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Tensor network states for gauge field theories:
m groundstates and particle excitations to real-time dynamics

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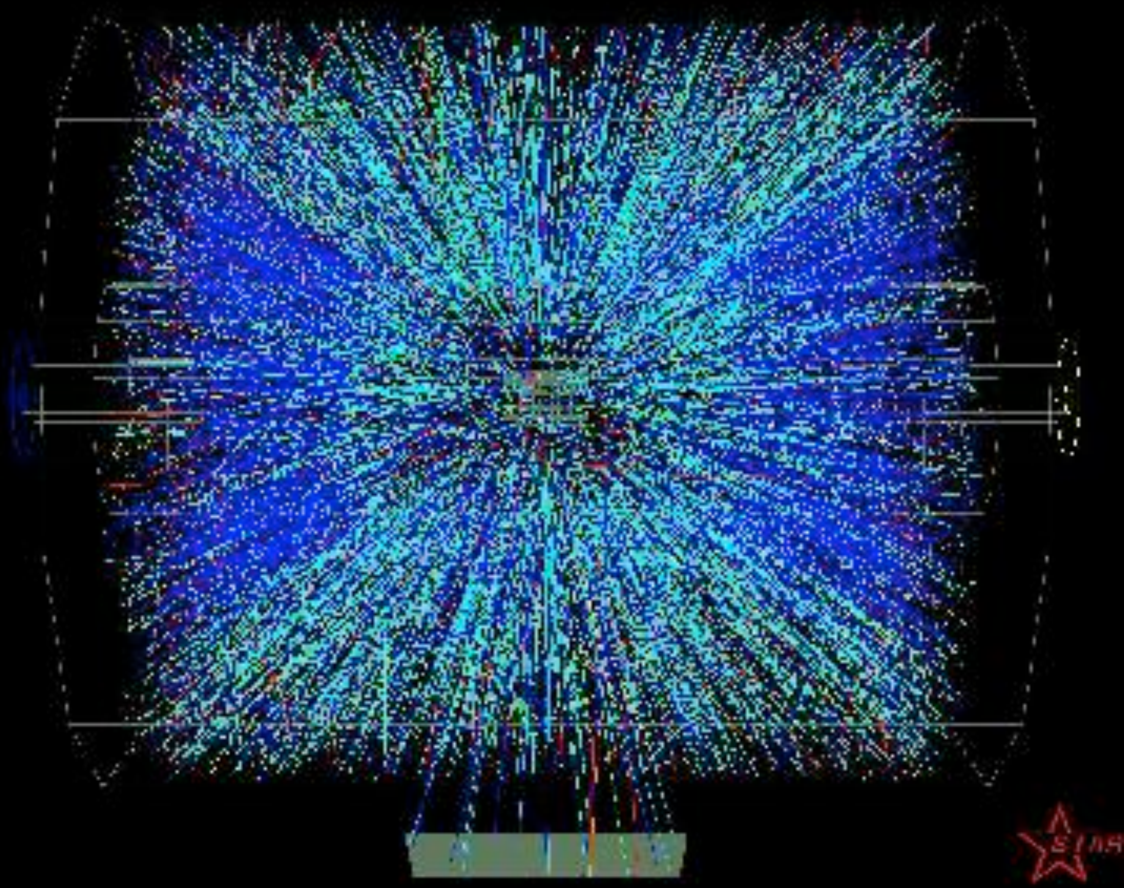
Desy Hamburg,
5 october 2017

Motivation

• Hamiltonian simulations, wave-functions, real-time physics

• No sign problem, finite fermionic chemical potential

• Understand gauge theories in the tensor network language i.e. in terms of their entanglement structure



Related work:

- T. M. Byrnes, P. Sriganesh, R.J. Bursill, C.J. Hamer Phys. Rev. D66, 13002 (2002)
- T. Sugihara, JHEP 07, 022 (2005)
- L. Tagliacozzo and G. Vidal, Phys. Rev. B 83, 115127 (2011)
- M.C. Bañuls, K. Cichy, K. Jansen, and J.I. Cirac, JHEP 11, 158 (2013)
- M.C Bañuls, K. Cichy, J.I. Cirac, K. Jansen, H. Saito, PoS (Lattice 2013) 332 and PoS(Lattice 2014) 302
- E.Rico, T. Pichler, M.Dalmonte, P. Zoller, S.Montangero, PRL 112, 201601 (2014)
- P. Silvi, E. Rico, T. Calarco and S. Montangero, New J. Phys. 16,10, 103015 (2014)
- L.Tagliacozzo, A. Celi and M. Lewenstein, Phys. Rev X4 (2014) 4,041024
- T. Osborne, A. Milsted, https://github.com/tobiasosborne/Latticegauge- theory-and-tensor-networks
- ....

Outline

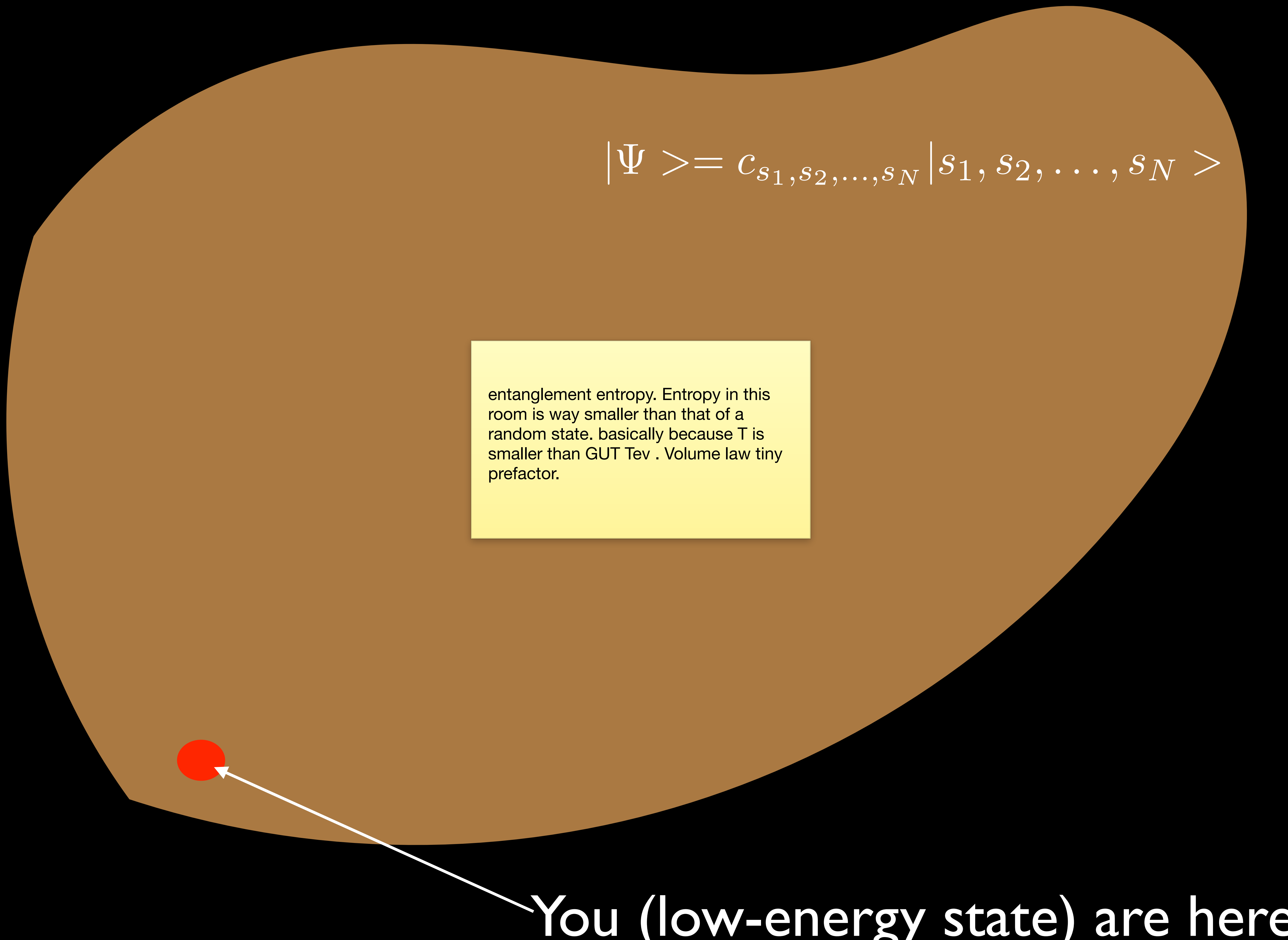
what turns your simulation of a gauge theory, into that of a gauge FIELD theory

• (very brief) Introduction to tensor network states

•Systematics (for d=1+1): gauge symmetry, local charge truncation, continuum limit

•Physics: static and real-time results

The tiny corner of Hilbert space



TNS for 1+1 dimensional gauge theories

(B. Buyens, J. Haegeman, K.V.A., H. Verschelde, F. Verstraete, Phys. Rev. Lett.: 113 (2014) 091601; proceedings Lattice 2014)

dim g = 1, superrenormalizable

Nice benchmarkmodel: weak and strong coupling expansion.

d=1+1 QED a.k.a. the Schwinger model:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\gamma^{\mu}(\partial_{\mu} - igA_{\mu})\psi + m\bar{\psi}\psi$$

- Can be solved exactly for $g \to \infty$ *(Schwinger '62, Coleman '76)*

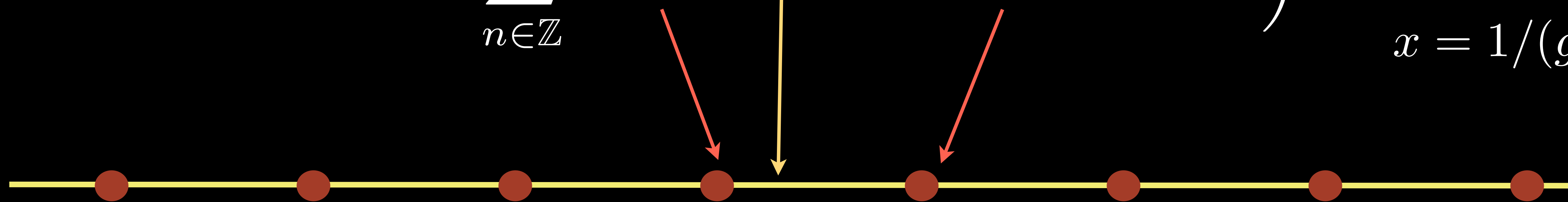
- Non-trivial physics, similar to QCD: e.g. confinement, (anomalous) chiral symmetry breaking

Kogut-Susskind ($A_0 = 0$ + staggered fermions) + Jordan-Wigner:

$$H = \frac{g}{2\sqrt{x}} \left(\sum_n L(n)^2 + \frac{\sqrt{x}}{g} m \sum_n (-1)^n \sigma_z(n) \right.$$

$$\left. + x \sum_{n \in \mathbb{Z}} (\sigma^+(n) e^{i\theta(n)} \sigma^-(n+1) + h.c.) \right).$$

$x = 1/(g^2$



fermions: $\sigma_z(n)|s_n>= s_n|s_n> \quad (s_n = \pm 1)$

gauge-fields: $L_n|p_n>= p_n|p_n> \quad p_n \in Z \qquad [\theta(n), L(m)] = i\delta_{nm}$

CT-symmetry: $T : n \to n+1 \quad \times \quad C : (s_n, L_n) \to (-s_n, -L_n) \quad (CT^2 = T^2)$

Extra ingredient: gauge invariance/Gauss law

$$G_n|\Psi>_{phys}= 0$$

$$G_n = L(n) - L(n-1) - \frac{1}{2}(\sigma_z(n) + (-1)^n) \qquad (\nabla.E = \rho)$$

gauge fi
conjugat

x is lattic
in the x t

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Gn is gen

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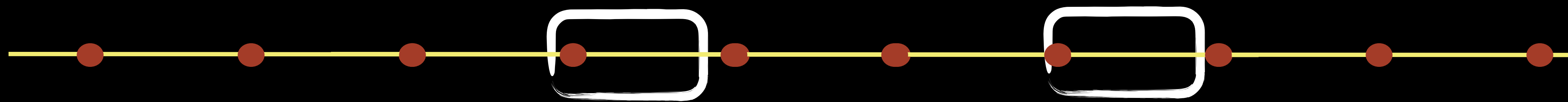
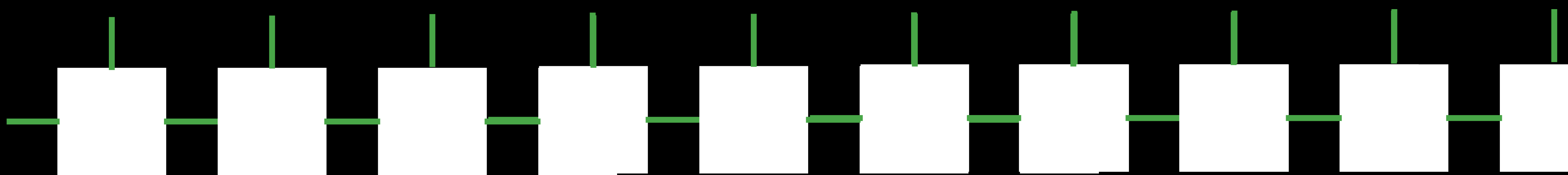
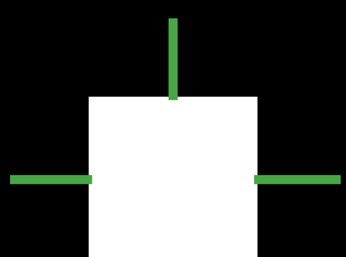
Matrix Product State

block: $|q_{2n}>=|s_{2n},p_{2n}>$ $|q_{2n+1}>=|-s_{2n},-p_{2n}>$





 $=[A^{s,p}]_{\alpha\beta}$

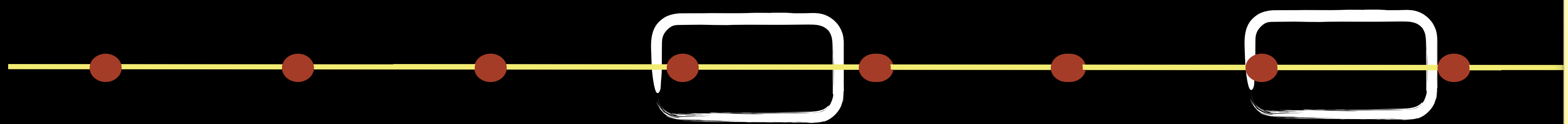
$$|\Psi>=v_L^{+}A_1^{q_1}A_2^{q_2}A_3^{q_3}\ldots A_n^{q_n}\ldots A_N^{q_N}v_R|q_1,q_2,q_3,\ldots q_n,\ldots q_N>$$

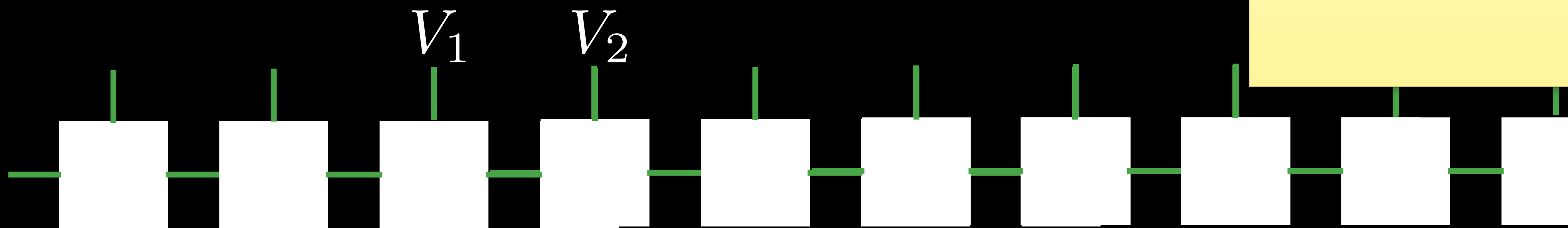
Per charge sector, some bond-dimension that we can choose.

Distribute bond-dimensions over different charge sectors , which given a total bond dimension will effectively truncate your Hilbert space

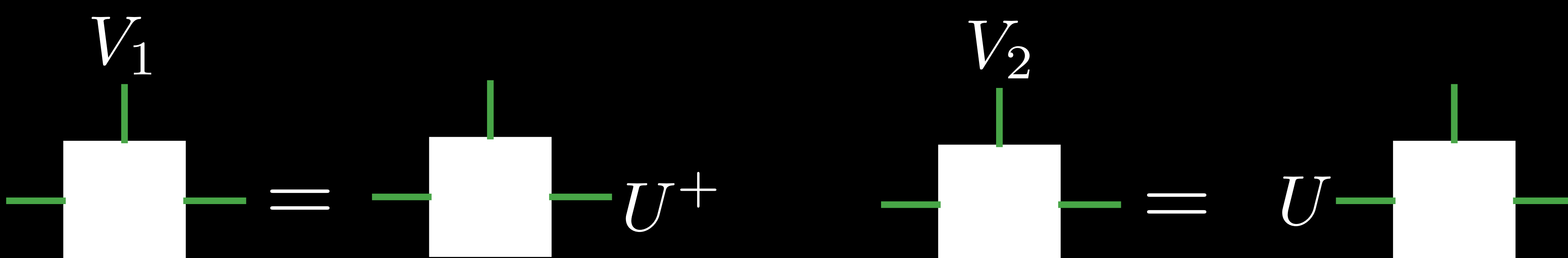
gauge-invariant Matrix Product State

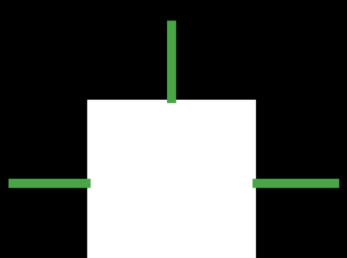
block: $|q_{2n}>=|s_{2n},p_{2n}>$ $|q_{2n+1}>=|-$





$$V_1 = e^{i\theta p_1} \qquad V_2 = e^{i\theta\left(p_2 - \frac{1}{2}(s_2+1)\right)}$$





$$= [A^{s,p}]_{(q,\alpha_q);(r,\beta_r)} = [a^{s,p}]_{\alpha_q,\beta_r}\delta_{p,q+(s+1)/2}\delta_{r,-p}$$

Explain, well, how to generalize to non-Abelian, to PEPS

Per charge
can choos

Distribute
sectors , w
effectively

e.g. $A_1^{1,p}$

$$
\begin{array}{cc}
q\backslash r & \begin{array}{ccccc} \cdots & p-1 & p & p+1 & \cdots \end{array} \\
\begin{array}{c} \vdots \\ p-1 \\ p \\ p+1 \\ p+2 \\ \vdots \end{array} &
\left(\begin{array}{c|c|c|c|c}
\ddots & \vdots & \vdots & \vdots & \iddots \\
\hline
\cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\
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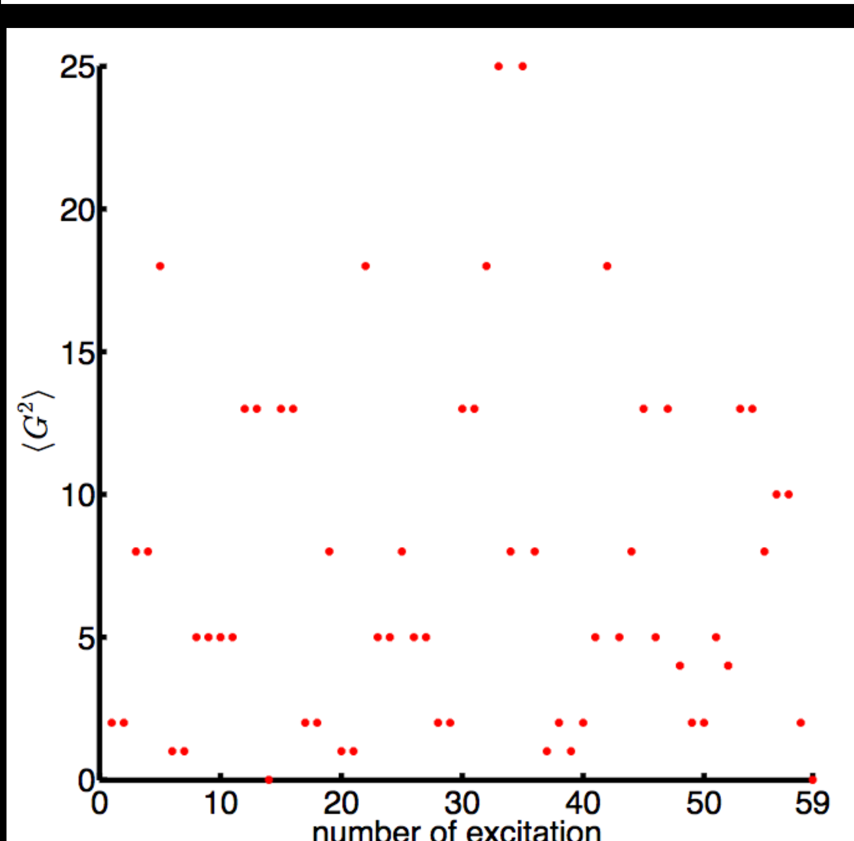
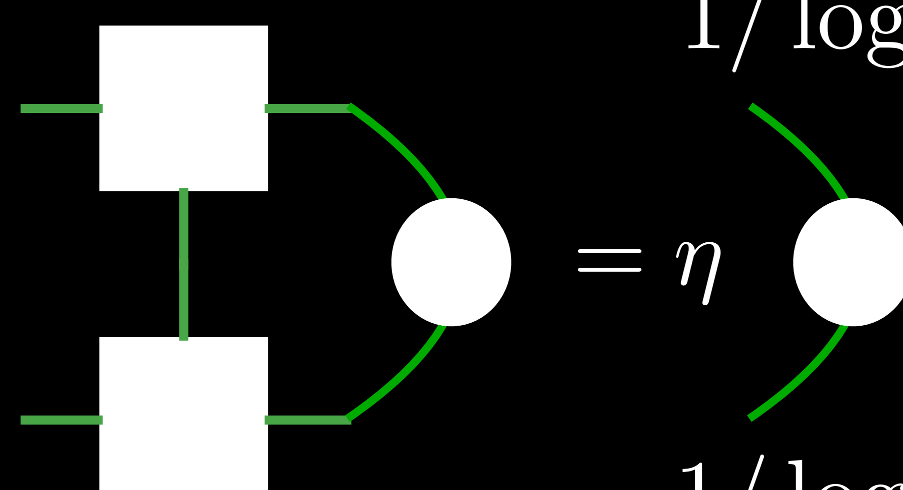
\cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\
\hline
\cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & a_1^{1,p} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\
\hline
\cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\
\hline
\iddots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
\end{array}
$$

verifying Elitzur's theorem
(and why you should exploit the block structure)

	without GI	with GI	without GI	with GI	with GI [1]
p_{max}	2	2	3	3	3
D	29	[2 6 9 8 4]	40	[2 3 7 11 10 4 2]	[5 20 48 70 62 34 10]
steps	9645	278	12417	561	
time	3 h 30 min	2 min	6 h 27min	5 min	
$\langle G^2 \rangle$	3×10^{-9}	0	3×10^{-9}	0	0
e	-3.048961	–3.048961	-3.048961	-3.048961	-3.048961
$E_{1,v}$	1.04252{10}	1.04254	1.04194 {14}	1.04209	1.04207
$E_{2,v}$	2.455 {37}	2.455	2.385 {59}	2.386	2.357
$E_{1,s}$	1.7719{20}	1.7719	1.7559 {31}	1.7565	1.7516

Table 1: Results of computations with and without imposing gauge invariance (GI). ($x = 100, m/g = 0.25$)



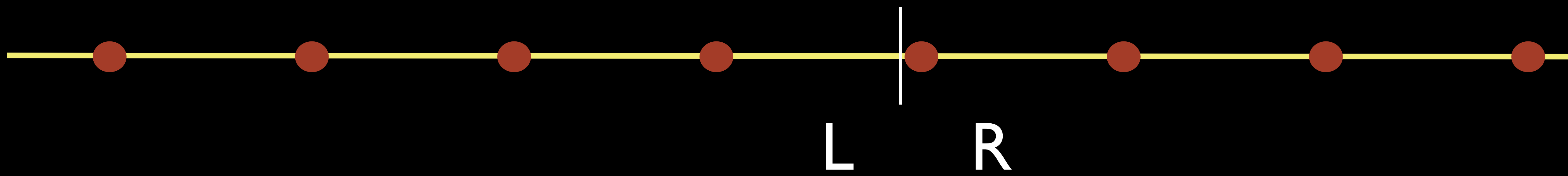


$$1/\log(\eta_0/\eta_1) \times p_{max} D^3$$

versus

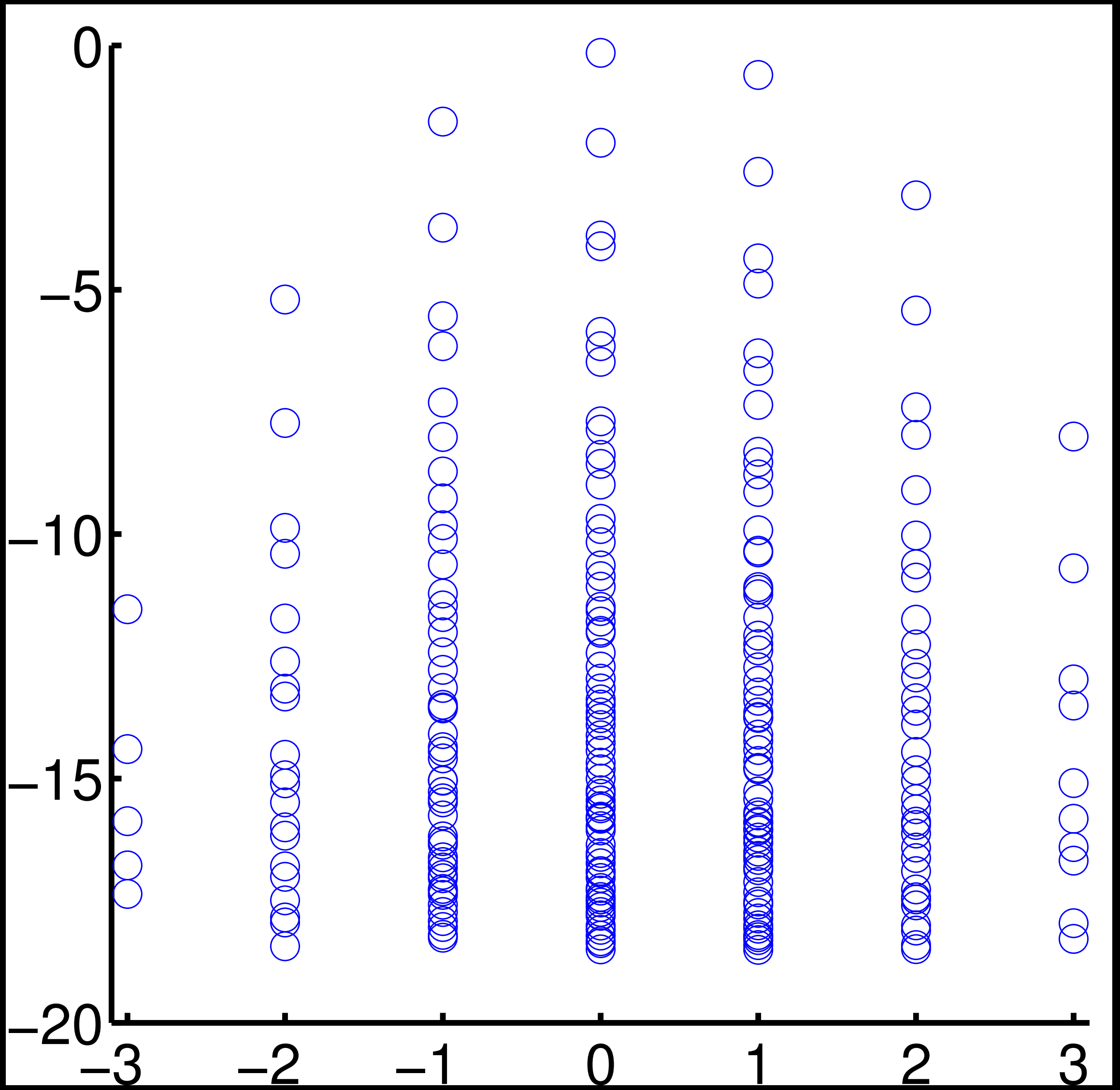
$$1/\log(\eta_0/\eta_1') \times p_{max} D_p^3$$

Effective truncation local Hilbert space

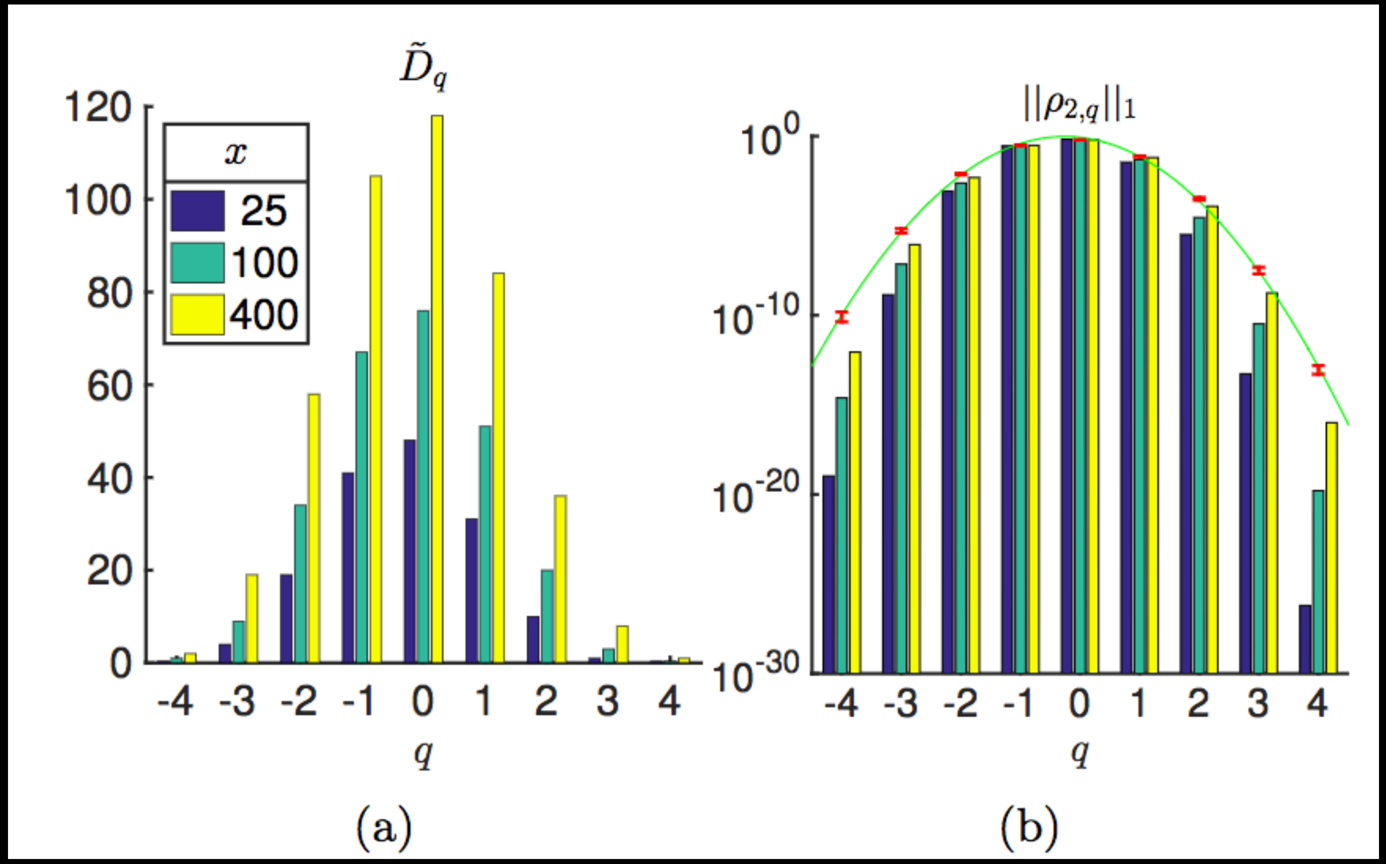


Schmidt-decomposition: $$|\Psi>=\sum_{p,i}\sqrt{\lambda_{(p,i)}}\,|\Psi_{p,i}>_L\,|\Psi_{-p,i}>_R$$

$\log_{10}\lambda_{p,i}$

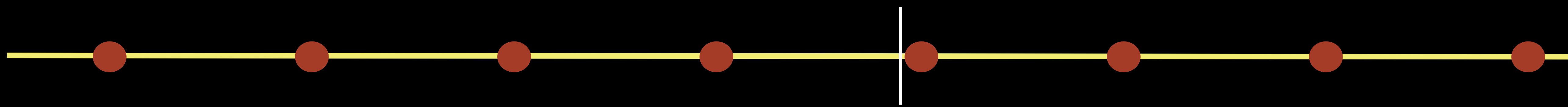


$D=(5,20,48,70,62,34,10)$ charge sector p



(B. Buyens, S. Montangero, J. Haegeman, F. Verstraete, K.V.A Phys. Rev. D 95, 094509 (2017))

Effective truncation local Hilbert space



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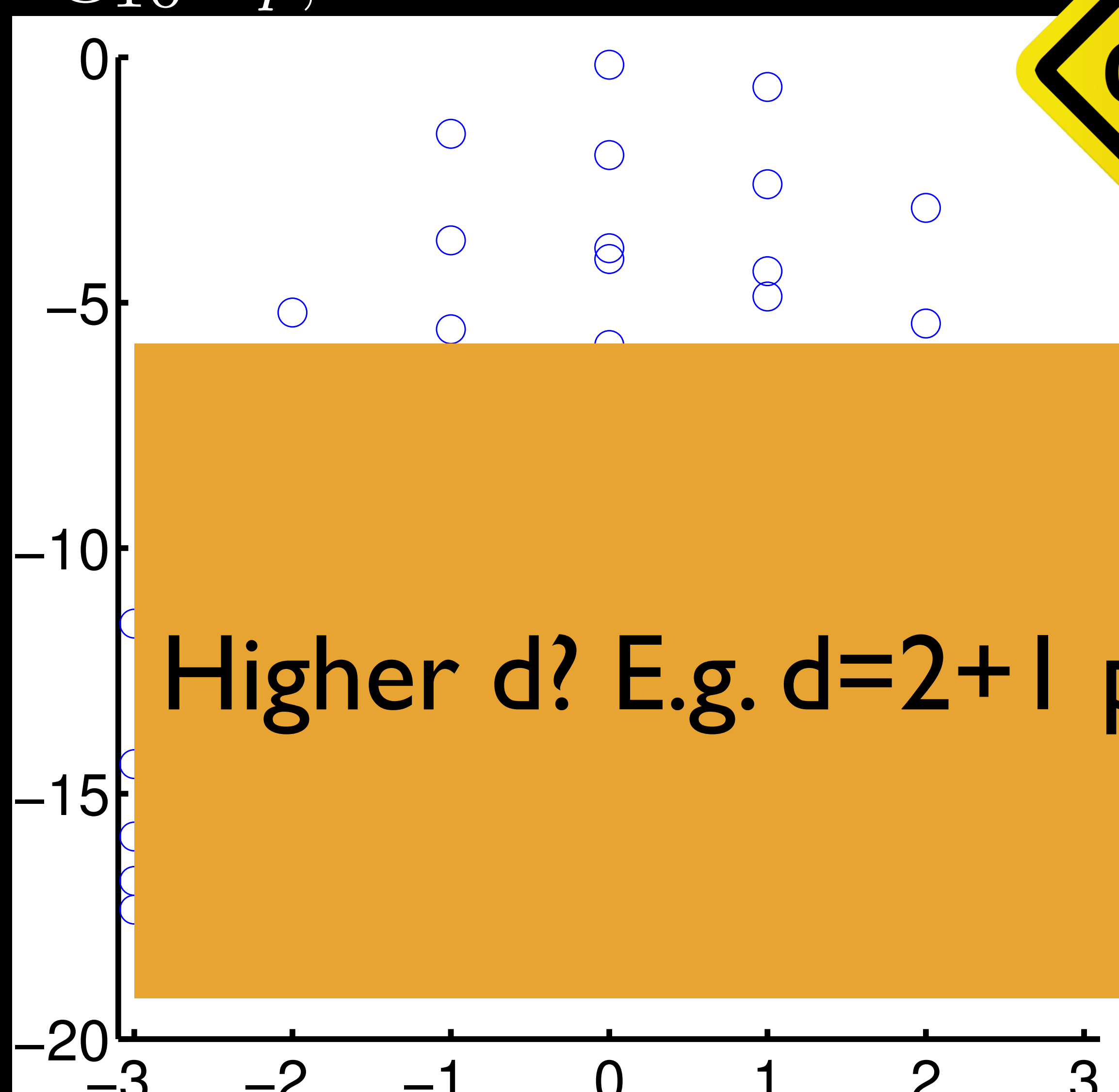
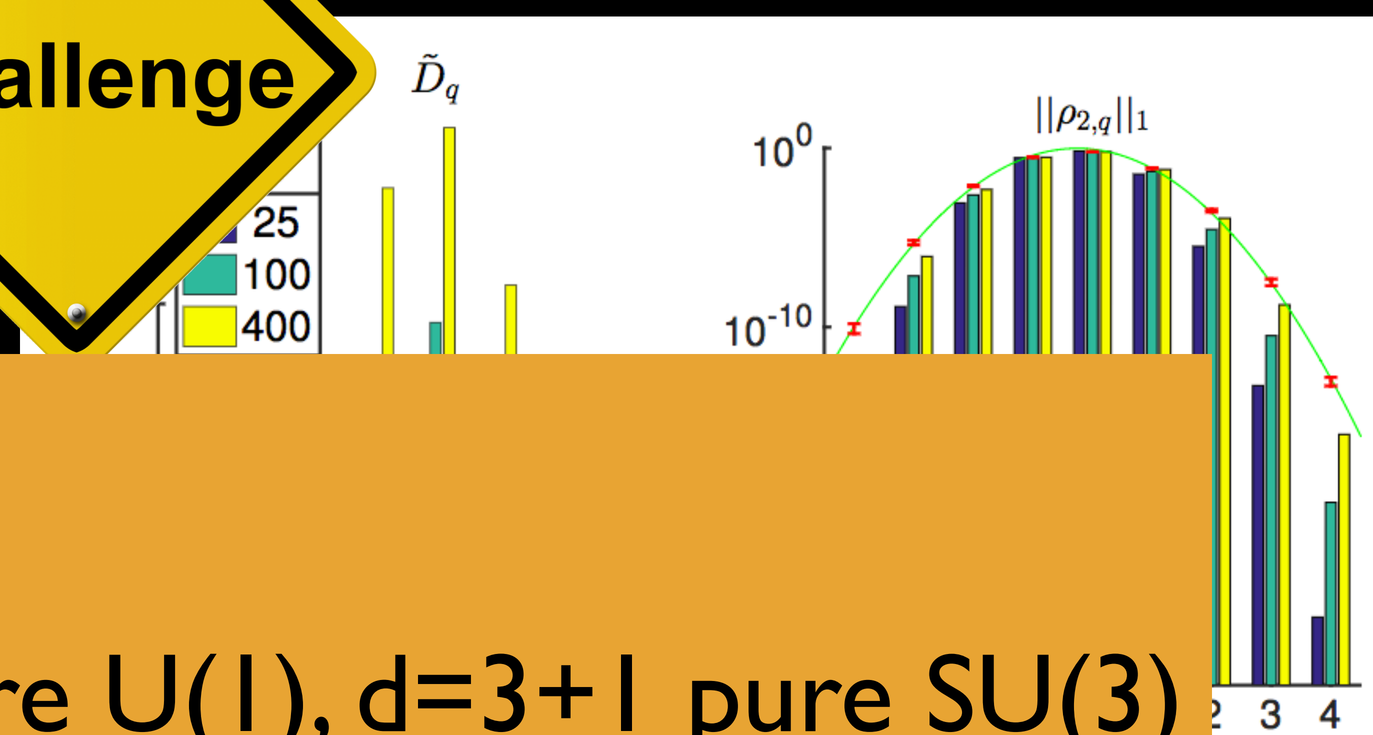
Schmidt-decomposition: $|\Psi> = \sum_{p,i} \sqrt{\lambda_{(p,i)}}\, |\Psi_{p,i}>_L \; |\Psi_{-p,i}>_R$

$\log_{10} \lambda_{p,i}$





Higher d? E.g. d=2+1 pure U(1), d=3+1 pure SU(3)

$D = (5, 20, 48, 70, 62, 34, 10)$ charge sector

increases with $\sqrt{x}$ in each of the eigenvalue sectors of $L(n)$, but falls of very fast with $|q|$. (b) By performing a polynomial extrapolation of $\log_{10}(||\rho_{2,q}||_1)$ in $1/\sqrt{x}$, we obtain estimates for the continuum value of $||\rho_{2,q}||_1$ (red

One-particle excitations

for gapped translation invariant local systems (also non-relativistic), the lowest lying exciting states can be obtained by applying a momentum superposition of a quasi-local operator on the ground-state:

$$|\Psi(k)> = \sum_n e^{ikan} T^n O_k T^{-n} |\Psi_0>$$

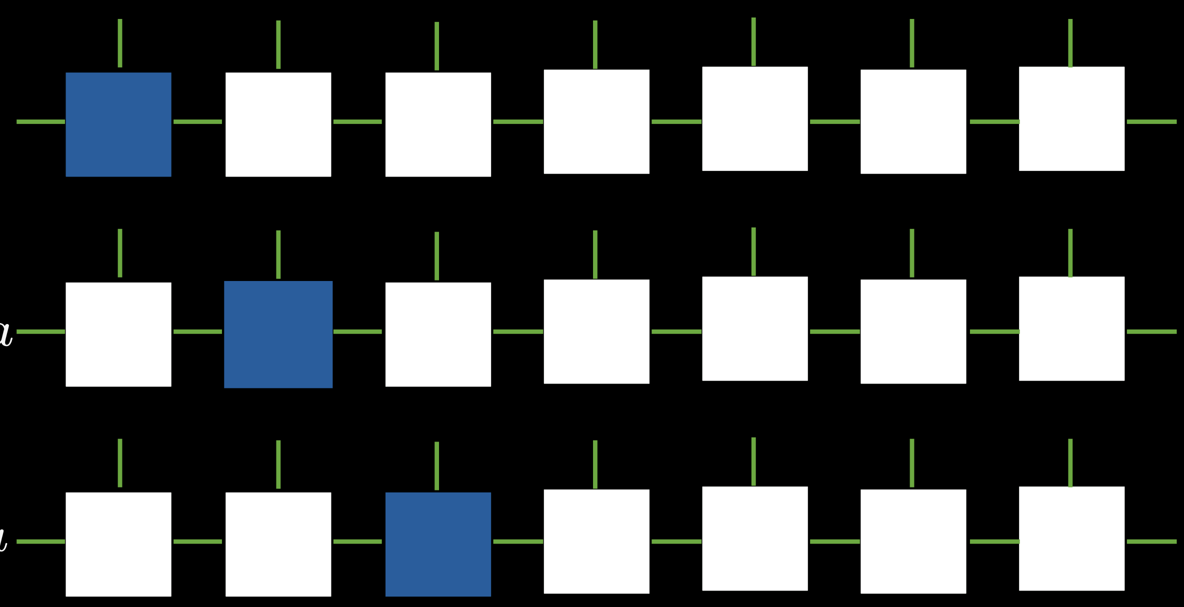
(Haegeman et al, Phys. Rev. B 85, 100408)

One-particle excitations on top of MPS vacuum approximation:

$$|\Psi(k,\gamma)> = \ldots + \gamma^m e^{ikma}$$

$$+\gamma^{m+1} e^{ik(m+1)a}$$

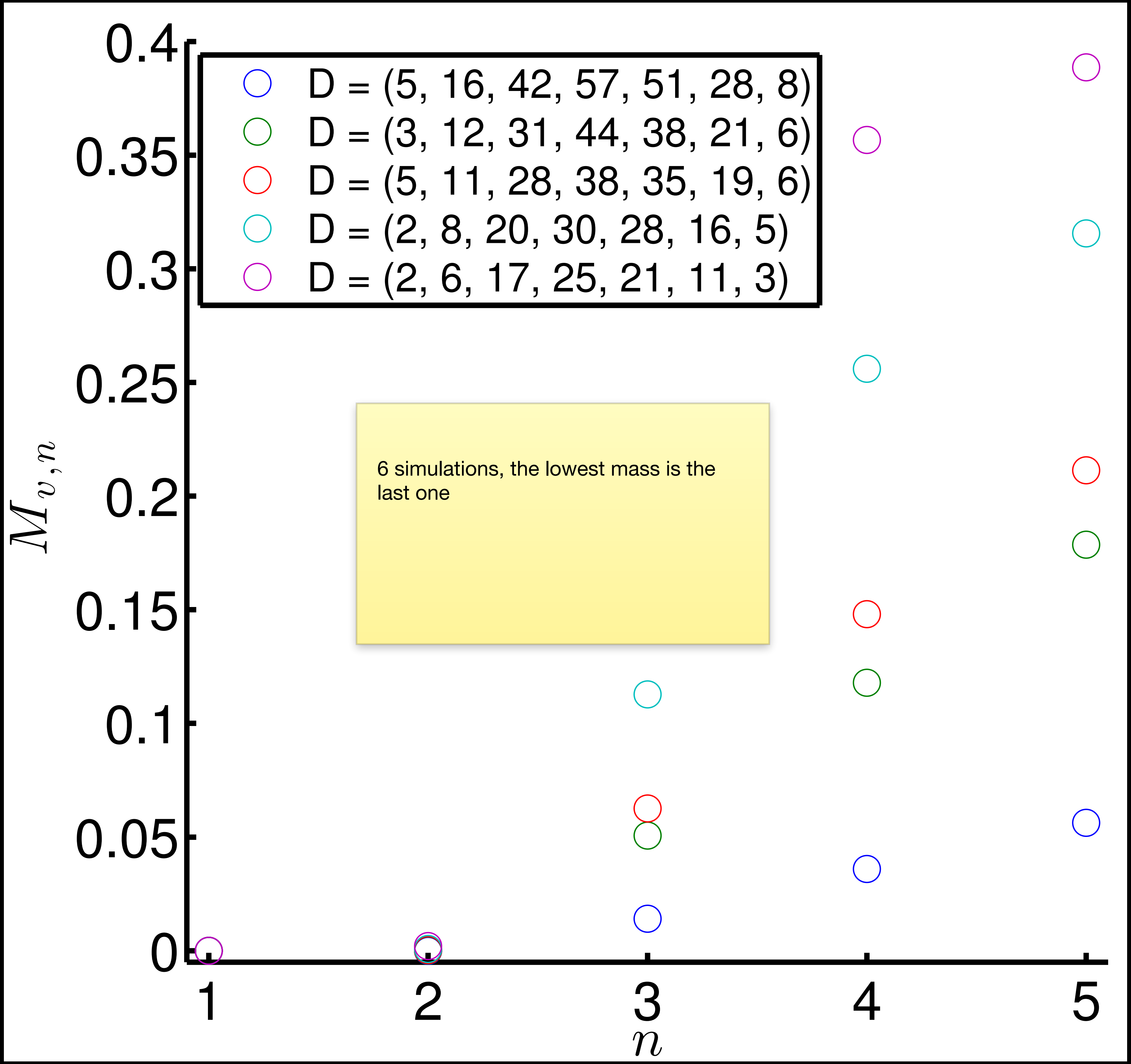
$$+\gamma^{m+2} e^{ik(m+2)a}$$



$$+\ldots$$

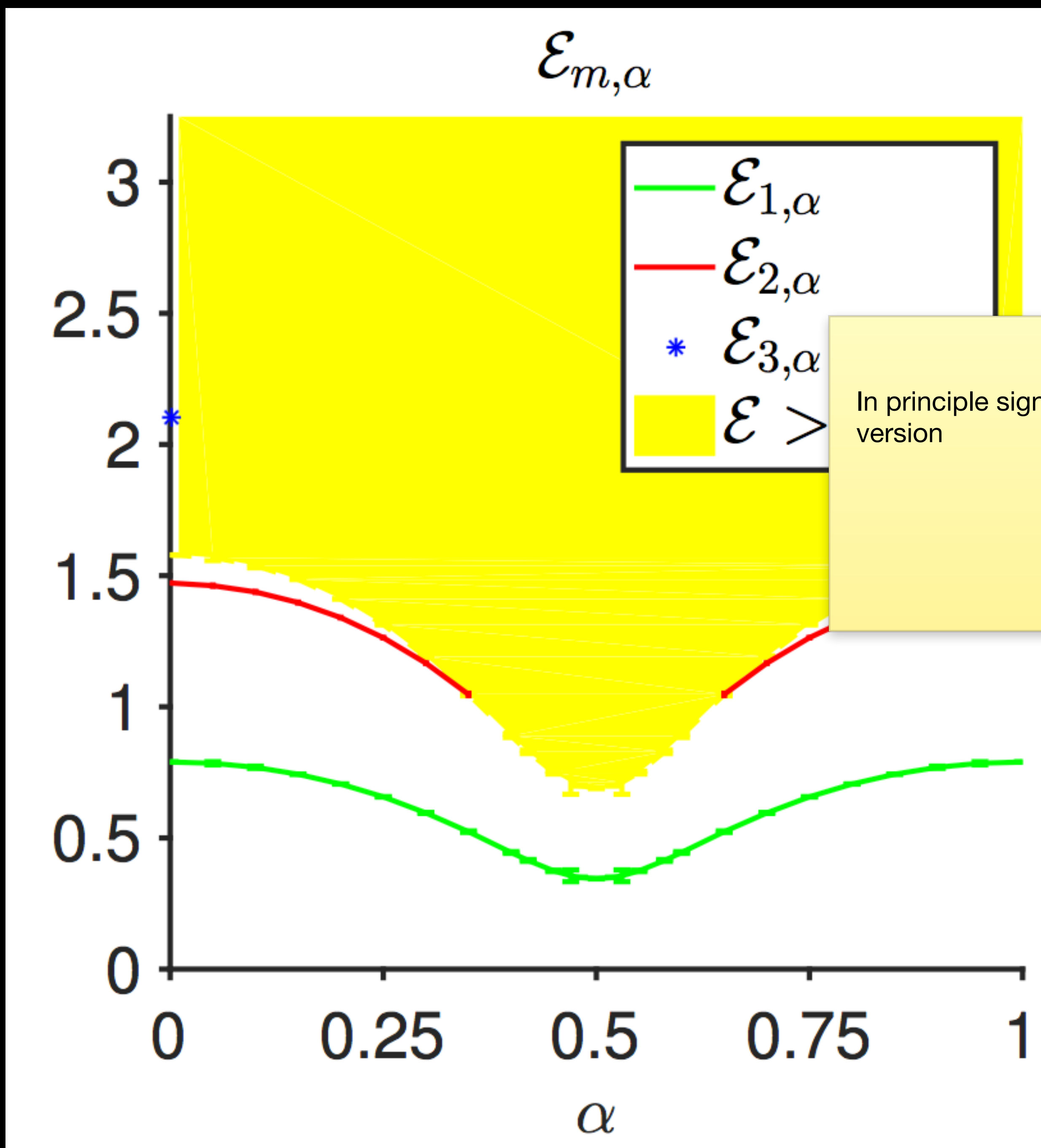
$$CT|\Psi(k,\gamma)> = \gamma e^{ika}|\Psi(k,\gamma)> \qquad (\gamma = \pm 1)$$

One-particle excitations: bond-dimension scaling



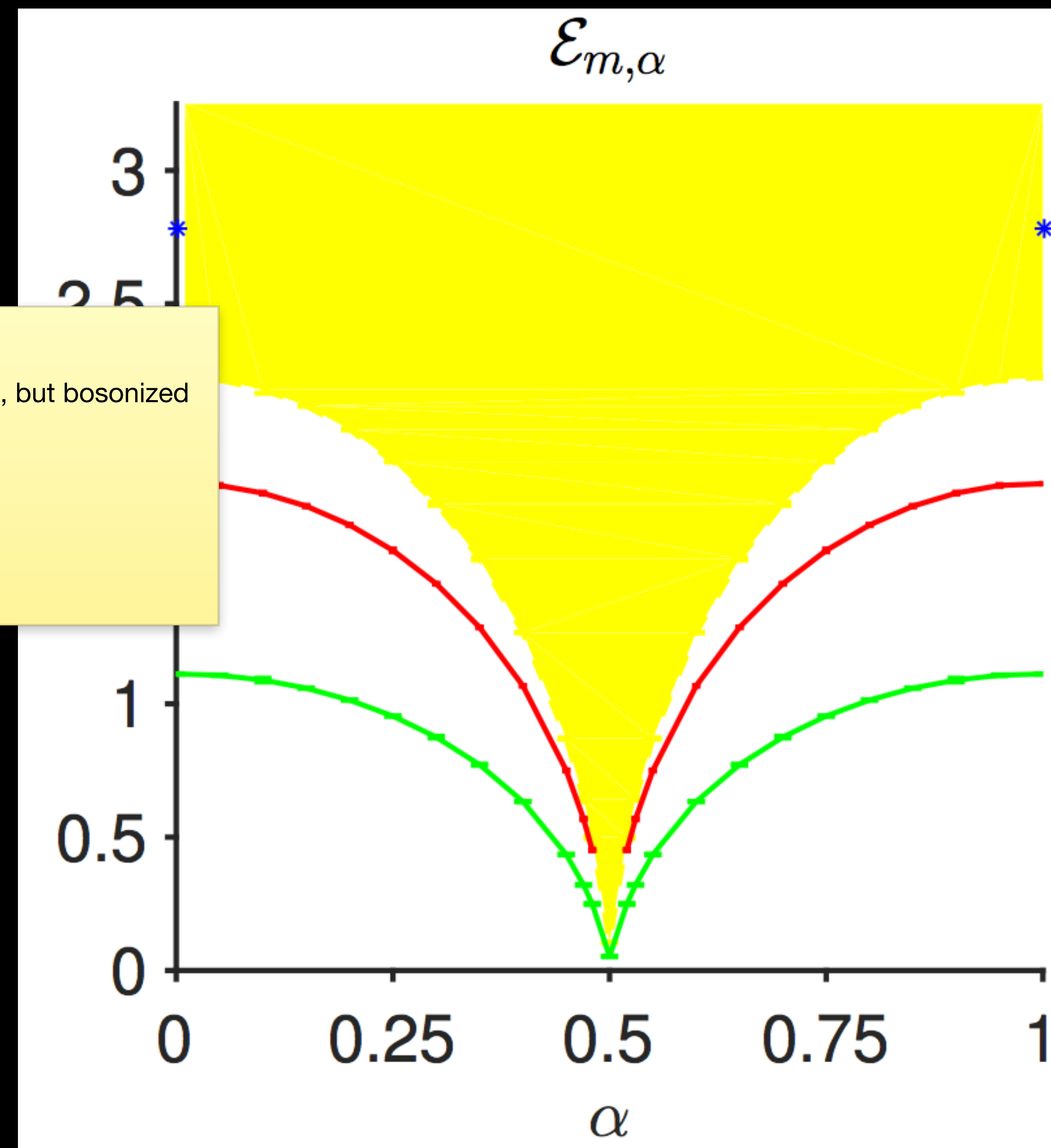
One-particle excitations in a background electric field.

$$L \to L + \alpha$$



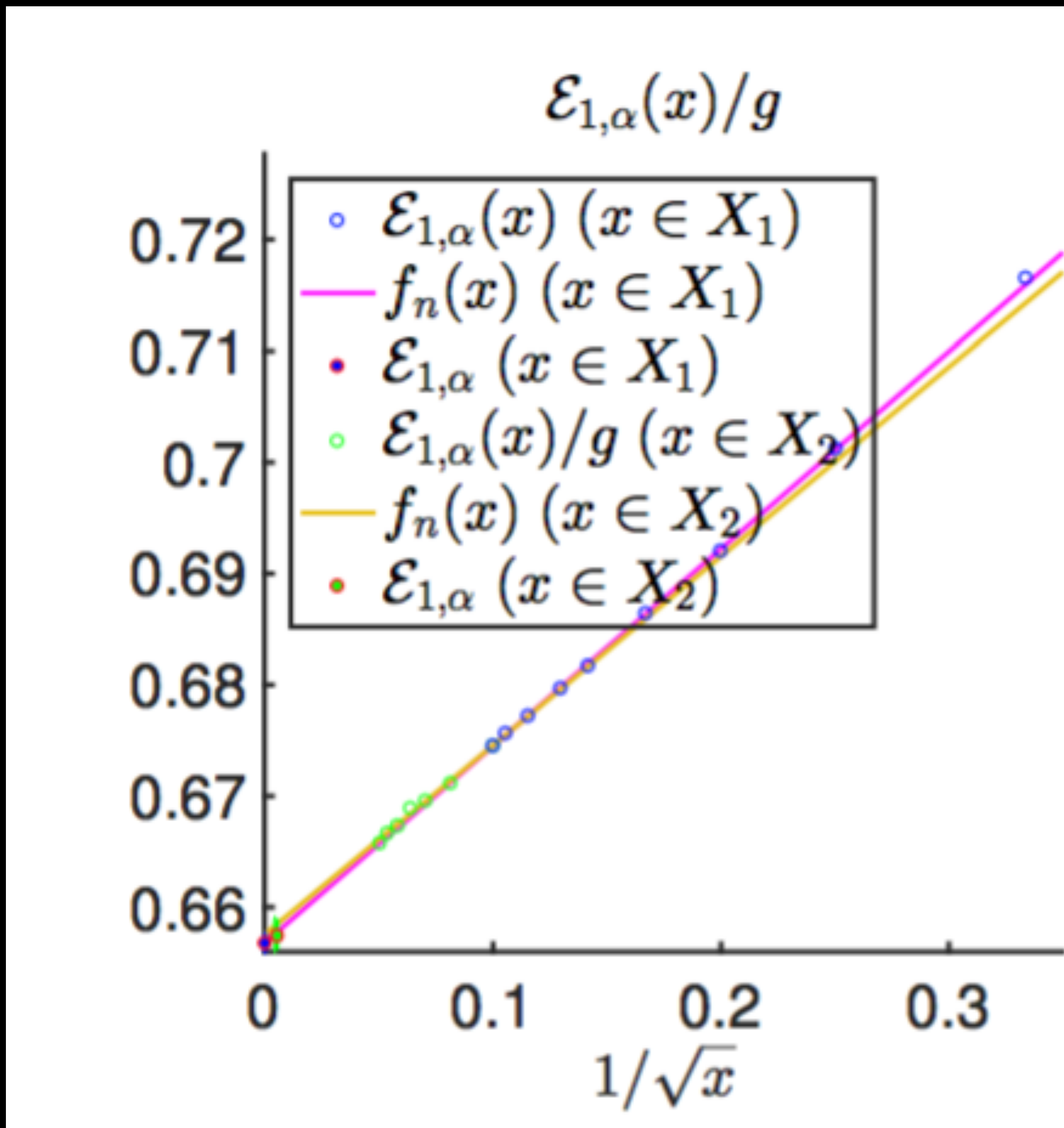
(a) $m/g = 0.125.$

In principle sign problem, but bosonized version



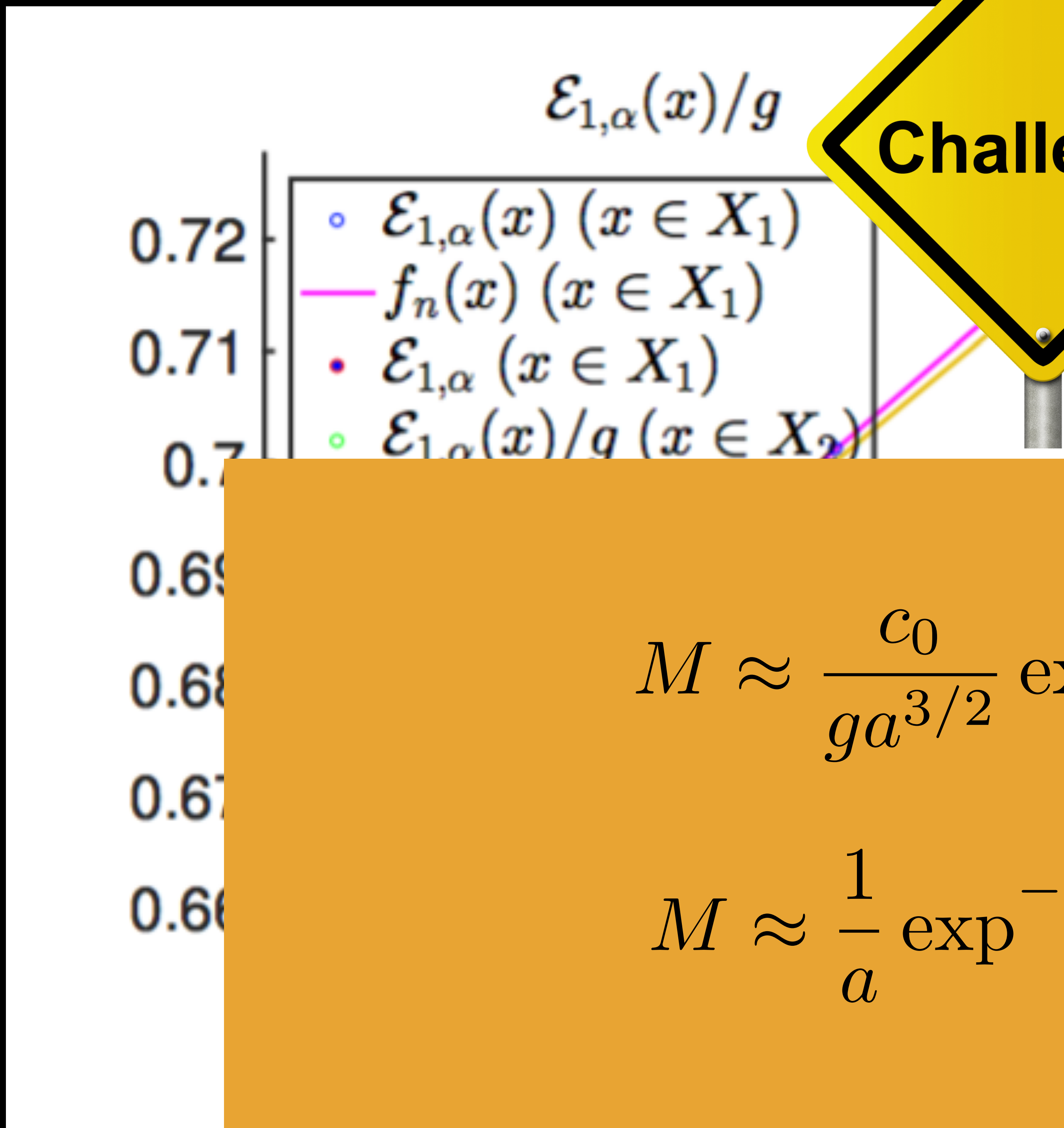
(c) $m/g = 0.3.$

One-particle excitations: continuum extrapolation



$$\mathcal{E}_1(a) = \mathcal{E}_1(0) + a\mathcal{E}_1'(0) + \ldots$$

One-particle excitations: continuum extrapolation



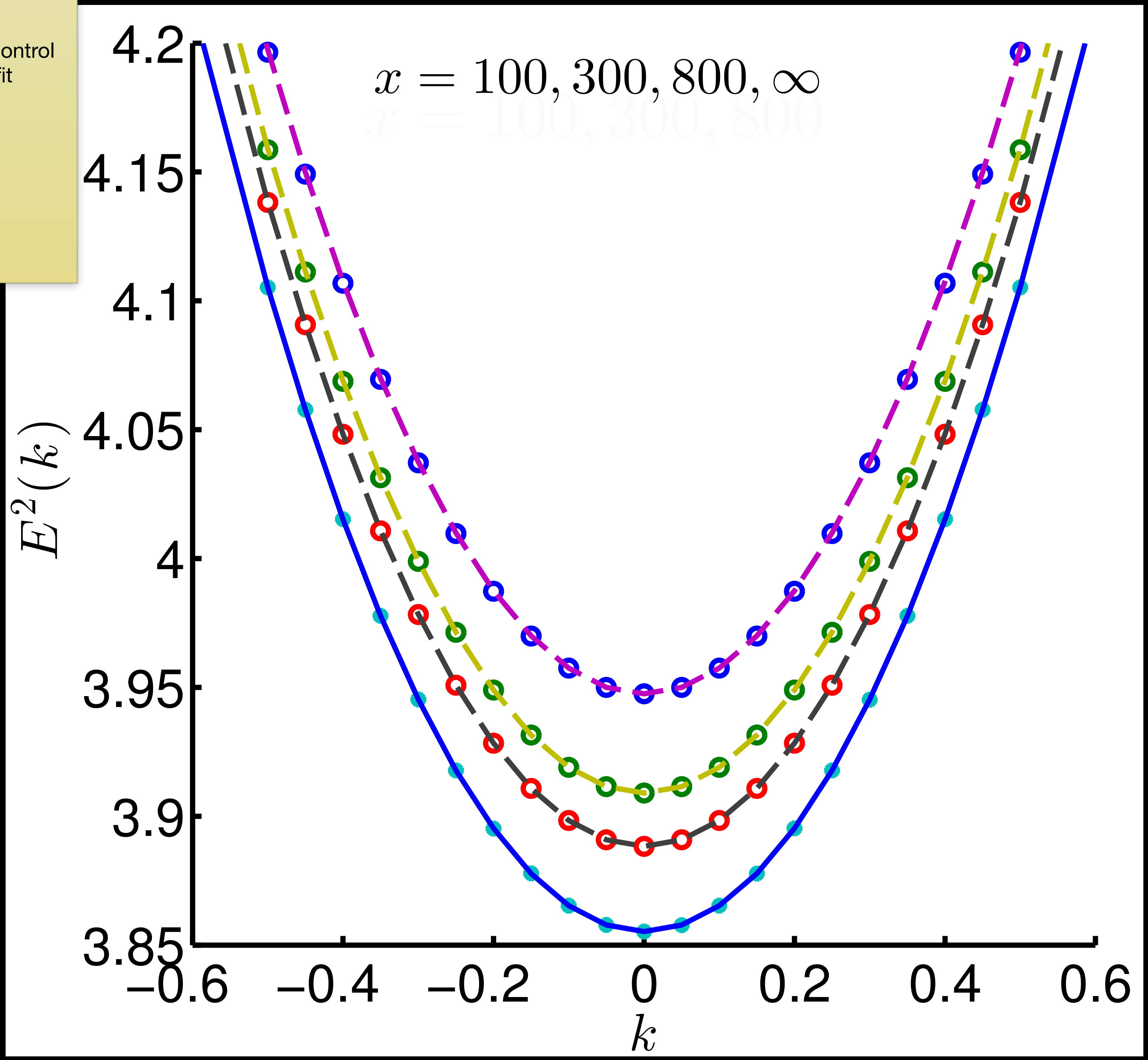
$$M \approx \frac{c_0}{g a^{3/2}} \exp^{-\frac{c_1}{g^2 a}}$$ U(1) d=2+1

$$M \approx \frac{1}{a} \exp^{-\frac{1}{\beta_0 g^2}}$$ QCD d=3+1

One-particle excitations: Lorentz-invariance

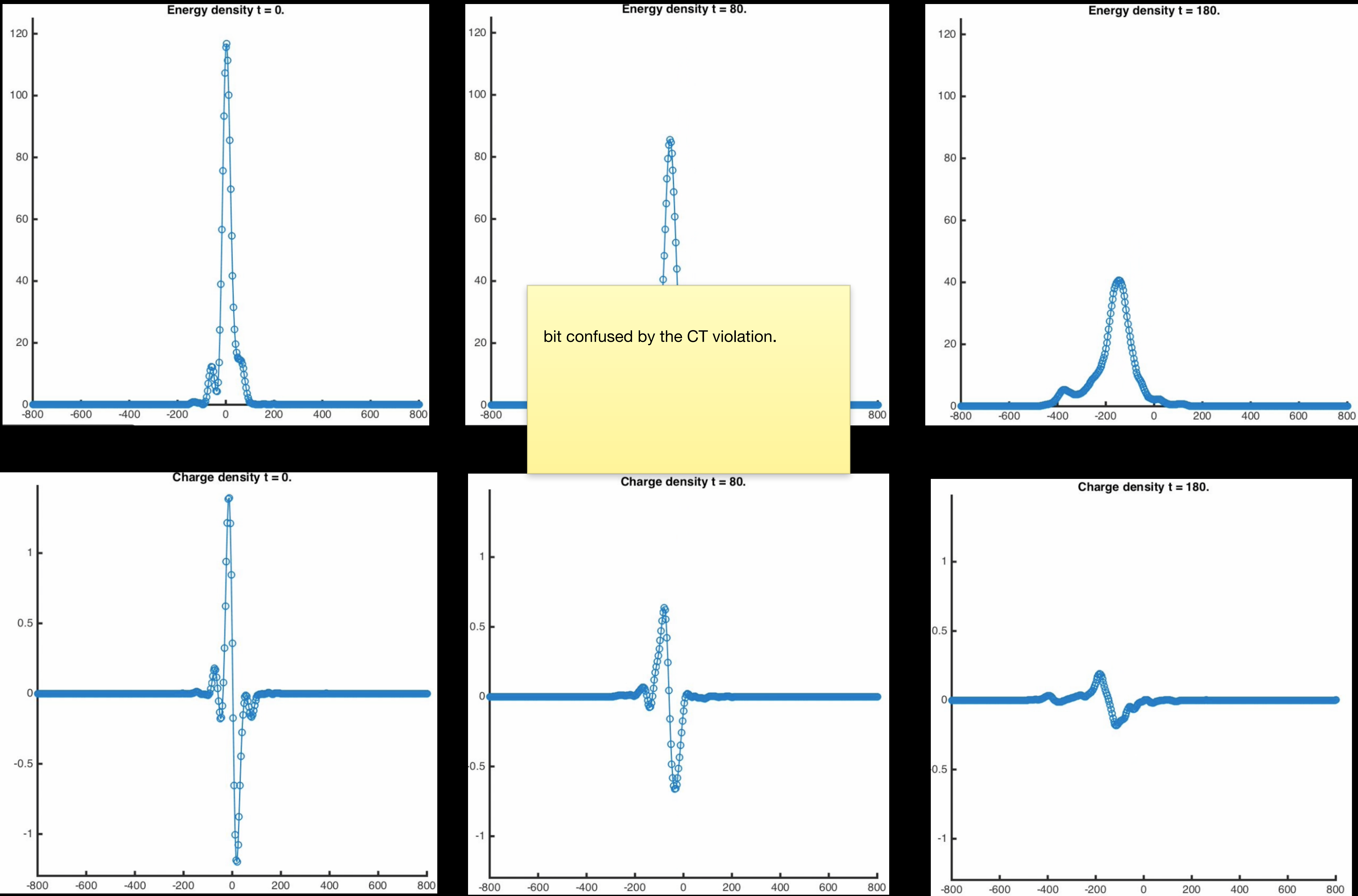
polynomial in a third order fit through largest five points. Control with quartic and polynomial fit through six point

CHALLENGE?



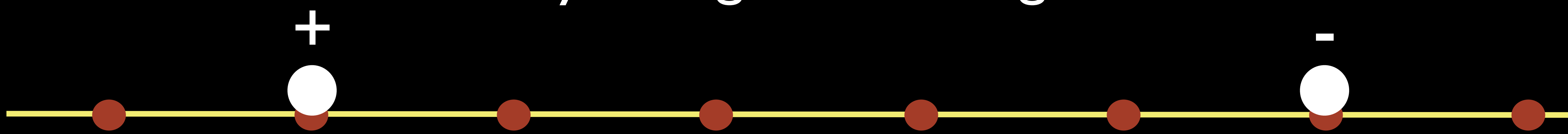
$$x = 1/(g^2a^2) = 100, 200, 300, 400, 600, 800$$

One-particle excitations: some snapshots of a meson wave-packet

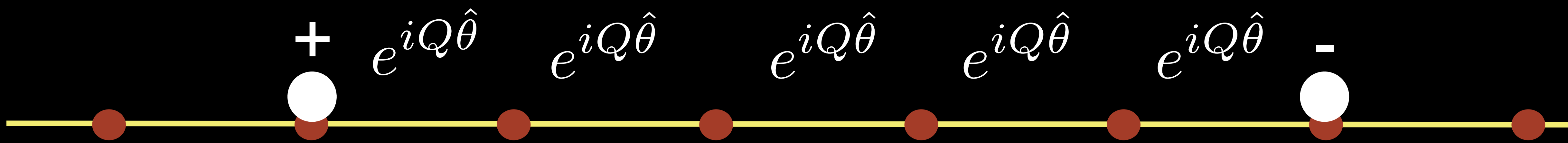


bit confused by the CT violation.

Heavy background charges



$$G_n = L(n) - L(n-1) - \frac{1}{2}(\sigma_z(n) + (-1)^n) \; - \pm Q$$



$$H = \frac{g}{2\sqrt{x}} \left(\sum_n (L(n) - Q)^2 + \frac{\sqrt{x}}{g} m \sum_n (-1)^n \sigma_z(n) \right.$$

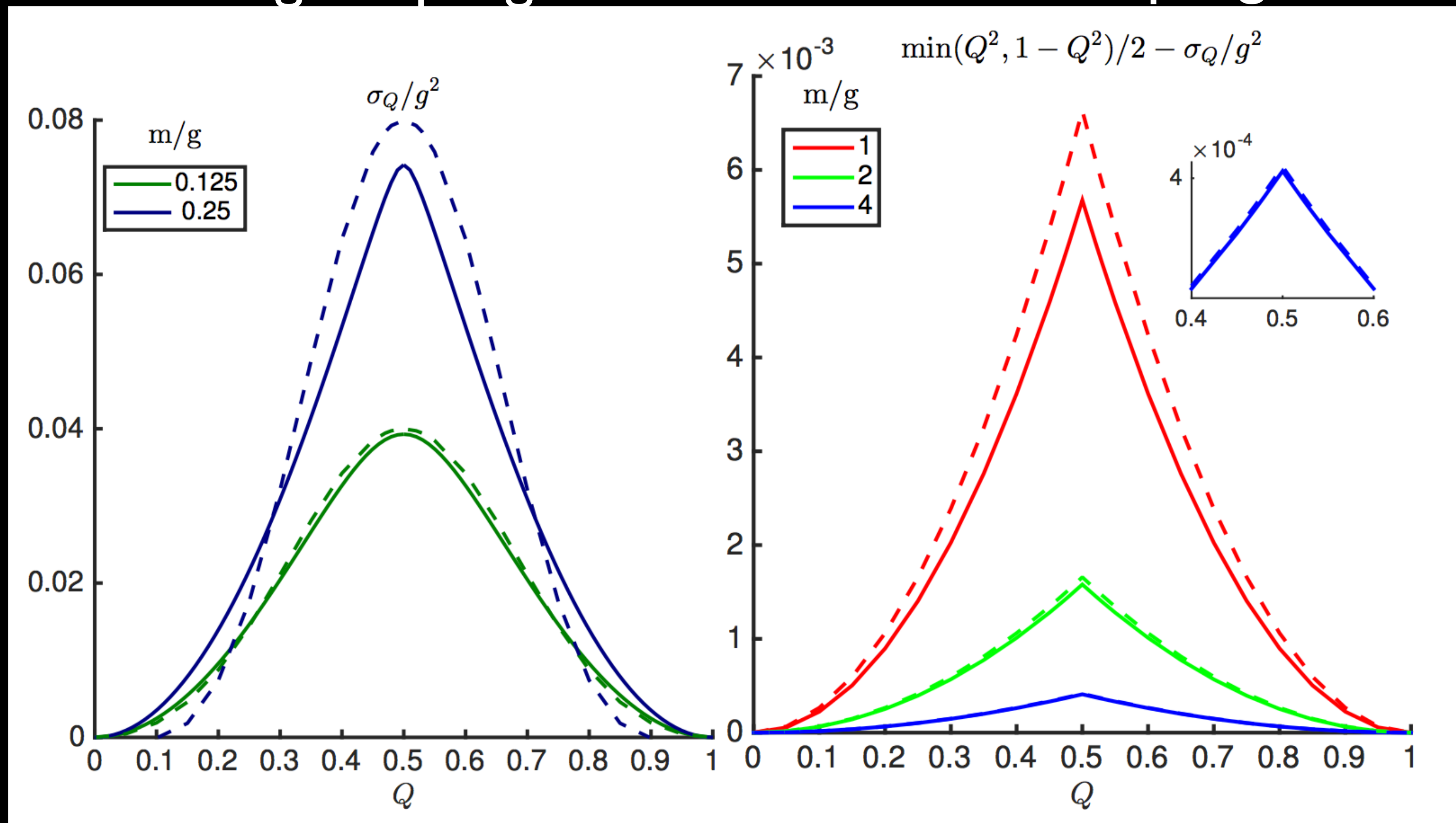
$$\left. + x \sum_n \sigma^+(n) e^{i\theta(n)} \sigma^-(n+1) + h.c. \right)$$

E=L-Q

fractional charges, integer valued charges

Heavy background charges: uniform case

strong coupling | weak coupling

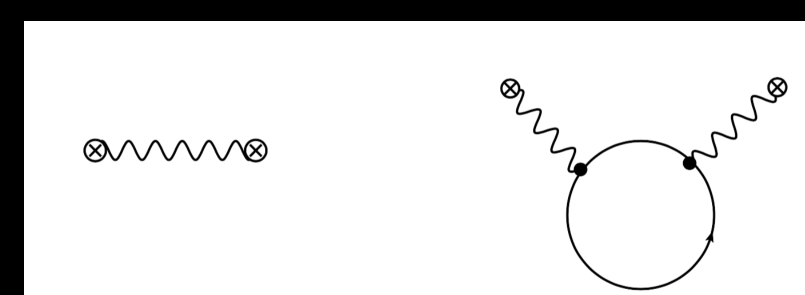


$$\frac{\sigma_Q}{g^2} \approx \frac{m}{g}\Sigma(1-\cos(2\pi Q)) + \frac{m^2\Sigma^2 E_+\pi}{4g^2}(1-\cos(4\pi Q))$$

$$\frac{\sigma_Q}{g^2} \approx \frac{Q^2}{2}\left(1 - \frac{g^2}{m^2}\frac{1}{6\pi}\right)$$

weak coupling computation:

$$S_{eff} = \int d^2x\ \mathcal{L}_{eff} = \int d^2x\ \ C_0(\frac{g}{m})\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + C_1(\frac{g}{m})\frac{(\bar{F}_{\mu\nu}\bar{F}^{\mu\nu})^2}{m^2} + \ldots$$



Heavy background charges: uniform case

UV finite von Neumann entropy

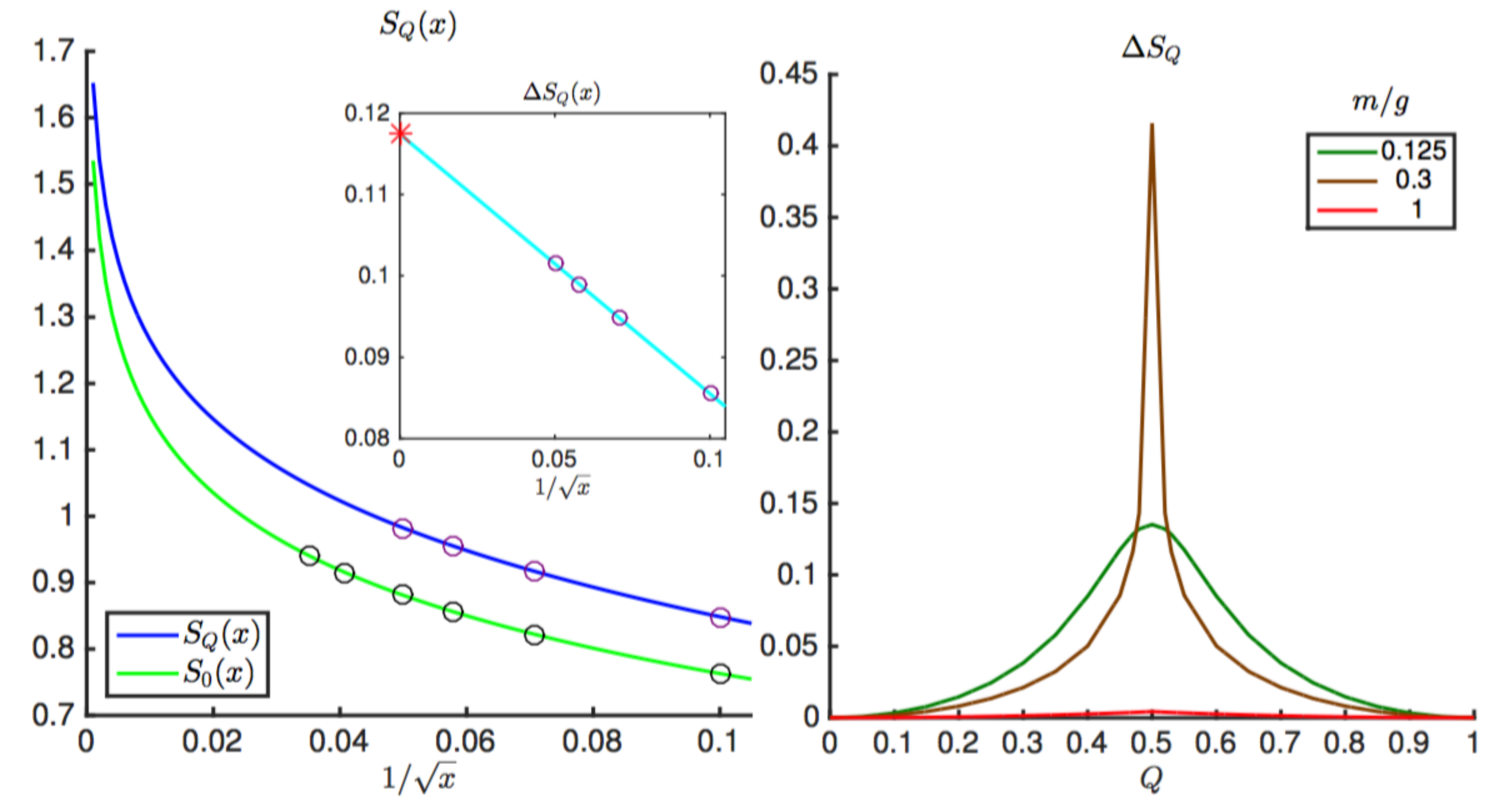


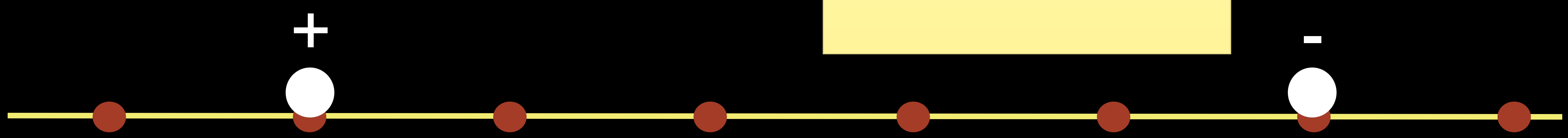
FIG. 2: (a): m/g = 0.25, Q = 0.45. Fit of the form $(-1/6)\log(1/\sqrt{x}) + A + C/\sqrt{x}$ to $S_Q(x)$ and $S_0(x)$. Inset: linear fit to $\Delta S_Q(x)$. (b): ΔS_Q for different values of m/g.

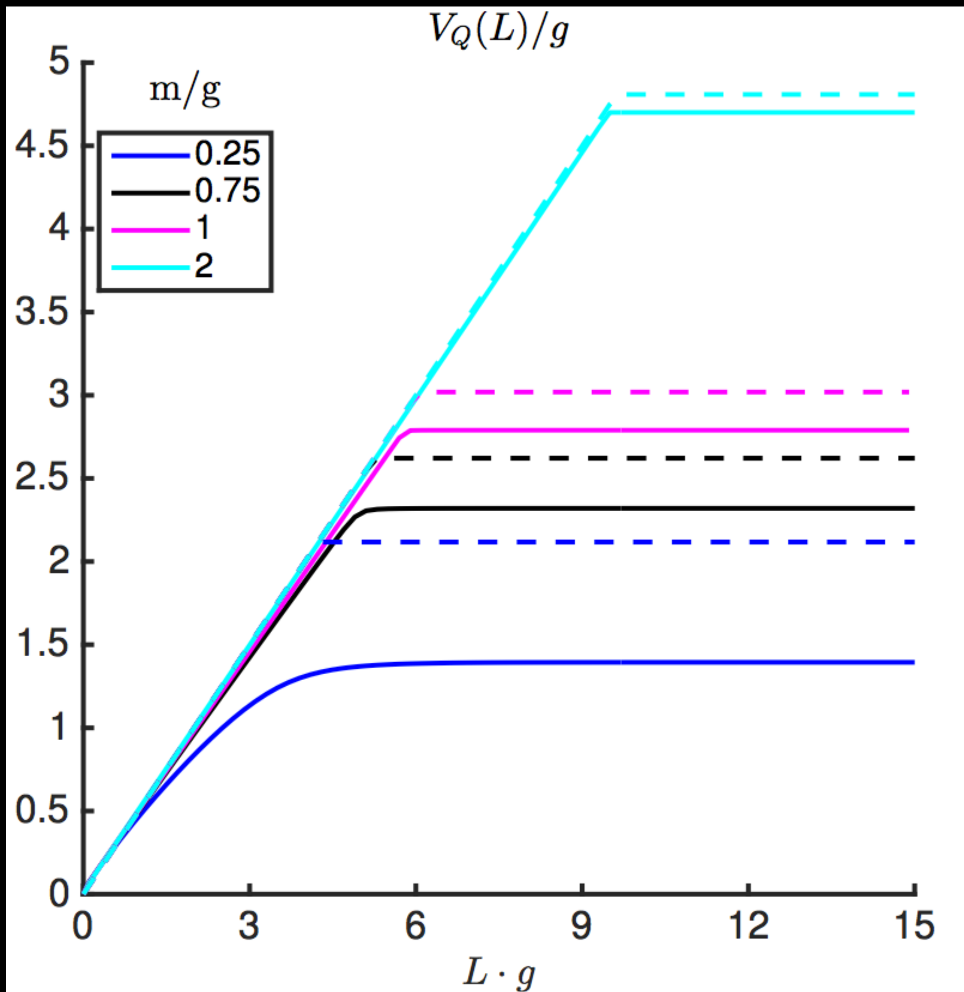
Probably something you can pro
perturbatively (since your theory
superrenormalizable)

Does entropy make sense as a p
quantity?

Heavy background charges: string breaking, the Q=1 case

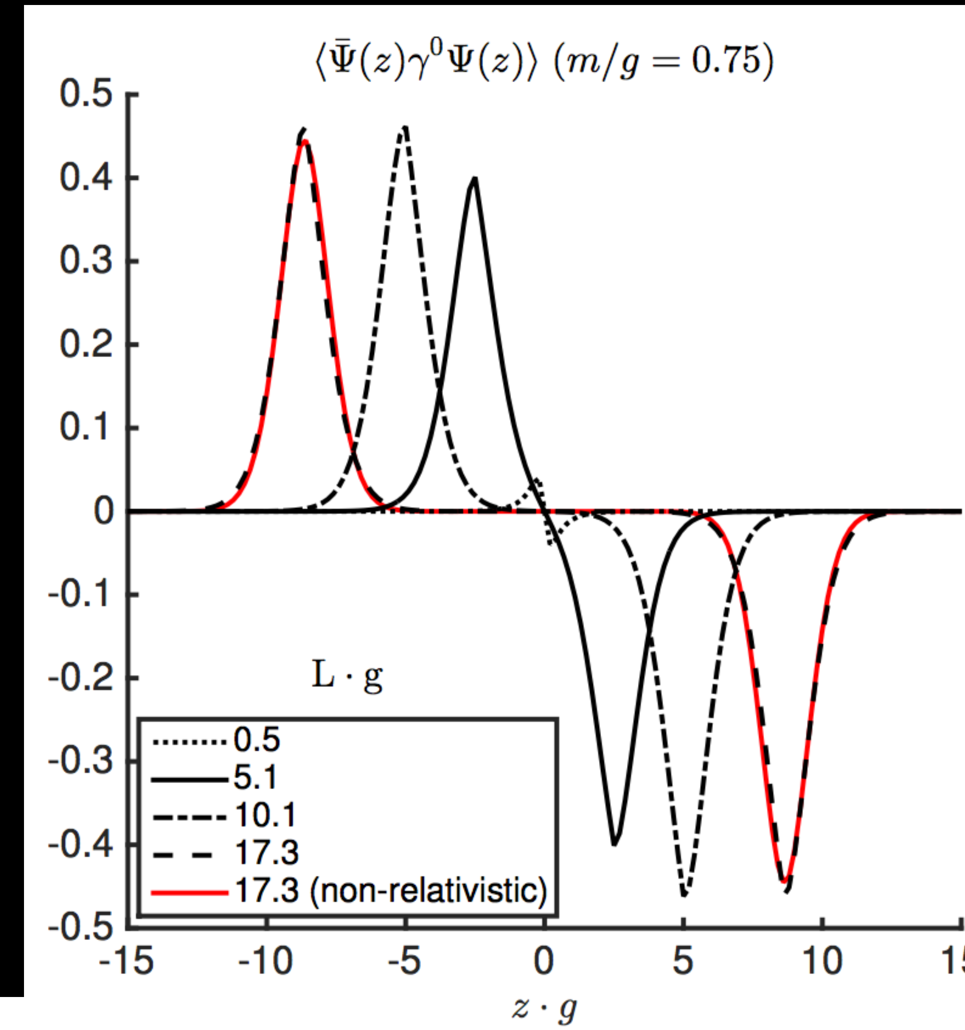
non-relativistic picture: electric field string, constant string tension, energy that grows linearly with the distance. At some point the energy is so large that it becomes energetically favorable to produce a light fermion pair out of the vacuum, which will screen the electric field

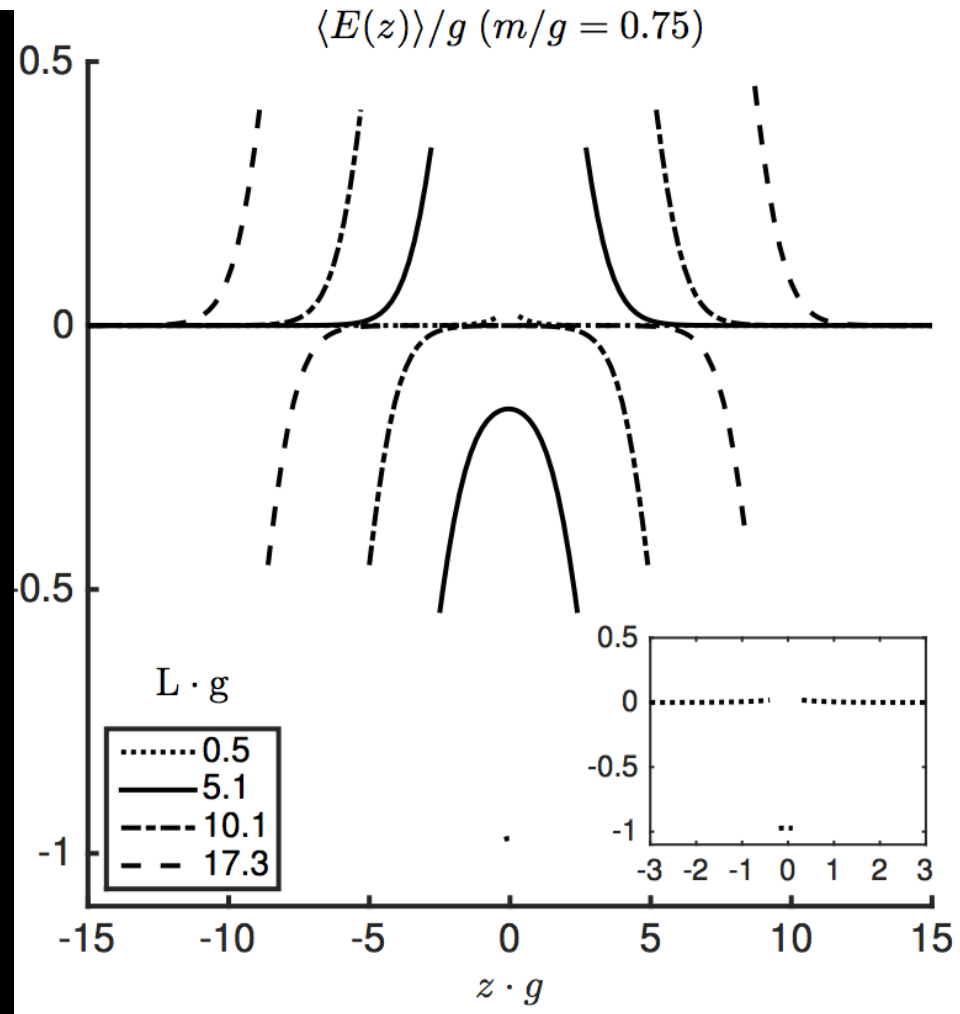




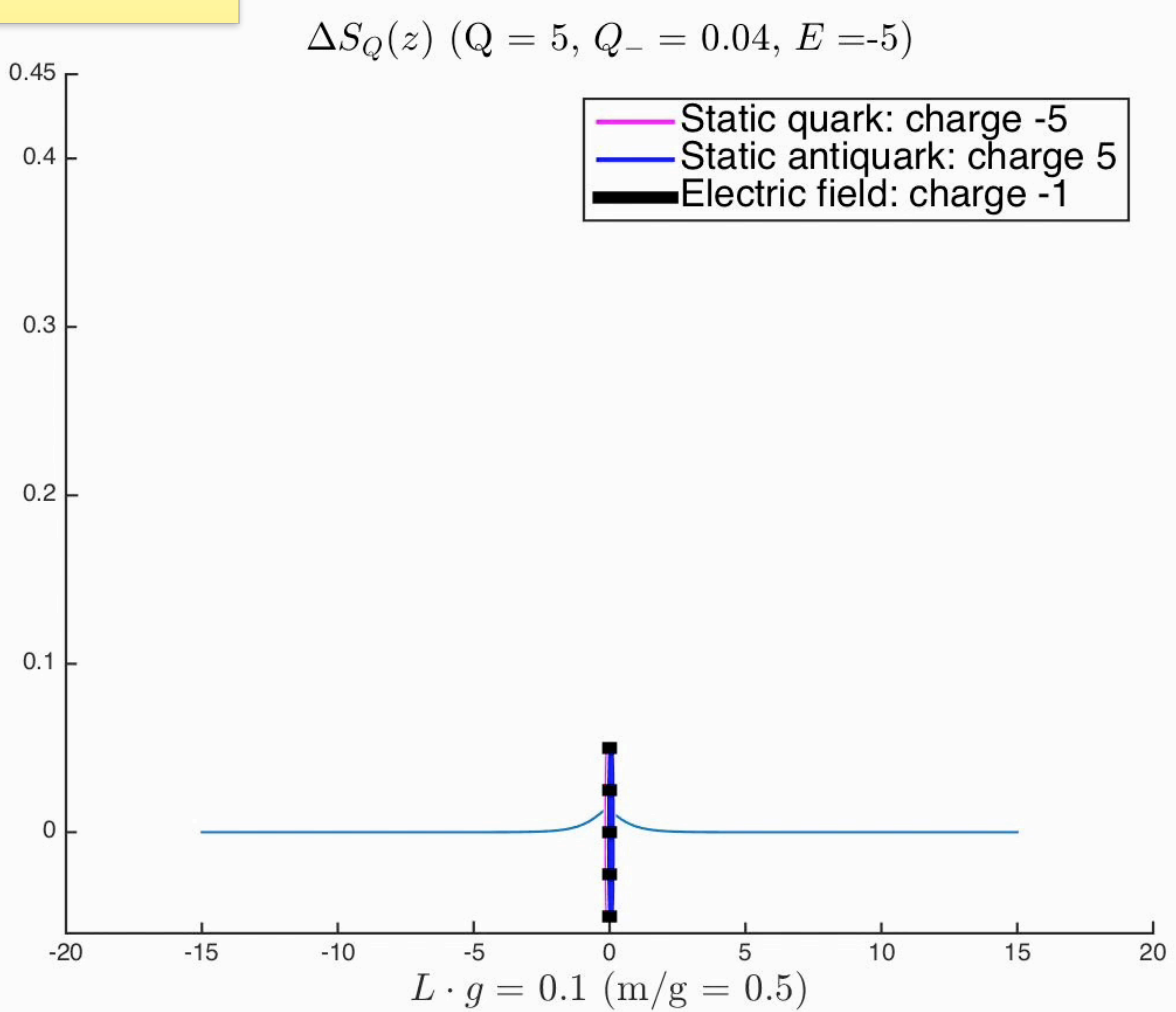
$$\mathcal{E}_{string} = \frac{Lg^2}{2}$$

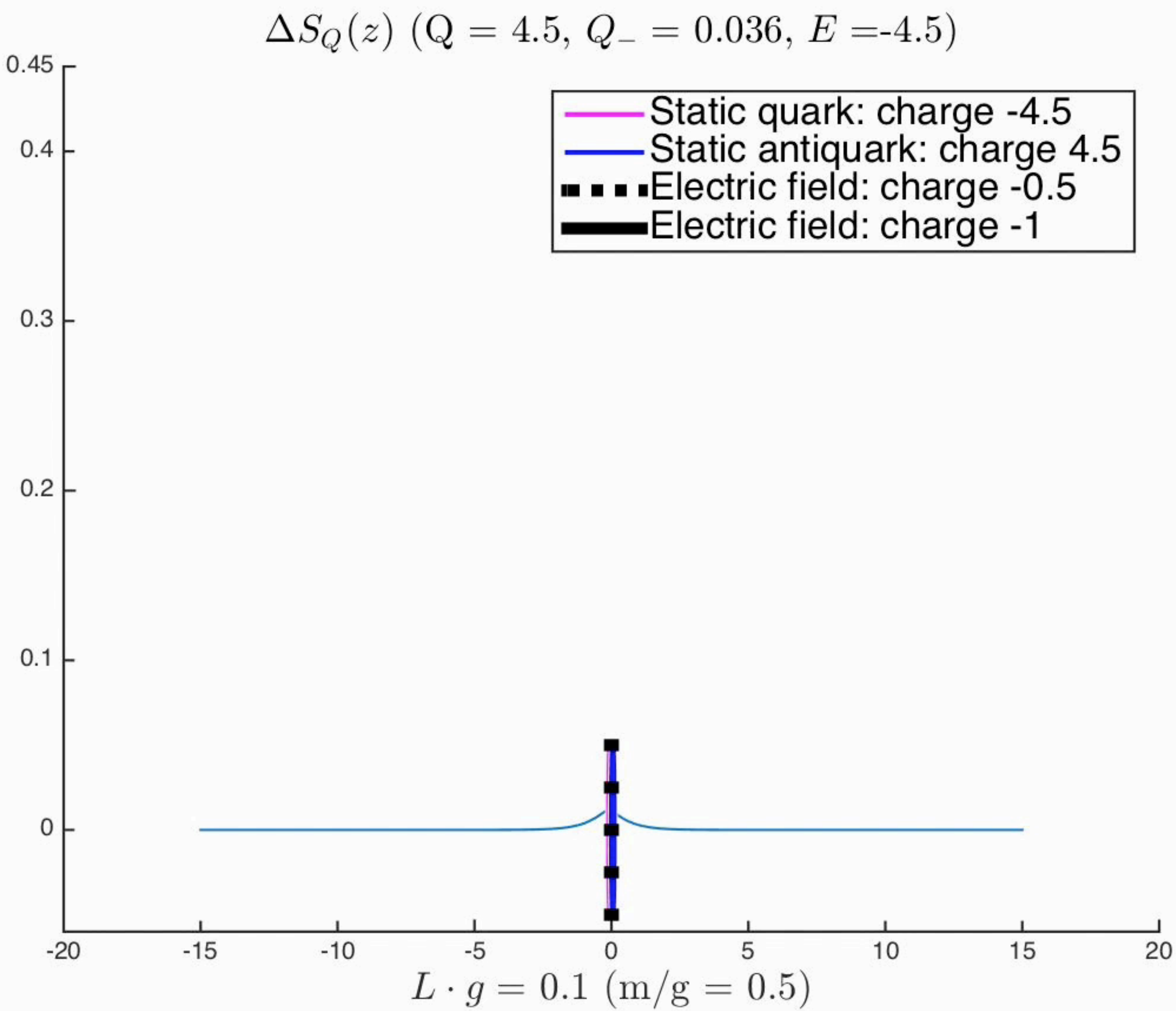
$$\mathcal{E}_{2meson} = 2m + 1.0188\frac{g^{4/3}}{m^{1/3}}$$





non-relativistic level crossing, correction on 2m, for smaller masses this transitions is more gradual, m=0.75 , between Lg=4 and 6 . Electric field profile and charge cloud. Airy function.


$\Delta S_Q(z)$ (Q = 5, Q_- = 0.04, E =-5)
Static quark: charge -5
Static antiquark: charge 5
Electric field: charge -1
0.45
0.4
0.3
0.2
0.1
0
-20
-15
-10
-5
0
5
10
15
20
$L \cdot g = 0.1$ (m/g = 0.5)


$\Delta S_Q(z)$ (Q = 4.5, Q_- = 0.036, E =-4.5)
Static quark: charge -4.5
Static antiquark: charge 4.5
Electric field: charge -0.5
Electric field: charge -1
0.45
0.4
0.3
0.2
0.1
0
-20
-15
-10
-5
0
5
10
15
20
$L \cdot g = 0.1$ (m/g = 0.5)

Real-time sin

(B. Buyens, J. Haegeman, K.V.A., H. Versc
Phys. Rev. Lett.: 113 (2014) 091601;   c

E^2/m

Energy of dipole 2m

Delta energy=2Eg/m>2m

E>m^2/g

classical, no backreaction

- Non-equilibrium evolution of a pure quantum state.
 Thermalization or not:-under which conditions do we have local relaxation?
 -which systems relax to local Gibbs states?

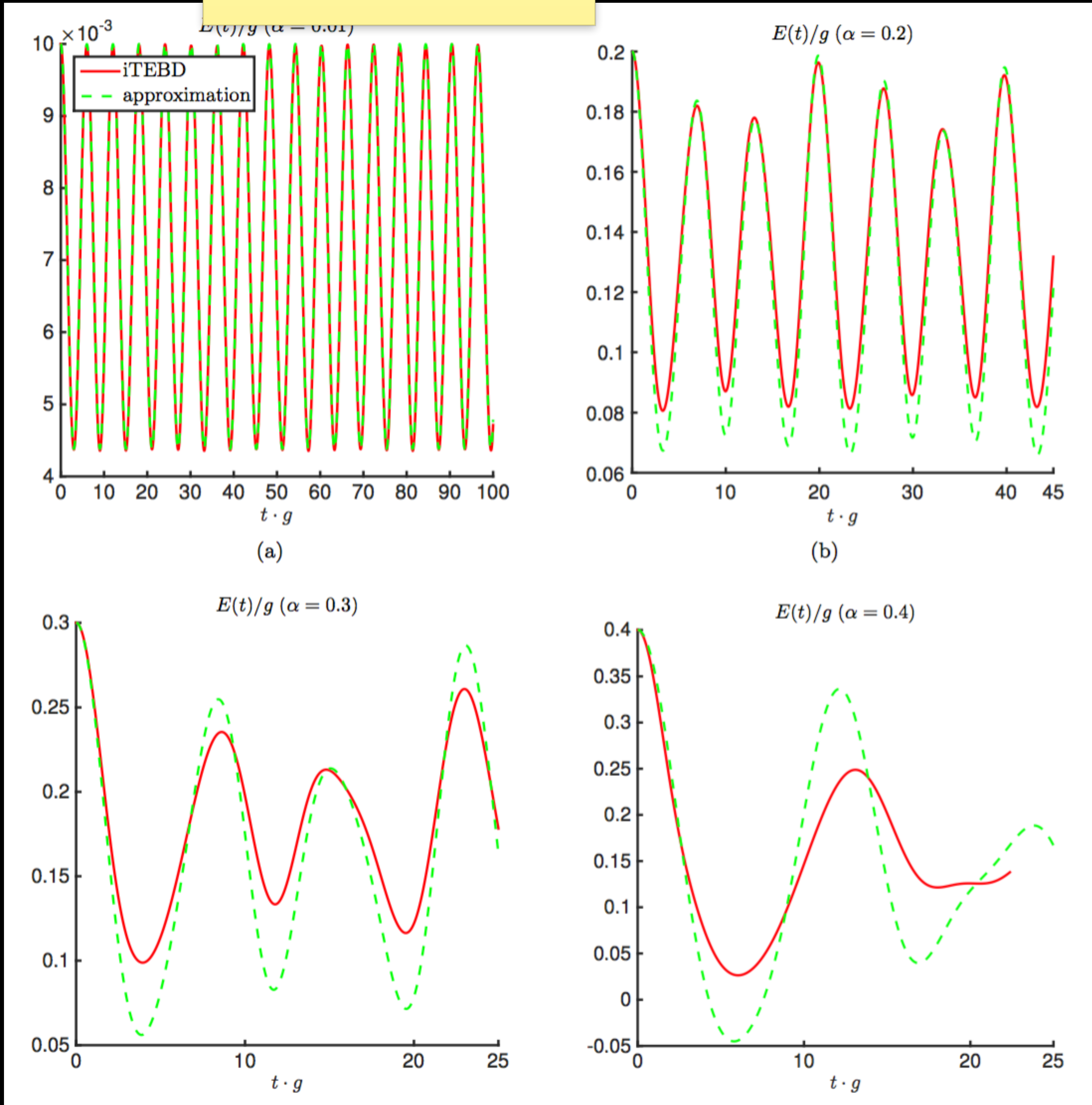
- We performed the real-time simulation of the Schwinger Mechanism: a classical background electric field $E > m^2/g$ induces (fermionic) particle pair production.

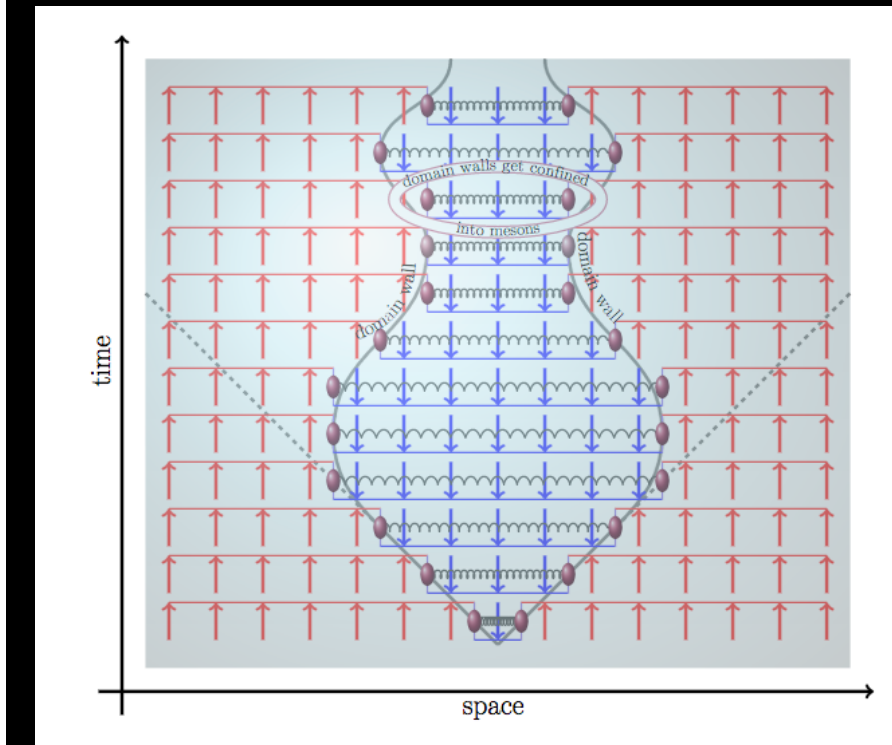
- This process was studied before in the semi-classical approximation *(Kluger et al, Phys. Rev. D, 45 (1992) 4659; Hebenstreit et al, Phys. Rev. D, 87, 105006 (2013))*

- We use iTEBD *Vidal, PRL 98 (2007), 070201* to perform the real-time evolution at the full quantum level.

quench: $L_n \to L_n + \alpha$

weak fields:





M. Kormos, M. Collura, G. Takacs, P. Calabrese Nature Physics 13 (2017)

α	$\rho_1\xi$	$\rho_2\xi$
0.01	2.6×10^{-5}	9.4×10^{-9}
0.1	2.7×10^{-3}	8.9×10^{-5}
0.2	1.2×10^{-2}	1.4×10^{-3}
0.3	3.5×10^{-2}	8.0×10^{-3}
0.4	8.9×10^{-2}	2.9×10^{-2}

arXiv:1612.00739

understanding weak field behavior from 1-particle states

1.define creation/annihilation operators for the 1- particle asymptotic states of H_α

$$a^+(k,i)|\Psi_0>_\alpha \equiv |k,i>_\alpha \qquad a(k,i)|\Psi_0>_\alpha \equiv 0$$

2.assume:

$$H_{\alpha=0} = c_0 + c_1^i a^+(0,i) + c_1^{i\,*} a(0,i) + c_2^{ij} a^+(0,i) a(0,j) + \ldots$$

$$a(k,i)|\Psi_0>_{\alpha=0} = d_i \delta(k)|k,i>_\alpha$$

3. this allows the self-consistent calculation of: c_0, c_1^i, c_2^{ij} and d_i

4. which in turn leads to a prediction for the time-evolution of an arbitrary observable

$$O = f_0 + f_1^i a^+(0,i) + f_1^{i\,*} a(0,i) + f_2^{ij} a^+(0,i) a(0,j) + \ldots$$

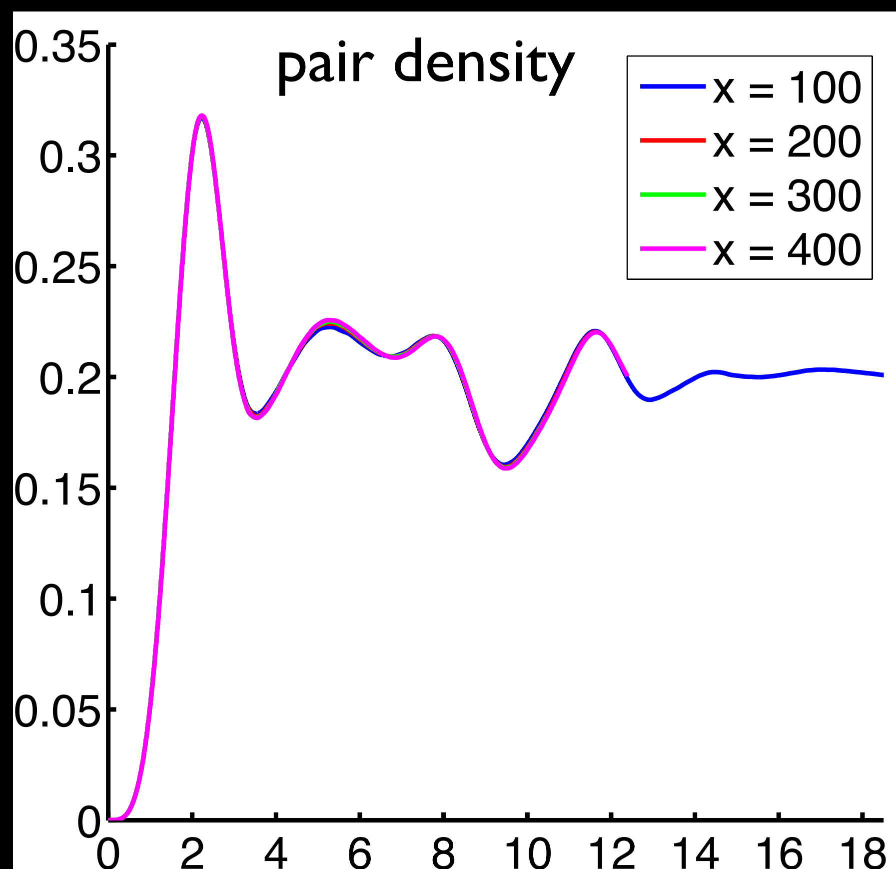
$${}_{\alpha=0}< \Psi_0 | e^{iH_\alpha t} O e^{-iH_\alpha t} |\Psi_0>_{\alpha=0}$$

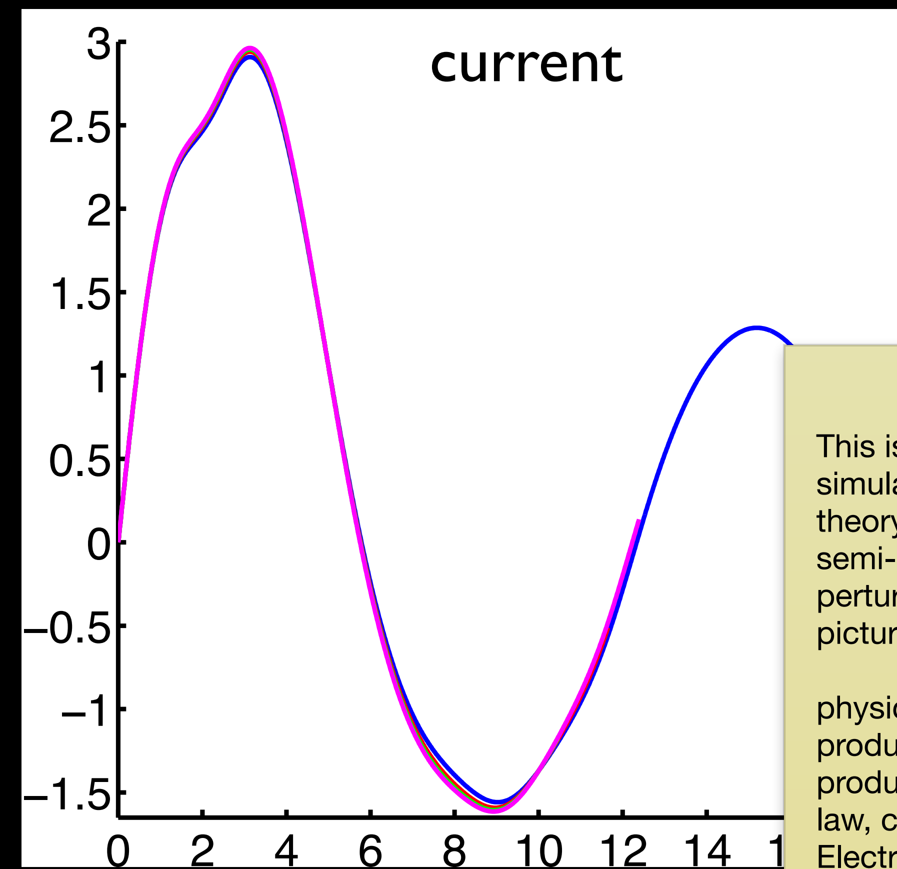
$$= f_0 + f_1^i d_i^* e^{iE_i t} + f_1^{i\,*} d_i e^{-iE_i t} + f_2^{ij} d_i^* d_j e^{i(E_i$$

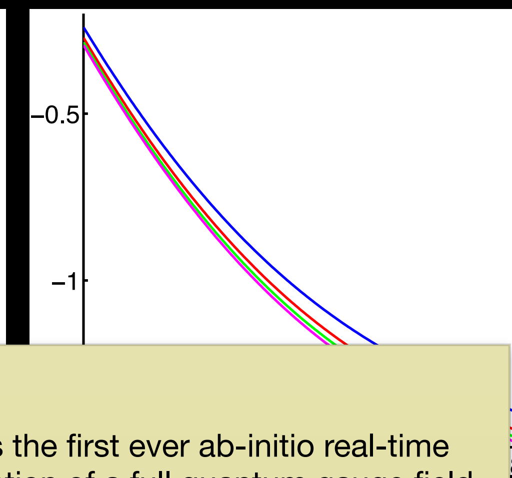
in-states, out-states, only defined at zero density. We are working at non-zero density.

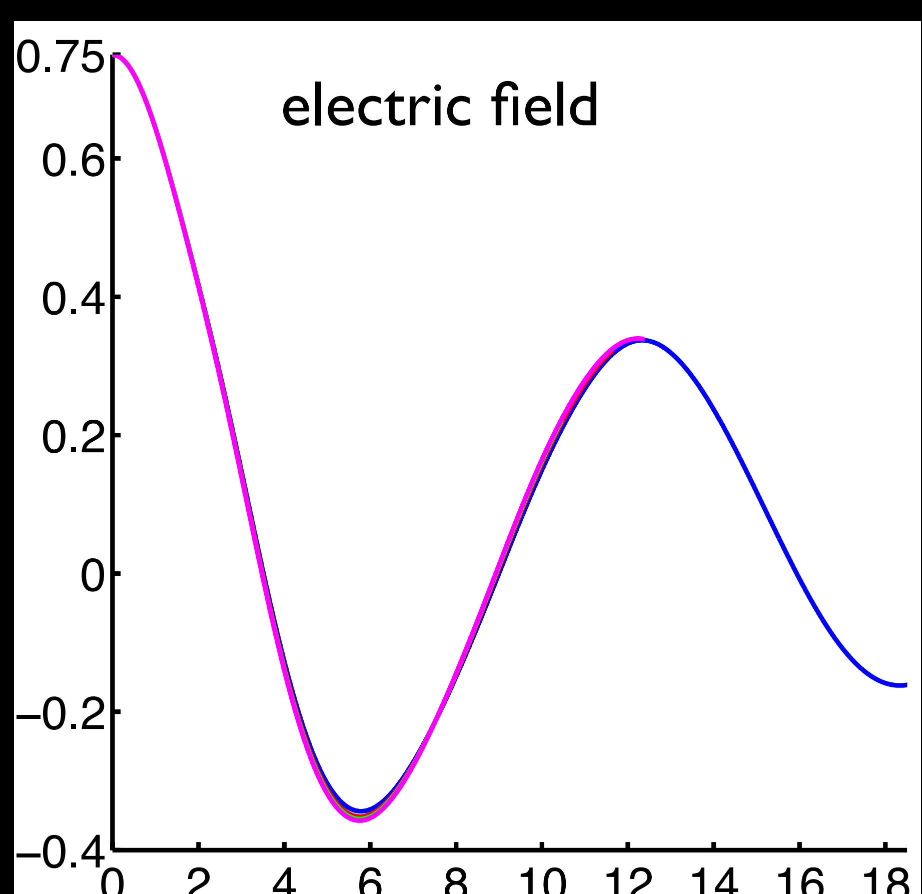
we're ignoring a^+a^+ terms in H, but this is part of the assumption

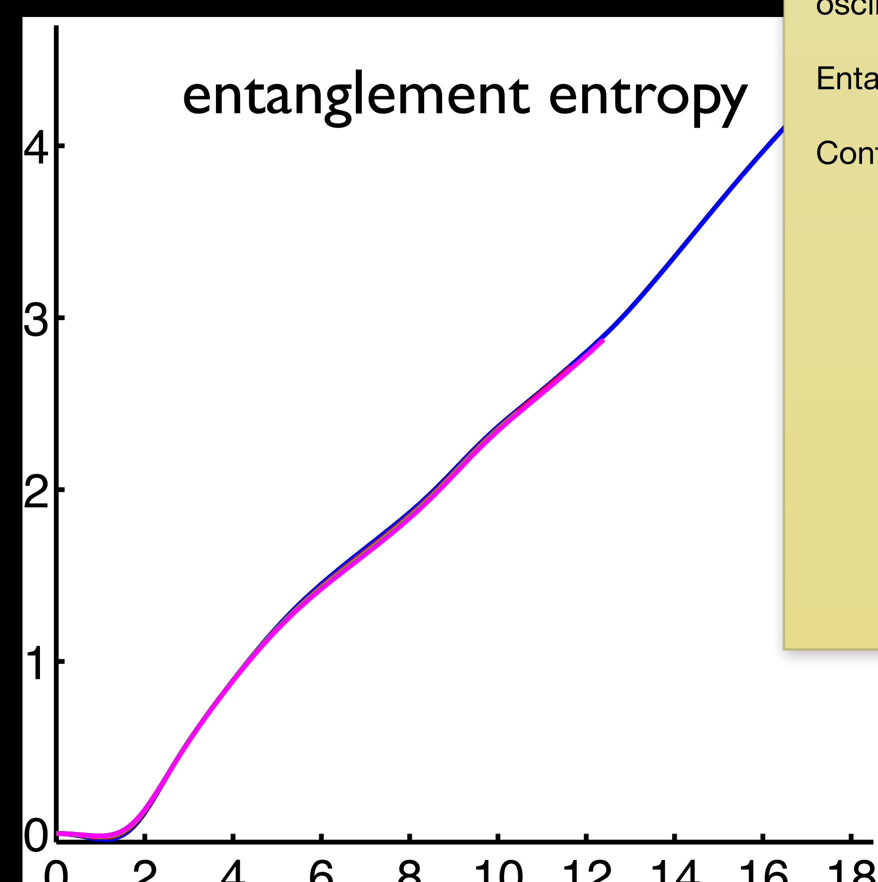
Real-time simulation Schwinger mechanism











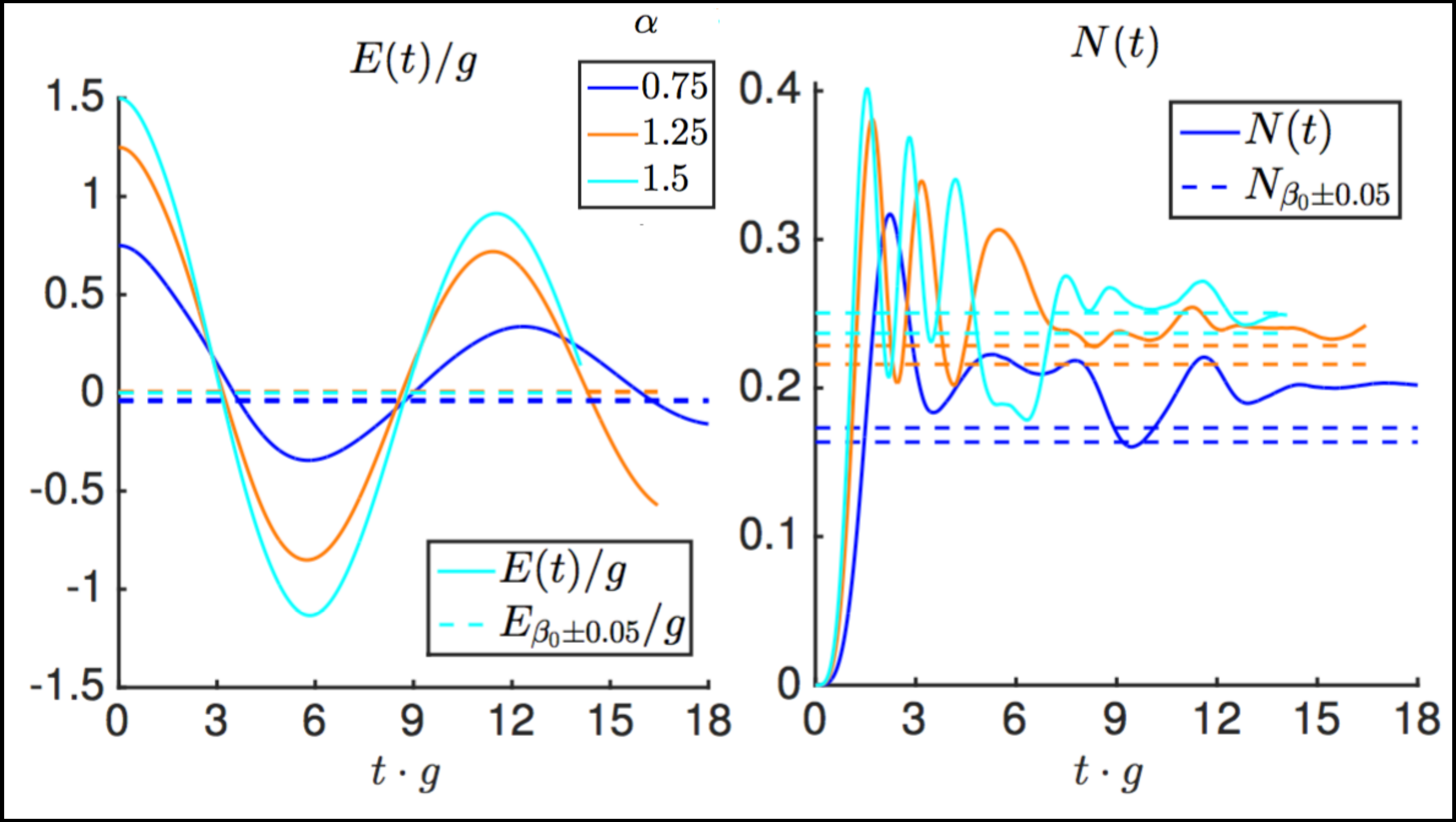
This is the first ever ab-initio real-time simulation of a full quantum gauge field theory. Long history, studied in different semi-classical approximations, perturbation theory, classical kinetic picture .

physical picture: initially particle pair production. Pairs in an electric field produce current. Through Ampere's law, current changes the electric field. Electric field goes down, particle pair production halts, and we get damped oscillations.

Entanglement is the problem.

Continuum limit.

Thermalization?



(B. Buyens et al, Phys. Rev. D 94, 085018 (2016))

Conclusions

- For d=1+1, MPS has produced almost anything you would dream of (at least regarding simulations of gauge (field) theories

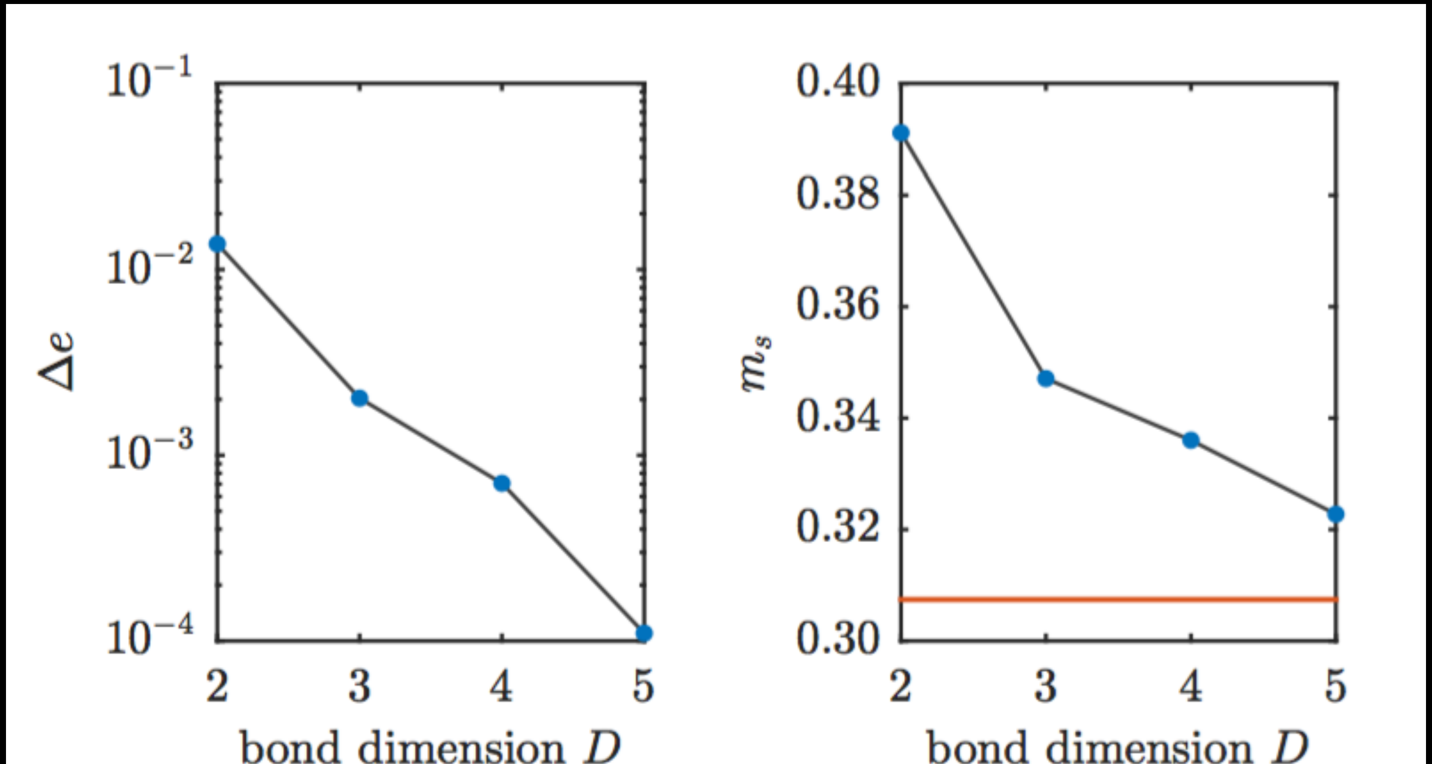
- For higher d:

1. parent-Hamiltonian philosophy

see.eg. Haegeman et al, Phys. Rev. X 5, 011024 (2015), Zohar et al, Annals of Physics (2015), pp. 385-439 and Ann. Phys 374, 84-137 (2016)

2. Variational approach for specific microscopic Hamiltonians.

(L. Vanderstraeten et al, 10.1103/PhysRevB. 94.155123)

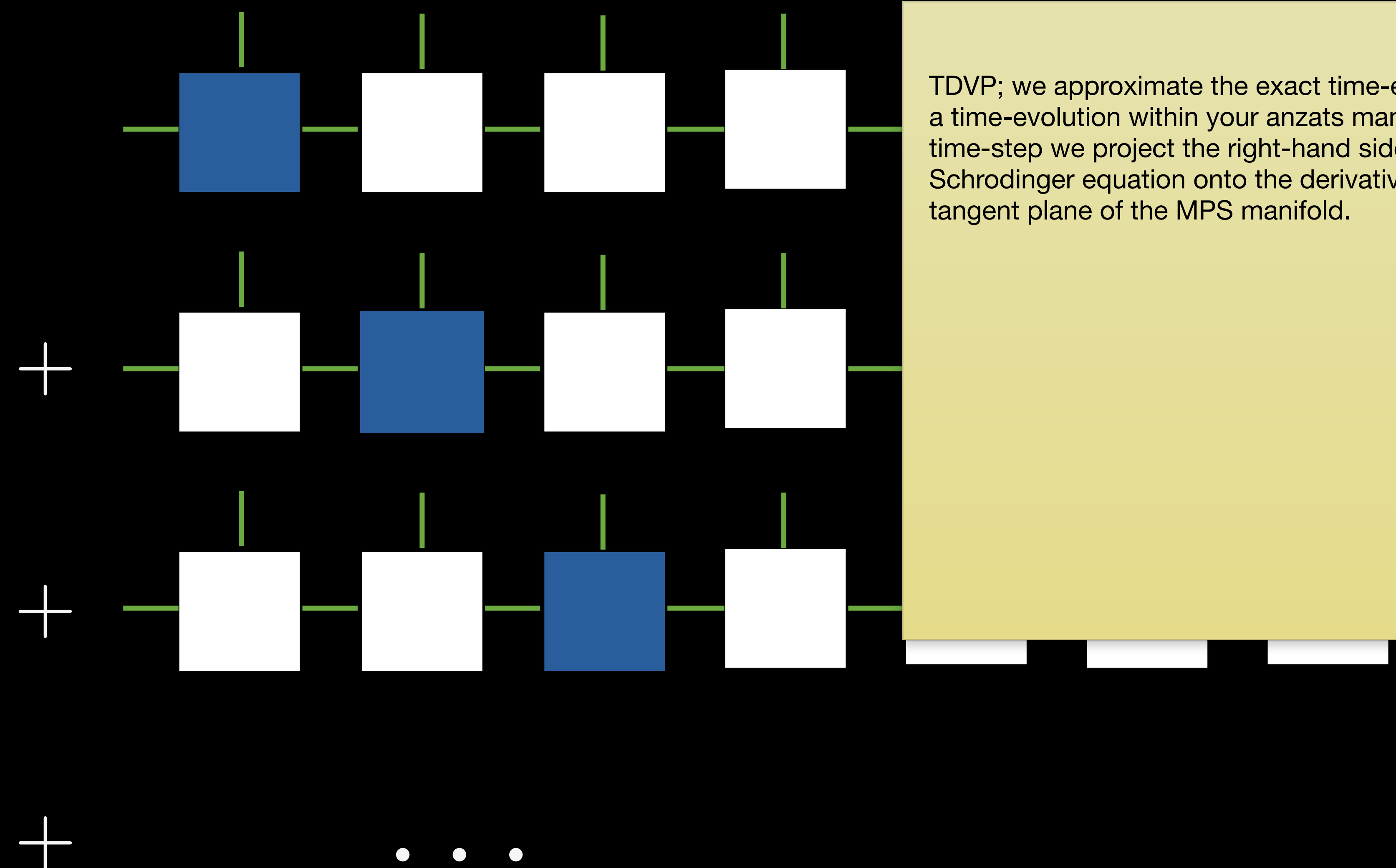


Extra slides

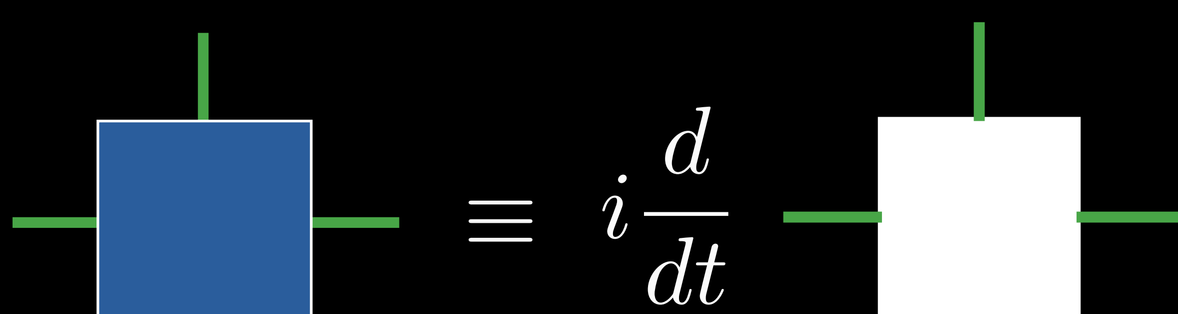
Time Dependent Variational principle

(Haegeman et al, PRL 107, 070601 (2011))

$$i\frac{d}{dt}|\Psi>= H|\Psi> \approx$$



TDVP; we approximate the exact time-evolution with a time-evolution within your anzats manifold. At each time-step we project the right-hand side of the Schrodinger equation onto the derivative of an MPS/ tangent plane of the MPS manifold.



imaginary time evolution

↓

ground-state approximation

Exponential growth bond-dimension during linear growth entropy (orange line):