Transport in nanostructures at the level of single electrons

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Simon Gustavsson
Roland Schleser
Martin Sigrist
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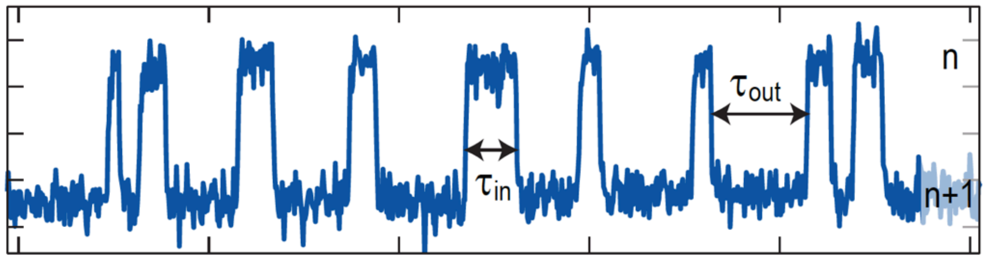
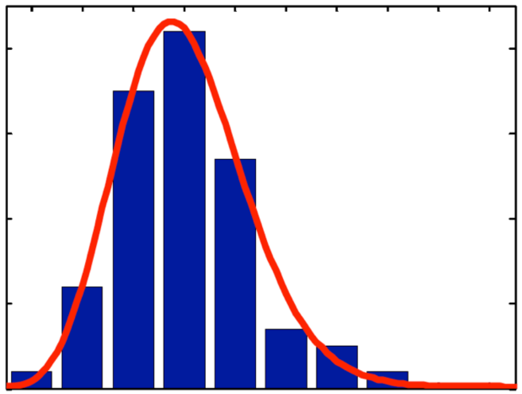
Wafers:

Werner Wegscheider
Christian Reichl

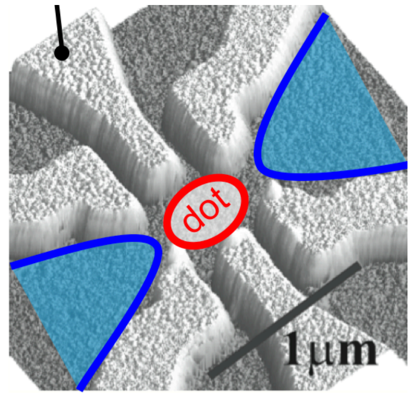




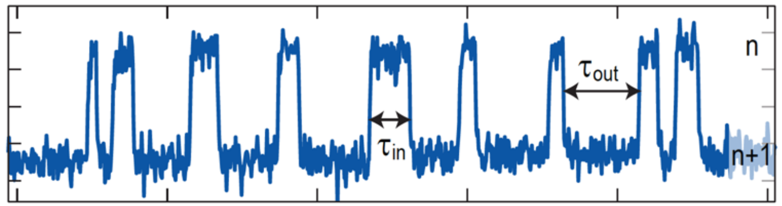
QSIT Quantum Science and Technology
National Centre of Competence in Research

ETH Eidgenössische Technische Hochschule Zürich

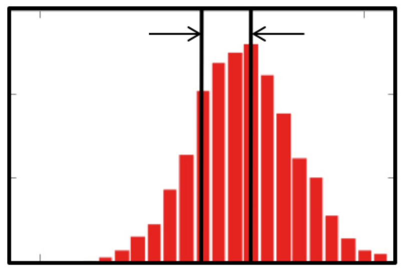
Our (tentative) program



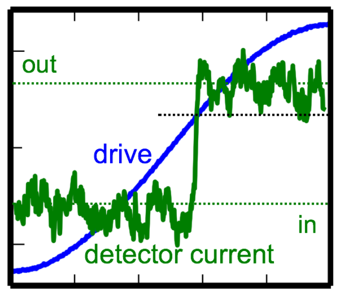
I. Handling single electrons



II. Single electrons in real time



III. Thermodynamic aspects of electron transport



IV. Non-equilibrium thermodynamics at the level of single electrons

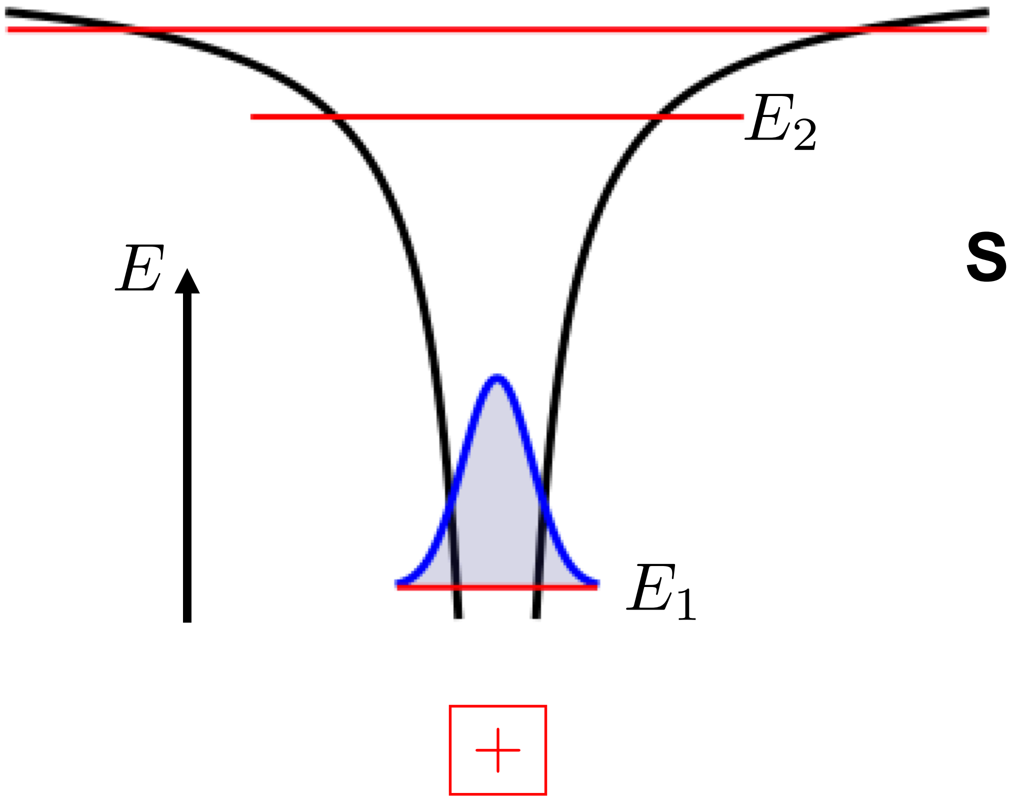
Part I

Handling single electrons

Which groundbreaking paper of physics was published 91 years ago?

The hydrogen atom

E. Schrödinger, Phys. Rev. **28**, 1049 (1926).
Quantized energy levels in the hydrogen atom

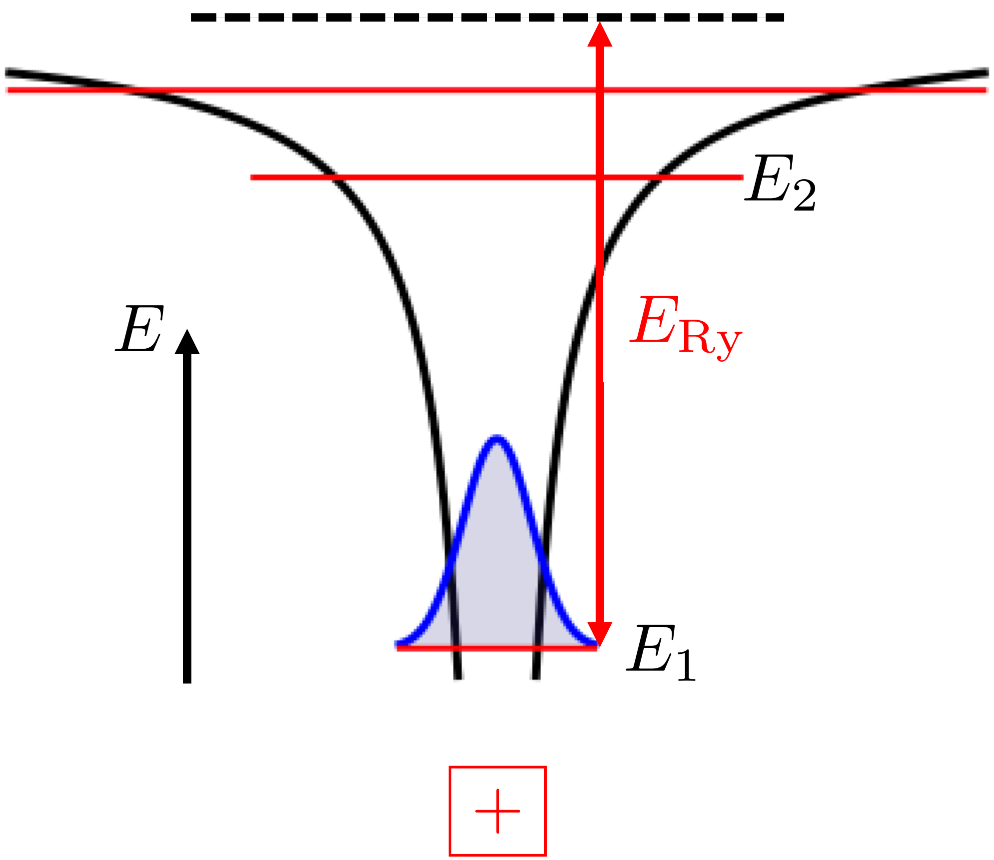


$$E_n = -\frac{E_{\mathrm{Ry}}}{n^2}, \qquad n = 1, 2, 3 \ldots$$

Single-particle level spectrum

The hydrogen atom

E. Schrödinger, Phys. Rev. **28**, 1049 (1926).
Quantized energy levels in the hydrogen atom



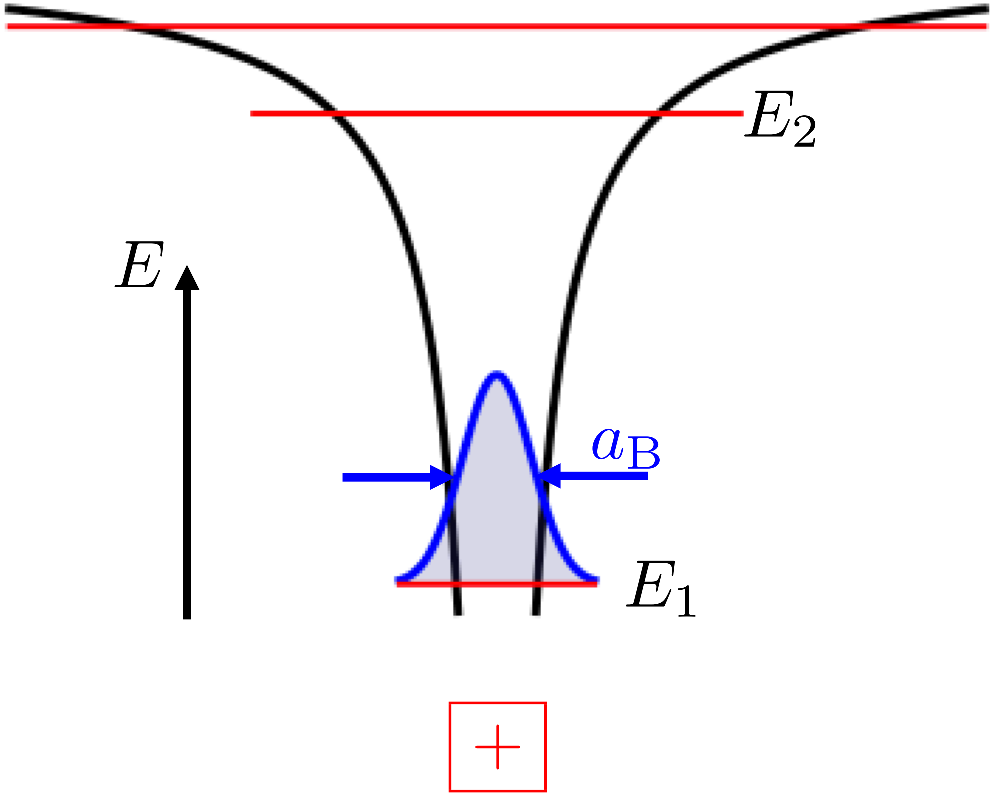
$$E_n = -\frac{E_{\mathrm{Ry}}}{n^2}, \qquad n = 1, 2, 3 \ldots$$

$$\boxed{E_{\mathrm{Ry}} = \frac{2m}{\hbar^2}\left(\frac{e^2}{8\pi\epsilon_0}\right)^2}$$

Atoms: 13.6 eV
GaAs: 6 meV

The hydrogen atom

E. Schrödinger, Phys. Rev. **28**, 1049 (1926).
Quantized energy levels in the hydrogen atom



$$E_n = -\frac{E_{\mathrm{Ry}}}{n^2}, \qquad n = 1, 2, 3 \ldots$$

$$\boxed{a_B = \frac{\hbar^2}{2m}\left(\frac{8\pi\epsilon_0}{e^2}\right)}$$

Atoms: 0.53 Å
GaAs: 10 nm

Lessons from the hydrogen problem

Binding energy scales like $$E^{\star}_{\mathrm{Ry}} \propto \frac{m^{\star}}{\epsilon^{2}}$$

Size scales like $$a^{\star}_{\mathrm{B}} \propto \frac{\epsilon}{m^{\star}}$$

Technology sets a lower limit on size.

Materials with small $m^{\star}$ and large ϵ are advantageous.

The price is a smaller energy scale.

→ Low-temperature experiments with nanostructures

Pauli's exclusion principle

Wolfgang Pauli:

- 1923 Privatdozent at the University of Hamburg
- Spring 1925: General formulation of the exclusion principle

W. Pauli, Z. Physik **31**, 765 (1925)

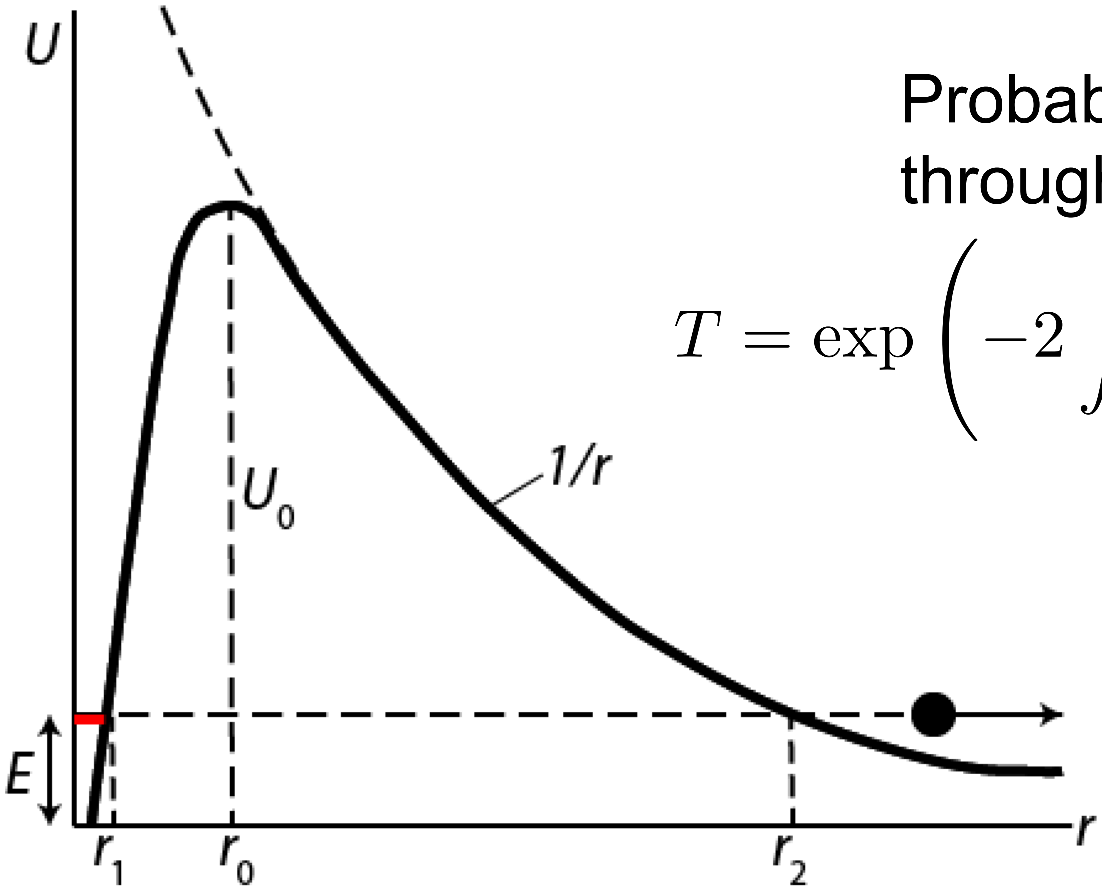
- Nobel Prize in Physics 1945

each level can be occupied with one spin-up electron and one spin-down electron

Could the fathers of quantum theory envision tunneling from discrete levels?

Tunneling from discrete levels

G. Gamov, Z. Phys. **51**, 204 (1928).
α-decay described by tunneling from quantized levels



Probability of transmission through barrier:

$$T = \exp\left(-2\int_{x_1}^{x_2}\sqrt{\frac{2m[U(x)-E]}{\hbar^2}}dx\right)$$

Statistical properties of tunneling decays

Processes with constant (in time) tunneling rate Γ.

Tunneling probability (at any time): $\Gamma \mathrm{d}t$

Statistical properties of tunneling decays

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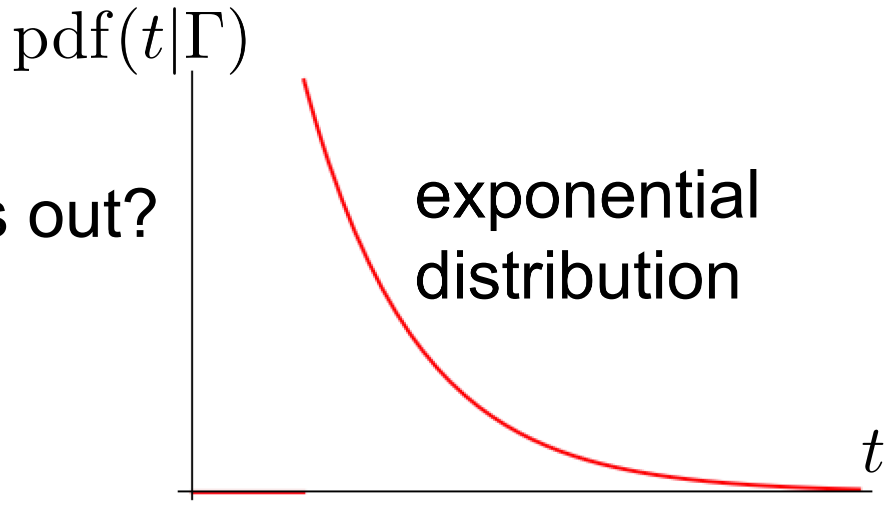
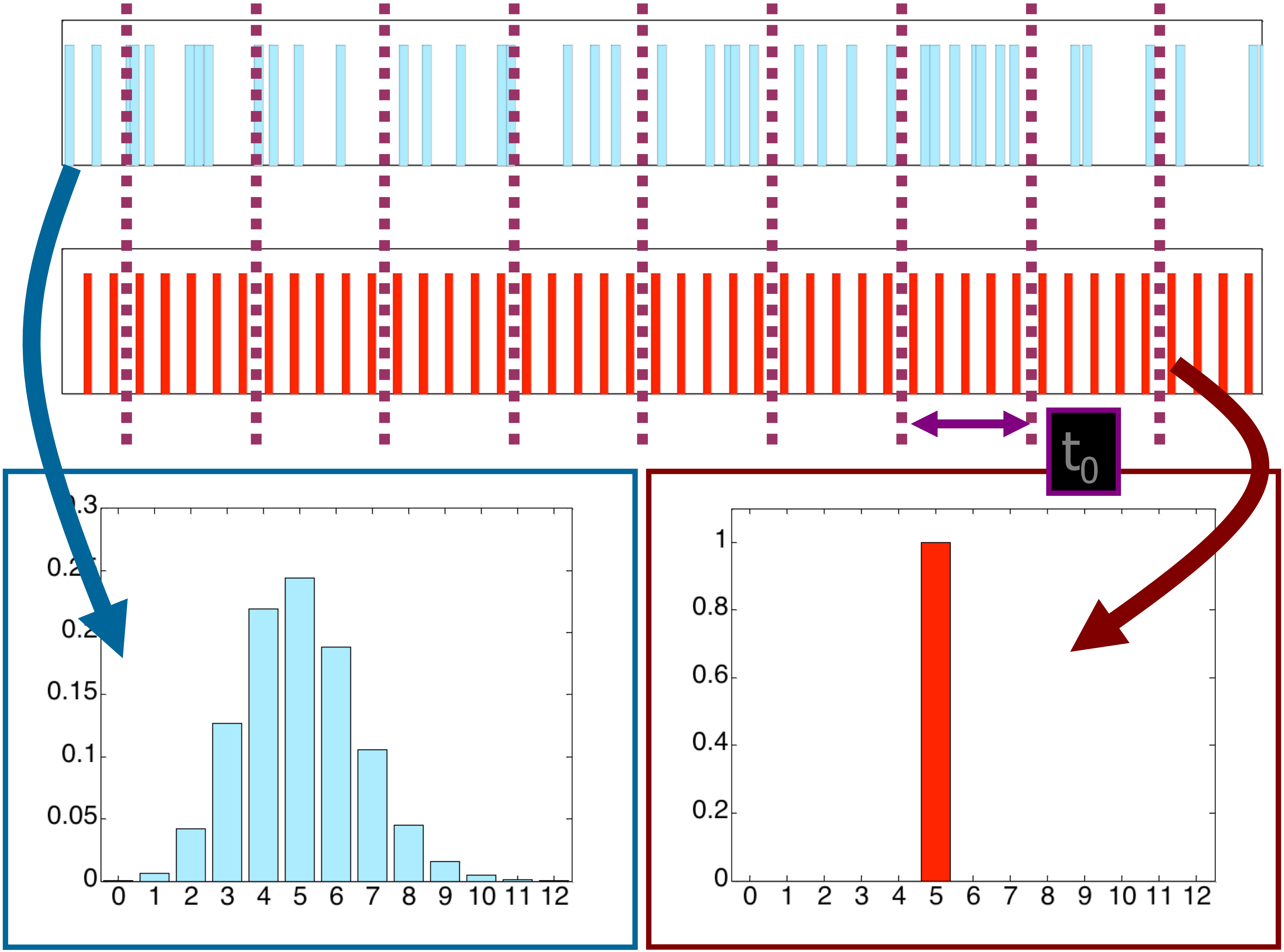
Waiting time statistics:

$$\boxed{\mathrm{prob}(t|\Gamma) = e^{-\Gamma t}\Gamma \mathrm{d}t}$$

Place a particle in a quantum level
How long does it take until it tunnels out?





t0
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9 10 11 12
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10 11 12

Statistical properties of tunneling decays

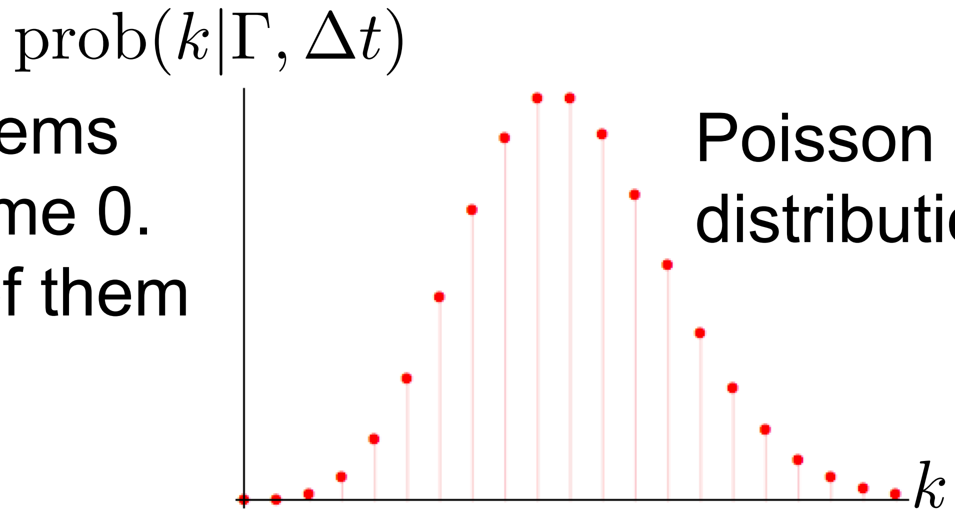
Processes with constant (in time) tunneling rate Γ.

Tunneling probability (at any time): $\Gamma \mathrm{d}t$

Full counting statistics:

$$\boxed{\mathrm{prob}(k|\Gamma,\Delta t) = \frac{(\Gamma \Delta t)^k}{k!} e^{-\Gamma \Delta t}}$$

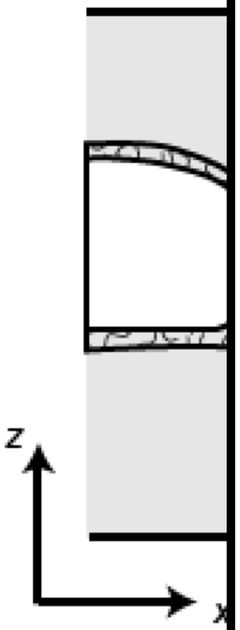
Take a large ensemble of systems that can decay starting from time 0. What is the probability that *k* of them have decayed within time Δt?



Poisson distribution

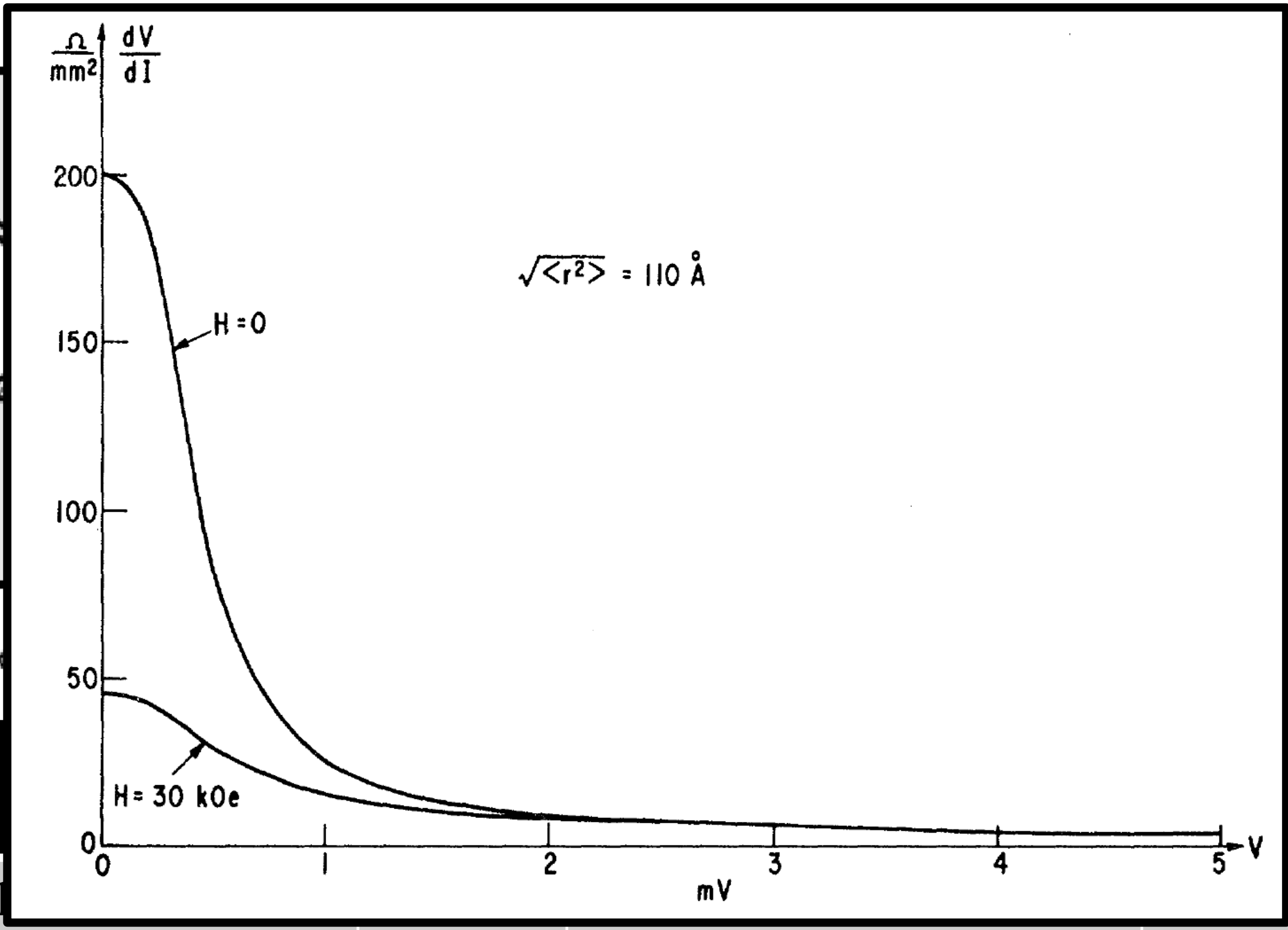
Electron tunneling through small grains of tin

Giaever and Zeller, Phys. Rev. Lett. **20**, 1504 (1968)



= 1.6 K

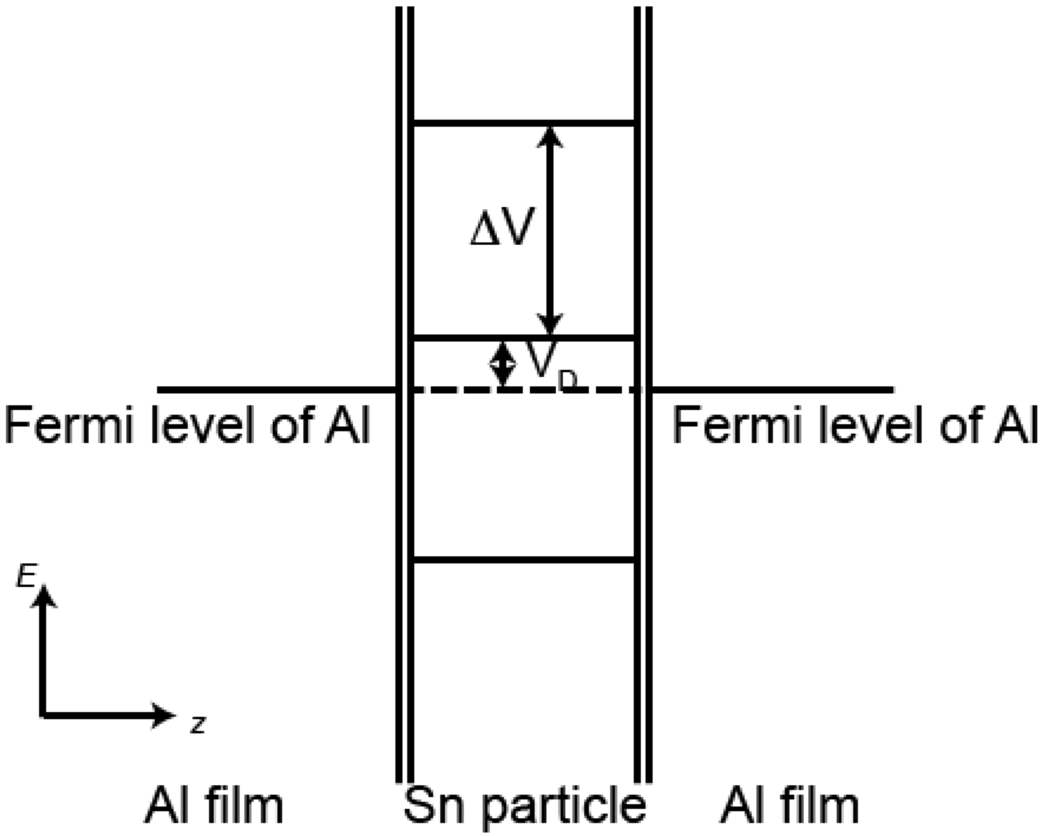
um



			v. temperature
sample wit			160 mK
sample with largest grains	100 nm	8 neV	20 nK

Coulomb gap in tunneling

Giaever and Zeller, Phys. Rev. Lett. **20**, 1504 (1968)



Charging a capacitor:

$$\Delta Q = C \Delta V$$

$$C = 4\pi\varepsilon\varepsilon_0 r \quad \text{(Sphere)}$$

Charging a single electron:

$$\boxed{\Delta V = \frac{e}{C} = \frac{e}{4\pi\epsilon\epsilon_0 r}}$$

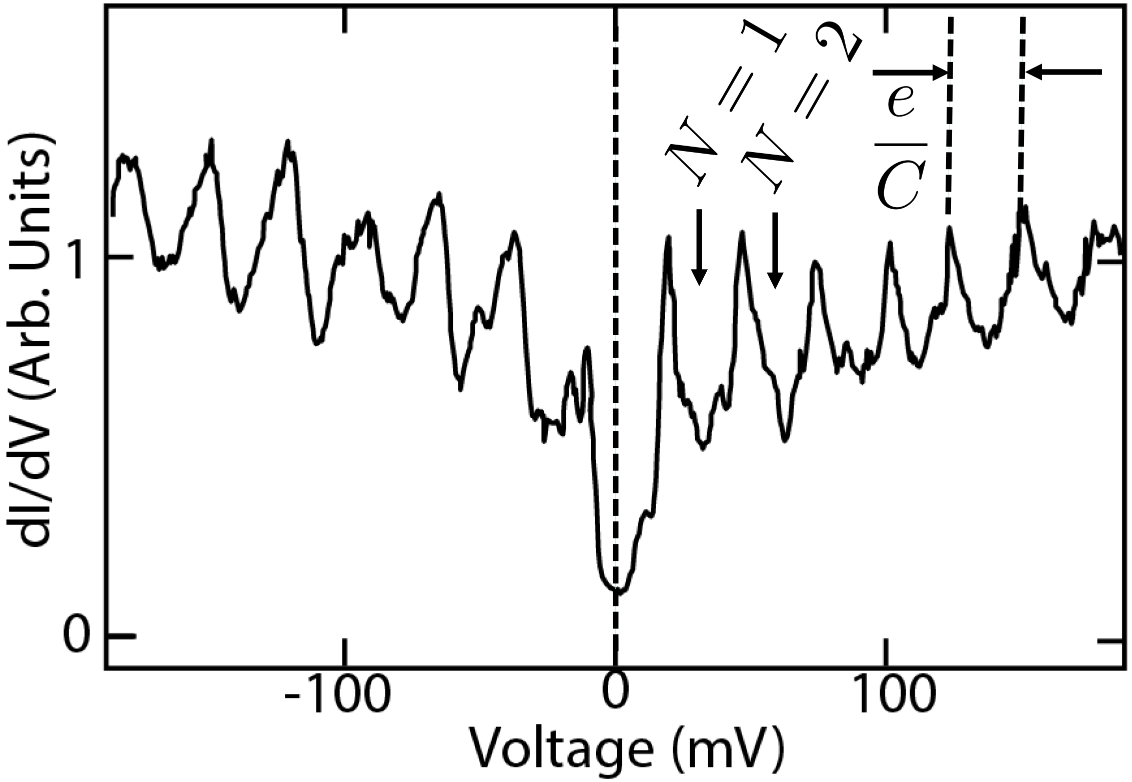
Grains: 1–10 mV

Atoms: 10 V

Coulomb staircase

Barner and Ruggiero, Phys. Rev. Lett. **59**, 807 (1987)

Addition spectrum (classical)



Silver grains 7.5 nm

aluminium oxide tunneling barriers

$$E(N) = \frac{Q^2}{2C} = \frac{e^2 N^2}{2C}$$

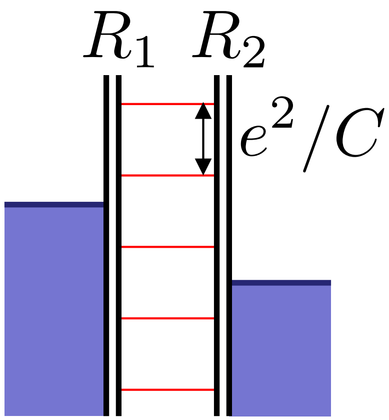
$$\boxed{\Delta V(N) = \frac{e}{C}\left(N + \frac{1}{2}\right)}$$

Single-electron tunneling current

Barner and Ruggiero, Phys. Rev. Lett. **59**, 807 (1987)

charging time of grain: $\tau_1 = R_1 C$

discharging time of grain: $\tau_2 = R_2 C$



Total single-electron tunneling time through one level:

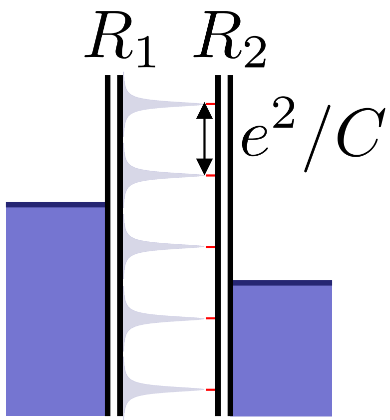
$$\tau = (R_1 + R_2)C = RC$$

Single-electron tunneling current through one level:

$$\boxed{I = \frac{e}{\tau} = \frac{e}{RC}}$$

How big is a tunneling resistance?

Condition on the tunneling resistance:
time-energy uncertainty



lifetime broadening of grain level:

$$\Delta E_\tau = \frac{h}{R_1 C} + \frac{h}{R_2 C} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{h}{C}$$

Resolving charging energy is possible, if

$$\Delta E_\tau \ll \frac{e^2}{C} \Rightarrow \boxed{\frac{1}{R_1} + \frac{1}{R_2} \ll \frac{e^2}{h} = \frac{1}{25.8\,\mathrm{k}\Omega}}$$

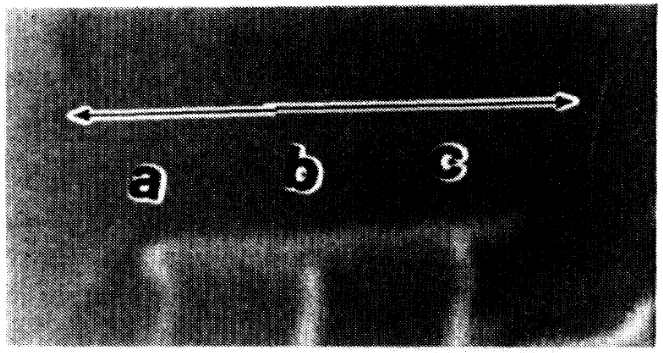
Progress in nanolithography

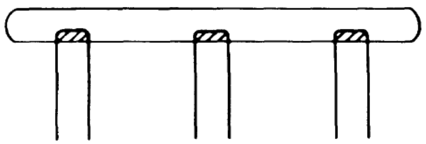
Lateral Metal
Single-Electron Transistor (SET)

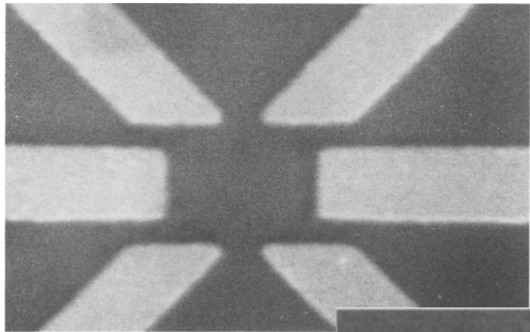
Fulton and Dolan,
Phys. Rev. Lett. **59**, 109 (1987)

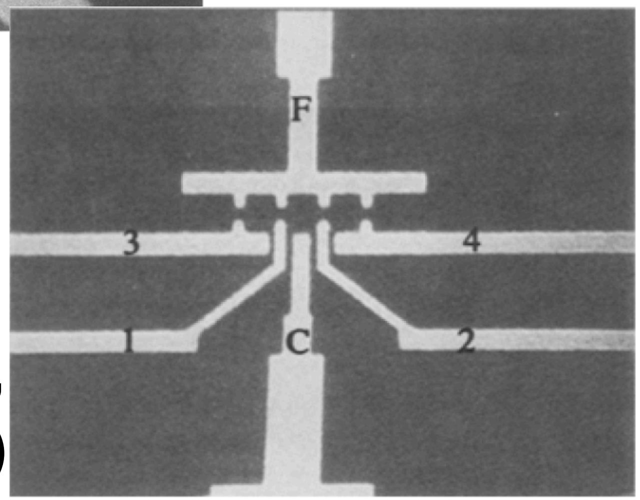
Lateral Semiconductor
Quantum Dot

Staring *et al*,
Physica B **175**, 226 (1991)









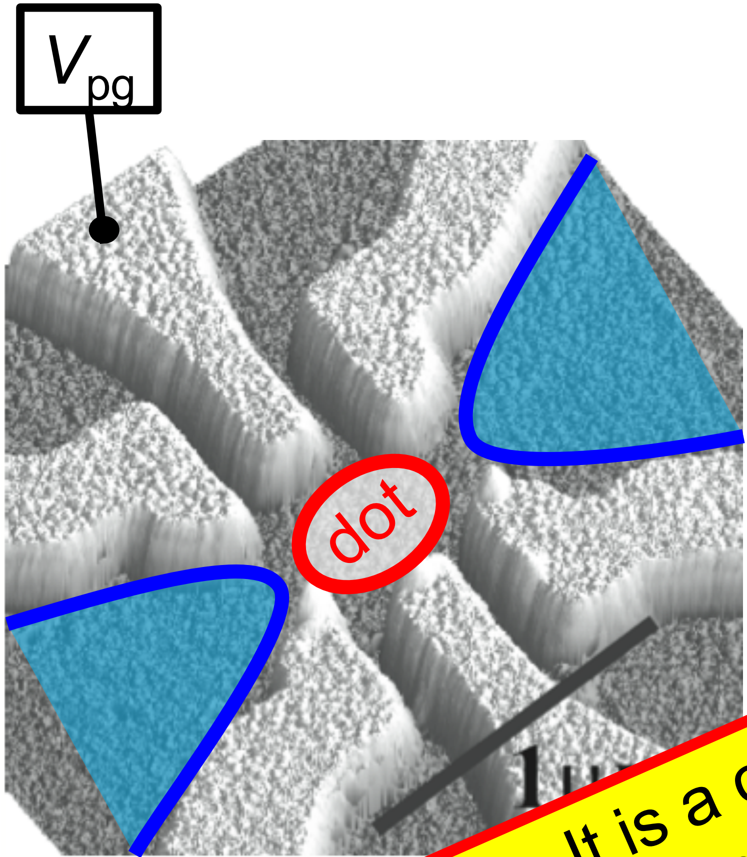
Kouwenhoven *et al*,
Z. Phys. B **85**, 367 (1991)

Atoms vs. quantum dots

	Atom	Quantum dot
Confinement	r^{-1},strong, rigid hard to tune	r^2, soft, parabolic tunable
Symmetry	perfect, given by nature	never perfect, hard to achieve
Electrical addressing	hard to achieve	well suitable tunable coupling
Optical addressing	well suitable	well suitable
Coupling to	thermal photons, …	photons, phonons, other electrons

Both systems give access to single electron/spin manipulation

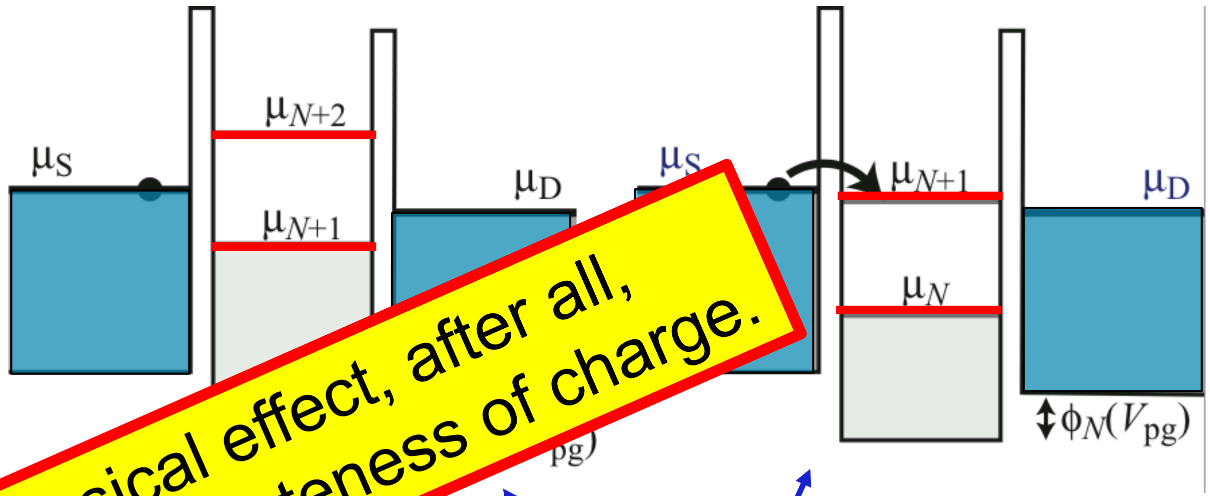
Coulomb blockade



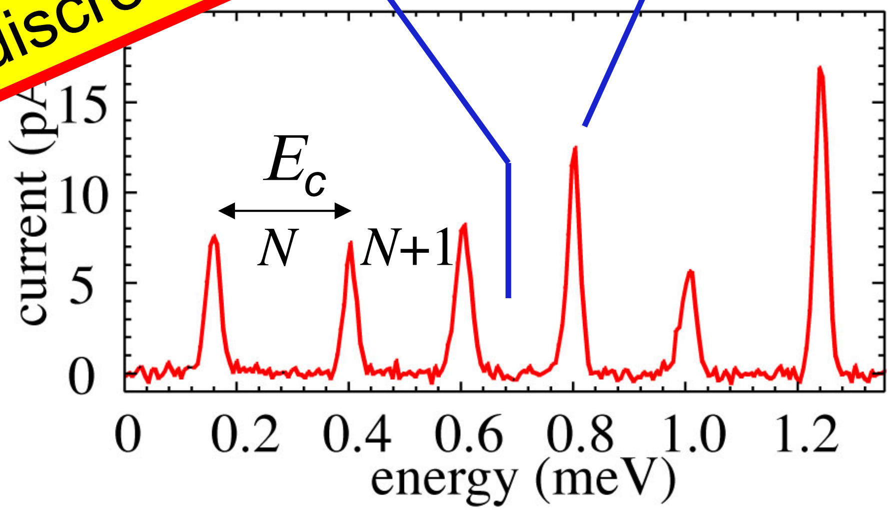
2D-electron g ow
the surface

Coulomb blockade

Resonant transport



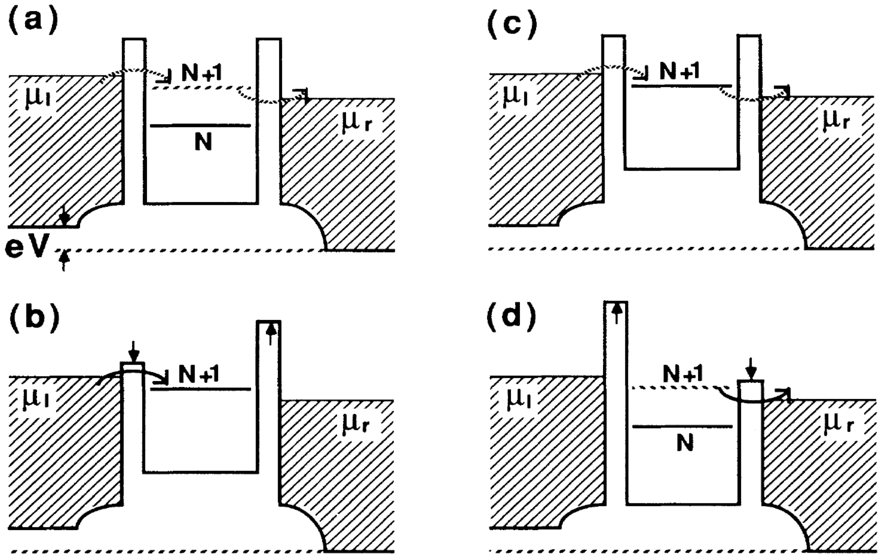
It is a classical effect, after all, based on the discreteness of charge.



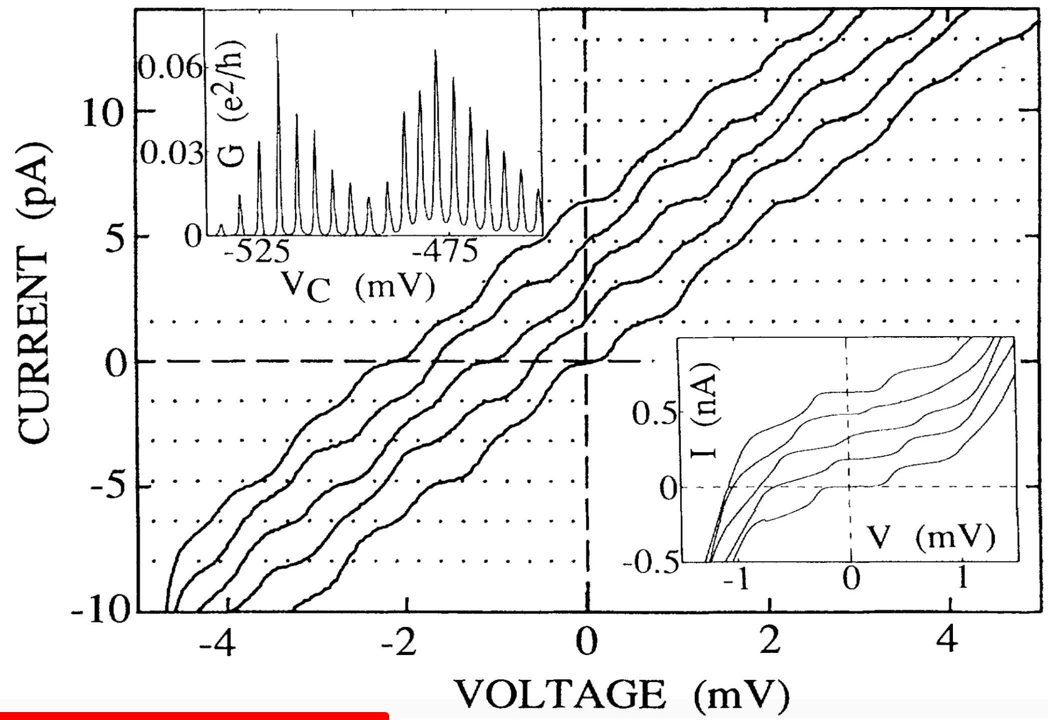
How do we know it is single electrons?

Kouwenhoven *et al*, Phys. Rev. Lett. **67**, 1626 (1991)

Quantum dot turnstile



f = 10 MHz

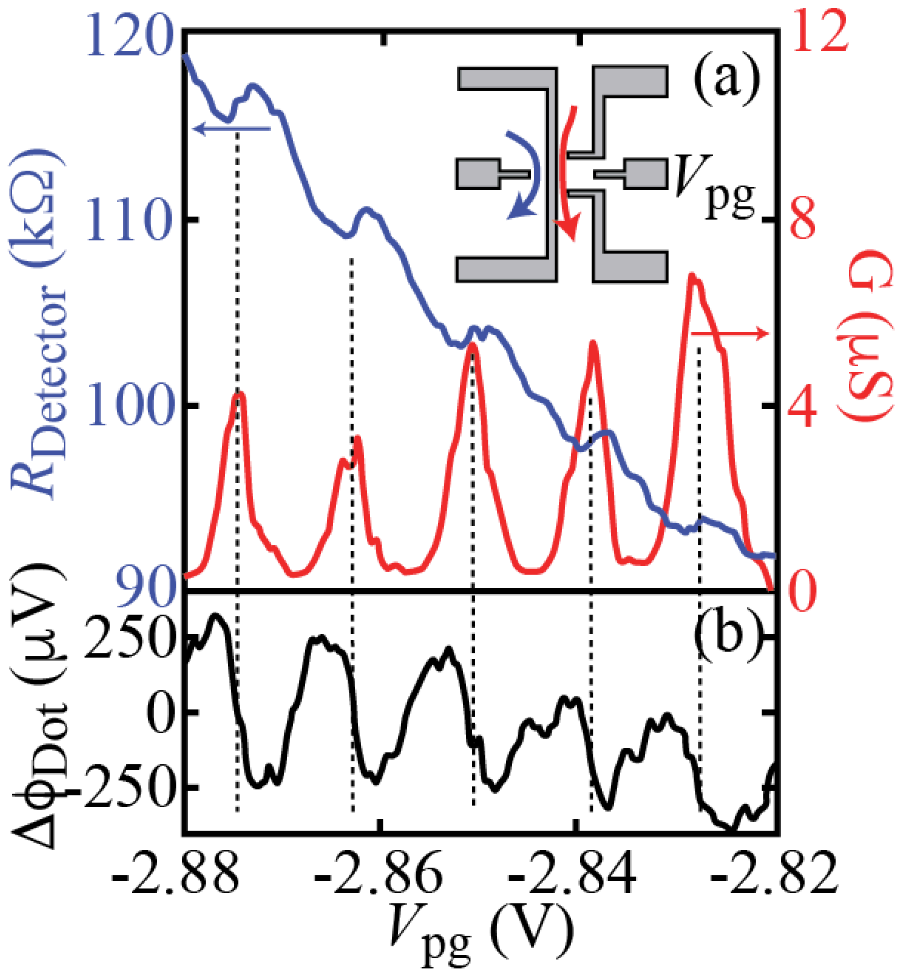


$$\boxed{I = nef, \quad n = 1, 2, 3 \ldots}$$

How do we know it is single electrons?

Field *et al*, Phys. Rev. Lett. **70**, 1311 (1993)

Single-electron charge detection



Part II

Single electrons in real time

Single-electron switching in MOSFETs

1/ f and random telegraph noise in silicon metal-oxide-semiconductor field-effect transistors

M. J. Uren, D. J. Day,[a] and M. J. Kirton
Royal Signals and Radar Establishment, Great Malvern, Worcestershire, United Kingdom

(Received 30 May 1985; accepted for publication 17 September 1985)

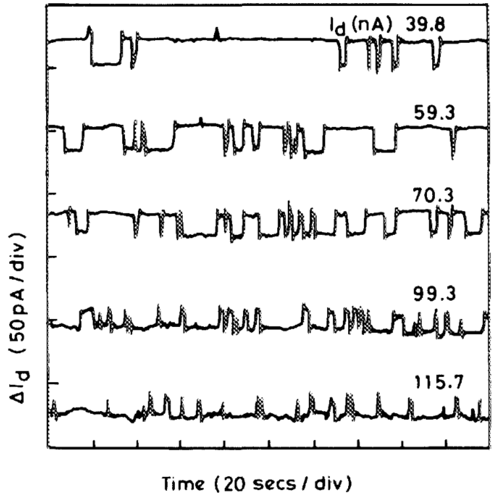


FIG. 2. Random telegraph signals. Change in current vs time measured at the indicated currents. Active area is 0.4 μm^2, $V_d = 4$ mV, $T = 293$ K.

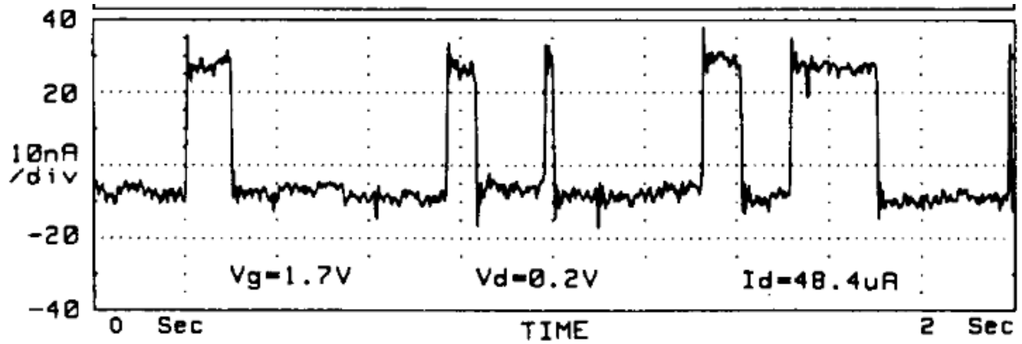


Fig. 1. The fluctuations of drain current at different gate voltages for a deep-submicrometer MOSFET ($W_{eff} = 1.2$ μm, $L_{eff} = 0.35$ μm, $T_{ox} = 8.6$ nm).

90 IEEE ELECTRON DEVICE LETTERS, VOL. 11, NO. 2, FEBRUARY 1990

Random Telegraph Noise of Deep-Submicrometer MOSFET's

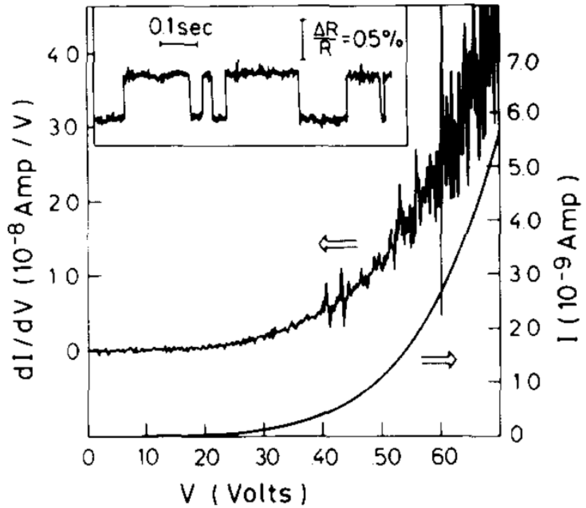
K.K. HUNG, P. K. KO, CHENMING HU, SENIOR MEMBER, IEEE, AND
YIU CHUNG CHENG, MEMBER, IEEE

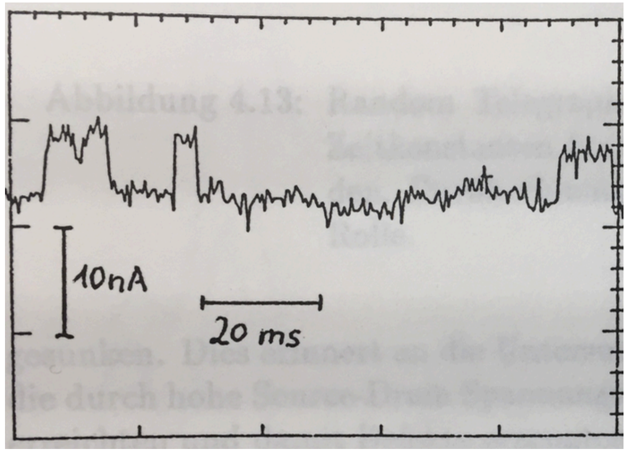
Single-electron switching in amorphous barriers

RANDOM TELEGRAPHIC NOISE IN LARGE AREA a-Si:H/a-$Si_{1-x}N_x$:H DOUBLE BARRIER STRUCTURES

Roberto ARCE[a], Lothar LEY[b], and Martin HUNDHAUSEN[b]

Max-Planck-Institut für Festkörperforschung, Heisenbergstr. 1, D-7000 Stuttgart 80, Federal Republic of Germany





Journal of Non-Crystalline Solids 137&138 (1991) 523–526
North-Holland

JOURNAL OF NON-CRYSTALLINE SOLIDS

CURRENT FLUCTUATIONS IN THIN a-SiC:H FILMS

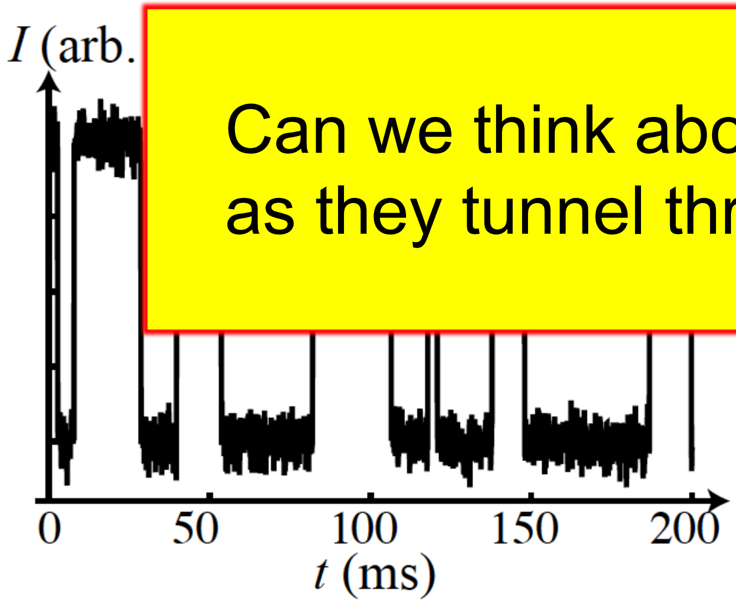
Th. Ihn, A.K. Savchenko*, M.E. Raikh, R. Schwarz

Technische Universität München, Physikdepartment E16, James-Franck-Straße, W-8046 Garching, Federal Republic of Germany

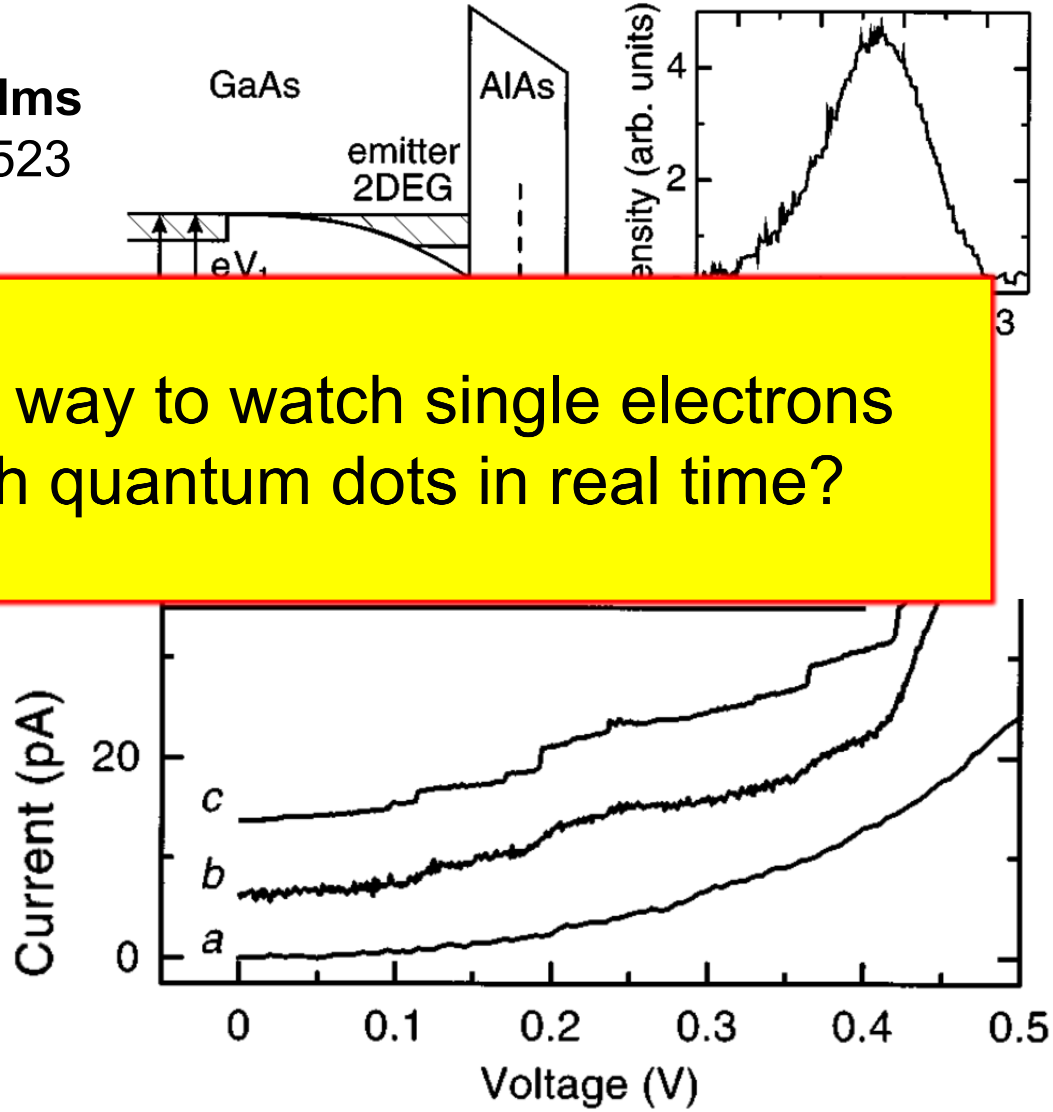
A physicist's dream

Tunneling through InAs quantum dots
I.E. Itskevich, TI *et al*, Phys. Rev. B **54**, 16401 (1996).

Current fluctuations in thin a-Si:H films
TI *et al,* J Non-Crystalline Solids **137**, 523 (1991).



Random telegraph noise



Can we think about a way to watch single electrons as they tunnel through quantum dots in real time?

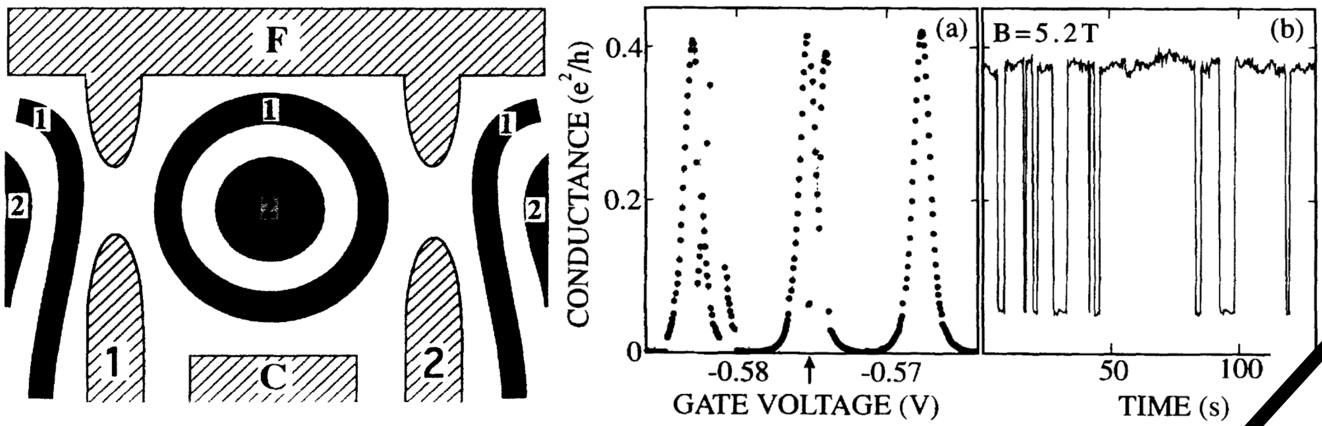
Historic work

VOLUME 73, NUMBER 2 PHYSICAL REVIEW LETTERS 11 JULY 1994

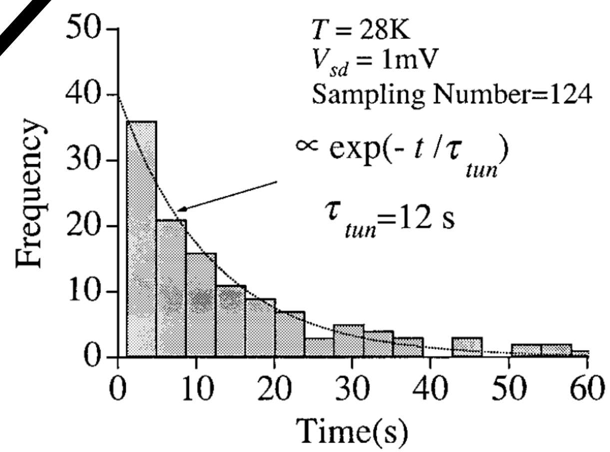
Time-Resolved Tunneling of Single Electrons between Landau Levels in a Quantum Dot

N. C. van der Vaart, M. P. de Ruyter van Steveninck, L. P. Kouwenhoven,*
A. T. Johnson,† Y. V. Nazarov, and C. J. P. M. Harmans
Department of Applied Physics, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands

C. T. Foxon‡
Philips Research Laboratories, Redhill, Surrey RH15HA, United Kingdom
(Received 21 March 1994)



Waiting time distribution



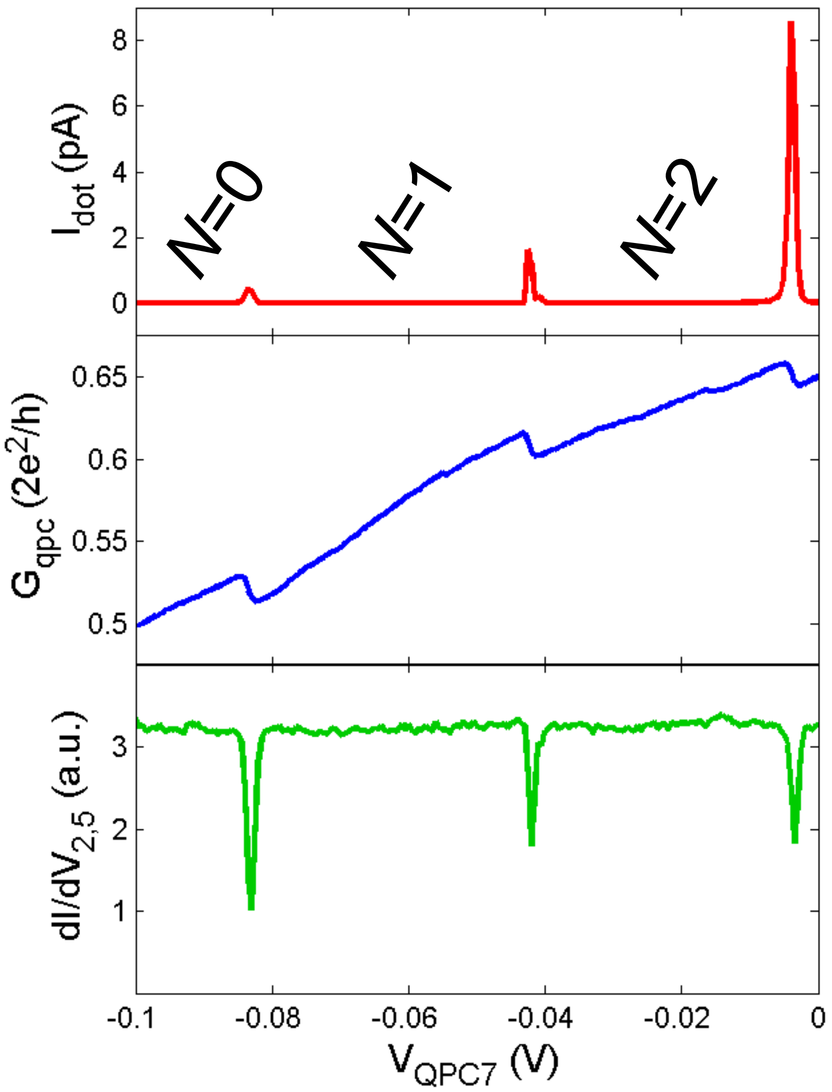
Time-resolved measurement of single-electron tunneling in a Si single-electron transistor with satellite Si islands

A. Fujiwara,[a] Y. Takahashi, K. Murase, and M. Tabe[b]
NTT LSI Laboratories, 3-1 Morinosato Wakamiya, Atsugi, Kanagawa 243-01, Japan

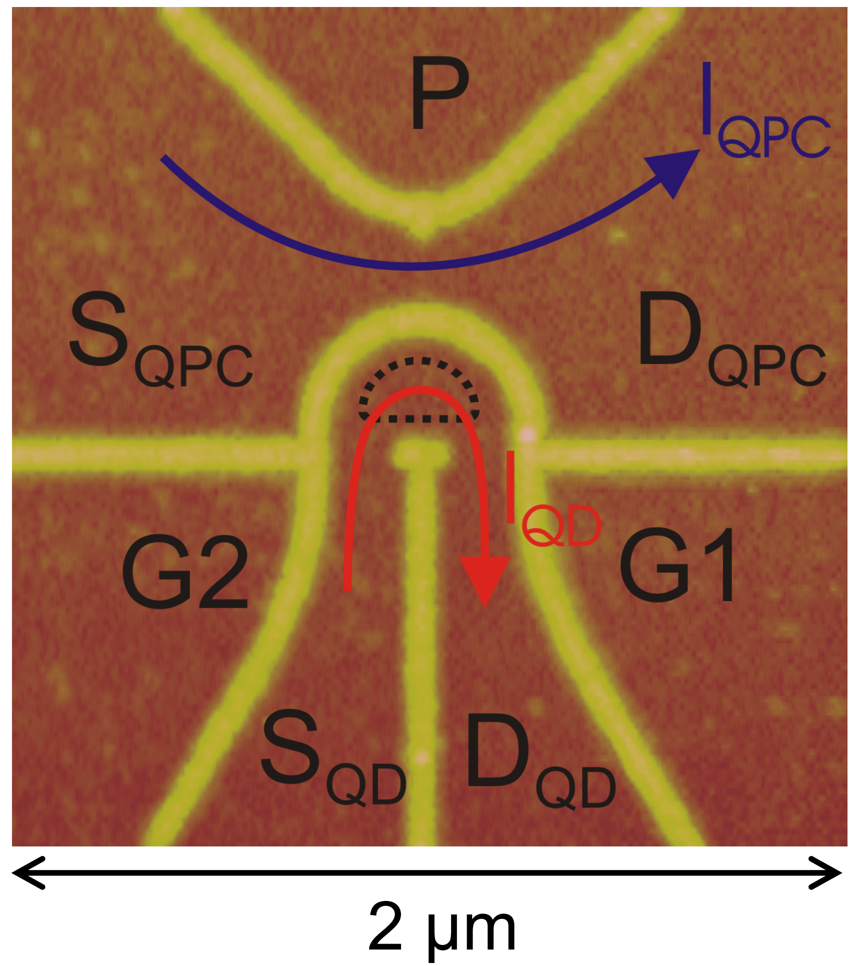
(Received 1 June 1995; accepted for publication 5 September 1995)

Can we see the first electron?

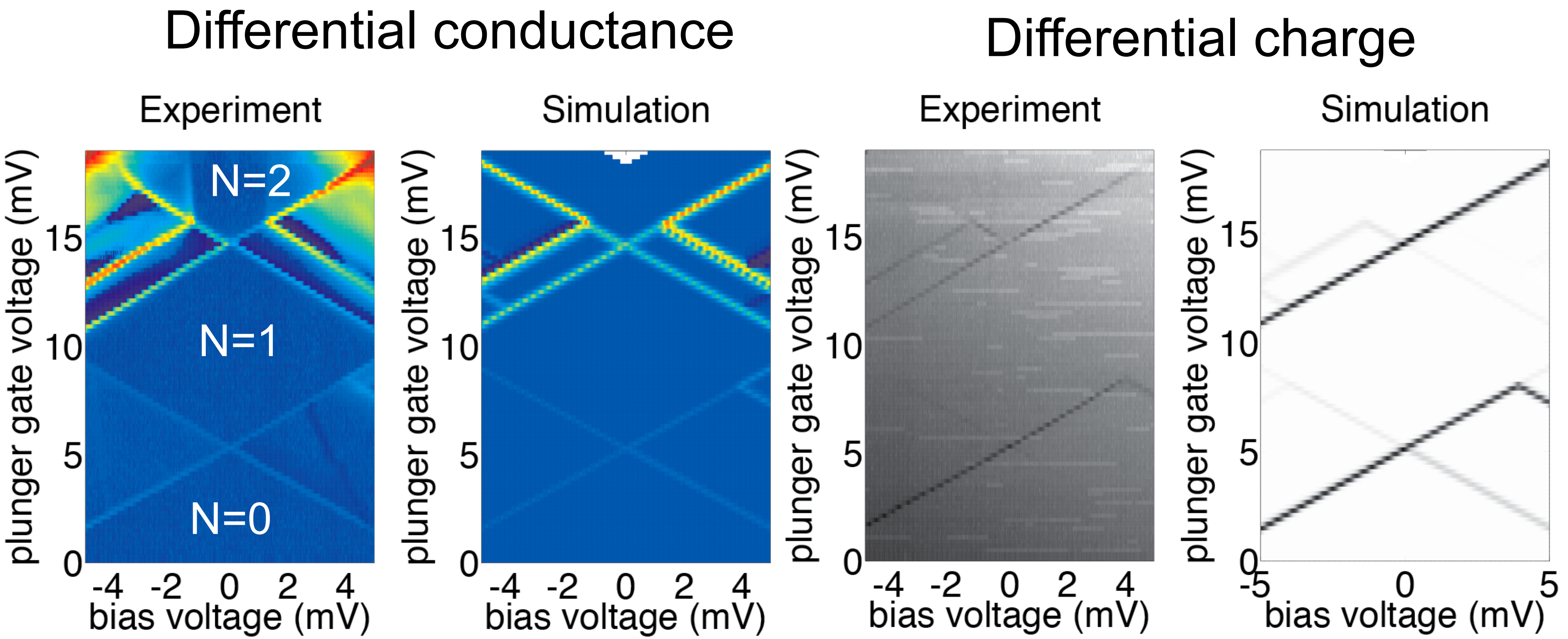
Schleser *et al*, Appl. Phys. Lett. **85**, 2005 (2004)



Semicircular quantum dot



Quantum mechanics reappears!

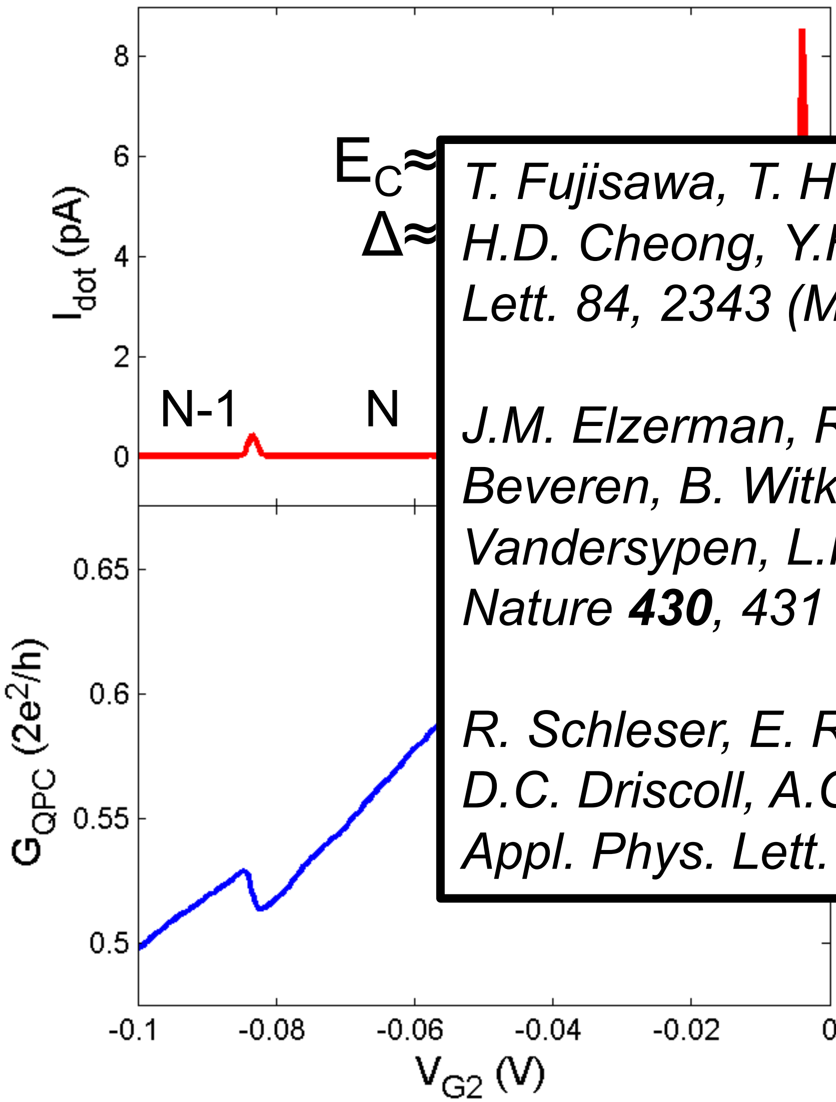


Martin Sigrist, PhD-thesis ETH (2005)

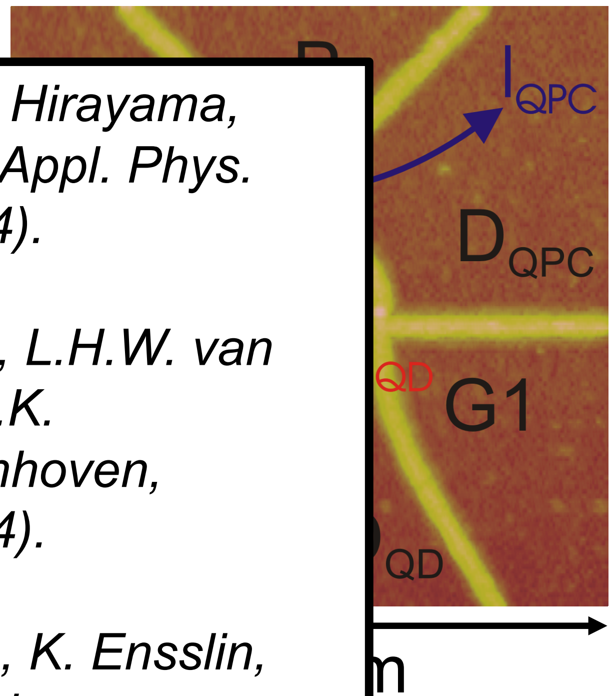
simulation rate equations: C.W. Beenakker, Phys. Rev. B **44**, 1646 (1991)

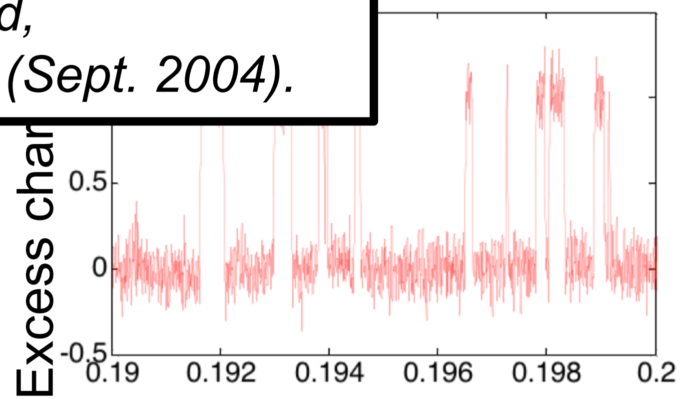
Single-electron tunneling in real time

Schleser *et al*, Appl. Phys. Lett. **85**, 2005 (2004)



Quantum dot sample





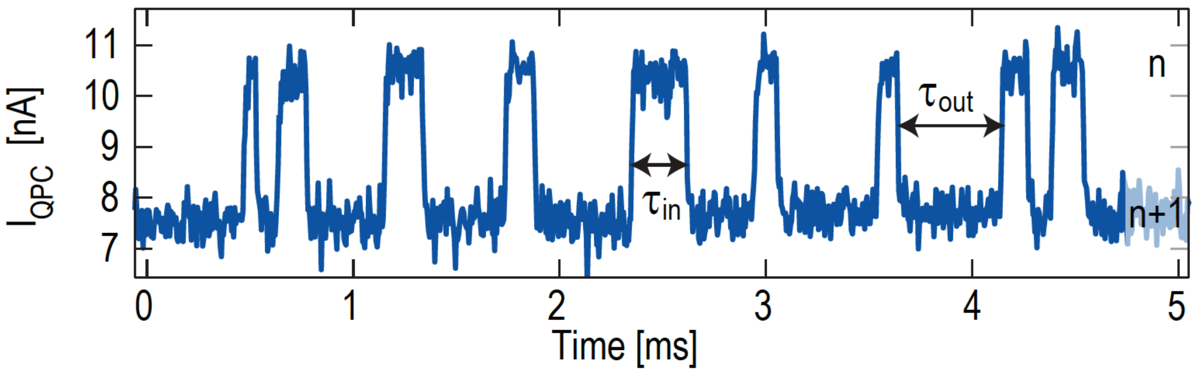
T. Fujisawa, T. Hayashi, Y. Hirayama, H.D. Cheong, Y.H. Jeong, Appl. Phys. Lett. 84, 2343 (March 2004).

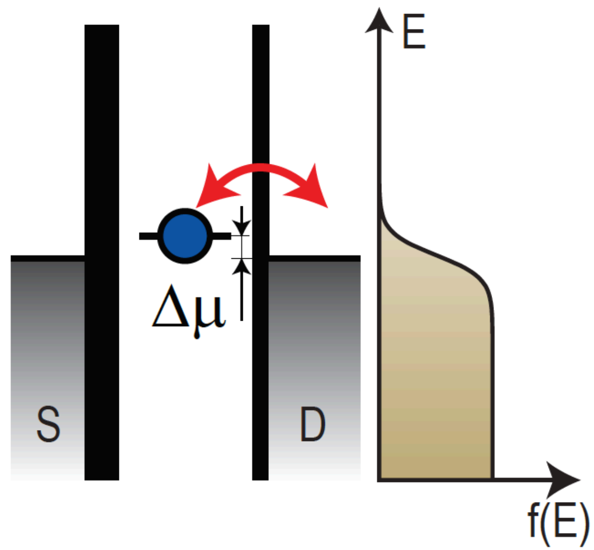
J.M. Elzerman, R. Hanson, L.H.W. van Beveren, B. Witkamp, L.M.K. Vandersypen, L.P. Kouwenhoven, Nature ***430****, 431 (July 2004).*

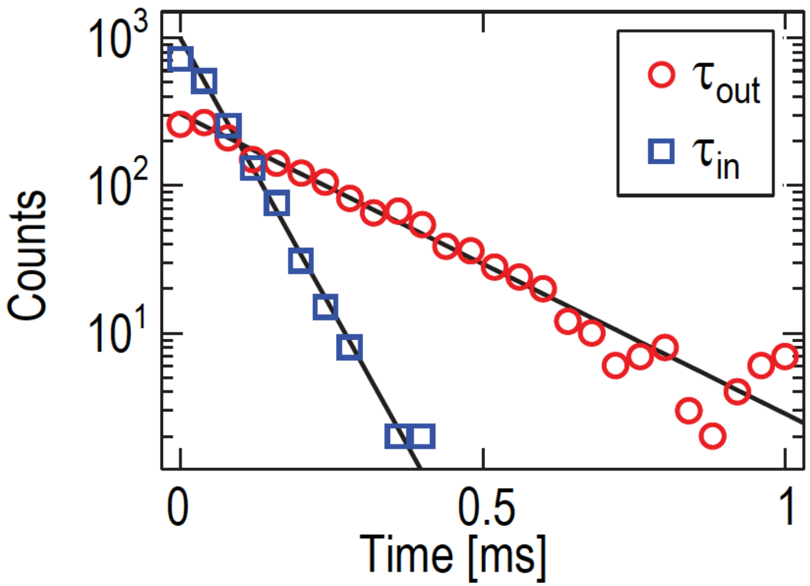
R. Schleser, E. Ruh, T. Ihn, K. Ensslin, D.C. Driscoll, A.C. Gossard, Appl. Phys. Lett. ***85****, 2005 (Sept. 2004).*

Waiting-time statistics for tunneling through a single barrier

Schleser *et al*, Appl. Phys. Lett. **85**, 2005 (2004)
S. Gustavsson, PhD-thesis ETH (2008)







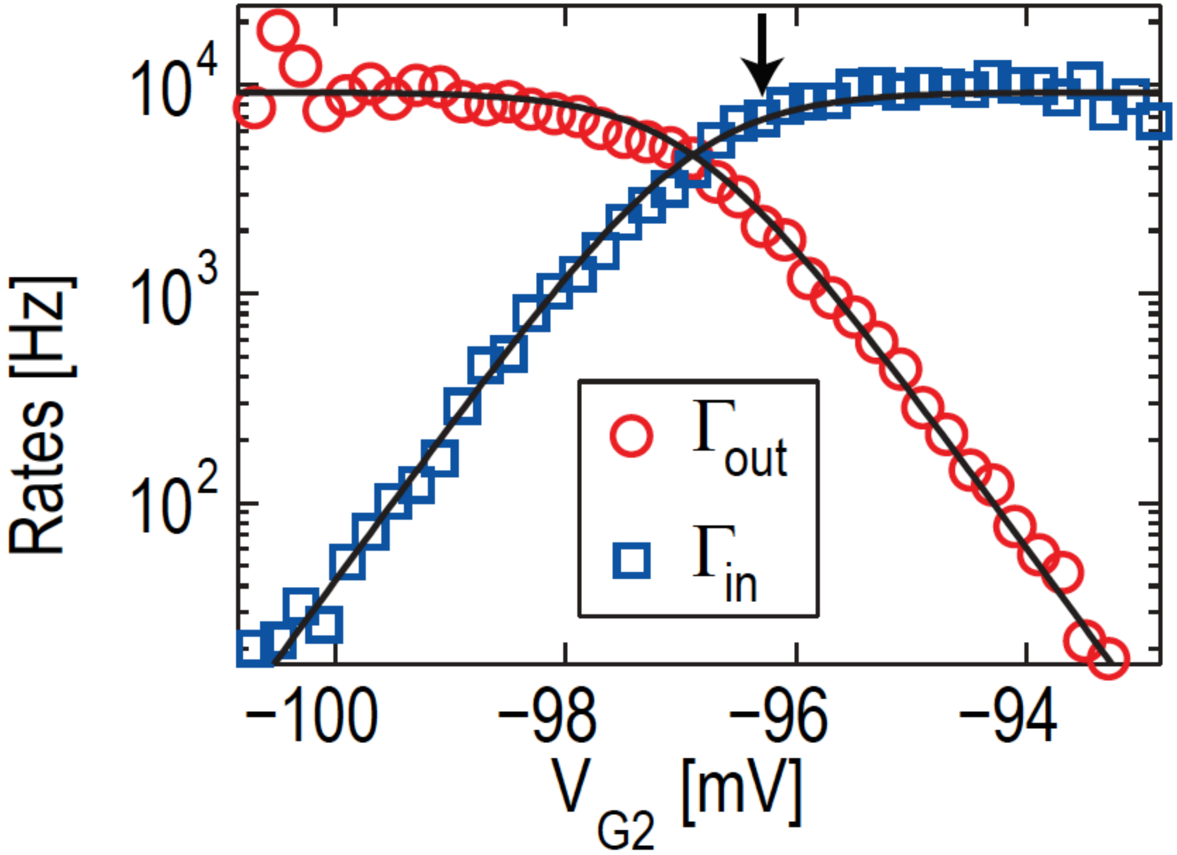
$$\mathrm{pdf}(\tau|\Gamma) = \Gamma e^{-\Gamma\tau}$$

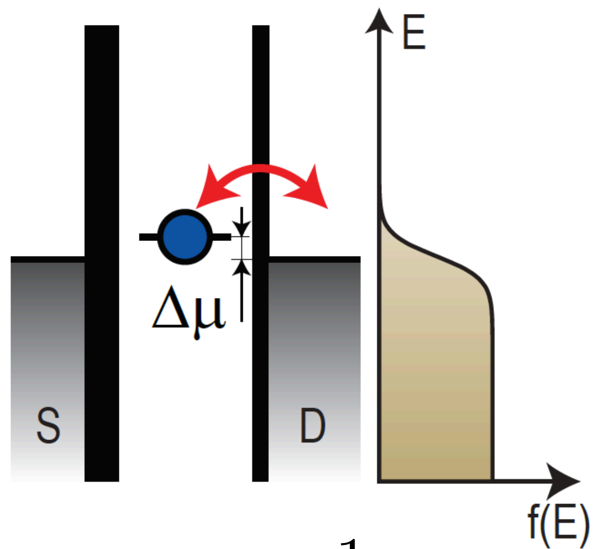
exponential distribution

$$\boxed{\langle\tau\rangle = \int_0^\infty d\tau\,\tau\,\mathrm{pdf}(\tau|\Gamma) = \frac{1}{\Gamma}}$$

Tunneling through a single barrier

Schleser *et al*, Appl. Phys. Lett. **85**, 2005 (2004)
Simon Gustavsson, PhD-thesis ETH (2008)



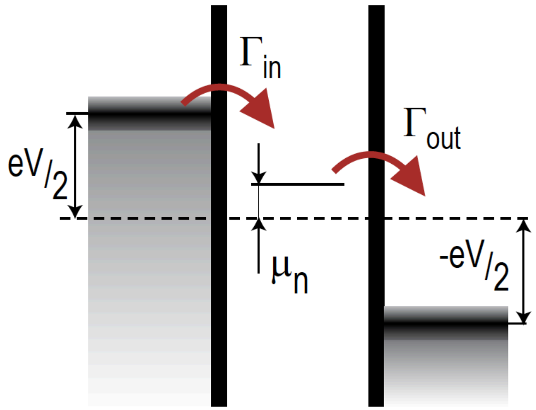


$$\Gamma = \frac{1}{\langle \tau \rangle}$$

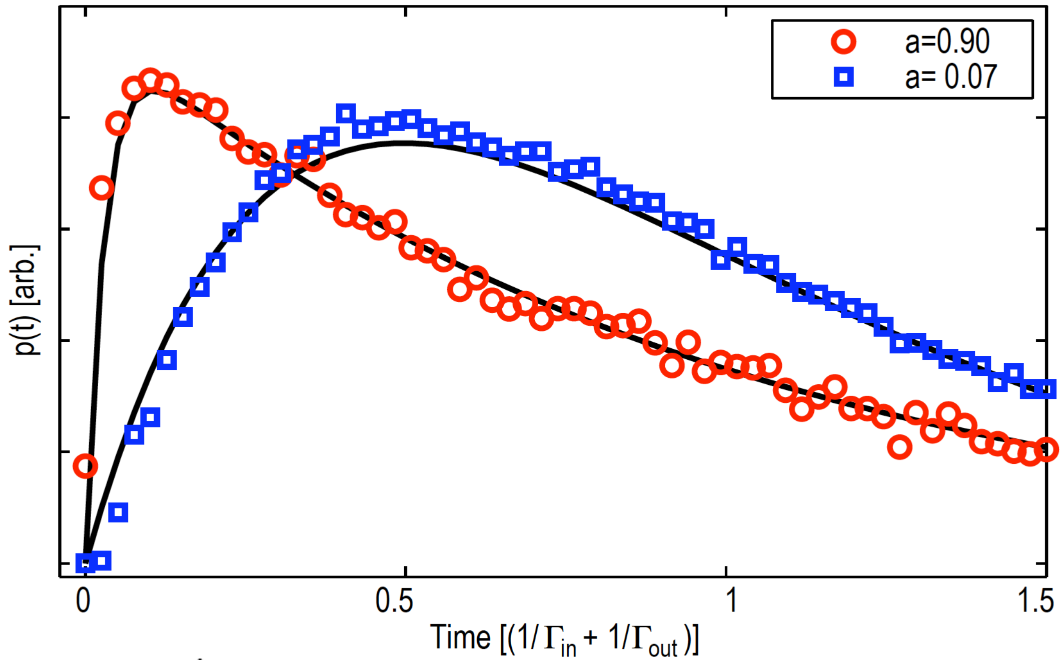
$$\boxed{\begin{aligned} \Gamma_{\rm in} &= \gamma f(E) \\ \Gamma_{\rm out} &= \gamma[1 - f(E)] \end{aligned}}$$

Measuring current electron by electron

Gustavsson *et al*, Phys. Rev. Lett. **96**, 076605 (2006)



Waiting time statistics



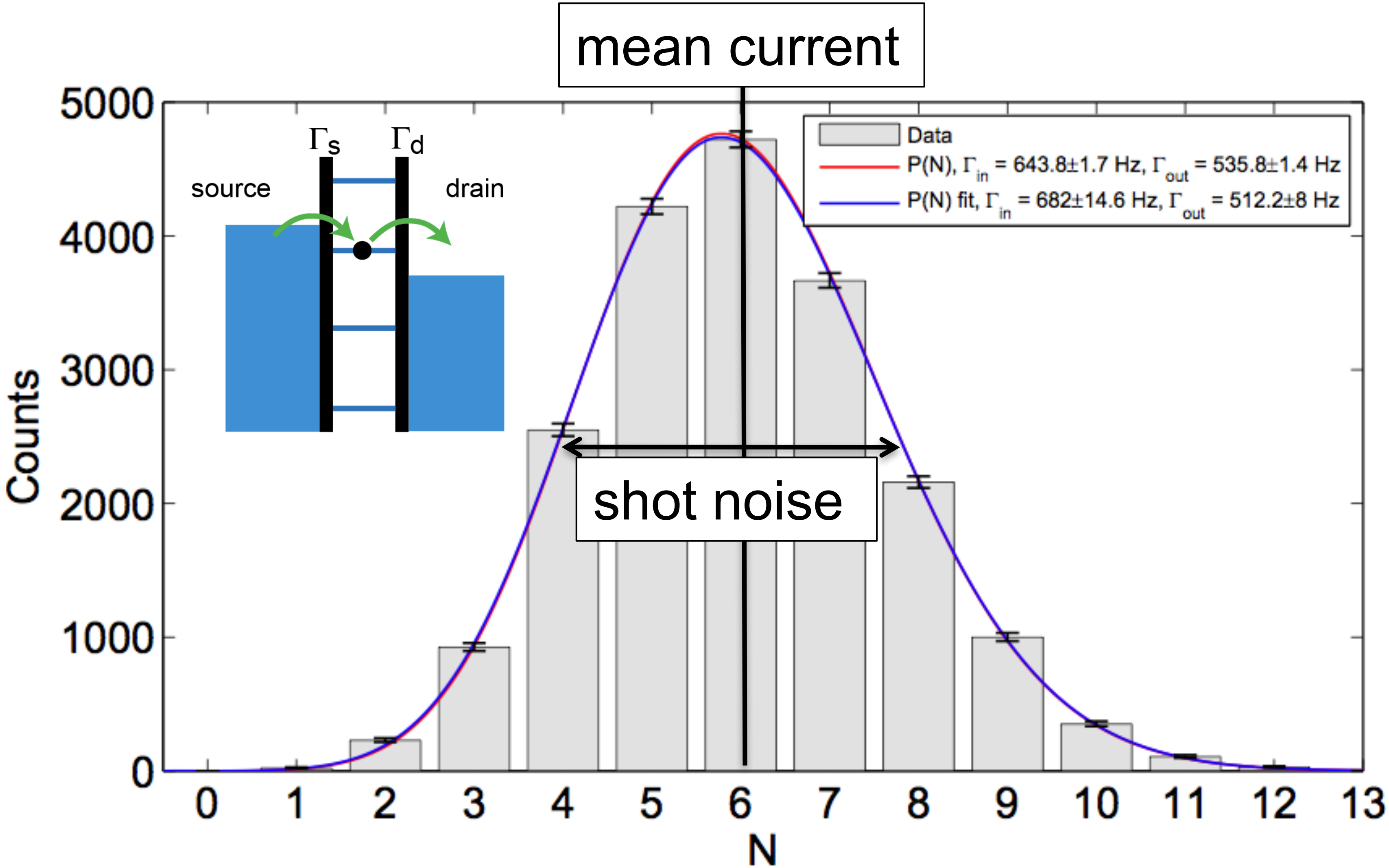
Waiting time between successive tunneling-in events

$$p(t)dt = \int_0^t e^{-\Gamma_{\rm out}t'}\Gamma_{\rm out}dt' e^{-\Gamma_{\rm in}(t-t')}\Gamma_{\rm in}dt$$

$$= \Gamma_{\rm in}\Gamma_{\rm out}\frac{e^{-\Gamma_{\rm out}t} - e^{-\Gamma_{\rm in}t}}{\Gamma_{\rm in} - \Gamma_{\rm out}}dt$$

$$a = \frac{\Gamma_{\rm in} - \Gamma_{\rm out}}{\Gamma_{\rm in} + \Gamma_{\rm out}}$$

Full counting statistics of the current



Measurement by Simon Gustavsson: PRL **96**, 076605 (2006)
Theory: Bagrets and Nazarov, PRB **67**, 085316 (2003);
see also: C. Fricke *et al*, PRB **76**, 155307 (2007)

Single particle interference

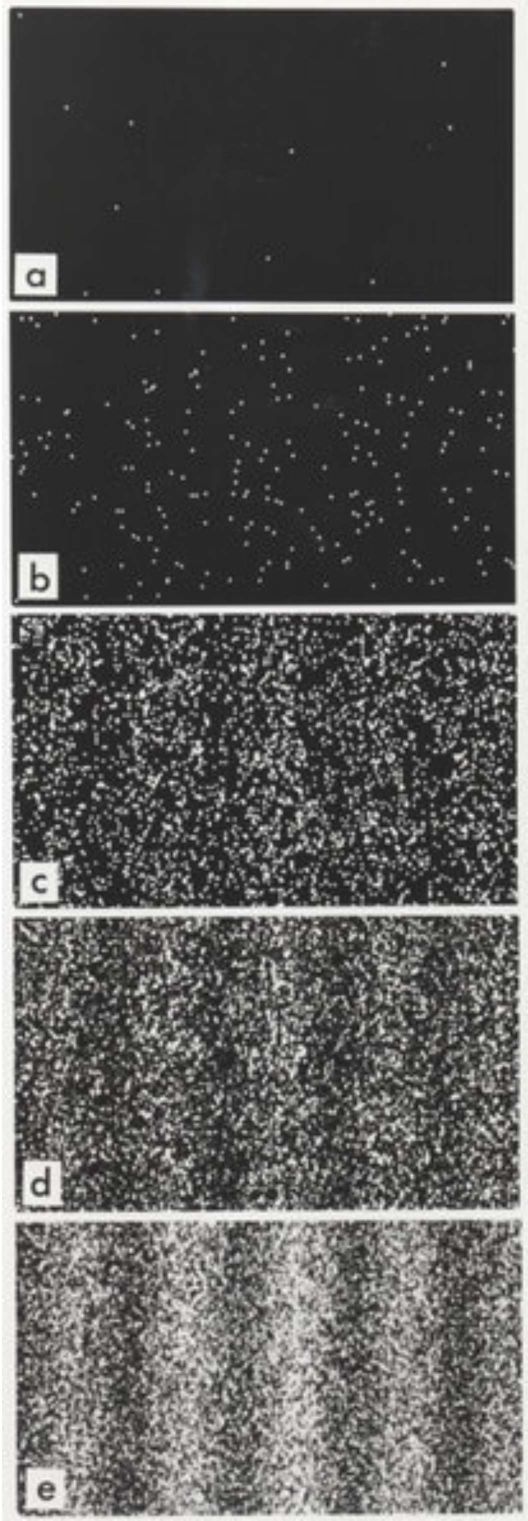
Each photon then interferes only with itself.

Paul Dirac,

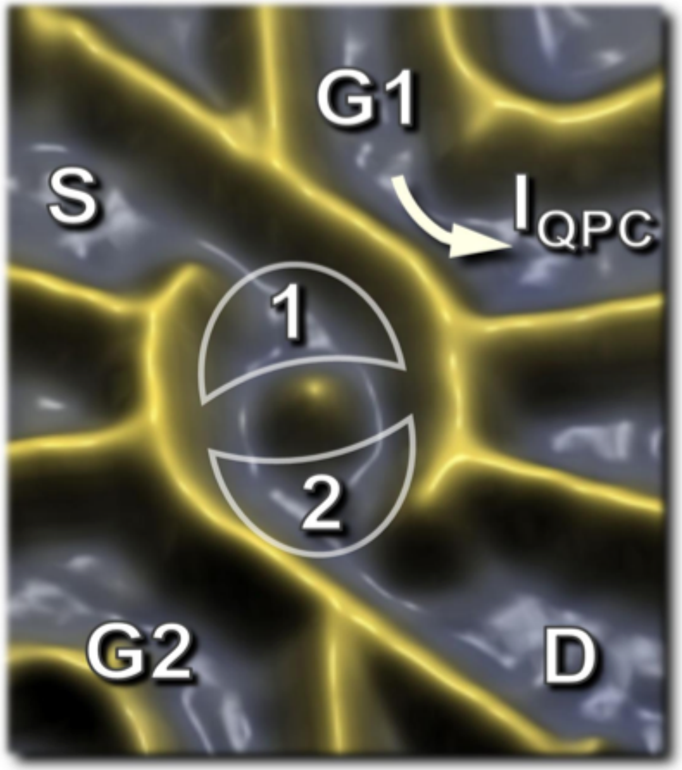
The Principles of Quantum Mechanics, 4th ed., OUP

Experiment with single electrons

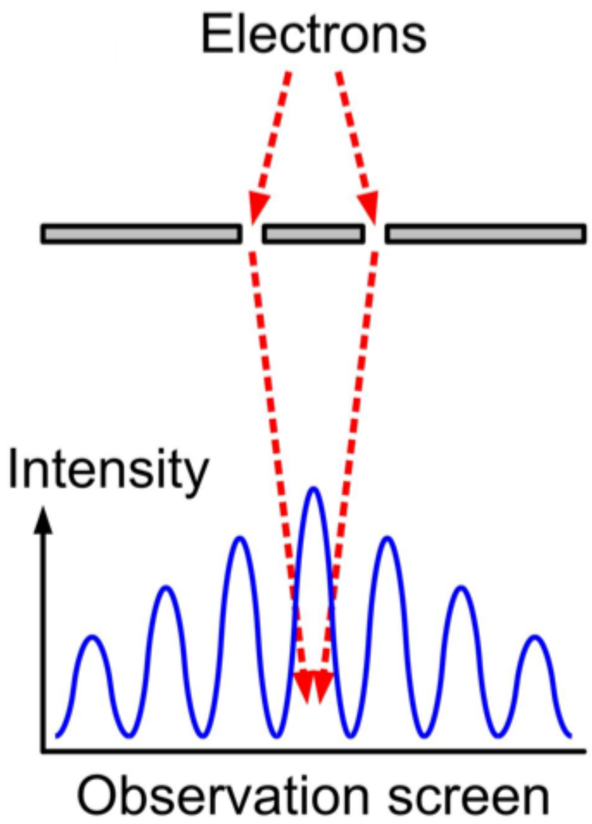
A. Tonomura et al.,
American Journal of Physics **57**, 117-120 (1989)



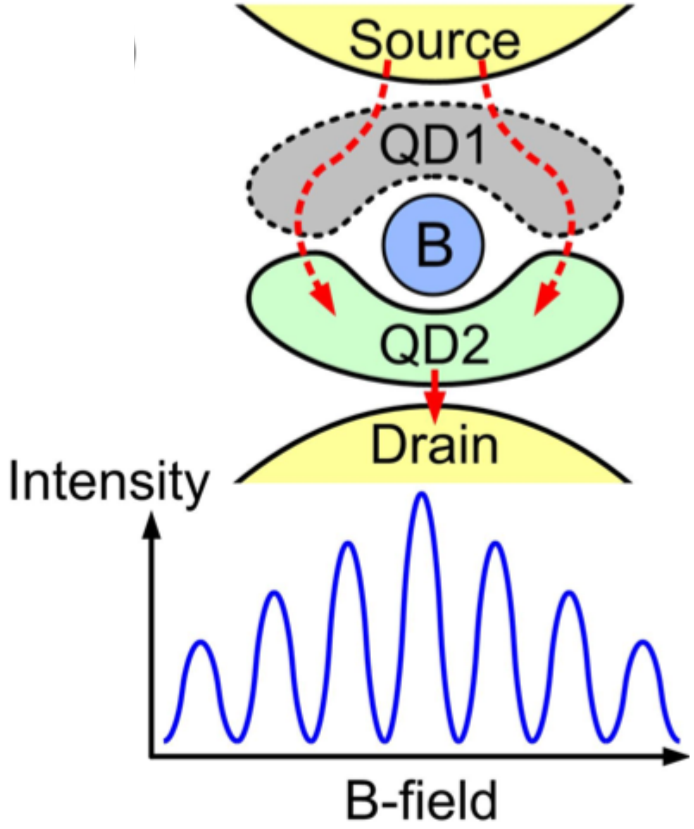
The setup



G1
S
I_{QPC}
1
2
G2
D

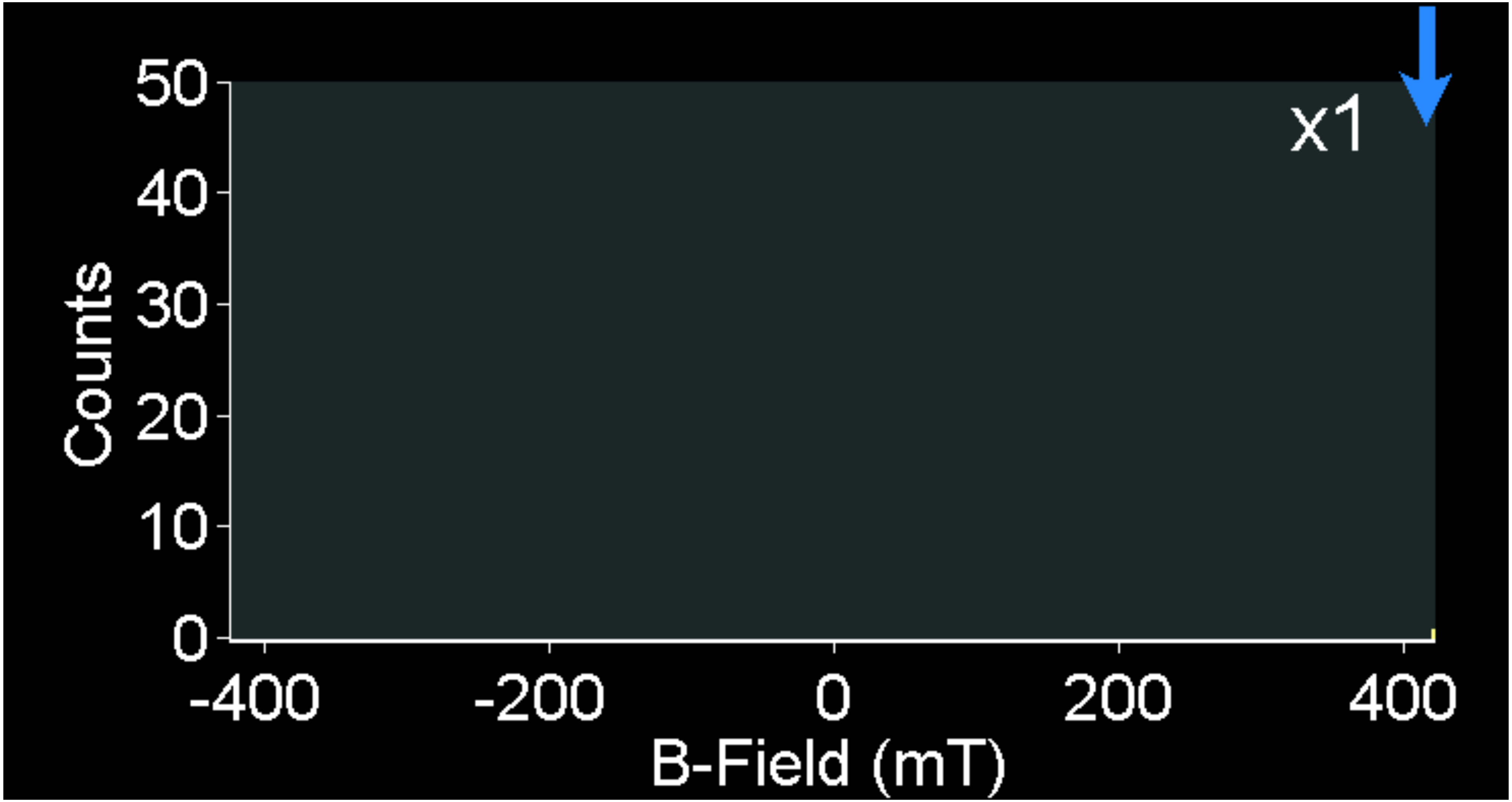


Electrons
Intensity
Observation screen



Source
QD1
B
QD2
Drain
Intensity
B-field

Building up an interference pattern



Conclusion

Here is the miracle:

QD extends over 100 x 100 x 10 nm
= 200 x 200 x 20 = 8 x 10^5 unit cells
= 6.4 x 10^6 atoms

They have 2.56 x 10^7 *valence* electrons
or 5 x 10^8 electrons in total.

We are able to control and observe a single one of them and its quantum properties in real time!

Part III

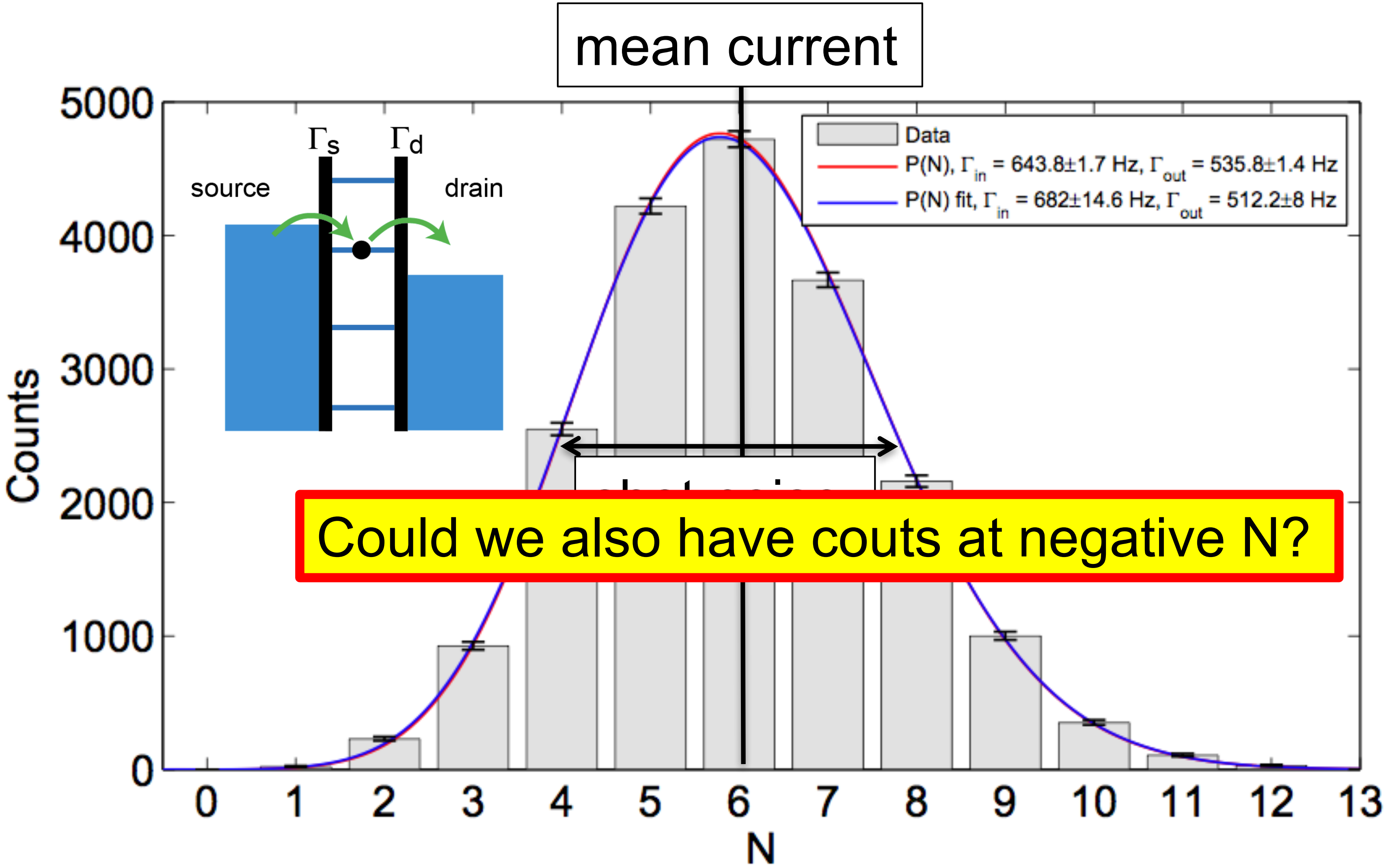
Thermodynamic aspects of electron transport

A wealth of counting experiments

S. Gustavsson *et al*, Surf. Sci. Rep. **64**, 191 (2009).
T. Ihn *et al*, Solid State Commun. **149**, 1419 (2009).
…

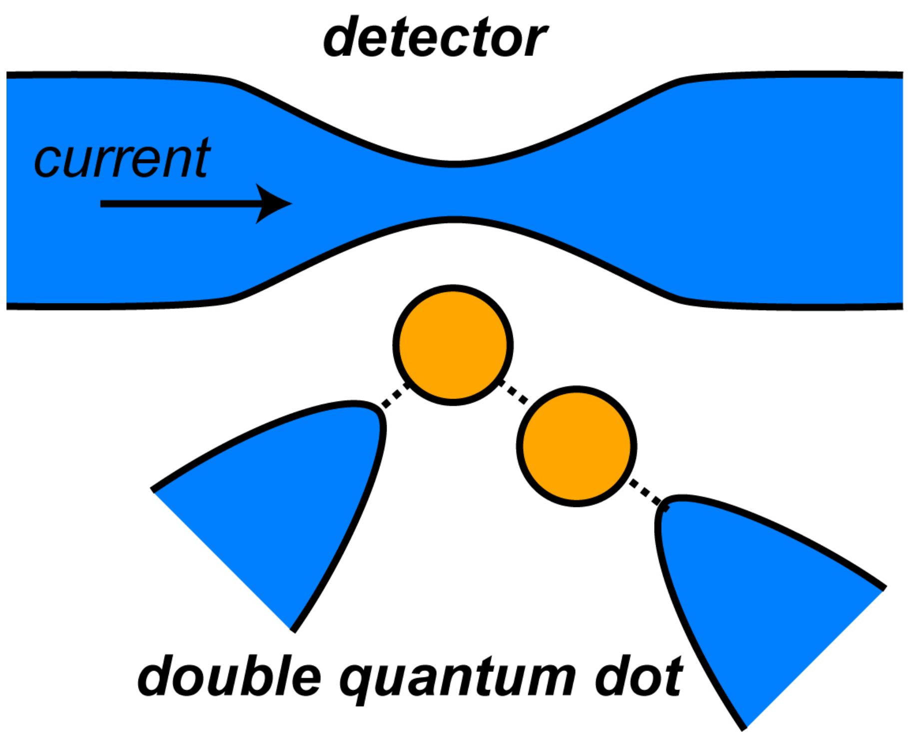
- Shot noise and super-poissonian noise
- excited state spectroscopy
- single-electron interference
- co-tunneling in counting
- photon detection
- detector back-action
- charge detection in graphene and InAs nanowires
- radio-frequency read-out
- irreversibility and tunneling
- fluctuation theorems, Jarzinski relation
- the arrow of time
- measurement of degeneracies
- spin-blockade and spin-orbit interaction

Full counting statistics of the current



Measurement by Simon Gustavsson: PRL **96**, 076605 (2006)
Theory: Bagrets and Nazarov, PRB **67**, 085316 (2003);
see also: C. Fricke *et al*, PRB **76**, 155307 (2007)

Charge detection in double quantum dots: the concept

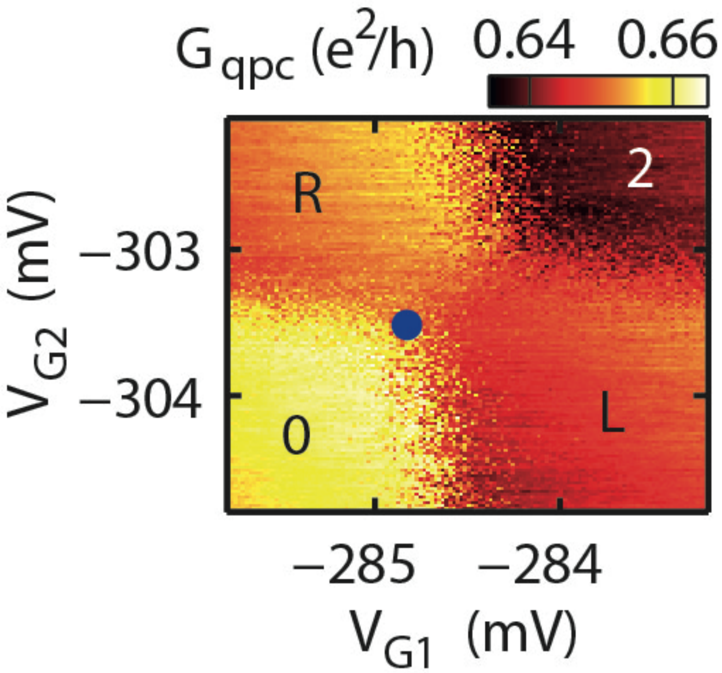


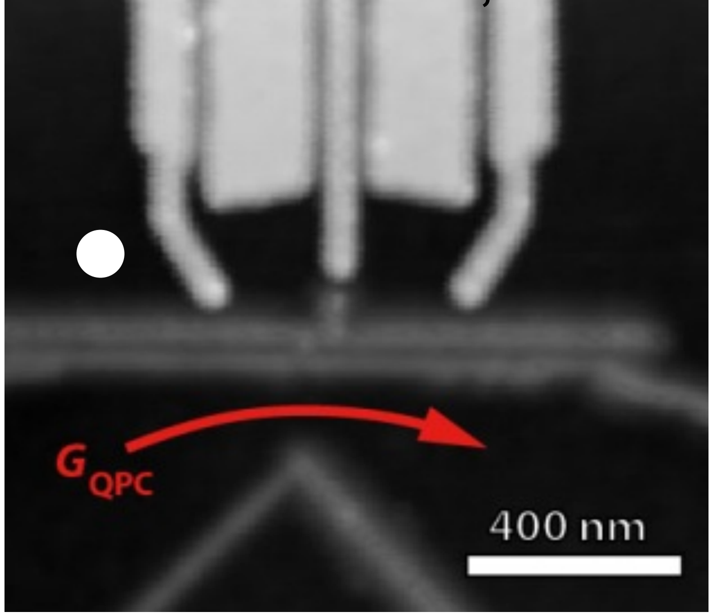
quantum point contact detector

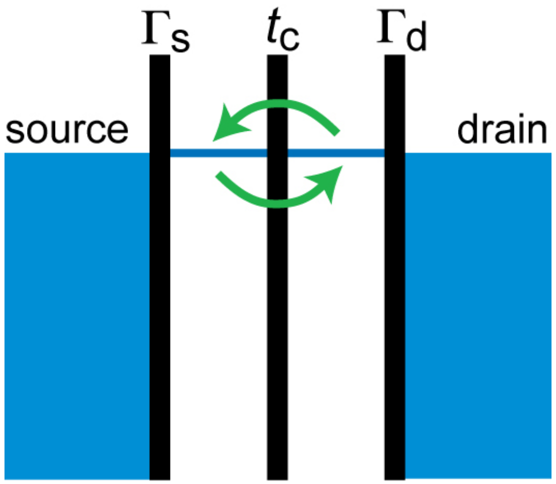
detects the position of electrons inside the DQD structure

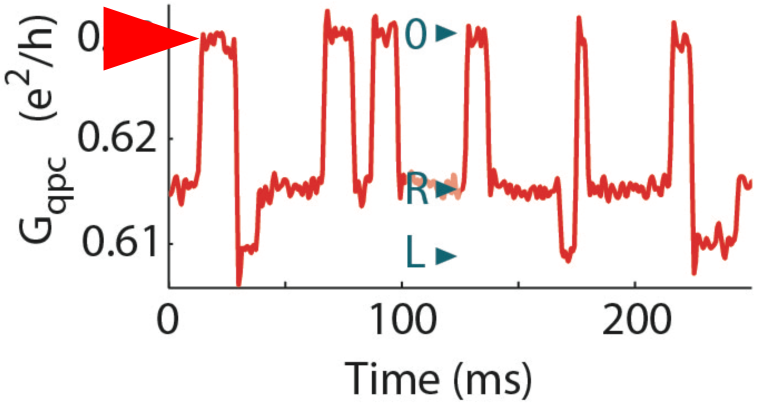
Charge detection in double quantum dots: an implementation

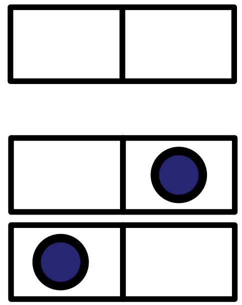
C. Rössler *et al*, APL **97**, 152109





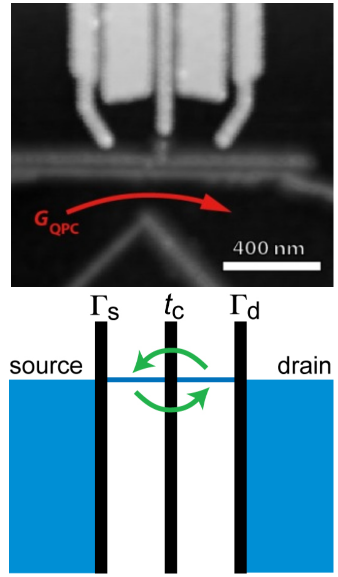


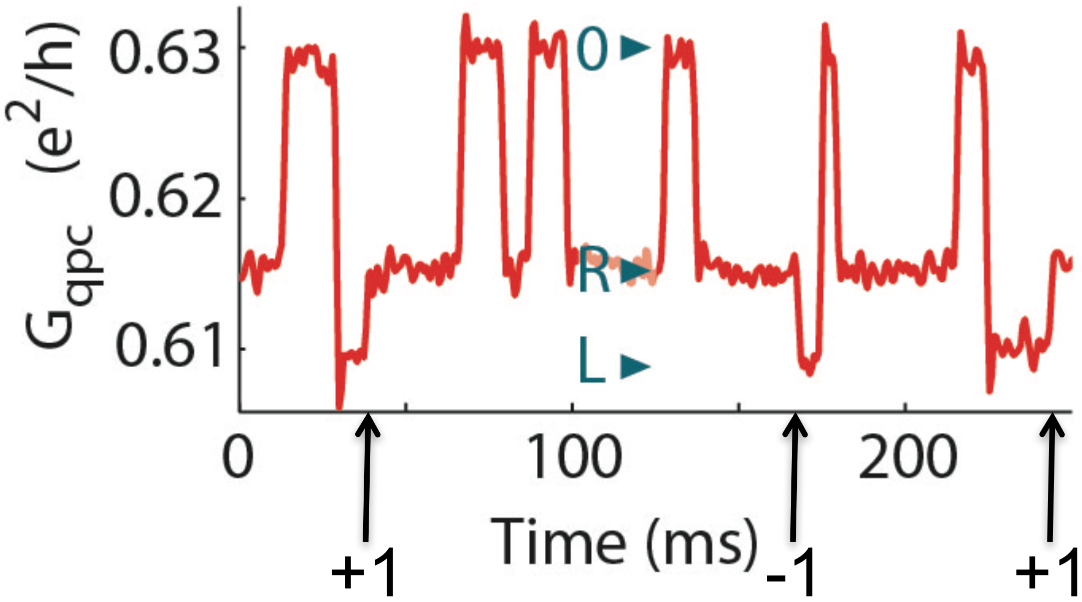




see also: T. Fujisawa *et al*, Science 3

How and what we count

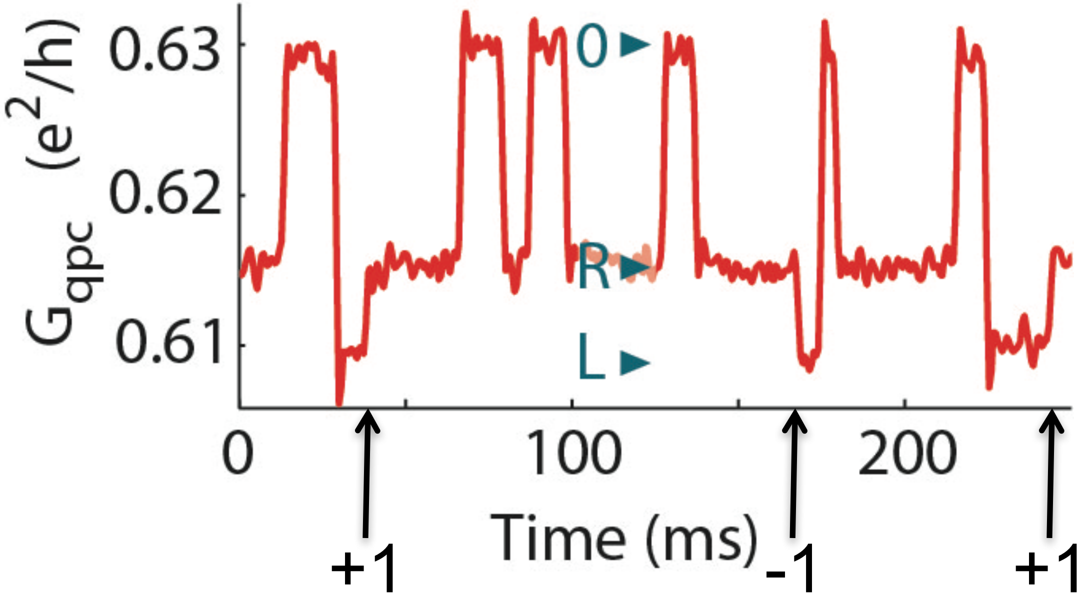


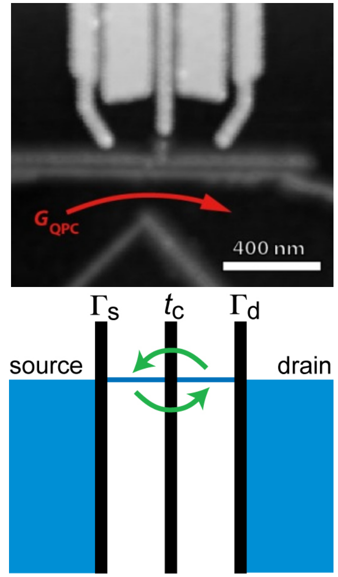


Count:
Transition from left to right dot: +1
Transition from right to left dot: -1

Sum of Counts per time interval: Full counting statistics

How and what we count



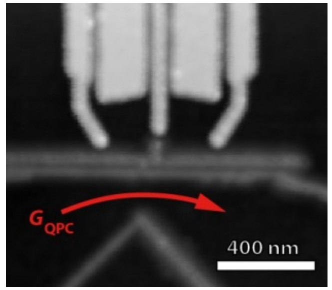


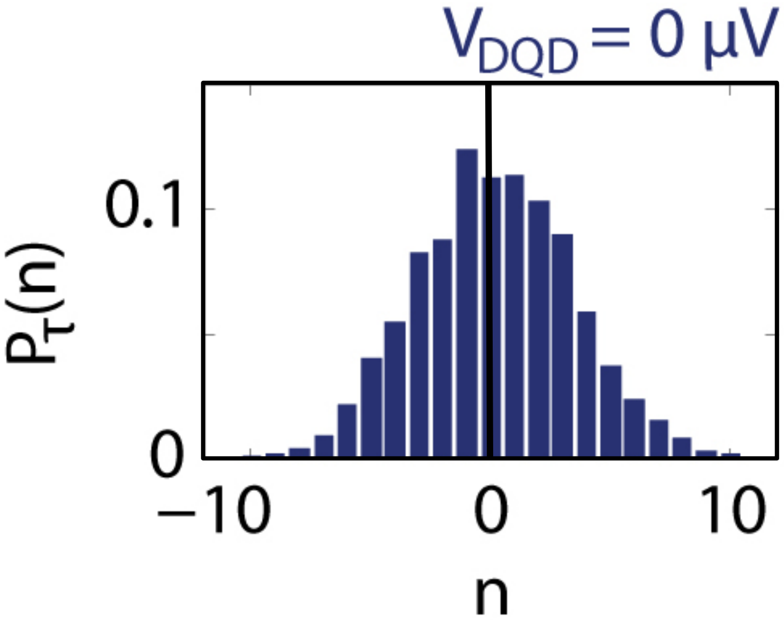
$$\frac{d}{dt}\begin{pmatrix} p_0 \\ p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} -(W_{10}+W_{20}) & W_{01} & W_{02} \\ W_{10} & -(W_{01}+W_{21}) & W_{12}e^{-i\chi} \\ W_{20} & W_{21}e^{i\chi} & -(W_{02}+W_{12}) \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \end{pmatrix}$$

Allows **bi-directional counting**

Direct access to full counting statistics of the current

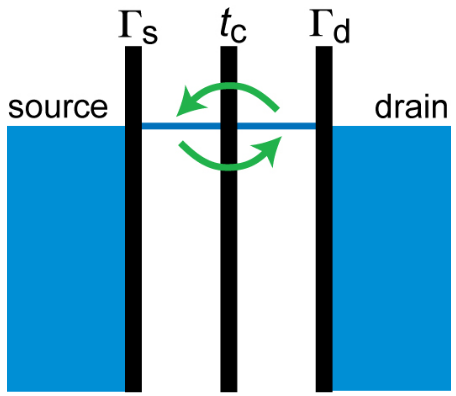
Full counting statistics of the current

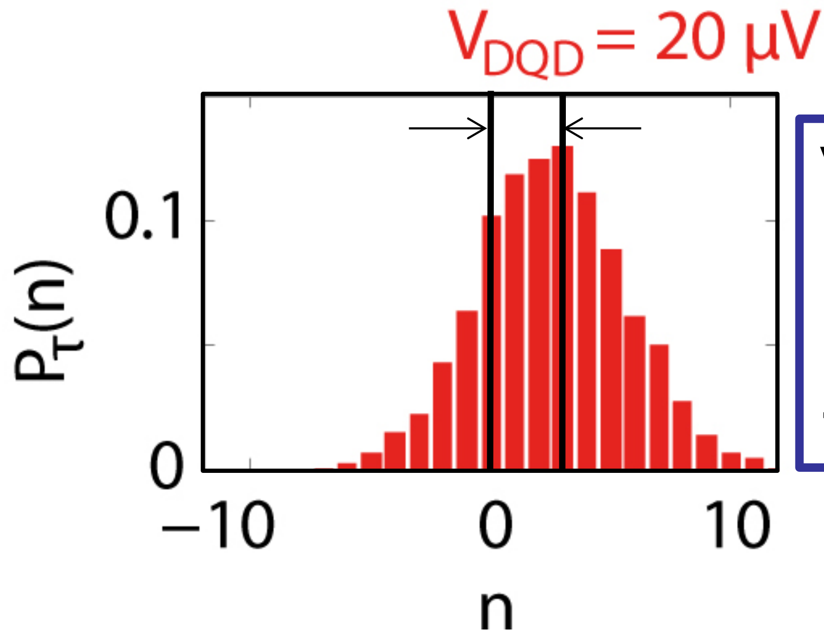




$V_{DQD} = 0\ \mu V$
→ no net charge flow

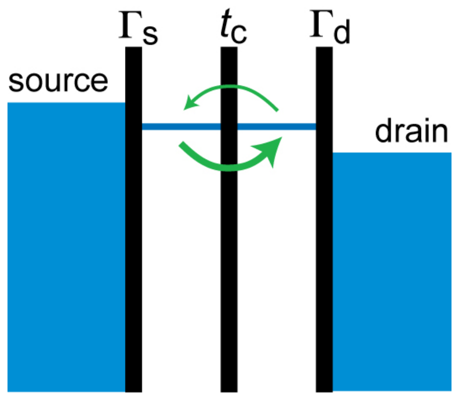
$\langle n \rangle = 0.09 \pm 0.09$





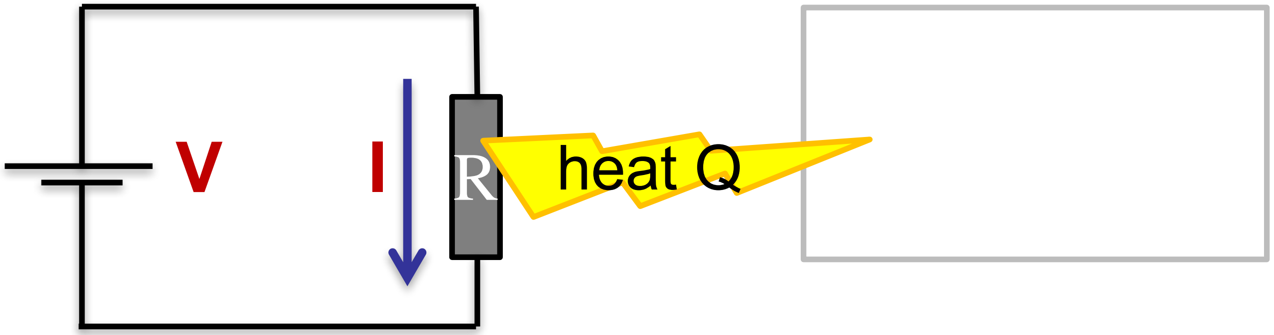
$V_{DQD} = 20\ \mu V$
→ finite charge flow

$\langle n \rangle = 2.39 \pm 0.09$



Thermodynamics of electron transport

Dissipative current: Macroscopically irreversible process



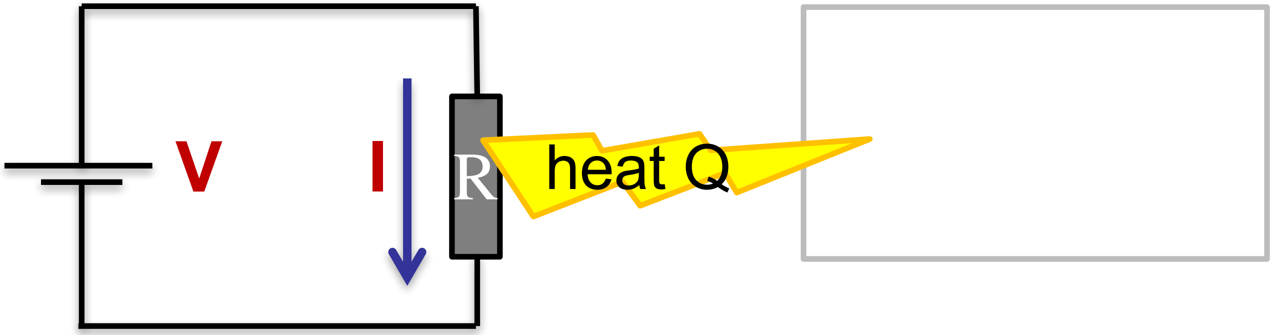
Non-equilibrium physics

2nd law of thermodynamics: $\Delta S = \Delta Q / T > 0$

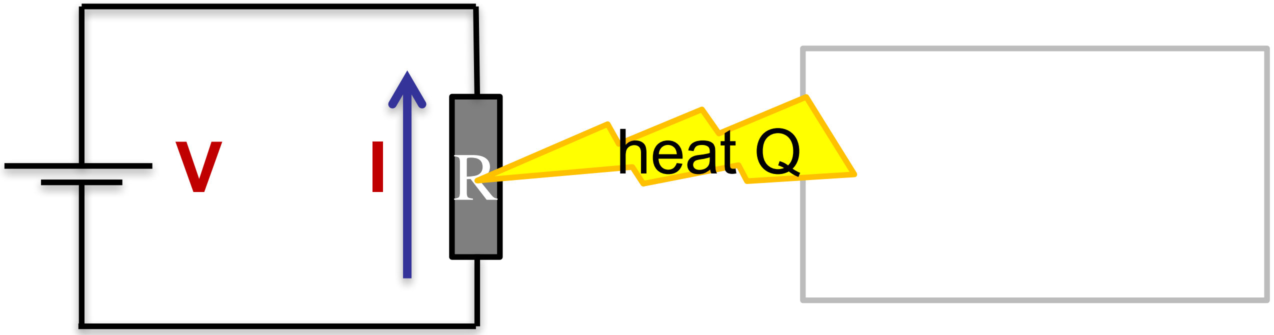
A macroscopic number of elementary processes average (large system or time scale) and give a net entropy increase

Taking a microscopic view

Reverse processes are not forbidden!



$P_\tau(\Delta S>0)$



$P_\tau(\Delta S<0)$

The second law can be violated on short time-scales τ and/or in small systems

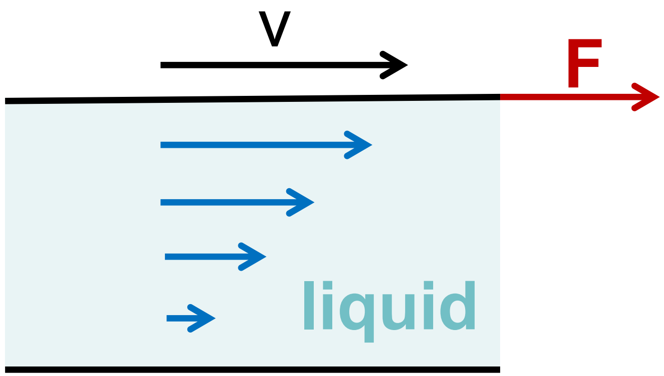
Entropy production $\Delta S = \Delta Q/T$

ΔS is a **fluctuating variable**, probability $P_\tau(\Delta S)$

The fluctuation theorem

Quantitative statement about $P_\tau(\Delta S)$

$$\boxed{\frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = e^{\Delta S/k_\mathrm{B}}}$$



D.J. Evans, E.G.D. Cohen, G.P. Morriss, PRL ***71****, 2401 (1993).*

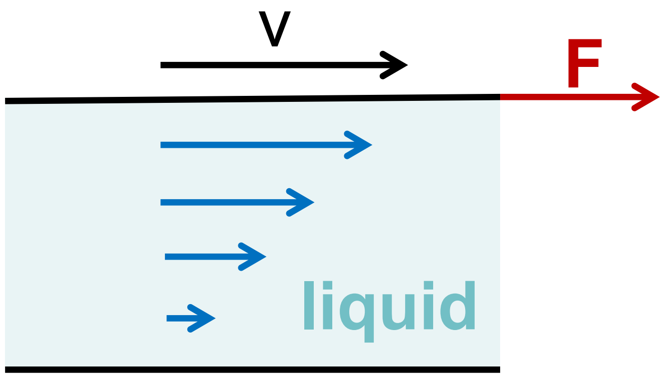
$$p_{\mathrm{S}\to\mathrm{D}}(\epsilon) = \mathcal{T} f_\mathrm{S}(\epsilon)[1 - f_\mathrm{D}(\epsilon)]$$
$$p_{\mathrm{S}\leftarrow\mathrm{D}}(\epsilon) = \mathcal{T} f_\mathrm{D}(\epsilon)[1 - f_\mathrm{S}(\epsilon)]$$

$$\frac{p_{\mathrm{S}\to\mathrm{D}}(\epsilon)}{p_{\mathrm{S}\leftarrow\mathrm{D}}(\epsilon)} = \frac{f_\mathrm{S}(\epsilon)}{1 - f_\mathrm{S}(\epsilon)} \frac{1 - f_\mathrm{D}(\epsilon)}{f_\mathrm{D}(\epsilon)}$$
$$= \exp\left(\frac{\epsilon - \mu_\mathrm{D}}{k_\mathrm{B} T_\mathrm{D}} - \frac{\epsilon - \mu_\mathrm{S}}{k_\mathrm{B} T_\mathrm{S}}\right) = \exp(\Delta S)$$

The fluctuation theorem

Quantitative statement about $P_\tau(\Delta S)$

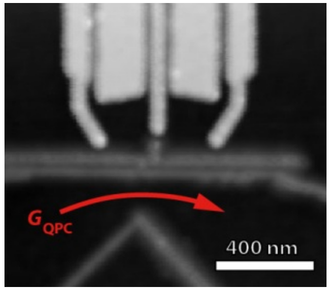
$$\boxed{\frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = e^{\Delta S/k_\mathrm{B}}}$$



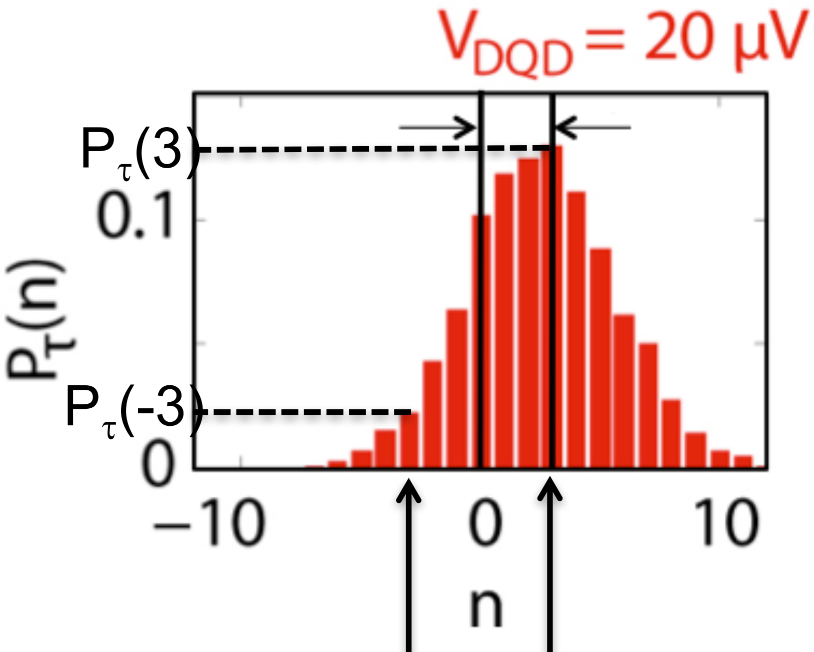
*D.J. Evans, E.G.D. Cohen, G.P. Morriss, PRL **71**, 2401 (1993).*

Fluctuation theorem…

…and full counting statistics



$$\frac{P_\tau(\Delta S)}{P_\tau(-\Delta S)} = \exp\left(\frac{\Delta S}{k_\mathrm{B}}\right) \Rightarrow \frac{P_\tau(n)}{P_\tau(-n)} = \exp\left(\frac{neV}{k_\mathrm{B}T}\right)$$



Example: -3 +3

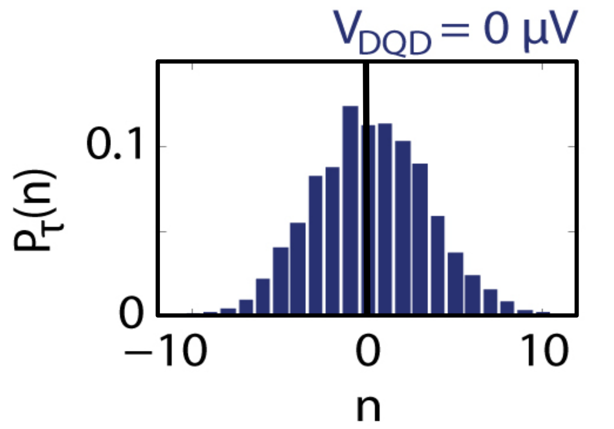
n: number of charges transferred from source to drain

see also: Y. Utsumi *et al*, PRB **81**,

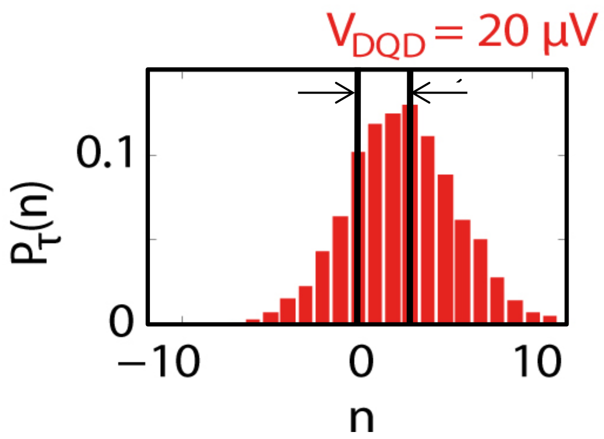
Experimental test...

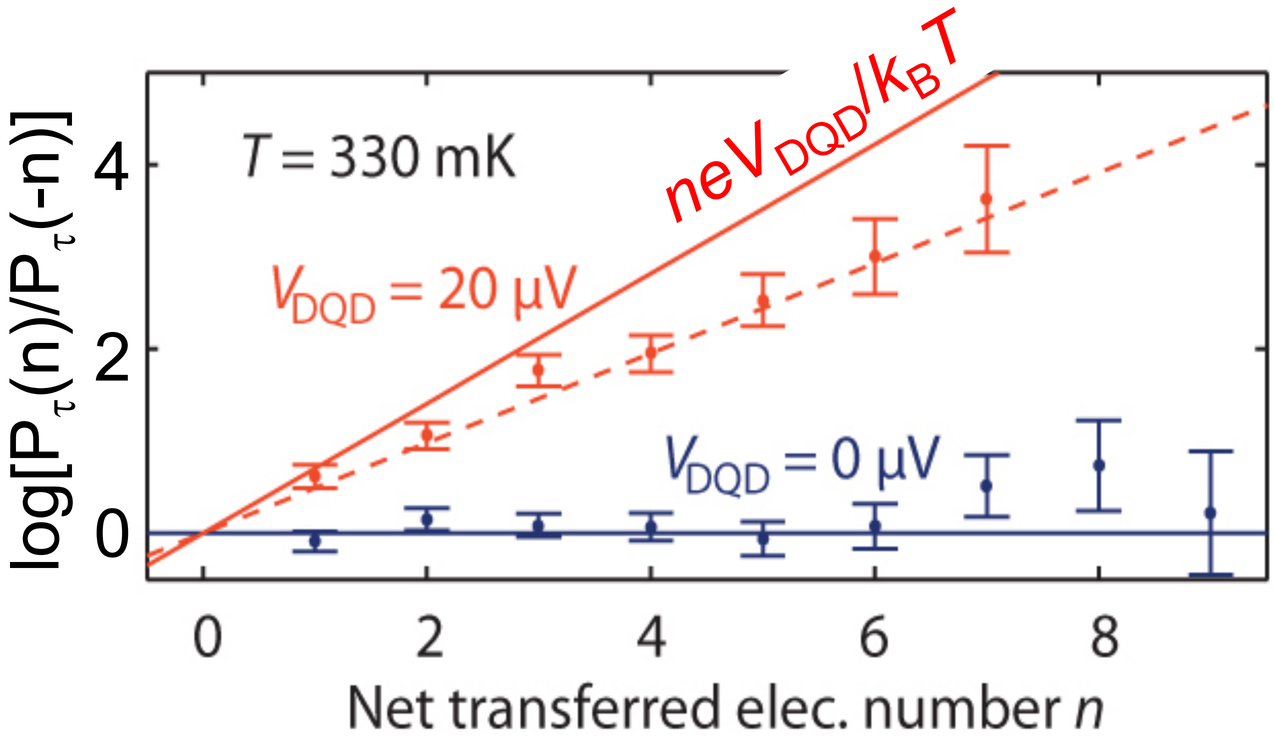
...of the fluctuation theorem

$$\frac{P_\tau(n)}{P_\tau(-n)} = \exp\left(\frac{neV_{\mathrm{DQD}}}{k_{\mathrm{B}}T}\right)$$



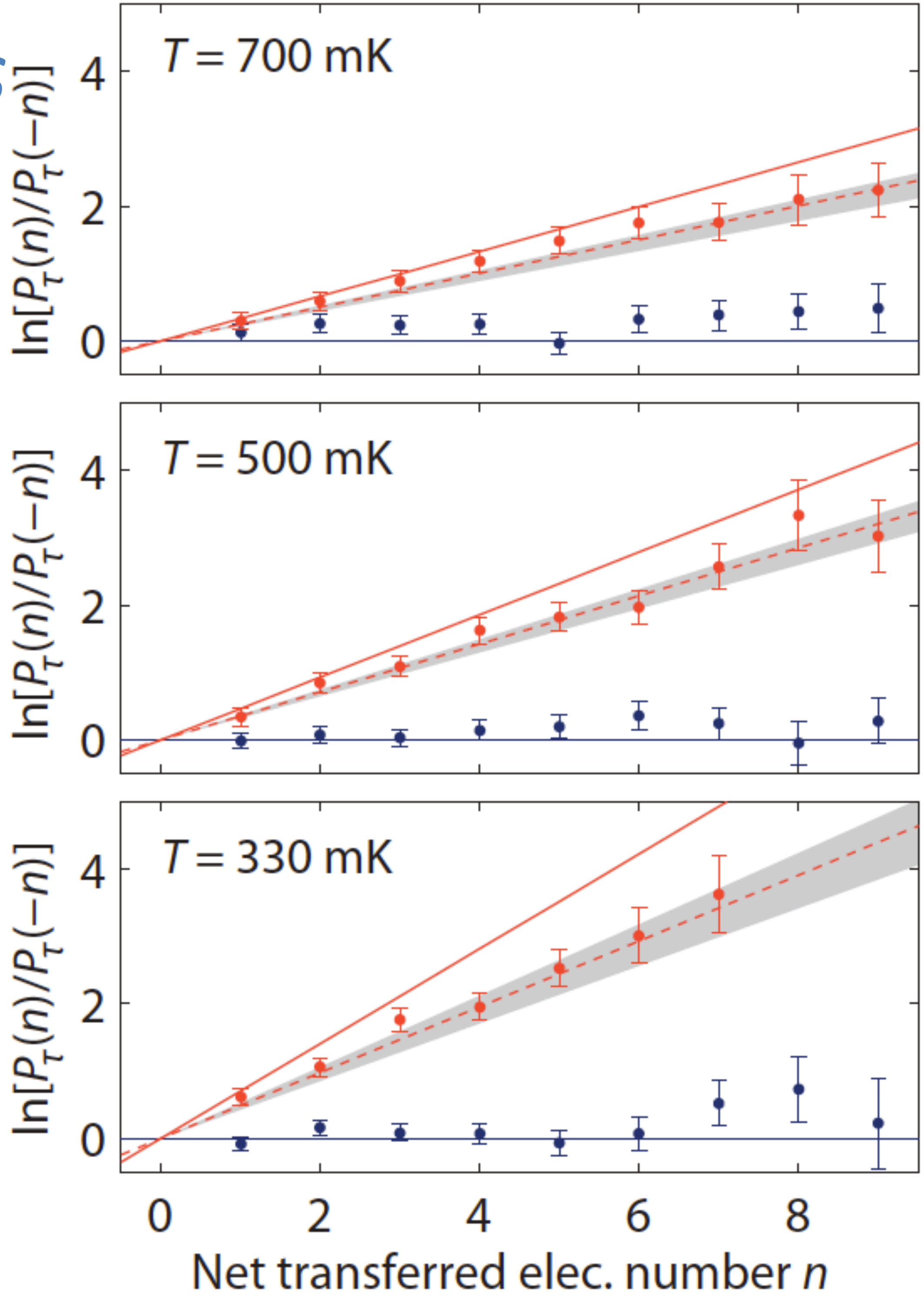
*B. Küng et al, PRX **2**, 011001 (2012).*





after finite bandwidth correction: dashed line

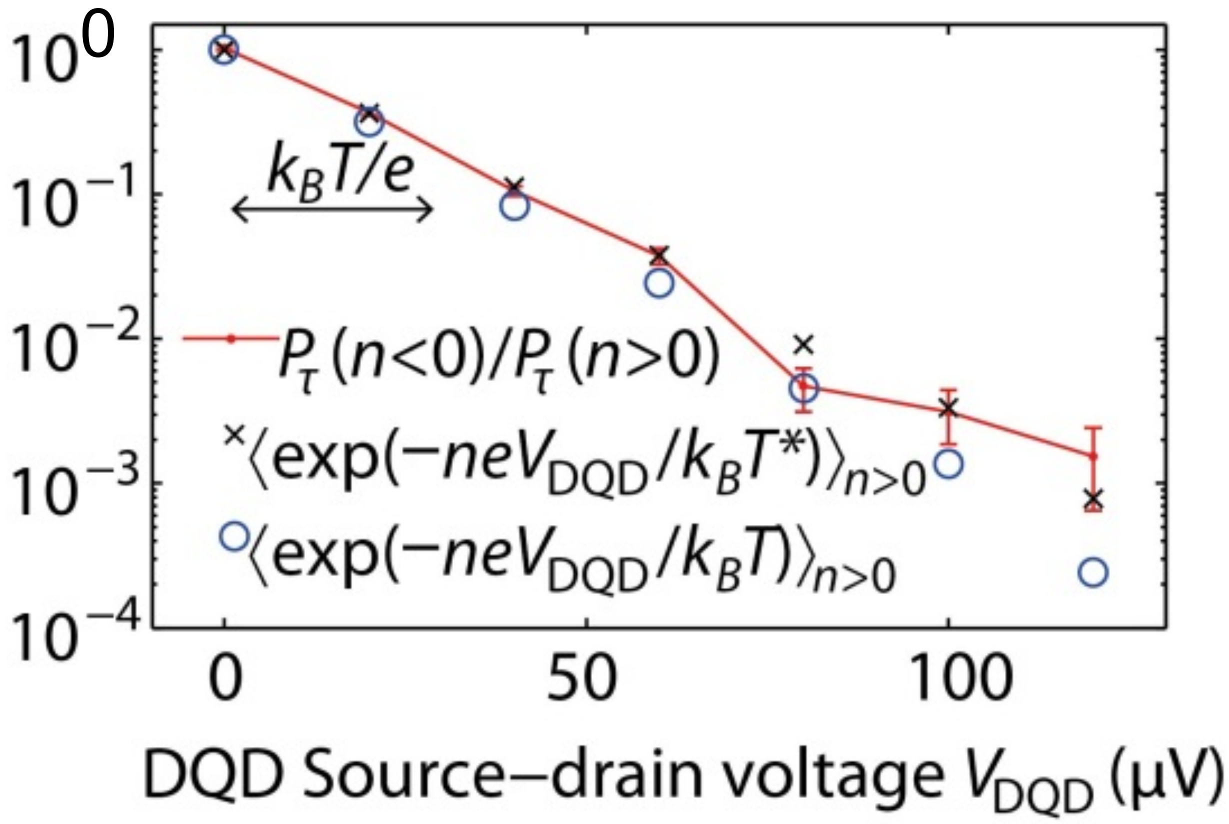
Higher temperatures



→ fluctuation theorem holds

Higher source–drain bias voltage

$$\frac{P_\tau(n)}{P_\tau(-n)} = \exp\left(\frac{neV_{\mathrm{DQD}}}{k_{\mathrm{B}}T}\right) \Rightarrow \frac{P_\tau(n<0)}{P_\tau(n>0)} = \frac{\sum_{n>0} P_\tau(n)\exp\left(\frac{-neV_{\mathrm{DQD}}}{k_{\mathrm{B}}T}\right)}{\sum_{n>0} P_\tau(n)}$$

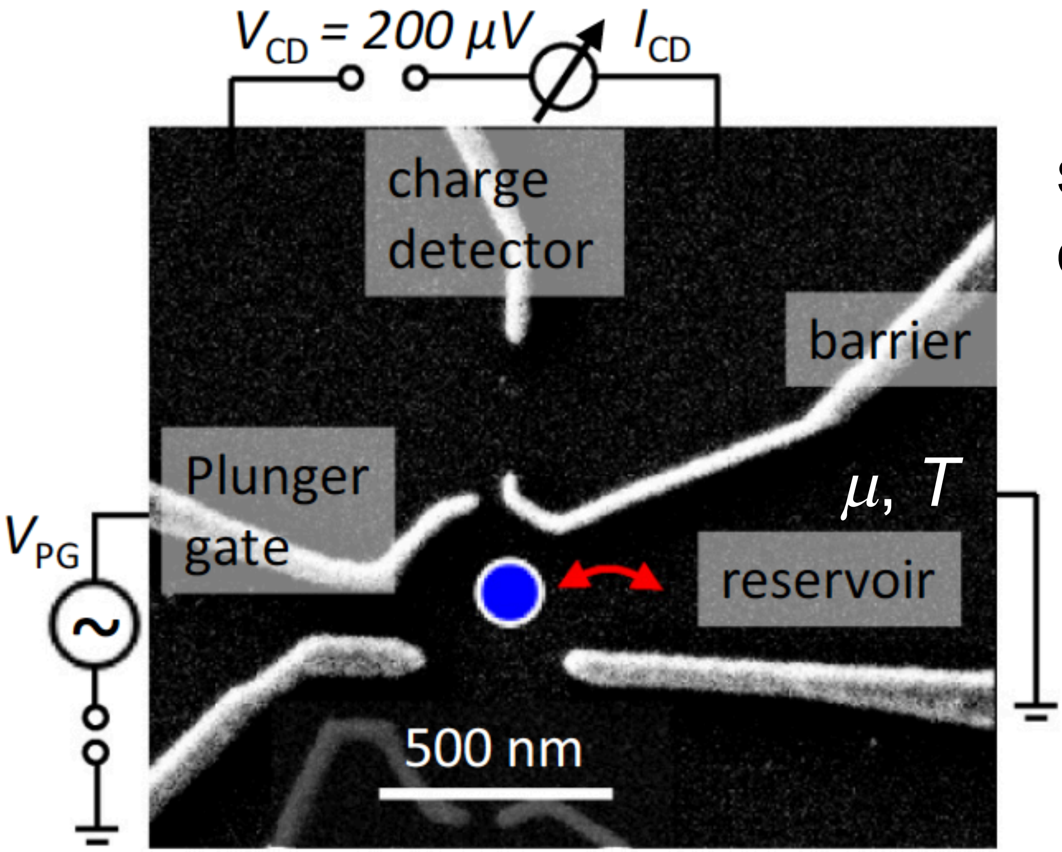


Plot integrated FT (smaller error)

Part IV

Non-equilibrium thermodynamics at the level of single electrons

Quantum dot thermodynamics



sample based on GaAs 2DEG

State variables: μ, T, V_{PG}

Thermodynamic process

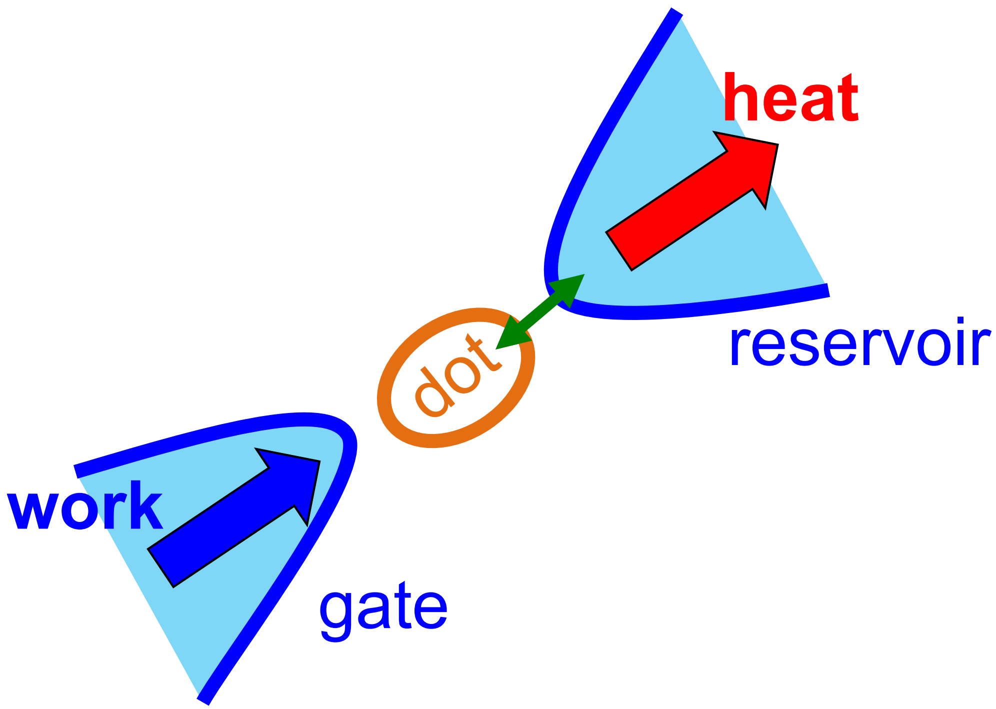
Elementary thermodynamic process:

State A

$V_G^{(1)}$, T, E_F

State B

$V_G^{(2)}$, T, E_F



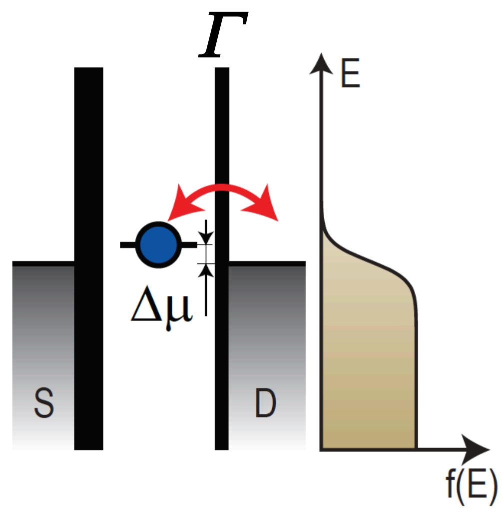
here:
work performed
on single electron

heat generated
by single electron

Non-equilibrium thermodynamics

Reversible processes:
system always close to equilibrium with reservoir

Quantum dot equilibration time:
tunneling rate Γ
fully tunable



Processes fast on the scale of Γ:

Drive quantum dot far from thermodynamic equilibrium with reservoir

Jarzinsky equality

Repeatedly drive a system that is in contact with a heat reservoir from initial equilibrium state to final non-equilibrium state

Statistical variable: Work performed on the system

Free energy difference between initial and final states in equilibrium

$$\left\langle \exp\left(-\frac{\Delta W}{k_{\mathrm{B}}T}\right)\right\rangle = \exp\left(-\frac{\Delta F}{k_{\mathrm{B}}T}\right)$$

Ensemble average

Reservoir temperature

C. Jarzynski, PRL **78**, 2690 (1997)

Related work:

J. Liphardt et al, Science **296**, 5574 1832-1835 (2002)

O.P Saira et al., PRL. **109,** 180601 (2012)

Shuoming An et al, Nature Physics **11**, 193–199 (2015)

T. B. Batalhao et al., PRL **113**, 140601 (2014)

Thermodynamics of small numbers

Thermodynamic (statistical) fluctuations irrelevant due to

- ensemble averaging (macroscopic system)
- time averaging (small bandwidth observation)

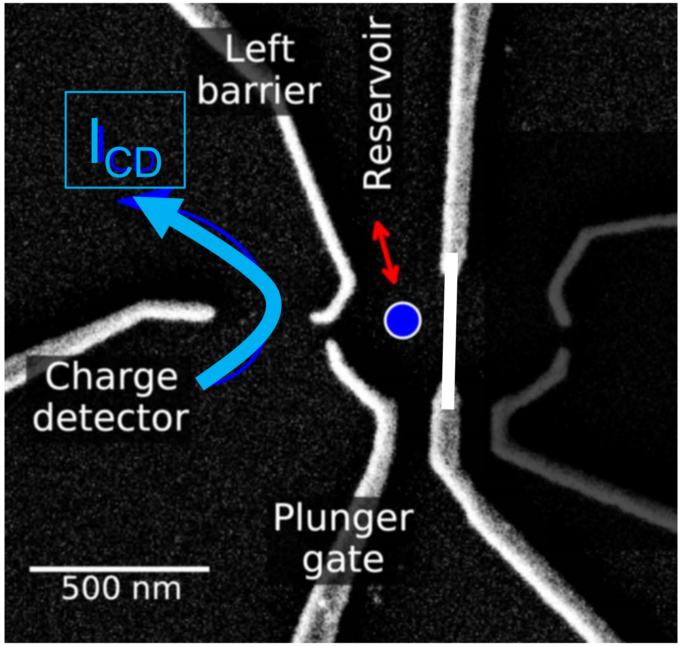
$$\boxed{\frac{\Delta W}{W_{\mathrm{tot}}} \propto \frac{1}{\sqrt{N}}}$$

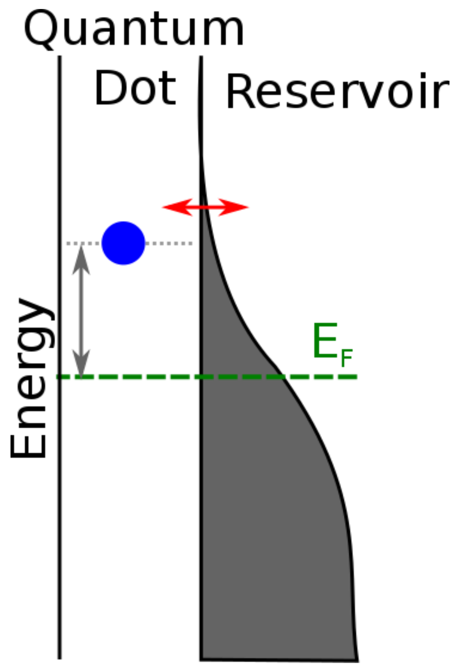
Particle number
or number of repetitions
or of tunneling events

$$W_{\mathrm{tot}} = \sum_{j=1}^{N} W_i \qquad \Delta W^2 = \sum_{j=1}^{N} (W_i - \langle W_i \rangle)^2$$

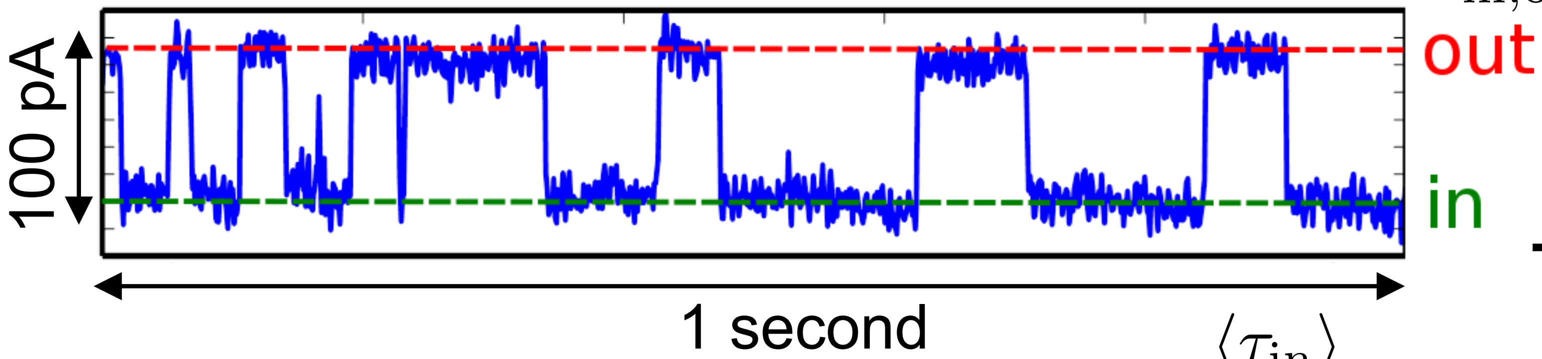
What happens, if $N \to 1$?
Fluctuations become very relevant!

Calibration at equilibrium





Electron tunneling between the quantum dot and the reservoir



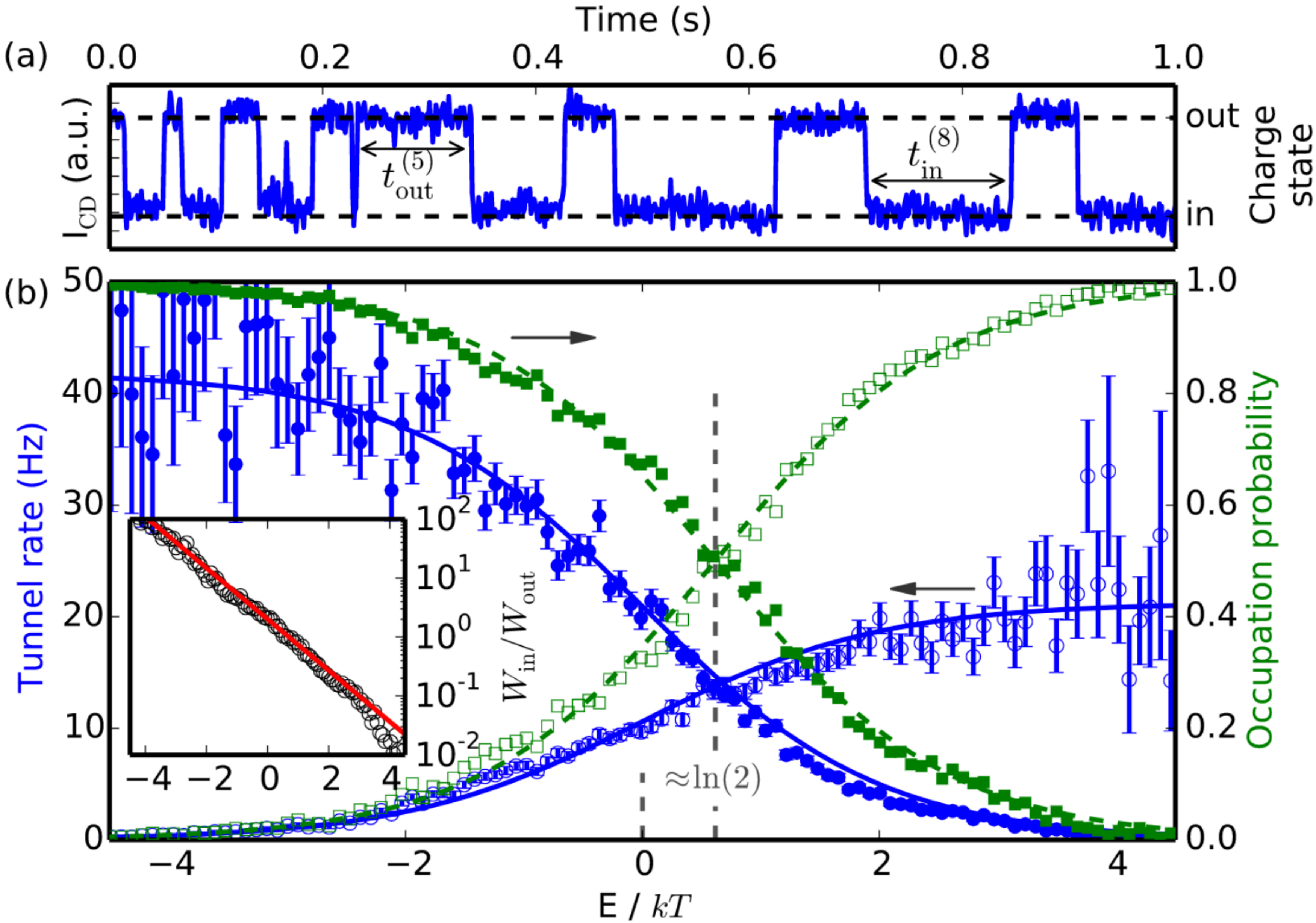
Tunneling rates:

$$\Gamma_{\mathrm{in,out}} = \frac{1}{\langle \tau_{\mathrm{in,out}} \rangle}$$

Temperature:

$$\frac{\langle \tau_{\mathrm{in}} \rangle}{\langle \tau_{\mathrm{out}} \rangle} = \exp\left(-\frac{E - E_{\mathrm{F}}}{k_{\mathrm{B}} T}\right)$$

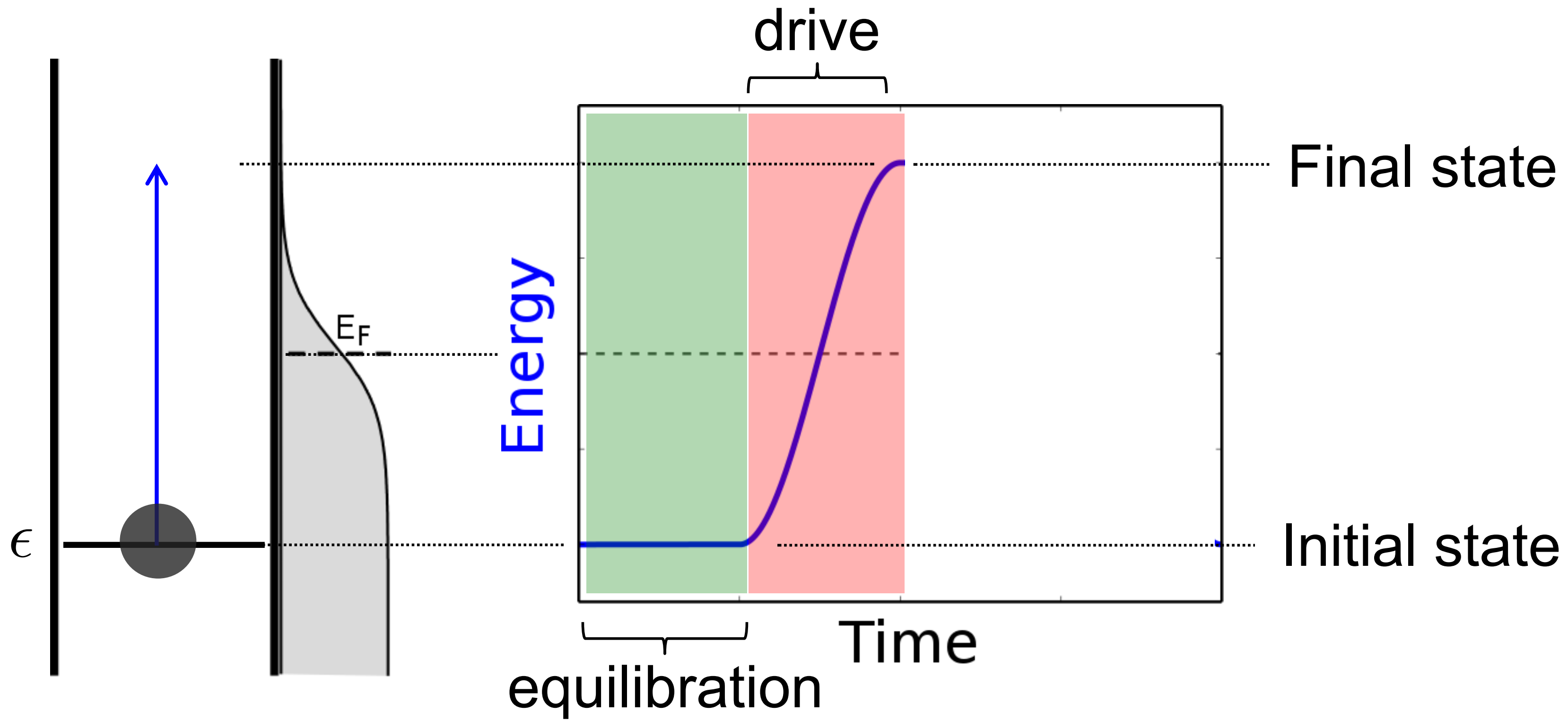
Calibration at equilibrium



Jarzinsky equality

$$\underbrace{\left\langle \exp\left(-\frac{\Delta W}{k_\mathrm{B}T}\right)\right\rangle}_{\text{Measurement}} = \underbrace{\exp\left(-\frac{\Delta F}{k_\mathrm{B}T}\right)}_{\text{Theory}}$$

$$E(t) = E_\mathrm{F} - A\cos(2\pi f t) \quad \text{for } 0 \leq t \leq 1/2f$$

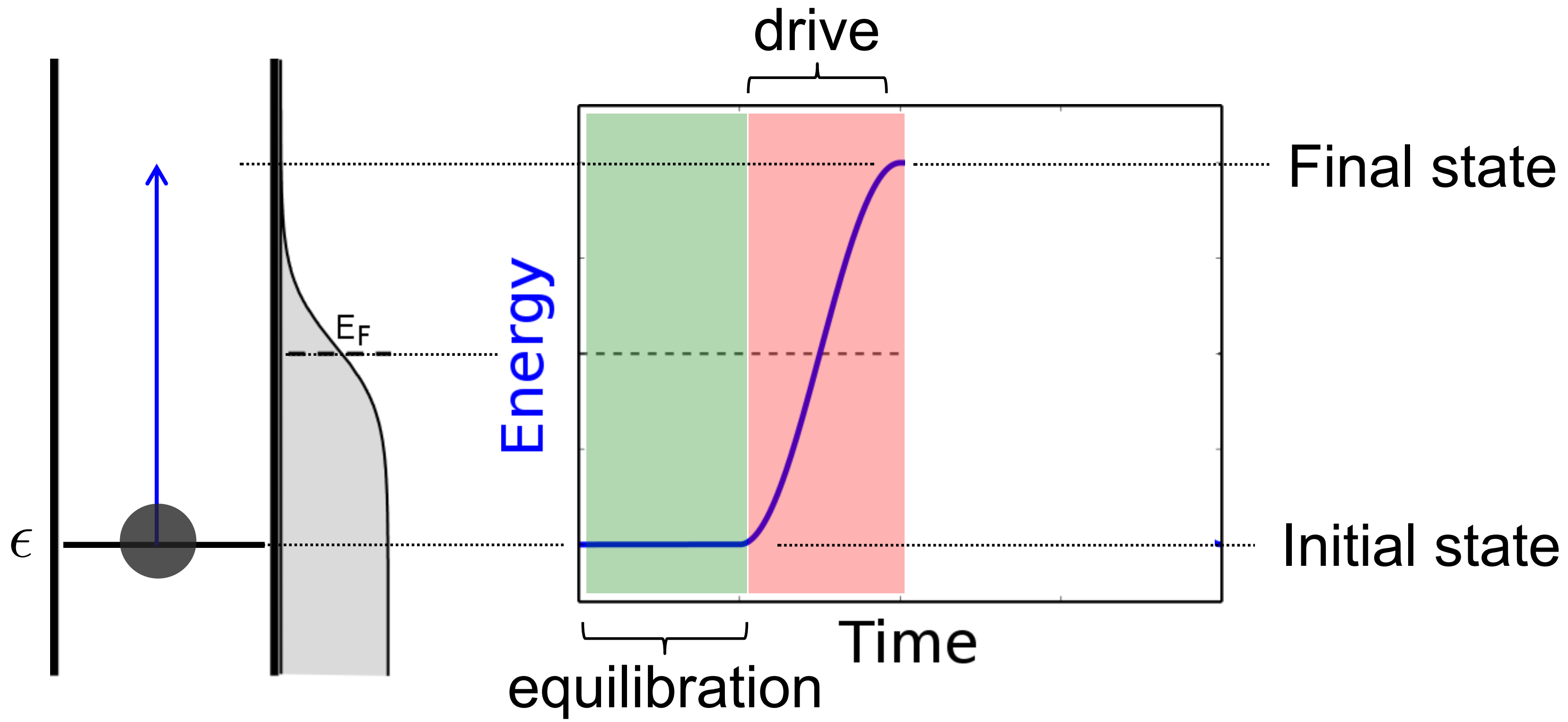


Jarzinsky equality

$$\underbrace{\left\langle \exp\left(-\frac{\Delta W}{k_\mathrm{B}T}\right)\right\rangle}_{\text{Measurement}} = \underbrace{\exp\left(-\frac{\Delta F}{k_\mathrm{B}T}\right)}_{\text{Theory}}$$

- measure ΔW during driving
- repeat the measurement 20'000 times

$$E(t) = E_\mathrm{F} - A\cos(2\pi f t) \quad \text{for } 0 \leq t \leq 1/2f$$



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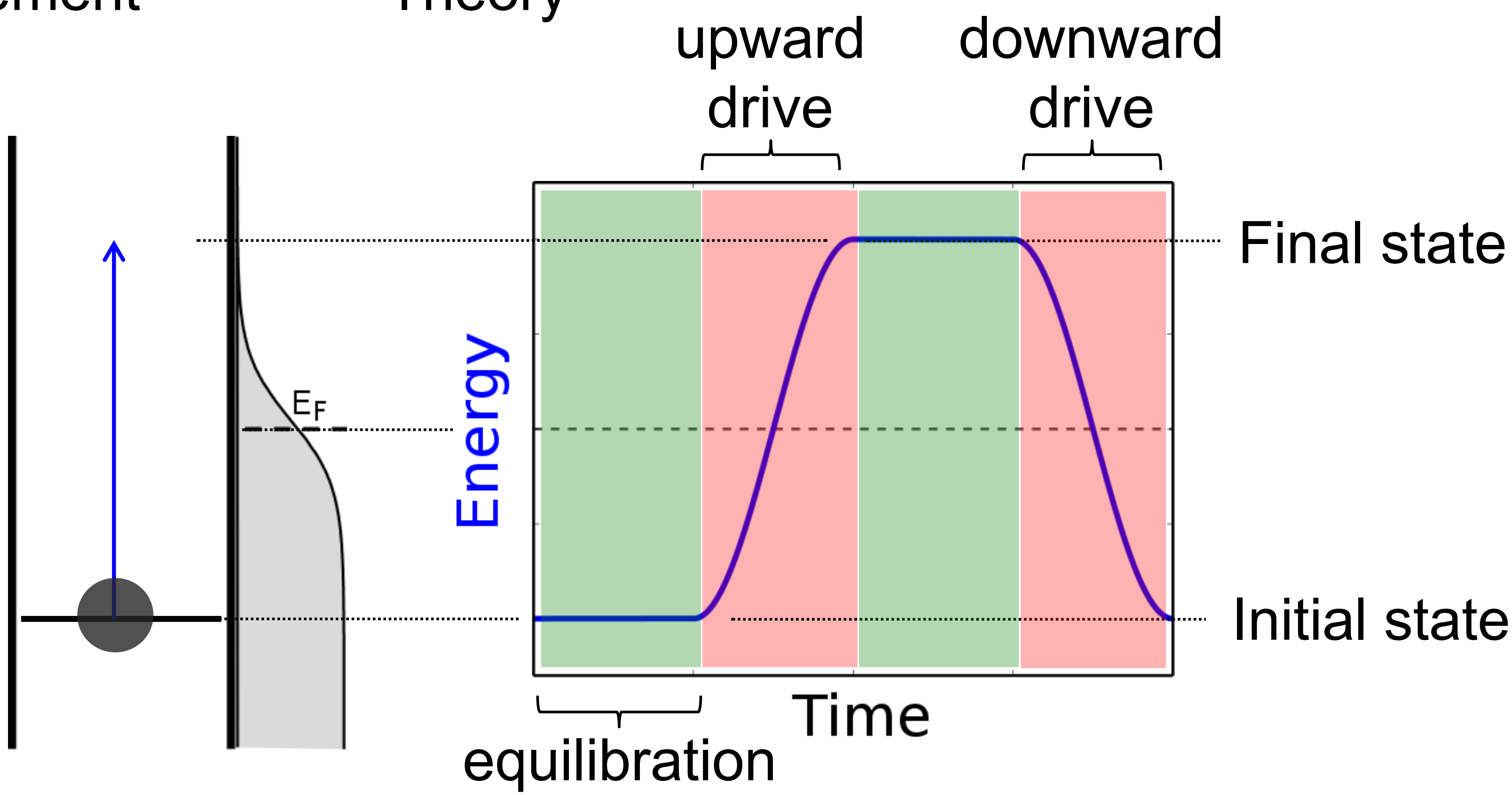
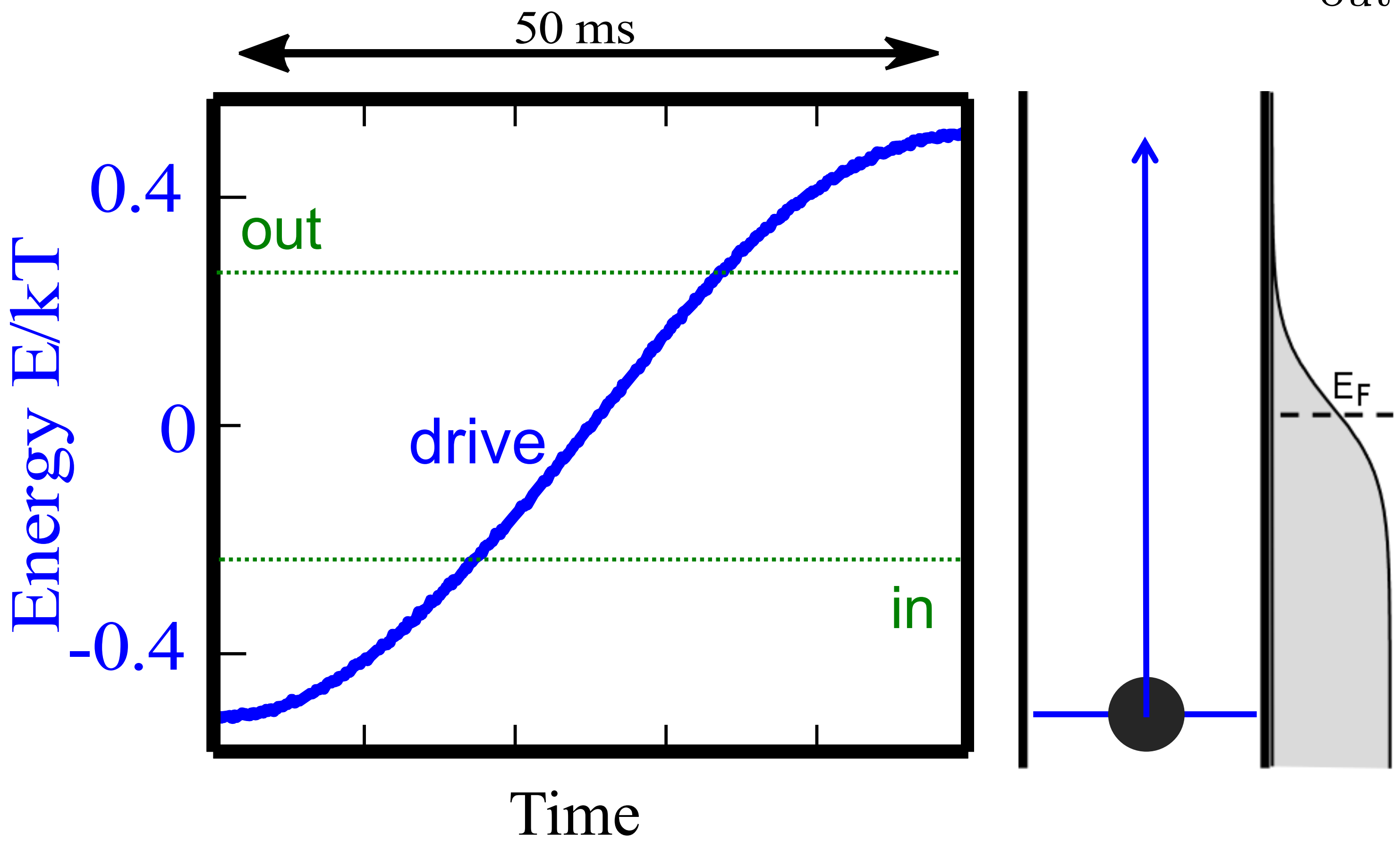


Upward drive

$$\Gamma_{\text{in}} = 42\,\text{Hz}$$

$$E(t) = E_{\text{F}} - A\cos(2\pi f t) \quad \text{for } 0 \leq t \leq 1/2f$$

$$\Gamma_{\text{out}} = 21\,\text{Hz}$$



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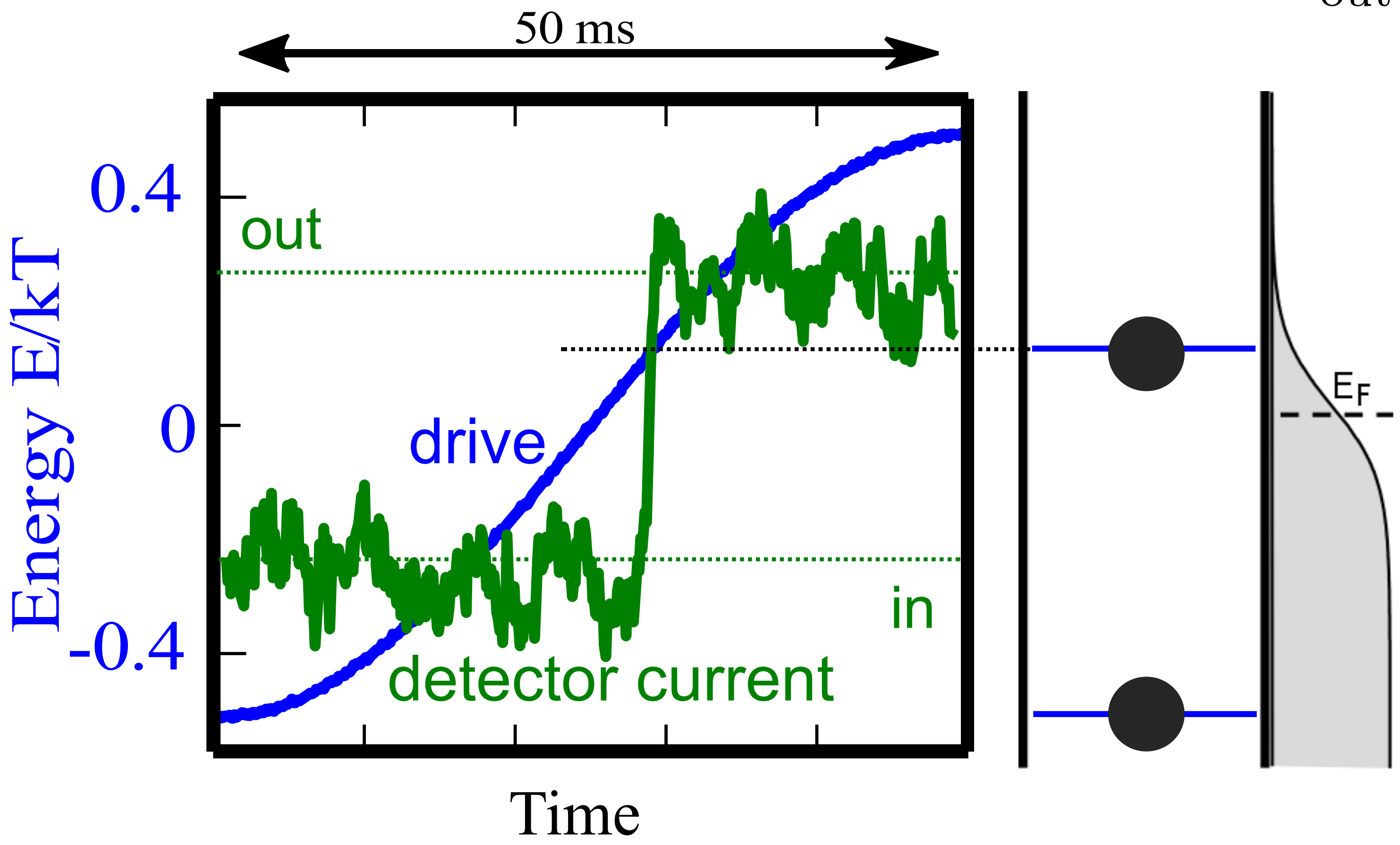

Upward drive

$$E(t) = E_{\mathrm{F}} - A\cos(2\pi f t) \quad \text{for } 0 \leq t \leq 1/2f$$

$$\Gamma_{\mathrm{in}} = 42\,\mathrm{Hz}$$
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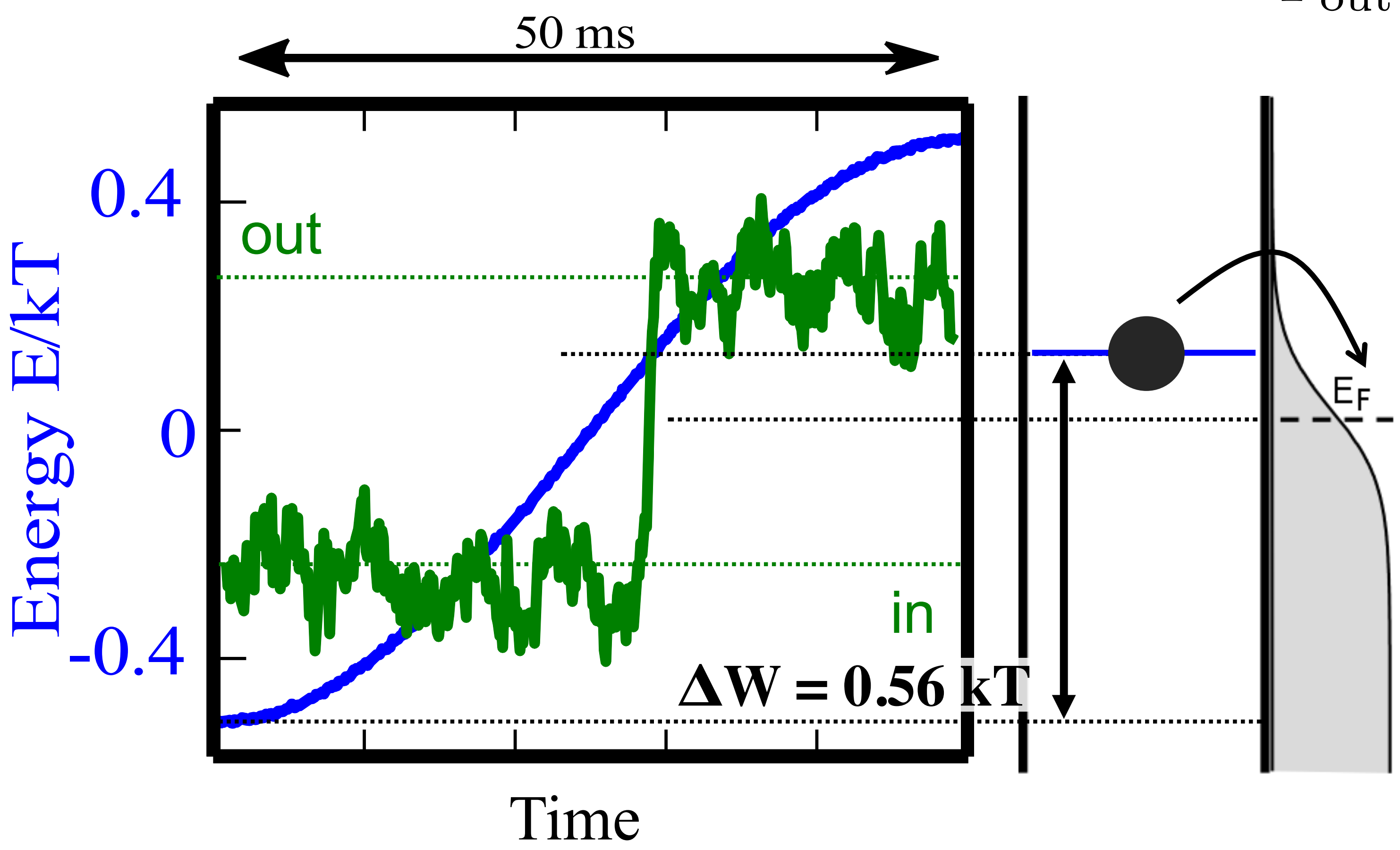
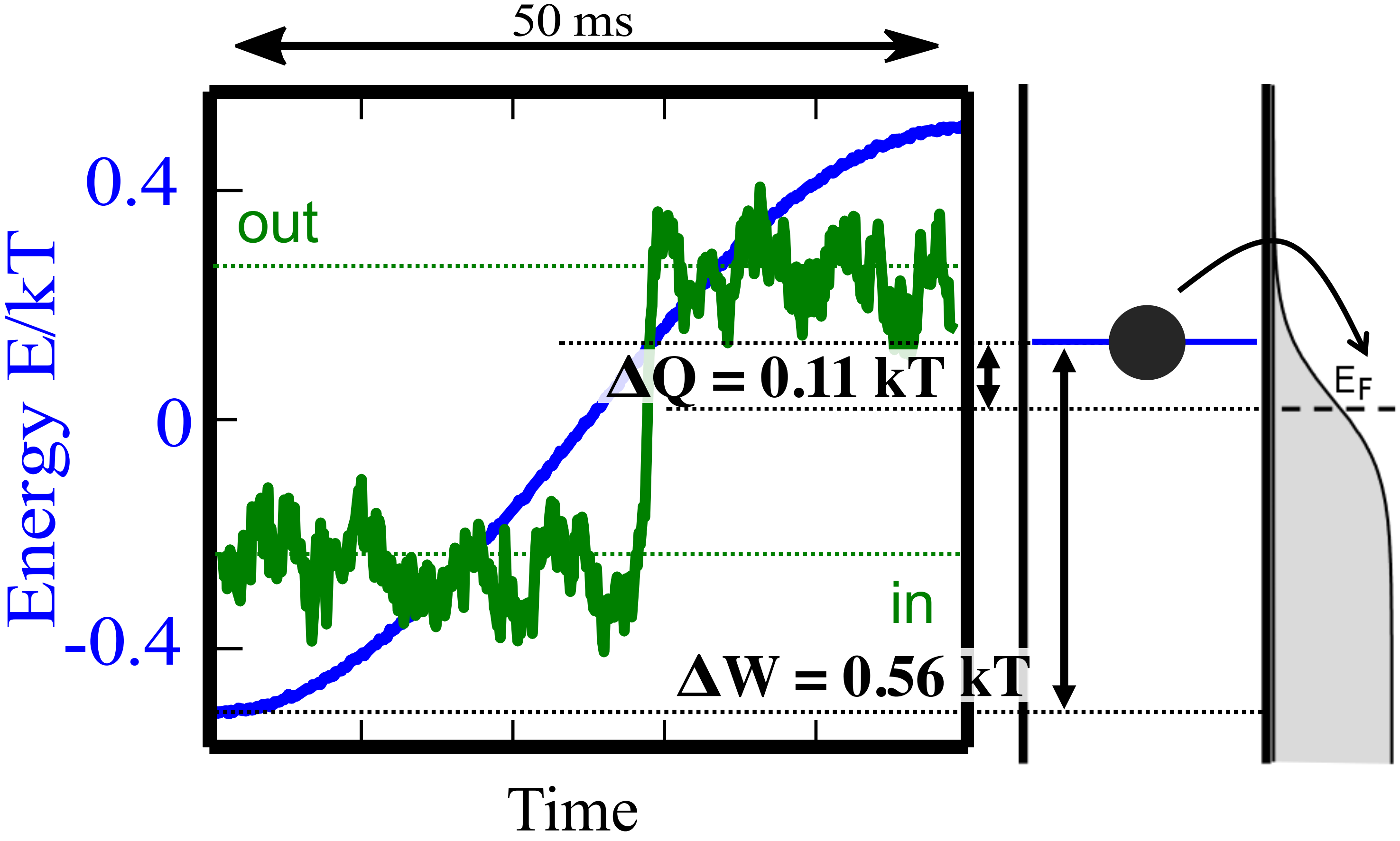


Upward drive

$$E(t) = E_F - A\cos(2\pi f t) \quad \text{for } 0 \leq t \leq 1/2f$$

$$\Gamma_{in} = 42\,\mathrm{Hz}$$
$$\Gamma_{out} = 21\,\mathrm{Hz}$$



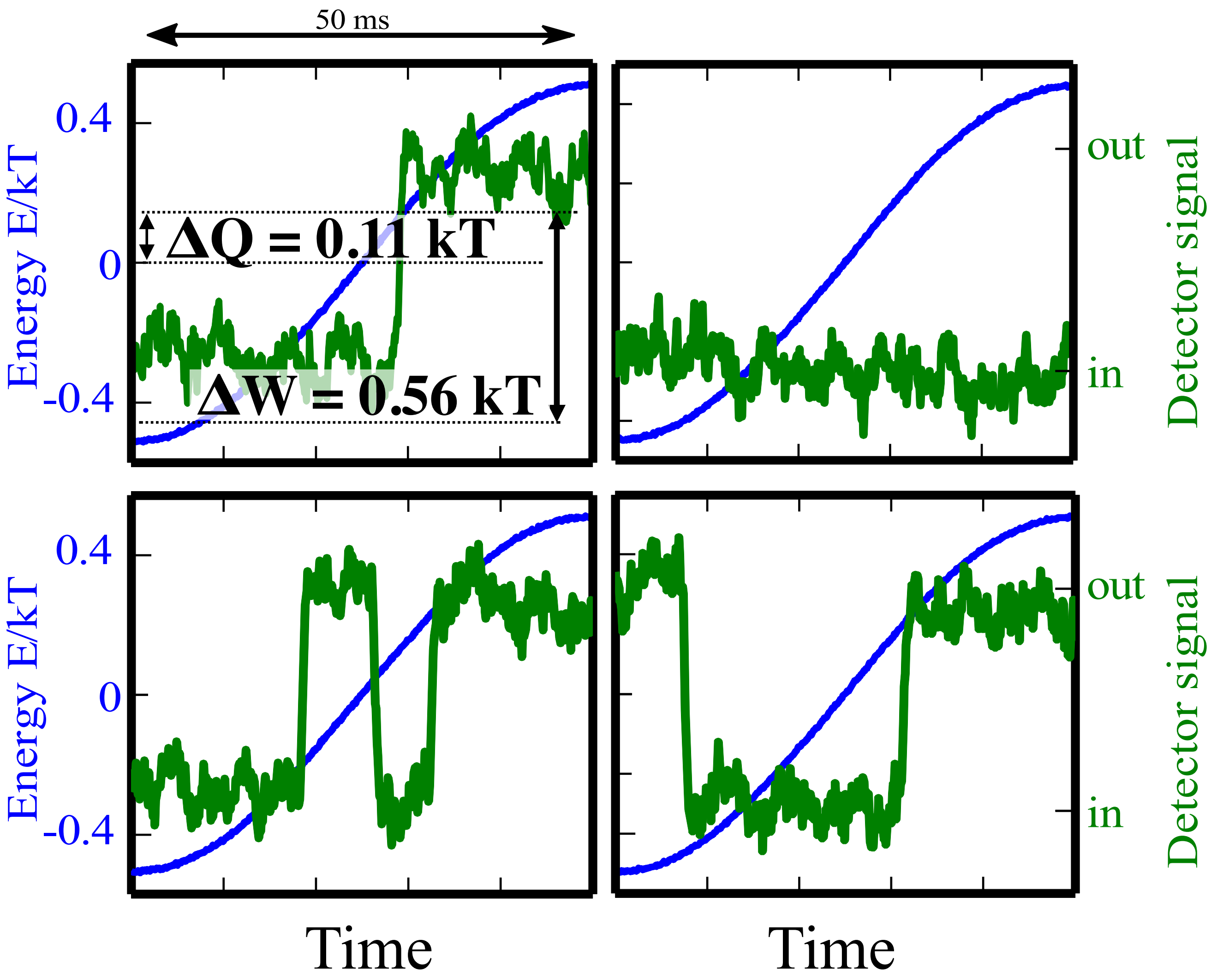
An ensemble of repetitions

$$\Gamma_{\mathrm{in}} = 42\,\mathrm{Hz}$$

$$\Gamma_{\mathrm{out}} = 21\,\mathrm{Hz}$$

~ 20'000 repetitions

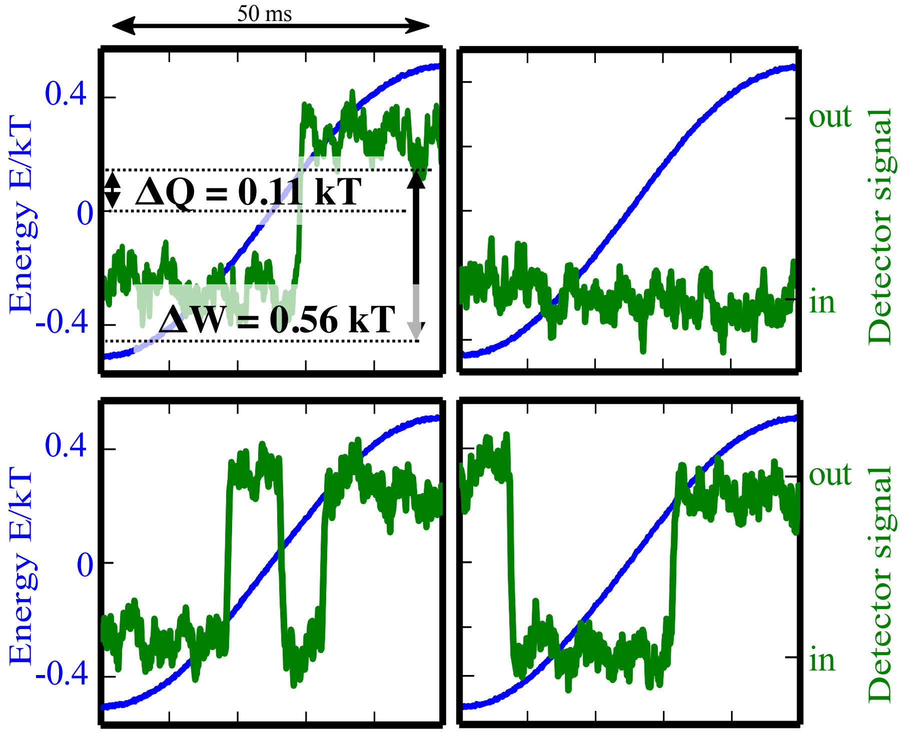
Examples:



An ensemble of repetitions

$$\Gamma_{\rm in} = 42\,\mathrm{Hz}$$

$$\Gamma_{\rm out} = 21\,\mathrm{Hz}$$



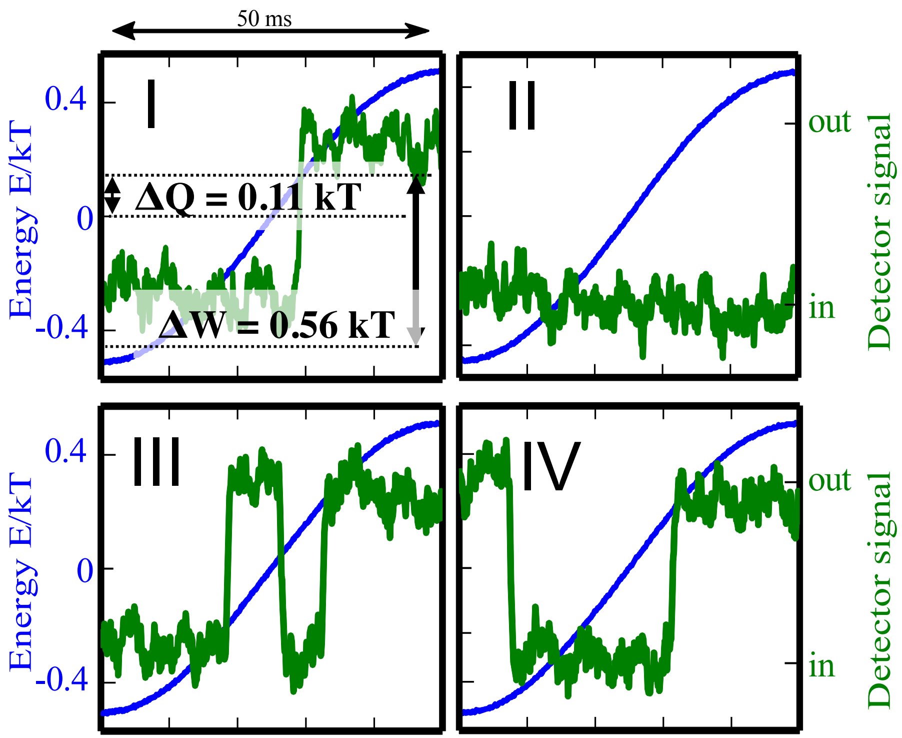
~ 20’000 repetitions

measured

$$\left\langle \exp\left(-\frac{\Delta W}{kT}\right)\right\rangle = \exp\left(-\frac{\Delta F}{kT}\right)$$

known from characterization

An ensemble of repetitions



~ 20’000 repetitions

equilibrium thermodynamics

$$\Delta F = kT \ln\left(\frac{Z_0}{Z_1}\right)$$

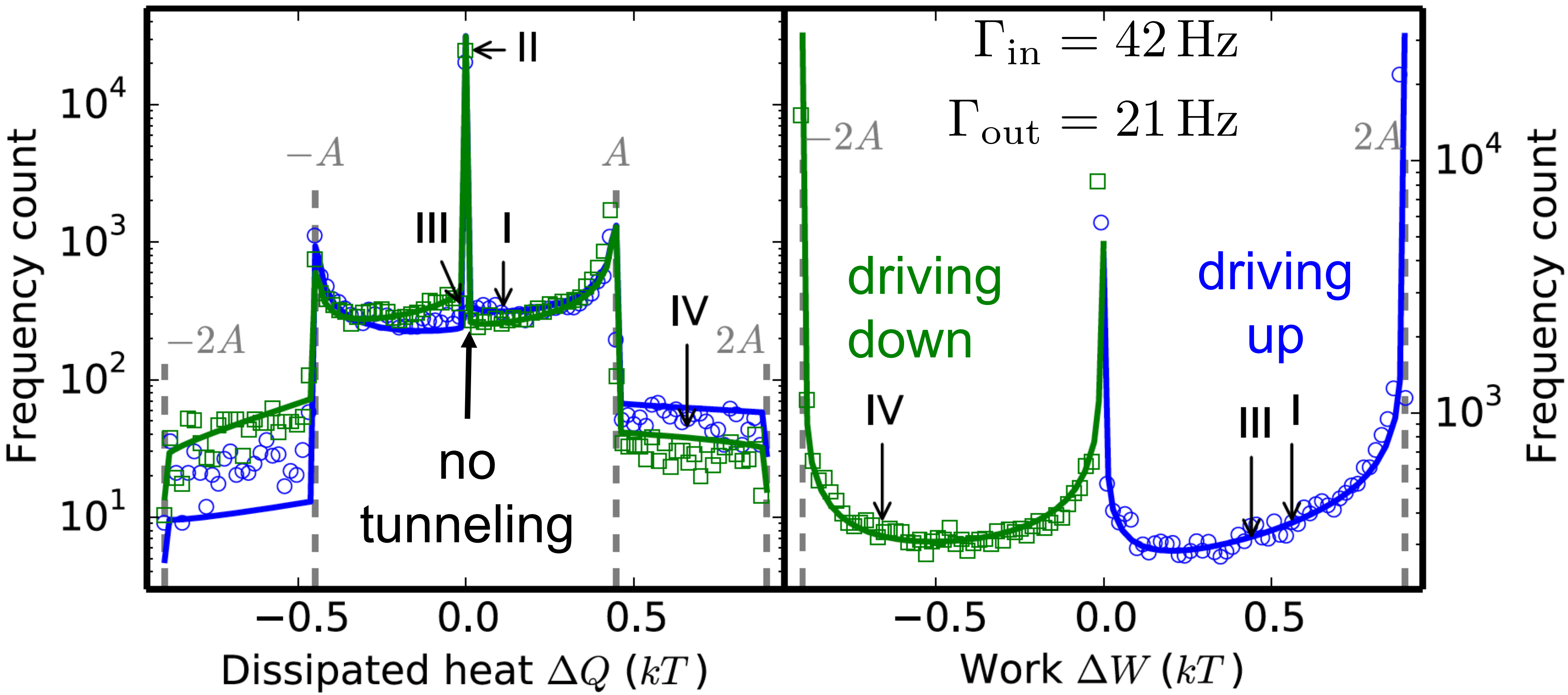
measured

$$\left\langle \exp\left(-\frac{\Delta W}{kT}\right)\right\rangle = \exp\left(-\frac{\Delta F}{kT}\right)$$

known from characterization

Extracted distribution functions

$$f = 10\,Hz, \quad A = 0.45kT$$

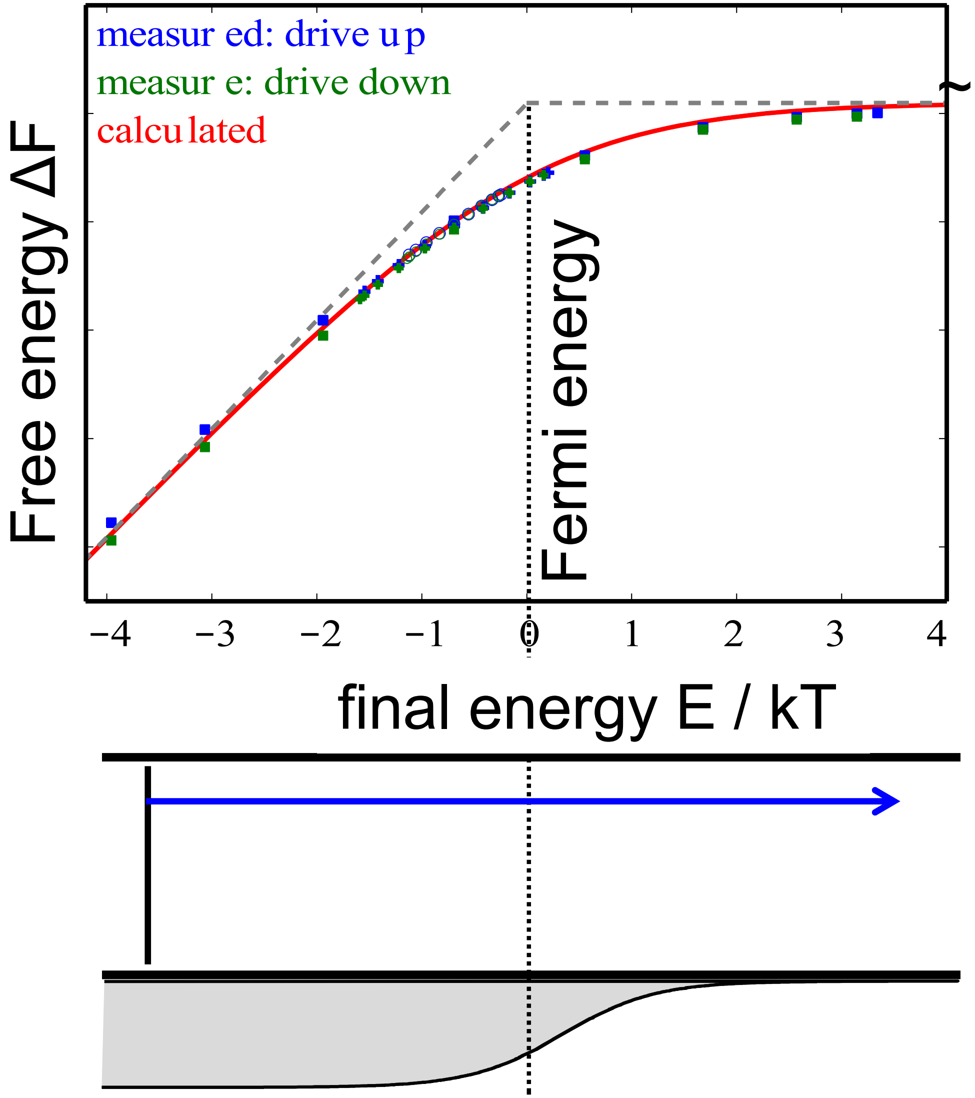


Distributions are far from gaussian => far from equilibrium
Solid lines: from rate equation model

A. Hofmann *et al*, PRB **93**, 035425 (2016).

An ensemble of repetitions

here: all measurements start at the same initial state



~ 20’000 repetitions

equilibrium thermodynamics

$$\Delta F = kT \ln\left(\frac{Z_0}{Z_1}\right)$$

measured

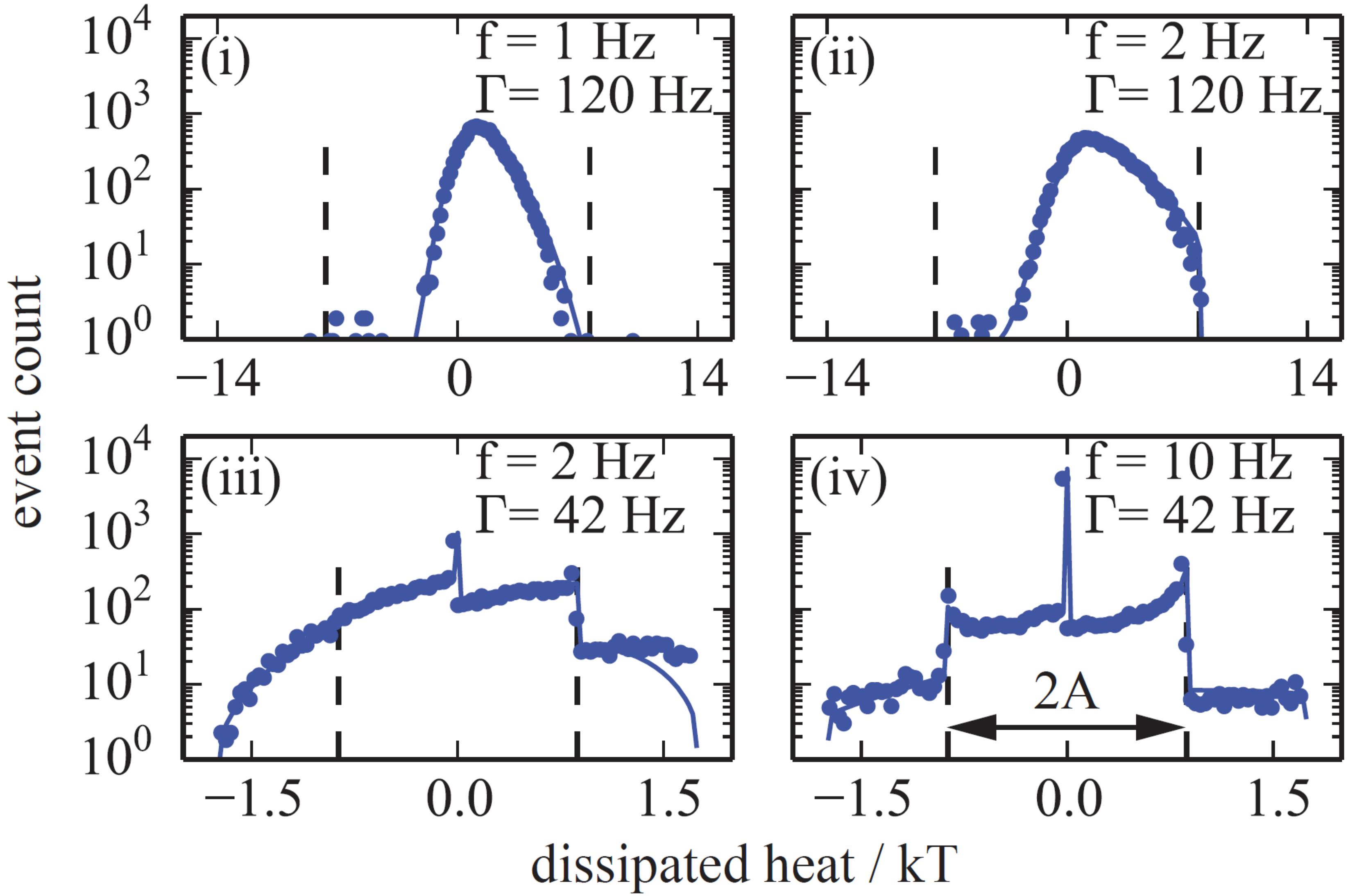
$$\left\langle \exp\left(-\frac{\Delta W}{kT}\right)\right\rangle = \exp\left(-\frac{\Delta F}{kT}\right)$$

known from characterization

$$Z(E) = 1 + 2e^{-E/kT}$$

A. Hofmann *et al*, PRB **93**, 035425 (2016).

From equilibrium to non-equilibrium



A. Hofmann *et al*, PSS B 1–5 (2016).

The arrow of time

Arthur Stanley Eddington,
British Astronomer
1927

Microscopic laws of physics are invariant under time-reversal

Thermodynamics:
as time progresses $\Delta S \geq 0$

Measuring ΔS allows us to infer the direction of time

The arrow of time

For a given ΔF we have two sets:

- 20’000 realizations driving up (forward in time)
- 20’000 realizations driving down (backward in time)

We pick one of the realizations randomly.

Can we tell the set, from which the measurement was taken?

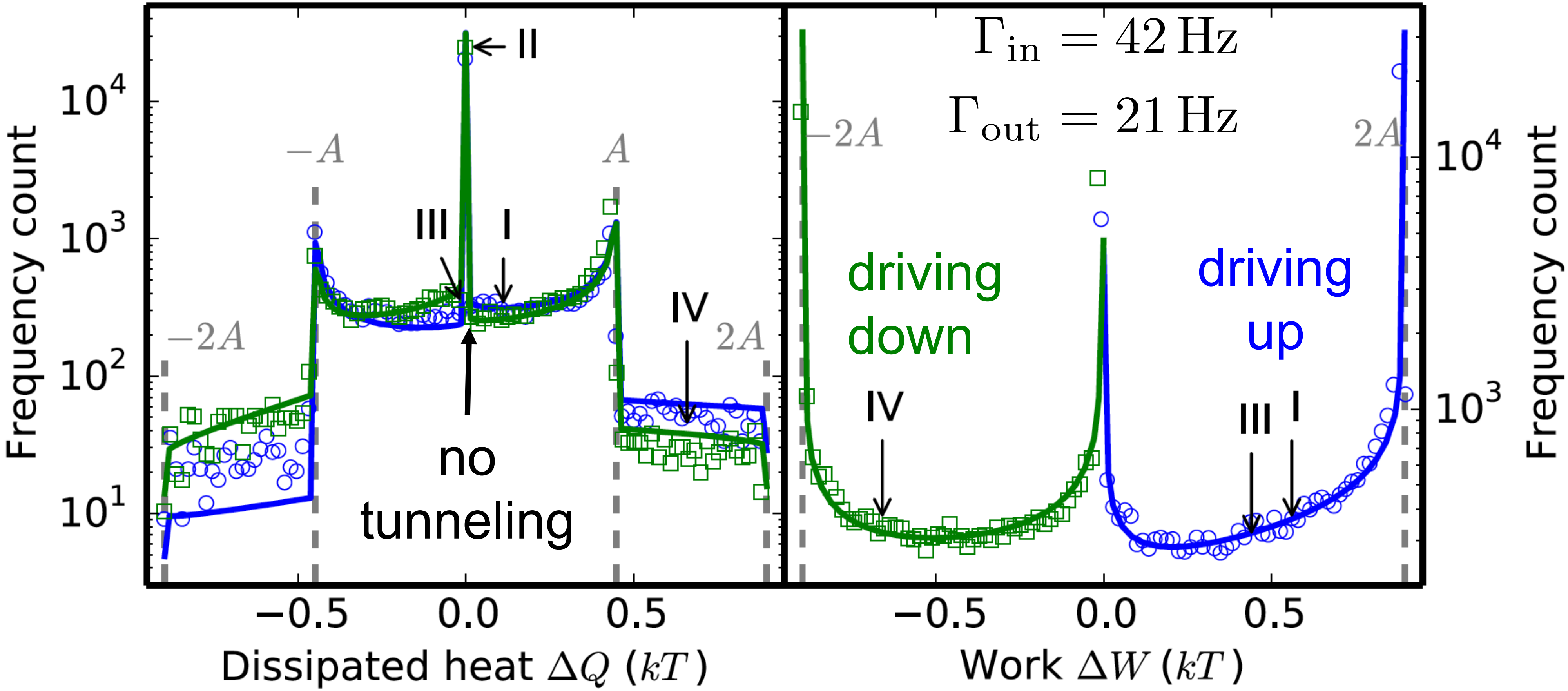
Key for the answer:

for a reversible process: $\Delta W - \Delta F = 0$

far from equilibrium: $\Delta W - \Delta F > 0$
(if forward in time)

Extracted distribution functions

$$f = 10\,Hz, \quad A = 0.45kT$$

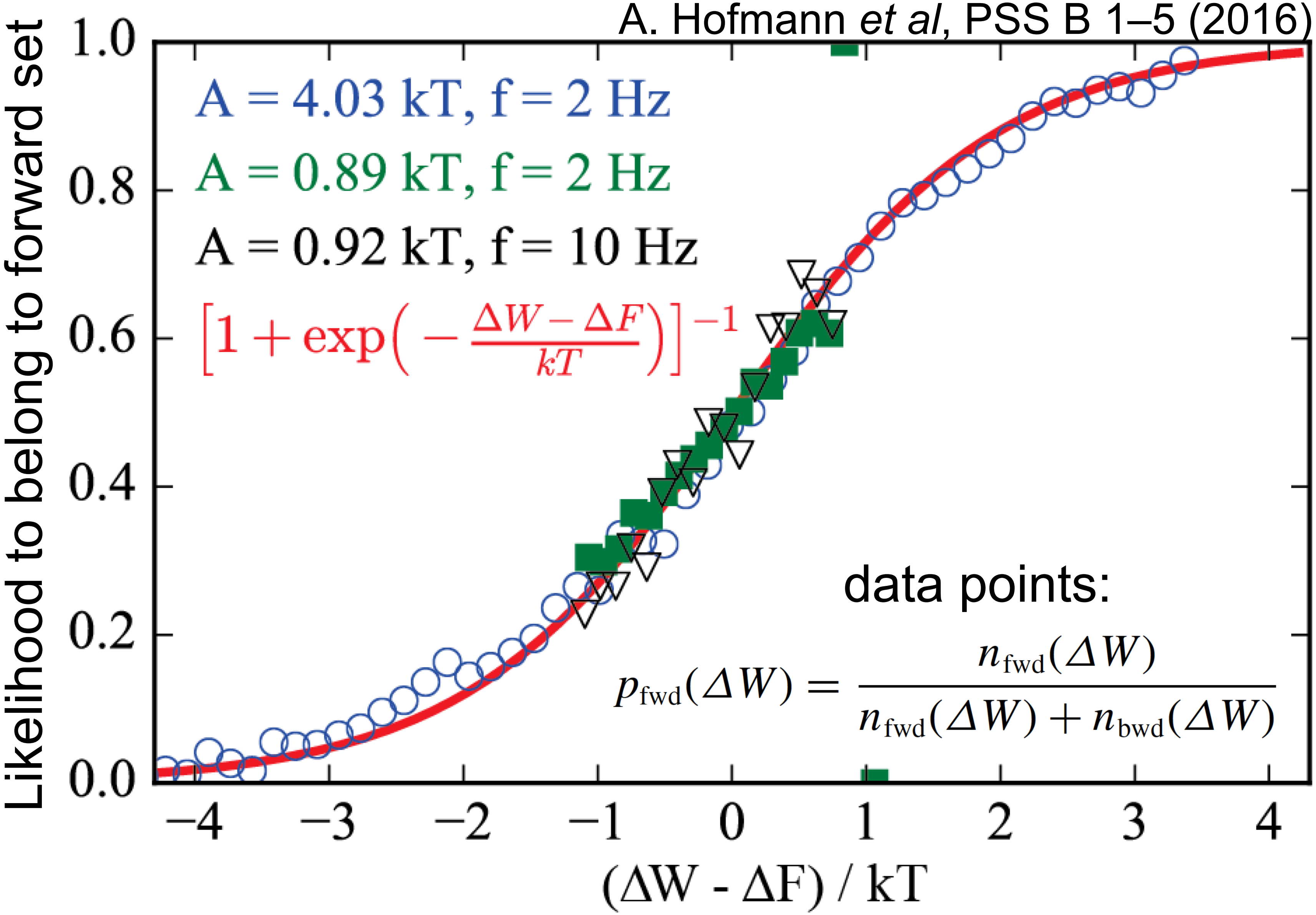


Distributions are not symmetric => “direction of time”

A. Hofmann *et al*, PRB **93**, 035425 (2016).

The arrow of time

A. Hofmann *et al*, PSS B 1–5 (2016).



Theory: Shirts *et al*, PRL **91**, 140601 (2003);
Maragakis *et al*, J. Chem. Phys. **129**, 024102 (2008).