Transverse momentum resummation for colour-singlet production

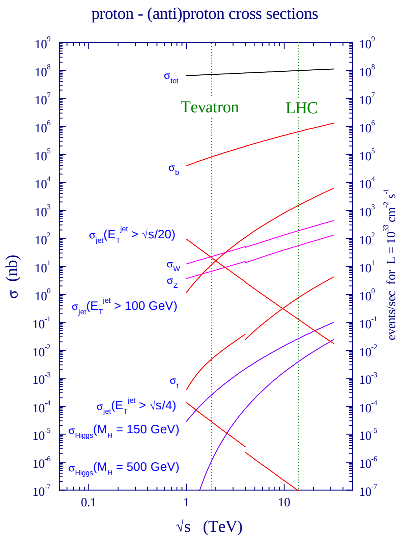
Anna Kulesza RWTH AACHEN

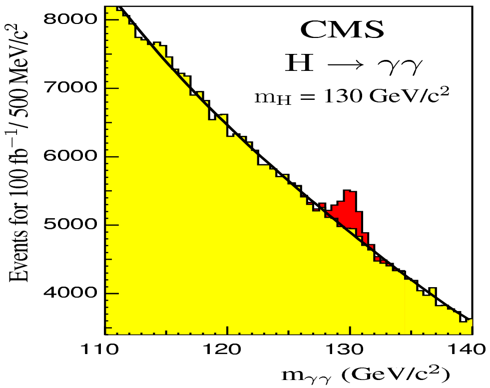
PHYSICS AT THE TERA SCALE
Helmholtz Alliance

PSRI, DESY Hamburg, 19.05.2009

Introduction

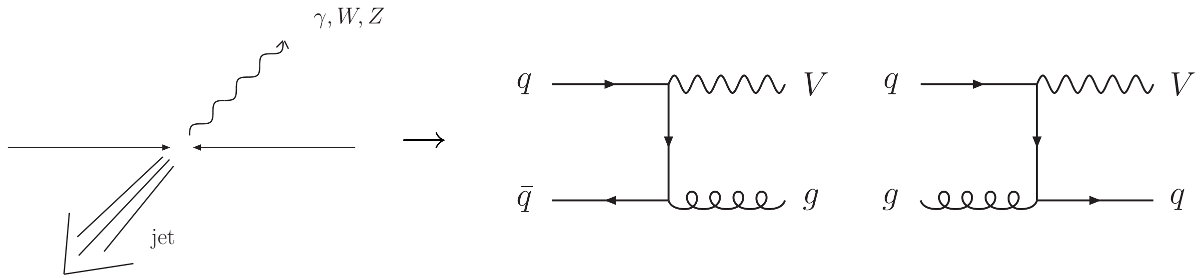
- LHC as W, Z factory
  - measurement of the transverse momentum (p_{T}) distribution
    $\Rightarrow$ one of the early benchmarks to be established
  - p_{T} distribution measurement important for
    $\Rightarrow$ determination of M_W
    $\Rightarrow$ determination of parton distribution functions
- LHC as Higgs discovery machine
  - important to fully exploit the physics potential of the production processes
    $\longrightarrow$ search and analysis strategies





Transverse momentum ($p_{\rm T}$) spectrum

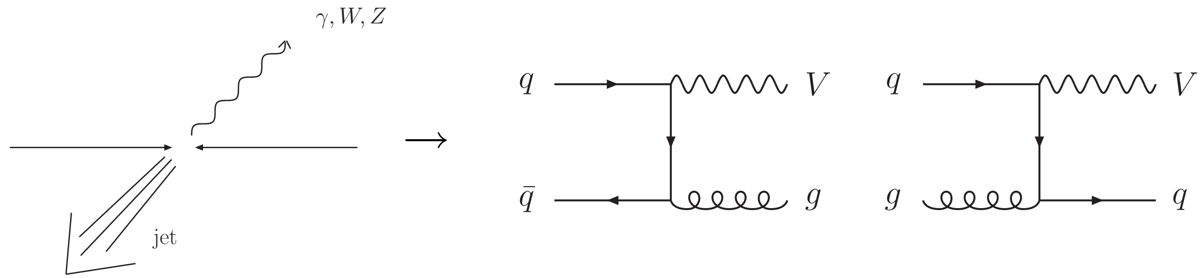
- Generated by production in association with (at least) one particle



$\longrightarrow$ Leading Order

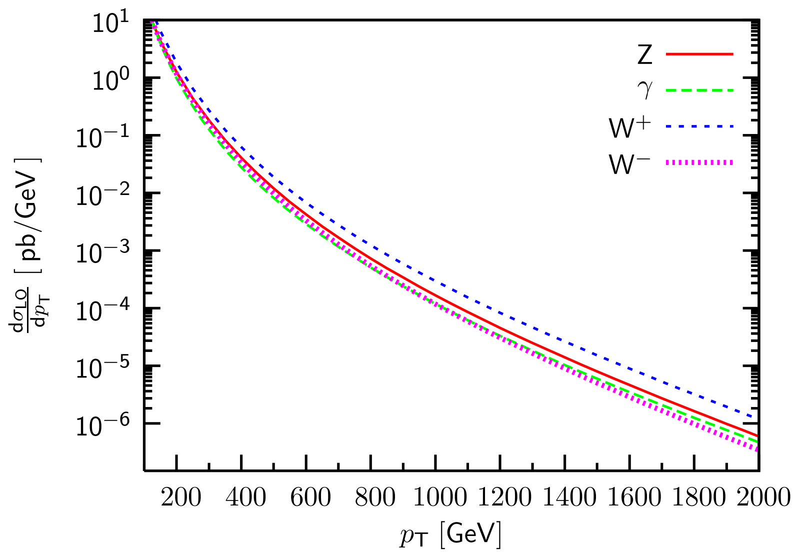
Transverse momentum (p_T) spectrum

- Generated by production in association with (at least) one particle



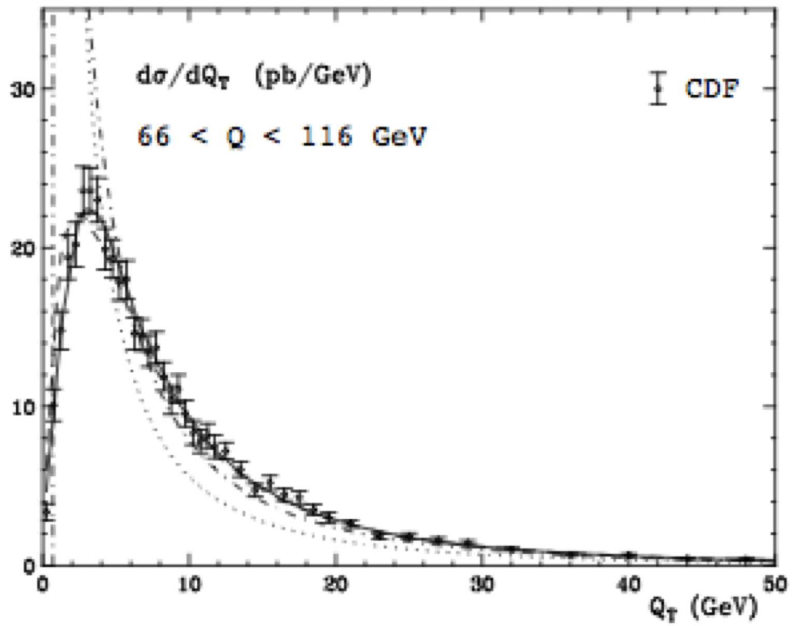
$\rightarrow$ Leading Order

- Most of the production at low transverse momentum (p_T)



LO W, Z, γ production at the LHC

Fixed-order distribution vs data



from [*AK, Sterman, Vogelsang'02*]

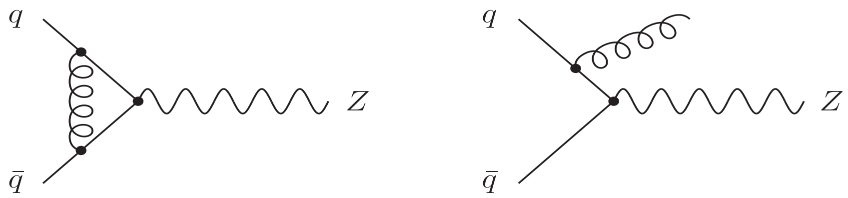
Run 1a + Run 1b data set, CDF [*Affolder et al.'00*]

LO: dotted

NLO: dot-dashed

NLO corrections

Virtual corrections: $d\sigma^V$ Real corrections: $d\sigma^R$



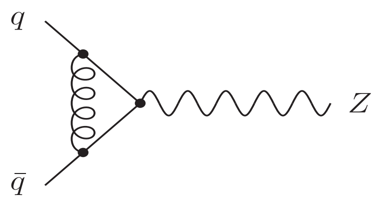
$$\sigma^{NLO} = \sigma^{LO} + \int d\sigma^V + \int d\sigma^R$$

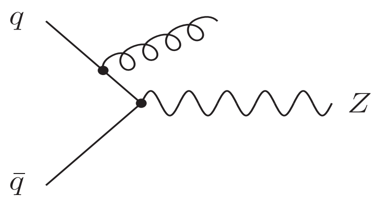
- Both type of corrections are infrared (IR) divergent: exchange/emission of soft and collinear gluons (photons)

NLO corrections

Virtual corrections: $d\sigma^V$

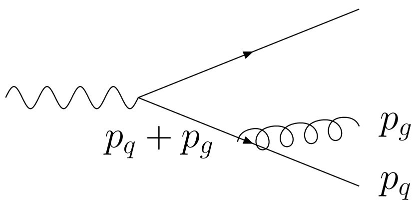
Real corrections: $d\sigma^R$





$$\sigma^{NLO} = \sigma^{LO} + \int d\sigma^V + \int d\sigma^R$$

- Both type of corrections are infrared (IR) divergent: exchange/emission of soft and collinear gluons (photons) e.g.



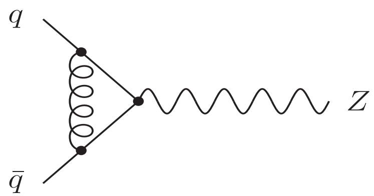
Propagator of the parent quark ($p_q^2 = 0$, $p_g^2 = 0$):

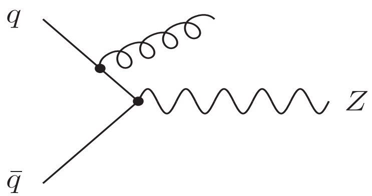
$$\frac{1}{(p_q + p_g)^2} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

Singular in the *soft* $E_g \to 0$ and *collinear* $\theta_{qg} \to 0$ limit

NLO corrections

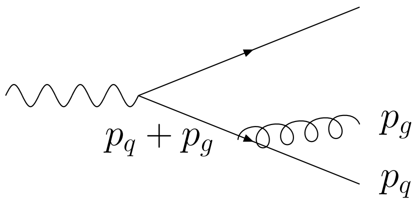
Virtual corrections: $d\sigma^V$ Real corrections: $d\sigma^R$





$$\sigma^{NLO} = \sigma^{LO} + \int d\sigma^V + \int d\sigma^R$$

- Both type of corrections are infrared (IR) divergent: exchange/emission of soft and collinear gluons (photons) e.g.



Propagator of the parent quark $(p_q^2 = 0,\ p_g^2 = 0)$:

$$\frac{1}{(p_q + p_g)^2} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

Singular in the *soft* $E_g \to 0$ and *collinear* $\theta_{qg} \to 0$ limit

- IR behaviour of QCD (QED) matrix elements is universal $\to$ factorization of IR singularities

Fixed order

Example: $pp \to H + X$ via gluon fusion

$$\frac{d\hat{\sigma}_{ij}}{dp_T^2 dy_H} = \frac{\sigma_0}{\hat{s}} \left[\frac{\alpha_S}{2\pi} G_{ij}^{(1)} + \left(\frac{\alpha_S}{2\pi} \right)^2 G_{ij}^{(2)} + \ldots \right]$$

$$\sigma_0 = \frac{\pi}{64} \left(\frac{\alpha_S}{3\pi v} \right)^2 \qquad v^2 = 1/(\sqrt{2} G_F) \qquad \hat{s} = s x_a x_b$$

$\uparrow$

calculated in the $m_t \to \infty$ limit

- $G_{ij}^{(1)}$, $G_{ij}^{(2)}$ known

Fixed order

Example: $pp \to H + X$ via gluon fusion

$$\frac{d\hat{\sigma}_{ij}}{dp_T^2 dy_H} = \frac{\sigma_0}{\hat{s}} \left[\frac{\alpha_S}{2\pi} G_{ij}^{(1)} + \left(\frac{\alpha_S}{2\pi}\right)^2 G_{ij}^{(2)} + \ldots \right]$$

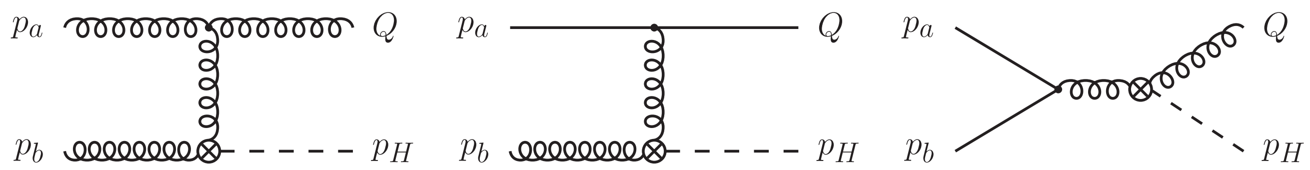
$$\sigma_0 = \frac{\pi}{64}\left(\frac{\alpha_S}{3\pi v}\right)^2 \qquad v^2 = 1/(\sqrt{2}G_F) \qquad \hat{s} = s x_a x_b$$

↑

calculated in the $m_t \to \infty$ limit

- $G_{ij}^{(1)}$, $G_{ij}^{(2)}$ known
- At LO contributions from $gg \to gH$, $gq \to Hq$, $q\bar{q} \to gH$

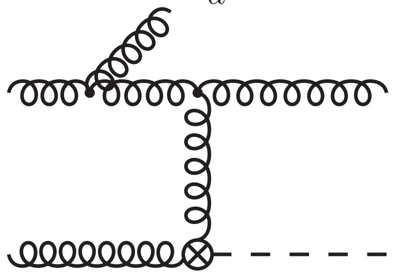
Sample diagrams



Fixed order: anatomy of the NLO

Consider $\mathcal{O}(\alpha_S)$ correction to the $p_{\rm T}$ distribution in the Drell-Yan type process

Sample diagram



Schematically: $G_{ij}^{(2)} = G_{ij,\rm sing}^{(2)} + G_{ij,\rm non-sing}^{(2)}$

$$\begin{aligned} G_{ij,\rm sing}^{(2)} \quad &= \quad a_{ij}^{\delta,\,z_a}(z_a, p_T)\delta(1-z_a) + a_{ij}^{(1),\,\rm z_a}(z_a, p_T)\,\mathcal{D}_1(z_a) \\ &+ \quad a_{ij}^{(0),\,\rm z_a}(z_a, p_T)\mathcal{D}_0(z_a) + a_{ij}^{\rm p_T,\,z_a}(z_a, p_T)\frac{1}{p_{\rm T}^2}\ln\frac{p_{\rm T}^2}{Q_T^2} + (z_a \leftrightarrow z_b), \end{aligned}$$

$$z_a = \frac{-\hat{t}}{Q^2 - \hat{t}} \qquad z_b = \frac{-\hat{u}}{Q^2 - \hat{u}} \qquad \mathcal{D}_i(z) = \left(\frac{\ln^i(1-z)}{1-z}\right)_+ \qquad Q_T^2 = p_{\rm T}^2 + Q^2$$

Fixed order: NLO

In the small p_T limit [*Ellis, Martinelli, Petronzio'81*]

$$\left.\frac{d\sigma}{dp_T^2\,dy_H}\right|_{p_T \ll m_H} = \alpha_S \left[c_{11}^{p_T} \left(\frac{1}{p_T^2} \ln\left(\frac{m_H^2}{p_T^2}\right)\right)_+ + c_{10}^{p_T} \left(\frac{1}{p_T^2}\right)_+ \right] + \alpha_S^2 \left[c_{23}^{p_T} \left(\frac{1}{p_T^2} \ln^3\left(\frac{m_H^2}{p_T^2}\right)\right)_+ + \ldots \right]$$

After integration over p_T^2 and y_H [*Hamberg, van Neerven, Matsuura'91*]($z = m_H^2/\hat{s}$)

$$\hat{\sigma}(z, m_H^2) = \alpha_S \left[c_{11}^z\, \mathcal{D}_1(z) + c_{10}^z\, \mathcal{D}_0 + c_{1\delta}^z\, \delta(1-z) + c_1^z \right] + \alpha_S^2 \left[c_{23}^z\, \mathcal{D}_3(z) + \ldots \right]$$

$$\mathcal{D}_i(x) = \left(\frac{\ln^i(1-x)}{1-x} \right)_+$$

Higher-order corrections

In general, in the IR region

$$\hat{\sigma}^{NLO} \sim \sum_{i,j} \int d\Phi^{LO} |M^{LO}|^2 \int d\, V_{i,j} \left[\Theta_{PS}^{R}(\ldots, p_i, p_j, \ldots) - \Theta_{PS}^{V}(\ldots, p_i + p_j, \ldots) \right]$$

Born level — singular, process independent — phase-space conditions

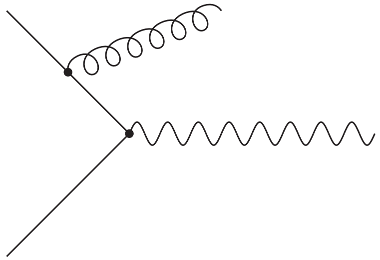
- $\Theta_{PS}^{R} \sim \Theta_{PS}^{V}$ in the soft and collinear region
- In other regions of phase space, real and virtual contributions can be highly unbalanced
- Single-gluon emission probability ($1 - z =$ fraction of energy carried out by the gluon, x=exclusive phase-space boundary):

$$\frac{d\omega(z)}{dz} \sim \alpha_s \left[\frac{1}{1-z} \ln \frac{1}{1-z} \right]_+ \quad \Rightarrow \quad \int_x^1 dz \frac{d\omega(z)}{dz} \sim \alpha_s \log^2(1-x)$$

$\Rightarrow$ if $x \to 1$ then double logarithmic divergences (soft and collinear limit)

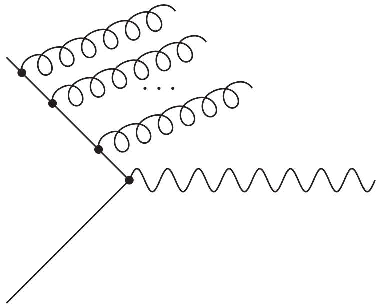
Small p_T at higher orders

The low end of the p_{T} spectrum



$$\frac{d\sigma}{dp_T^2} \sim \alpha_s \frac{1}{p_T^2} \log\left(\frac{M^2}{p_T^2}\right)$$

Factorization properties in the IR limit: double logarithmic structure carries to all orders



terms with $\alpha_s^n \log^m \left(\frac{M^2}{p_T^2}\right)$ $\quad 0 \leq m \leq 2\,n-1$

If $\alpha_s \log^2\left(\frac{M^2}{p_T^2}\right) \sim 1$: breakdown of the perturbation theory

Resummation

Recap

After cancellation of real and virtual singularities:

- Large logarithmic corrections near boundaries of phase space ($z \to 1$, $p_{\mathrm{T}} \to 0$):
  - "Invisible" $\ln(1-z) \longrightarrow$ large corrections to the cross section
    (z measures distance from the partonic production threshold

    Threshold corrections
  - "Visible" $\ln(p_{\mathrm{T}}^2/m_H^2) \longrightarrow$ explicitly divergent cross sections

    Recoil corrections

Resummation

Recap

After cancellation of real and virtual singularities:

- Large logarithmic corrections near boundaries of phase space ($z \to 1$, $p_{\mathrm{T}} \to 0$):
  - "Invisible" $\ln(1-z)$ $\longrightarrow$ large corrections to the cross section
    (z measures distance from the partonic production threshold

    Threshold corrections
  - "Visible" $\ln(p_{\mathrm{T}}^2/m_H^2)$ $\longrightarrow$ explicitly divergent cross sections

    Recoil corrections
- Origin: soft/collinear emission
  $\longrightarrow$ radiative tail of the emission strongly suppressed, large logarithms appear

Resummation

Recap

After cancellation of real and virtual singularities:

- Large logarithmic corrections near boundaries of phase space ($z \to 1$, $p_{\mathrm{T}} \to 0$):
  - "Invisible" $\ln(1-z) \longrightarrow$ large corrections to the cross section
    (z measures distance from the partonic production threshold

    Threshold corrections
  - "Visible" $\ln(p_{\mathrm{T}}^2/m_H^2) \longrightarrow$ explicitly divergent cross sections

    Recoil corrections
- Origin: soft/collinear emission

  $\longrightarrow$ radiative tail of the emission strongly suppressed, large logarithms appear
- Fixed-order perturbation theory bound to fail in particular kinematical configurations

Resummation

Recap

After cancellation of real and virtual singularities:

- Large logarithmic corrections near boundaries of phase space ($z \to 1$, $p_{\mathrm{T}} \to 0$):
  - "Invisible" $\ln(1-z) \longrightarrow$ large corrections to the cross section
    (z measures distance from the partonic production threshold

    Threshold corrections
  - "Visible" $\ln(p_{\mathrm{T}}^2/m_H^2) \longrightarrow$ explicitly divergent cross sections

    Recoil corrections
- Origin: soft/collinear emission
  $\longrightarrow$ radiative tail of the emission strongly suppressed, large logarithms appear
- Fixed-order perturbation theory bound to fail in particular kinematical configurations
- Solution: all-order treatment of the perturbative expressions $\longrightarrow$ resummation

Resummation: "bottom-top"

Generic hard scattering process:

$$\hat{\sigma}(z) \sim \hat{\sigma}_0 \left[1 + \sum_{n=1}^{\infty} \int_0^1 dz_1 ... dz_n \frac{d\omega_n(z_1, \ldots, z_n)}{dz_1 \ldots dz_n} \Theta_{PS}^{(n)}(z, z_1, ..., z_n)\right]$$

- Dynamical factorization (universal, process independent)
  QCD: gluon emission correlated (colour charge) but in the soft limit QED-like factorization [*Ermolaev, Fadin '81*][*Bassetto, Ciafaloni, Marchesini '83*]

$$\frac{d\omega_n(z_1, \ldots, z_n)}{dz_1 \ldots dz_n} = \frac{1}{n!} \prod_{i=1}^{n} \frac{d\omega(z_i)}{dz_i} \, .$$

- Phase–space factorization depends on the process: Θ_{PS} contains kinematical constraints defining physical cross section

$$\Theta_{PS}^{(n)}(z, z_1, ..., z_n) = \prod_{i=1}^{n} \Theta_{PS}(z, z_i)$$

$$\begin{aligned} \hat{\sigma}(z) &\sim \hat{\sigma}_0 \left\{ 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\int_0^1 dz_i \frac{d\omega(z_i)}{dz_i} \Theta_{PS}(z, z_i) \right]^n \right\} \\ &\sim \hat{\sigma}_0 \exp \left[\int_z^1 dz' \frac{d\omega(z')}{dz'} \Theta_{PS}(z, z') \right] \sim \hat{\sigma}_0 \exp \left[\alpha_S L^2 + \ldots \right] \end{aligned}$$

[*Catani'96*]

Leading logarithms in p_T distribution

In the approximation of strongly ordered gluons momenta

$$p^2_{T_{i1}} \ll p^2_{T_{i2}} \ll \ldots \ll p^2_{T_{iN}} \sim p^2_T(V) \ll Q^2$$

the dominant contribution is

$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}}{dp^2_T} = \frac{1}{p^2_T}\left[\alpha_s \frac{A^{(1)}}{2\pi}\ln\left(\frac{Q^2}{p^2_T}\right) - \alpha_s^2 \frac{(A^{(1)})^2}{8\pi^2}\ln^3\left(\frac{Q^2}{p^2_T}\right)\right.$$
$$\left. +\ldots + \alpha_s^N \frac{(-1)^{N-1}(A^{(1)})^N}{2^{2N-1}(N-1)!\pi^N}\ln^{2N-1}\left(\frac{Q^2}{p^2_T}\right) + \ldots\right]$$

with $A^{(1)} = 2C_F$

These terms can be summed giving Sudakov factor (DLLA approximation)

$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}}{dp^2_T} = \frac{\alpha_s A}{2\pi p^2_T}\ln\left(\frac{Q^2}{p^2_T}\right)\exp\left(\frac{-\alpha_s A}{4\pi}\ln^2\left(\frac{Q^2}{p^2_T}\right)\right)$$

[*Dokshitzer, Dyakonov, Troyan '80*]

Note: cross section supressed as $p_T \to 0$: result of strong ordering, p_T not conserved

Resummation

- In practice phase–space factorization often occurs in the space conjugate to the space of kinematic variables
- Resummed cross sections exist for both:
  - recoil resummation (in Fourier conjugate to $p_{\rm T}$, b space) [Collins, Soper '83][Collins, Soper, Sterman '85]

$$\delta\left(\mathbf{p}_{\rm T} - \sum_i \mathbf{k}^{\mathbf{i}}_{\mathbf{T}}\right) = \frac{1}{2\pi^2}\int d^2b e^{i\mathbf{b}(\mathbf{p}_{\rm T} - \sum_i \mathbf{k}^{\mathbf{i}}_{\mathbf{T}})} \qquad \ln(Q^2/p_{\rm T}^2) \leftrightarrow \ln(Q$$

$\Rightarrow$ resummation takes place in the Fourier conjugate to p_T, b-space

$$\frac{1}{\hat{\sigma}_0}\frac{d\sigma}{dp_T^2} = \frac{1}{4\pi^2}\int d^2b e^{-i\mathbf{b}\mathbf{p}_{\rm T}}\hat{\sigma}(b)$$

[Parisi, Petronzio'79]

  - threshold resummation (in Mellin N space) [Sterman '87][Catani, Trentadue'89]

$$\delta\left(1 - z - \sum_i z_i\right) = \frac{1}{2\pi i}\int_C dN e^{-N(1-z-\sum_i z_i)} \qquad \ln(1-z) \leftrightarrow \ln N$$

Resummation

Resummed cross sections are of the form

$$\hat{\sigma} = \hat{\sigma}_0 \int_{\mathrm{inv}} \mathcal{C} \exp(\mathcal{S})$$

where

$$\mathcal{S} \quad = \quad L f_1(\alpha_S\, L) + f_2(\alpha_S\, L) + \alpha_S f_3(\alpha_S\, L) + ...$$

$$LL \qquad NLL \qquad NNLL \qquad \ldots$$

$$\alpha_S^n L^{(n+1)} \quad \alpha_S^n L^n \qquad \alpha_S^n L^{(n-1)} \qquad \ldots$$

$L = \ln\left(b^2 Q^2\right)$ $(\mathrm{or}\ \ln(N))$

$\mathcal{C}$ contains finite contributions

All singular dependence on L exponentiates

Recoil resummation

[Collins,Soper,Sterman'85] $\frac{d\sigma^{\rm res}}{dp_T^2} = \frac{\tau}{2}\int db J_0(p_{\rm T}b)\, W(b,M) + Y_{\rm fin.}$ $\tau = M^2/s$

$$W(b,M) = \sum_{a,b}\sigma_0^H \int_0^1 dx_a dx_b \delta(x_a x_b - \tau) \int_{x_a}^1 \frac{dz_a}{z_a}\int_{x_b}^1 \frac{dz_b}{z_b} \mathcal{C}_{g/a}(\frac{x_a}{z_a}, b_0/b) f_{a/A}(z_a, b_0/b)$$
$$\mathcal{C}_{g/b}(\frac{x_b}{z_b}, b_0/b) f_{b/B}(z_b, b0/b)\ \exp\left[\mathcal{S}_{gg}(b,M)\right]$$

$$\mathcal{S}_{gg}(b,M) = -\int_{\frac{b_0^2}{b^2}}^{M^2} \frac{d\bar\mu^2}{\bar\mu^2}\left[\ln\left(\frac{M^2}{\bar\mu^2}\right) A_g(\alpha_S(\bar\mu^2)) + B_g(\alpha_S(\bar\mu^2))\right]$$

Functions $\mathcal{F} = A, B, \mathcal{C}$ have perturbative expansion $\mathcal{F} = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \mathcal{F}^{(n)}$

Known coefficients (NLL): $A_i^{(1)} = C_i$ $C_i = C_{F/A}$ for $i = q/g$

$A_i^{(2)} = \frac{1}{2}C_i K$ $K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9}T_R N_F$

$B_q^{(1)} = -\frac{3}{2}C_F$ $B_g^{(1)} = -\frac{1}{6}(11C_A - 4T_R N_f)$

[Kodaira, Trentadue '82][Catani, D'Emilio, Trentadue '88]

Also known NNLL coefficients $B_i^{(2)},\ A_i^{(3)}$ [Davies, Stirling'84][de Florian, Grazzini '00]

Applications (in ca. last 10 years)

- Drell-Yan lepton pairs at low Q
  [*Balázs, Yuan'97*] [*Fai,Qiu,Zhang'03*]
- W and Z boson production
  [*Balázs, Yuan, Ladinsky, Qiu, Landry, Brock, Nadolsky, Berge, Olness, Konychev'97-'05*][*Ellis, Ross, Veseli'97-'98*][*AK, Stirling'99-'01*] [*Laenen,Sterman,Vogelsang'00*][*Qiu,Zhang'01*] [*AK, Sterman, Vogelsang'02*][*Bozzi, Catani, Ferrera, de Florian, Grazzini'09*]
- SM Higgs boson production
  [*Bozzi, Catani, de Florian, Grazzini'03-05*][*Berger, Qiu'03*][*Balázs, Yuan'00*] [*Cao, Chen'07*][*AK, Stirling'03*][*AK, Sterman, Vogelsang'04*]
- Di-photon production
  [*Balázs, Berger, Mrenna,Yuan, Nadolsky,Schmidt'98-'07*]
- Z-pair production
  [*Balázs, Yuan'00*] [*Frederix, Grazzini'08*]
- W-pair production
  [*Grazzini'06*]

Applications cntd.

- SUSY Higgs production
  [*Field'04*] [*Belyaev, Nadolsky, Yuan'06*]
- Slepton-pair production
  [*Bozzi, Fuks, Klasen'06-'08*]
- Z' production
  [*Fuks, Klasen, Ledroit, Li, Morel'08*]
- Upsilon production
  [*Berger, Qiu, Wang'05*]
- Polarized vector boson production (RHIC)
  [*Nadolsky, Yuan'03*]
- Transversely polarized Drell-Yan
  [*Kawamura, Kodaira, Shimizu, Tanaka'06-'08*]
- ...

b-space resummation

- Resummation involves integration of the running coupling over the Landau pole: Integration over b from 0 to $\infty \Rightarrow \alpha_s(1/b)$ large when $b \to 1/\Lambda$

  : non-perturbative effects

  - Ambiguity in definition of perturbative series: pdf's and the Sudakov factor defined at

    $$b_* = \frac{b}{\sqrt{1+(b/b_{\rm lim})^2}} \qquad b_* < b_{\rm lim}$$

  - introduce an additional function $F^{NP}(Q, b, x_A, x_B)$ in the integrand to suppress the Sudakov factor

    $$\exp(\mathcal{S}(b,Q)) * F^{NP}(Q, b, x_A, x_B)$$
    $$F^{NP}(Q, b, x_A, x_B) = e^{-g(Q,b,x_A,x_B)b^2}$$

    [*Collins, Soper, Sterman'85*]

    fits from Drell-Yan/Z data

- Matching prescription

$$\frac{d\sigma}{dp_T^2 dQ^2} = \frac{d\sigma}{dp_T^2 dQ^2}_{\rm resummed} + \frac{d\sigma}{dp_T^2 dQ^2}_{\rm fixed-order} - \frac{d\sigma}{dp_T^2 dQ^2}_{\rm resummed\ expanded}$$

$\Rightarrow$ resummed part oscillates about 0 due to $J_0(bp_T)$ for large p_T

Developments

- Methods proposing other ways to avoid the Landau pole
  - return to p_{T}-space [*Ellis, Veseli'97*][*Frixione,Nason, Ridolfi '99*][*A.K., Stirling'99-'03*](approximation: expansion of the exponent, inverse b space transform done analytically)
  - deforming b-integration into contour over the complex b-plane [*Laenen, Sterman, Vogelsang'00*][*A.K., Sterman, Vogelsang'02*][*Bozzi, Catani, de Florian, Grazzini'05*]
  - smooth extrapolation of the perturbative result to the large b region [*Qiu, Zhang'01*]
  - using Borel resummation [*Bonvini, Forte, Ridolfi'08*]
- Improved matching [*Bozzi, Catani, de Florian, Grazzini'03-09*]

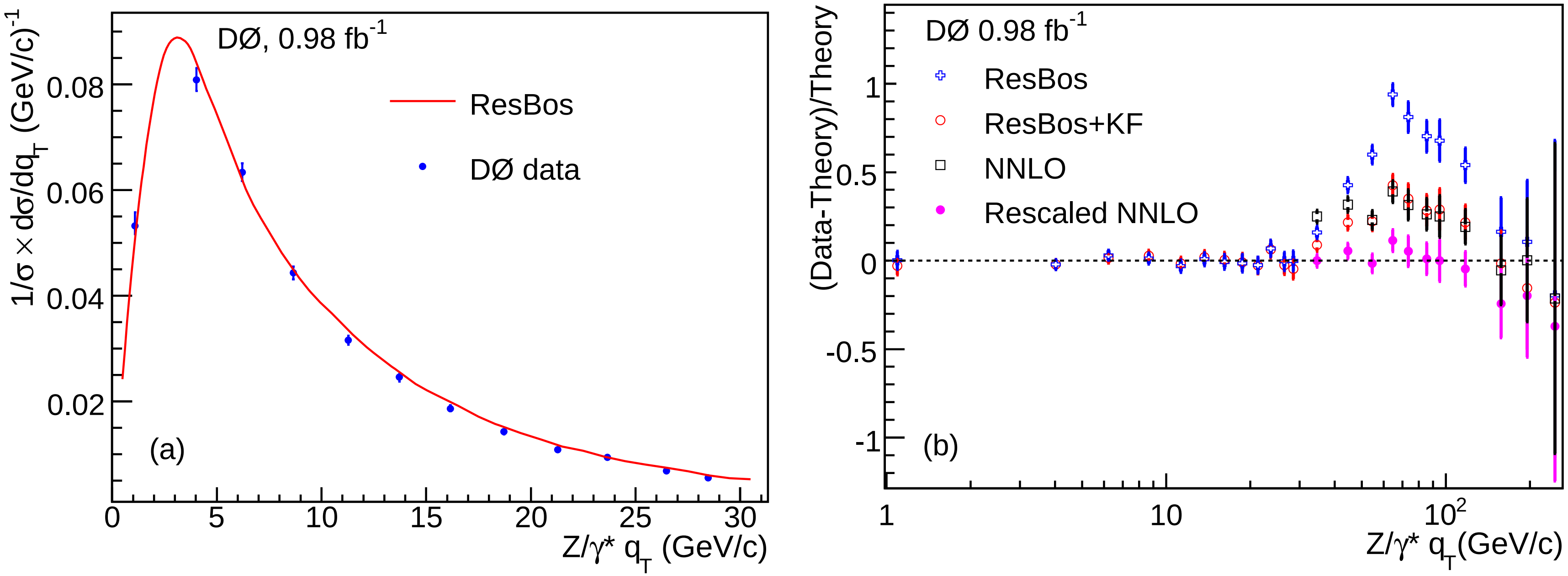
$$\tilde{L} = \ln\left(\frac{bQ}{b_0} + 1\right) \Rightarrow \int dp_{\mathrm{T}} \frac{d\sigma^{\mathrm{NLO}}}{dp_{\mathrm{T}}} = \sigma^{\mathrm{NNLO}}$$

- Related approaches
  - joint resummation [*Laenen, Sterman, Vogelsang'00*]: threshold logarithms resummed together with recoil logarithms up to NLL
  - resummation for double differential distributions in p_T and y [*Bozzi, Catani, de Florian, Grazzini'08*]

CSS resummation vs Tevatron data

ResBos [*Balázs et al.*]: 'standard CSS' implementation
depending on the process either NLL or (almost) NNLL accuracy, matching with NLO, decays implemented for some processes

Tevatron Run 2, $Z/\gamma^* \to e^+e^-$ data vs NNLL+NLO resummed predictions from ResBos



[*Abazov et al.'08*]

$$F^{\rm NP}(b, Q, x_a, x_b) = \exp\left[\left(-g_1 - g_2 \log\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \log(100 x_a, x_b)\right) b^2\right]$$

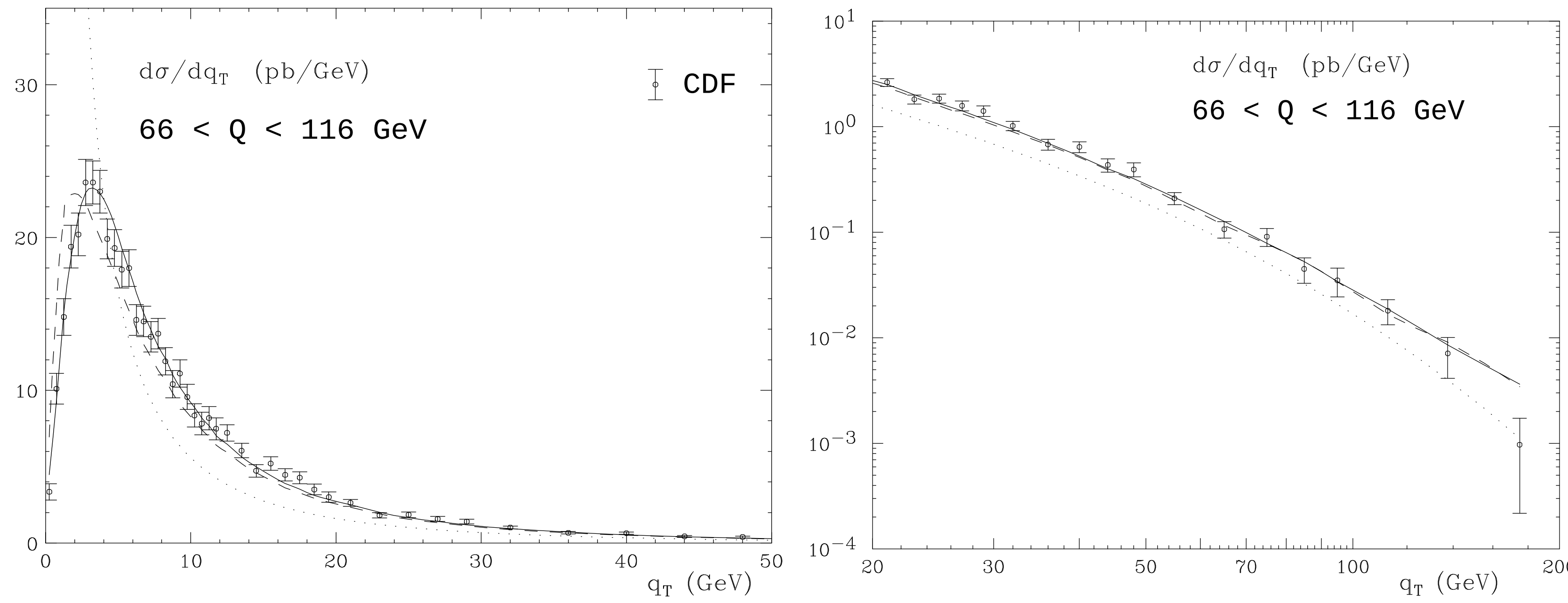
$$g_1 = 0.21 {\rm GeV}^2, g_2 = 0.68 {\rm GeV}^2, g_3 = -0.6, Q_0 = 1.6 GeV$$

Joint resummation vs Tevatron data

Joint resummation

- combined resummation of threshold and recoil logarithmic corrections up to NLL
- conservation of energy and transverse momentum implemented
- both threshold and recoil resummation formulae recovered in the appropriate limits

Z production at the Tevatron, CDF data, Run 1a+b



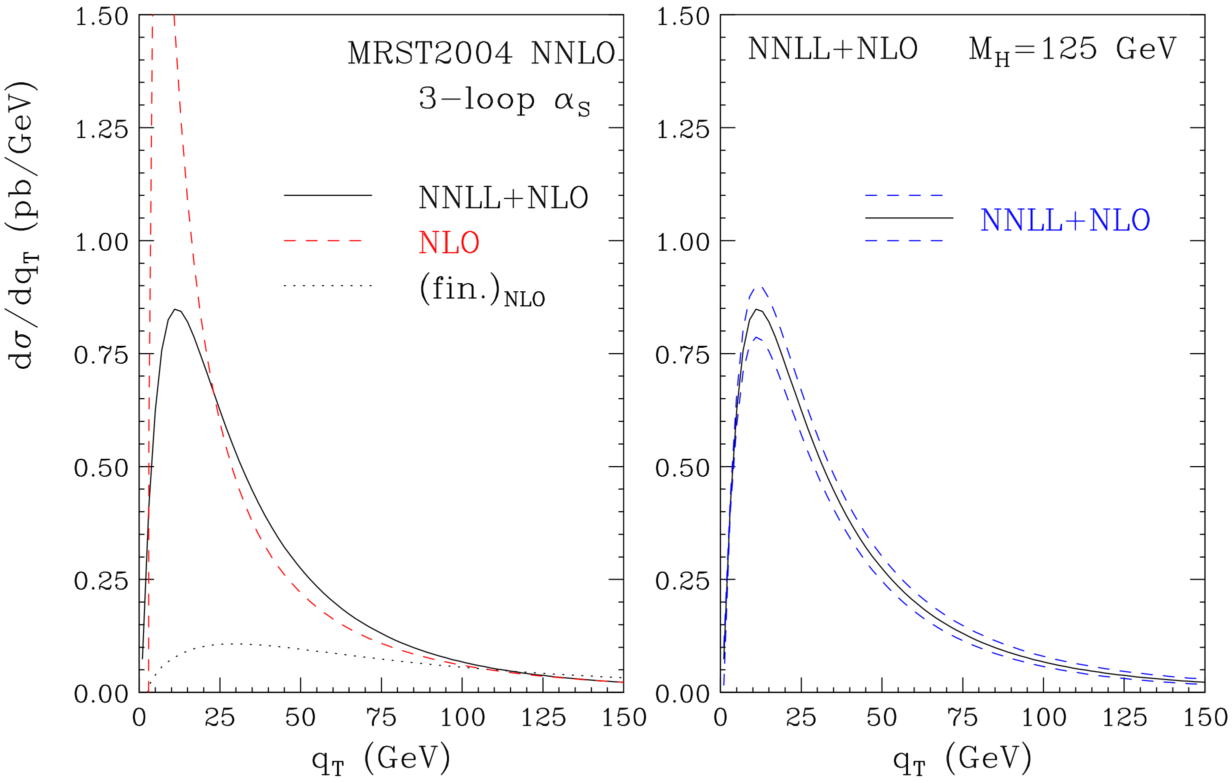
[*AK, Sterman, Vogelsang '02*]

NLL (with NNLL coefficients in the Sudakov exponent) jointly-resummed result matched

to LO, $F^{\mathrm{NP}}(b) = \exp\left(-gb^2\right)$; $g = 0.8\ \mathrm{GeV}^2$

p_T distribution for SM Higgs at the LHC

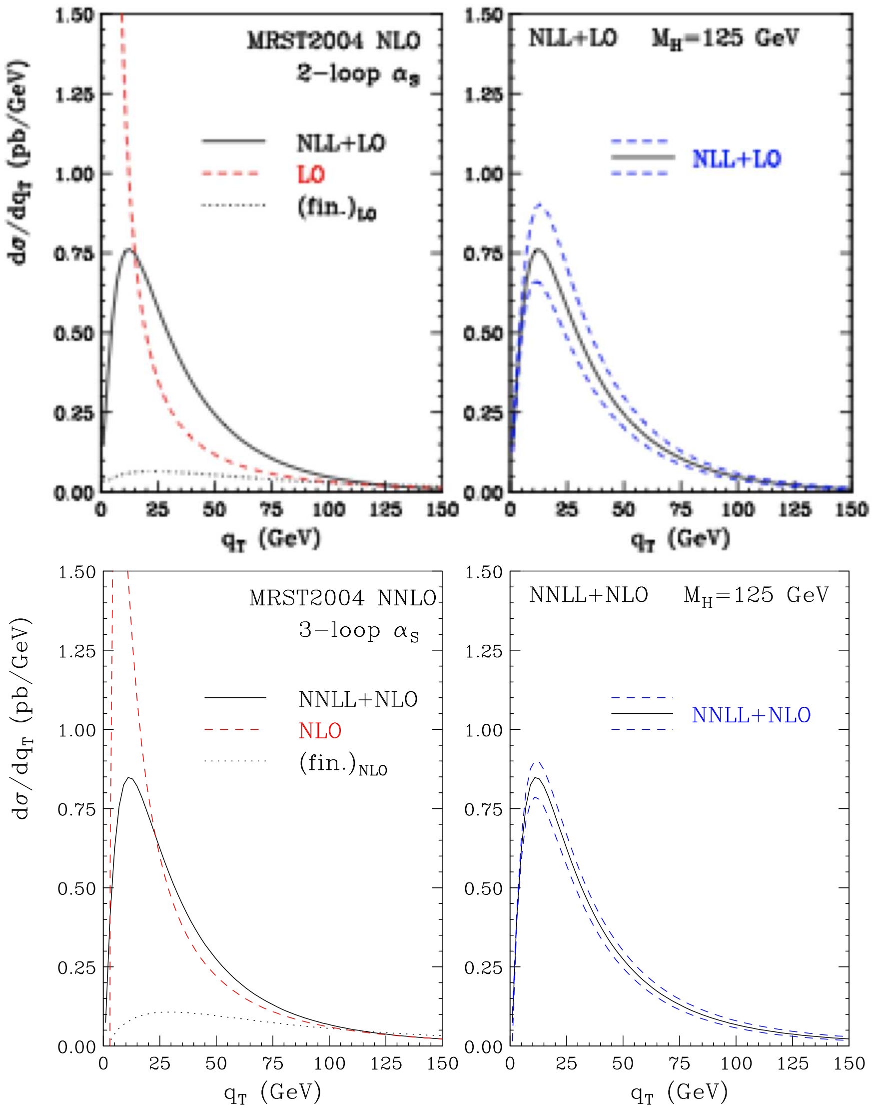
Recoil resummation predictions at NNLL + NLO [*Bozzi et al.'03,'05*]

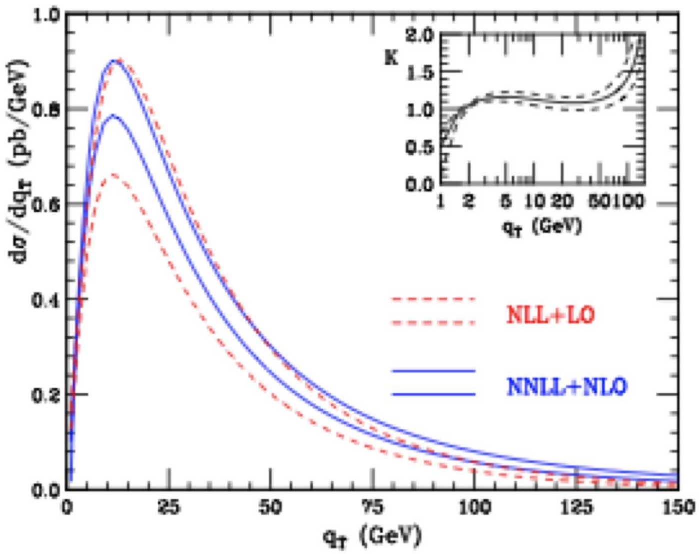


- Study of renormalization, factorization, resummation scale variation effects: perturbative QCD uncertainty of the NNLL+NLO results uniformly of $\mathcal{O}(10\%)$ over the range of small p_T $(10\ \text{GeV} \lesssim p_T \lesssim \frac{m_H}{3})$

p_T distribution for SM Higgs at the LHC

How big is the difference between NLL and NNLL?

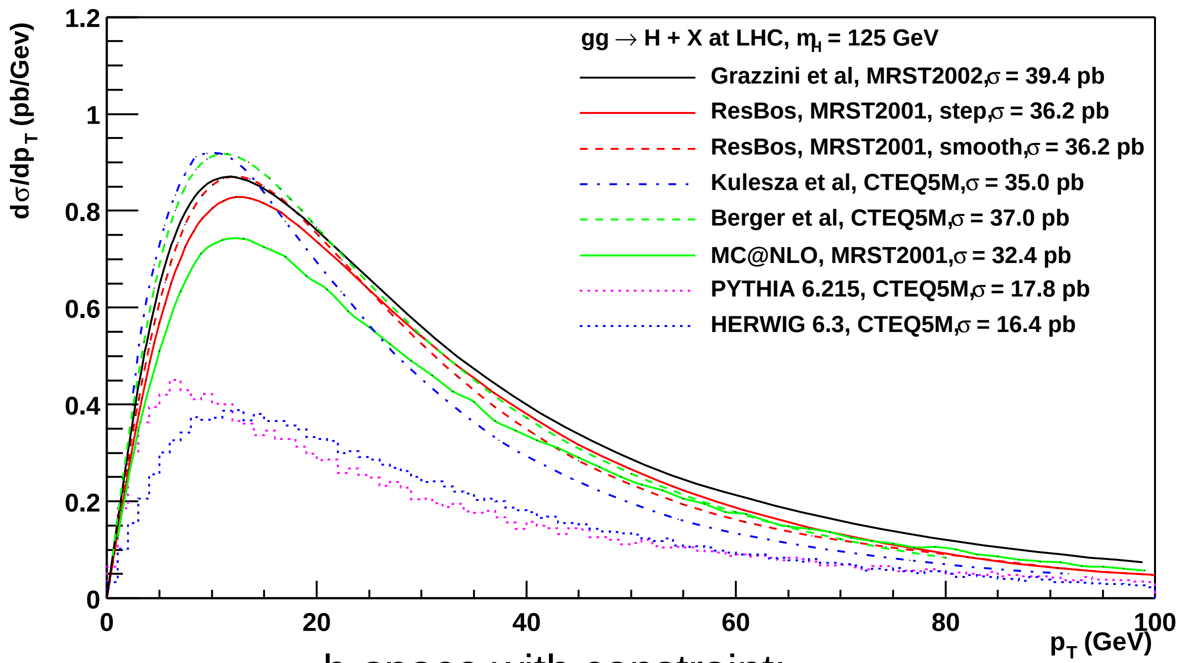




[*Bozzi et al.'05*]

Higgs production via gluon fusion at the LHC

[*Balazs, Grazzini, Huston, AK, Puljak'04*]



NNLL+NLO	b-space with constraint: $\int dp_{\rm T} \frac{d\sigma^{\rm NLO}}{dp_{\rm T}} = \sigma^{\rm NNLO}$	[*Bozzi et al.'03'05*]
"Sudakov" NNLL + LO	b-space	[*Berger, Qiu'02*]
"Sudakov" NNLL + LO	joint	[*A.K., Sterman, Vogelsang'03*]
"Sudakov" NNLL + (N)LO	b-space	[*Balazs, Yuan'00*]
MC@NLO	LO $p_{\rm T}$-distribution + parton shower	[*Frixione, Webber'02*]
PYTHIA	with hard matrix el. corrections	
HERWIG	without hard matrix el. corrections	

p_{T}-space method

- Starting point: the CSS formula in b space
- Expansion of the exponent makes it possible to take the inverse transform back to p_T space analytically, i.e. (schematically)

$$\frac{d\hat{\sigma}}{dp_T^2} = \frac{\hat{\sigma}_0}{2} \int_0^\infty b\, db\, J_0(p_T b) \exp\left[-\frac{\alpha_S C_i}{2\pi} \log^2\left(\frac{Q^2 b^2}{b_0^2}\right)\right]$$

$$\frac{d\hat{\sigma}}{dp_T^2} = \frac{\alpha_S C_i}{p_{\mathrm{T}}^2 \pi} \exp\left(-\frac{\alpha_S C_i}{2\pi} L\right) \sum_{N=1}^{\infty} \left(-\frac{2\alpha_S C_i}{\pi}\right)^{N-1} \frac{1}{(N-1)!}$$

$$\times \sum_{m=0}^{N-1} \begin{pmatrix} N-1 \\ m \end{pmatrix} L^{N-1-m} (2\tau_{N+m} + L\tau_{N+m-}$$

with $L = \log(Q^2/p_{\mathrm{T}}^2)$, $\tau_m = \int_0^\infty dy J_1(y) \log^m(y/b_0)$

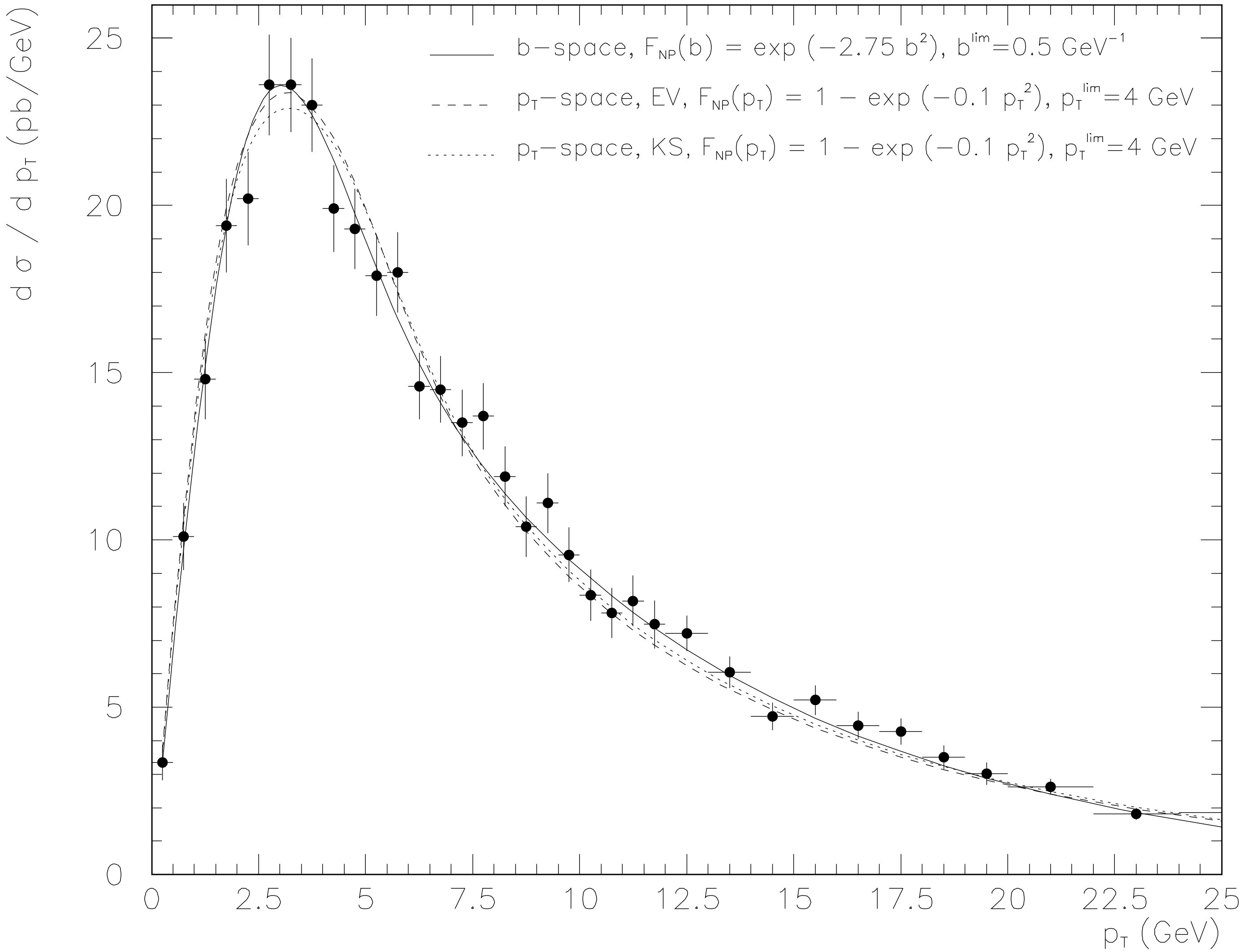
$\Rightarrow$ can be extended to deal with the full CSS formula [*AK, Stirling'99*]

- if some subleading terms neglected, possible to sum up $\log(Q^2/p_{\mathrm{T}}^2)$ to all orders into an exponential $\longrightarrow$ 'extended DLLA formula' [*Ellis, Veseli'98*]

p_{T}- and b-space at the Tevatron

Z production at the Tevatron: theory vs data (CDF, Run 1a+b), no matching

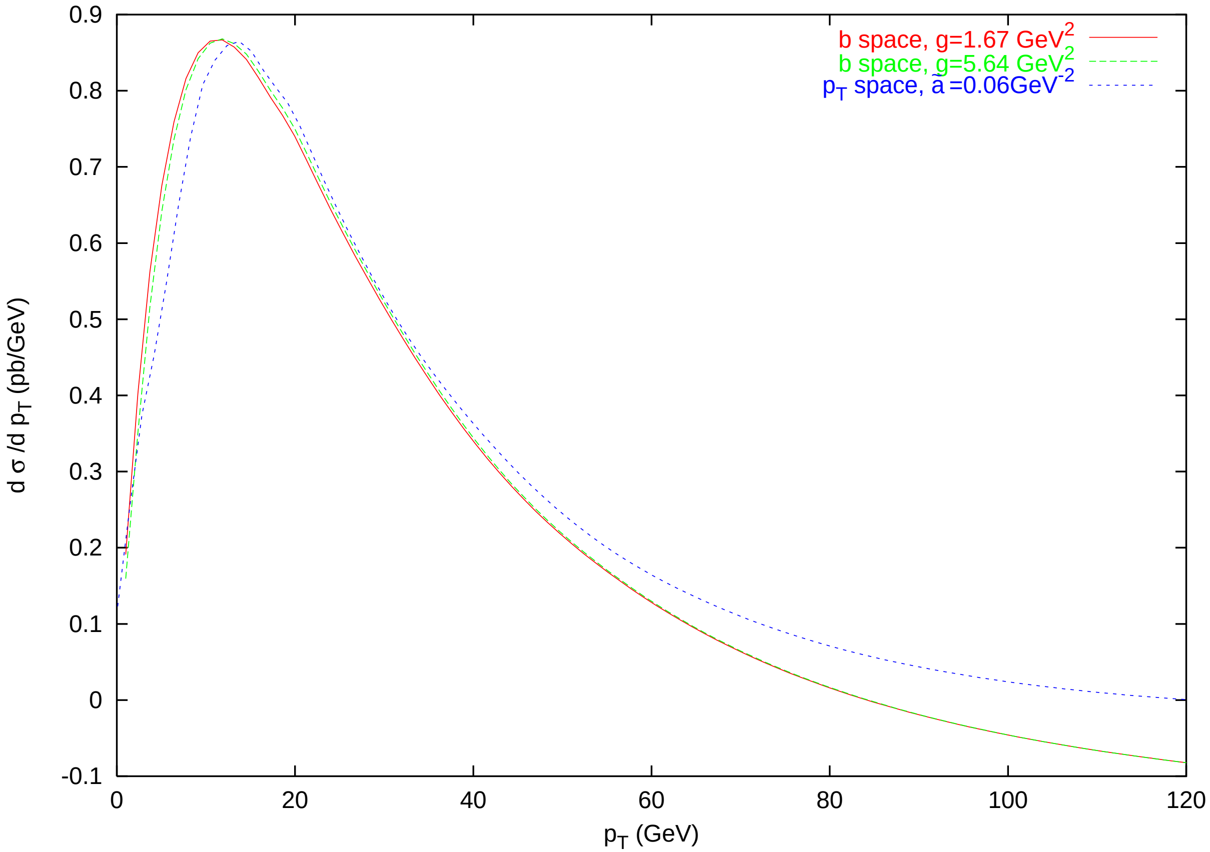
[*A.K., Stirling'01*]



p_{T}-and b-space at the LHC

Higgs production via gluon fusion at the LHC

[*A.K., Stirling'03*]



(N)NLL resummed predictions matched to LO, M_H=125 GeV

Summary

Transverse momentum distribution for colour singlet state production in hadronic collisions

- Fixed order calculations not sufficient; resummed calculations necessary
- CSS (and variants) recoil resummation method: well tested and reliable
- Technical advances
  - NNLL+NLO accuracy being reached
  - Many new developments, e.g.
    - treatment of the Landau pole
    - N(N)LL + LO joint resummation of recoil and threshold corrections
- Precision needed for theoretical predictions at the LHC !