm s}(\mu_{\rm R}^2)}{2\pi} \sum_c f_{a/{\rm A}}\big(\eta_{\rm a}/z, \mu_{\rm F}^2\big)\, P_{a,c}(z, y) \\ & \times \left[H_{c,b}\left(\boldsymbol{p} + \frac{x_{\rm A}}{\eta_{\rm a}}\boldsymbol{k}, Y; \eta_{\rm a}, \eta_{\rm b}; t\right) - H_{c,b}(\boldsymbol{p}, Y; \eta_{\rm a}, \eta_{\rm b}; t)\right] \\ & + (\text{when parton ``b'' is the emitter}) \quad .\end{aligned}$$

It would be nice to have an equation for the partonic cross section

Evolution Eq.: Collinear Part

The evolution of the PDF function can be approximated by

$$
\begin{aligned}
\frac{d}{dt} f_{a/\mathrm{A}}\left(x, \mu_{\mathrm{F}}^2\right) \approx - \int_0^{\frac{1}{1+y}} \frac{dz}{z} \, \frac{\alpha_{\mathrm{s}}(\mu_{\mathrm{R}}^2)}{2\pi} \\
\times \sum_{\hat{a}} \Bigg\{ P_{\hat{a},a}(z,y) \, f_{\hat{a}/\mathrm{A}}\left(x/z, \mu_{\mathrm{F}}^2\right) \\
- \delta_{\hat{a},a} \left[\frac{2C_a}{1-z} - (1+y)\gamma_{\hat{a}} \right] f_{\hat{a}/\mathrm{A}}\left(x, \mu_{\mathrm{F}}^2\right) \Bigg\} \ .
\end{aligned}
$$

Note, the scale of the strong coupling is the transverse momentum of the emitted parton

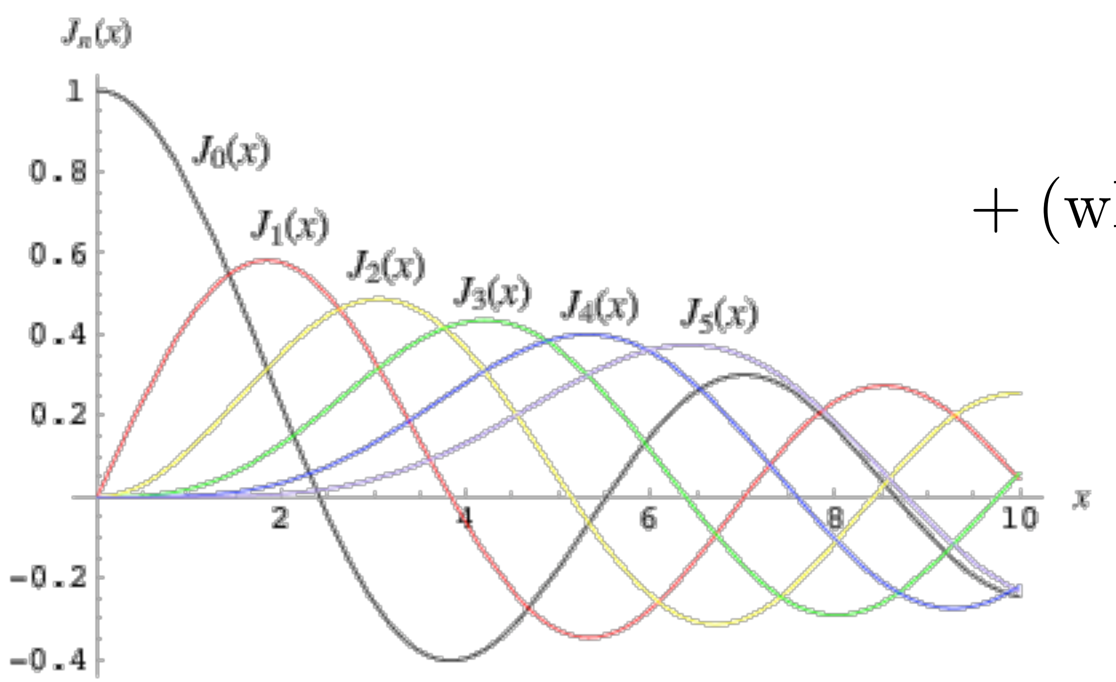
$$
\mu_{\mathrm{R}}^2 = \boldsymbol{k}^2 = [1 - z(1+y)]\mu_{\mathrm{F}}^2
$$

... might be a good PDF4MC ....

Evolution Eq.: Collinear Part

After Fourier transform we have

$$\frac{d}{dt}\tilde{H}_{a,b}(\boldsymbol{b},Y;\eta_{\rm a},\eta_{\rm b};t)\Big|_{\rm coll} = \int_0^{\frac{1}{1+y}} dz\,\frac{\alpha_{\rm s}(\mu_{\rm R}^2)}{2\pi} \times \Bigg\{ \sum_c P_{a,c}(z,y)\,\tilde{H}_{c,b}(\boldsymbol{b},Y;z\eta_{\rm a},\eta_{\rm b};t)\,J_0\!\left(\frac{x_{\rm A}}{z\eta_{\rm a}}|\boldsymbol{k}||\boldsymbol{b}|\right) - \left[\frac{2C_a}{1-z} - (1+y)\gamma_a\right]\tilde{H}_{a,b}(\boldsymbol{b},Y;\eta_{\rm a},\eta_{\rm b};t)\Bigg\} + (\text{when parton "b" is the emitter}) \quad .$$



$$J_0\!\left(\frac{x_{\rm A}}{z\eta_{\rm a}}|\boldsymbol{k}||\boldsymbol{b}|\right) - 1 \approx -\theta\!\left(z < 1 - \frac{2e^{-\gamma_E}}{\boldsymbol{b}^2 Q^2 y}\frac{\eta_{\rm a}^2}{x_{\rm A}^2}\right) = -\theta\!\left(z < 1 - \frac{y_c}{y}\right)$$

Evolution Eq.: Collinear Part

Finally we have

$$
\begin{aligned}
&\frac{d}{dt}\tilde{H}_{a,b}(\boldsymbol{b},Y;\eta_{\rm a},\eta_{\rm b};t)\big|_{\rm coll} \\
&\qquad = \theta(t>t_c)\int_0^1 dz\,\frac{\alpha_{\rm s}(\mu_{\rm R}^2)}{2\pi}\sum_c P^{a,c}(z)\,\tilde{H}_{c,b}(\boldsymbol{b},Y;z\eta_{\rm a},\eta_{\rm b};t) \\
&\qquad\quad + \theta(t>t_c)\int_0^1 dz\,\frac{\alpha_{\rm s}(\mu_{\rm R}^2)}{2\pi}\sum_c P^{b,c}(z)\,\tilde{H}_{a,c}(\boldsymbol{b},Y;\eta_{\rm a},z\eta_{\rm b};t) \\
&\qquad\quad + \theta(t\le t_c)\,[K_a(y,y_c)+K_b(y,y_c)]\,\tilde{H}_{a,b}(\boldsymbol{b},Y;\eta_{\rm a},\eta_{\rm b};t)
\end{aligned}
$$

where

$$
\begin{aligned}
K_a(y,y_c) &= \int_0^1 dz\,\frac{\alpha_{\rm s}(\mu_{\rm R}^2)}{2\pi}\left[\frac{2C_a}{1-z+y}-\gamma_a\right]\left[J_0\!\left(\frac{x_{\rm A}}{z\eta_{\rm a}}|\boldsymbol{k}||\boldsymbol{b}|\right)-1\right] \\
&\approx -\int_{y_cQ^2}^{yQ^2} d\boldsymbol{k}^2\,\frac{\alpha_{\rm s}(\boldsymbol{k}^2)}{2\pi}\left[\frac{2C_a}{\boldsymbol{k}^2+y^2Q^2}-\frac{\gamma_a}{yQ^2}\right]
\end{aligned}
$$

If there is no soft interference contribution then this equation provides the desired cross section formulae.

Evolution Eq.: Soft Part

Soft part of the evolution equation is

we need this at LL level

$$\frac{d}{dt} H_{a,b}(\boldsymbol{p}, Y; \eta_{\rm a}, \eta_{\rm b}; t)\Big|_{\rm soft} = -\int_0^1 d\lambda\, \tilde{H}^{(\rm a)}_{a,b}(\boldsymbol{b}, Y, \lambda; \eta_{\rm a}, \eta_{\rm b}; t)$$

$$\times \int_{y_c Q^2}^{y_\lambda Q^2} d\boldsymbol{k}^2\, \frac{\alpha_{\rm s}(\boldsymbol{k}^2)}{2\pi}\, \frac{2}{\boldsymbol{k}^2 + y^2 Q^2}$$

$$+\ (\text{when parton ``b'' is the emitter})$$

with the upper limit

$$y_\lambda \xrightarrow{y \to 0} \frac{1-\lambda}{\lambda}\, y^2 \quad .$$

What I expect that is

$$\tilde{H}^{(\rm a)}_{a,b}(\boldsymbol{b}, Y, \lambda; \eta_{\rm a}, \eta_{\rm b}; t) \sim \delta(1-\lambda) \times \mathcal{O}(LL) + \mathcal{O}(NLL)$$

Conclusion

- We are working on the Drell-Yan pT distribution.
- We studied the so called "Quantum shower" algorithm

JHEP 0709:114,2007
JHEP 0803:030,2008
JHEP 0807:025,2008

- We have promising preliminary result that THIS shower model can sum up the LL and NLL logarithms.
- I think it is a good idea to validate the parton shower against know analytic results. We have found that small details DOES matter.