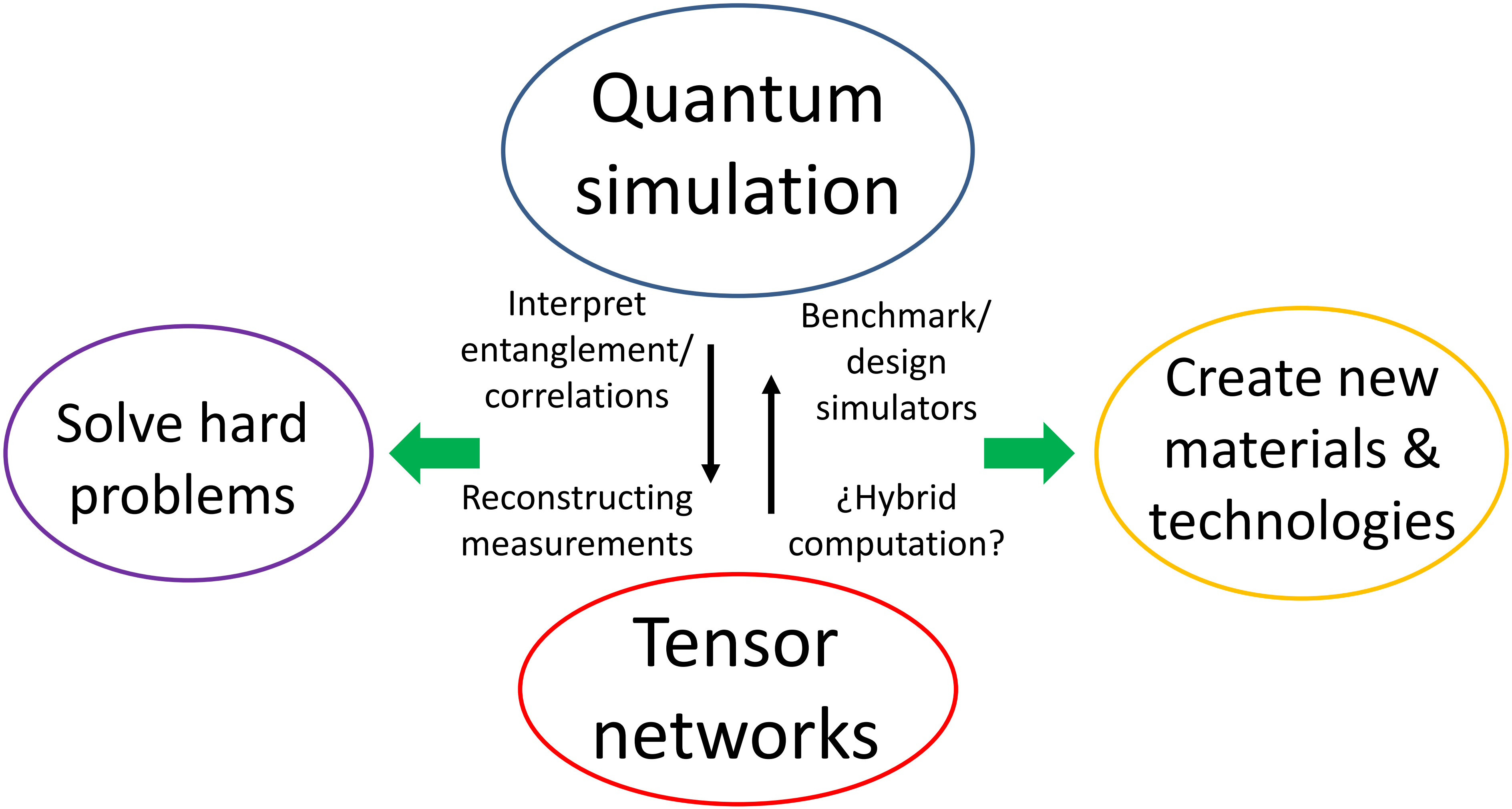
Simulation of background and dynamical gauge fields in quasi-1D quantum systems

ICFO Institut de Ciències Fotòniques

Alessio Celi

Tensor Networks from Simulation to Holography – DESY 4-6/10/2017

Philosophy: parallel approach



Philosophy: parallel approach

Quantum simulation

Examples: High-Tc superconductors & quantum magnetism, gauge theories

Interpret entanglement/ correlations

Benchmark/ design simulators

Solve hard problems

Create new materials & technologies

Reconstructing measurements

¿Hybrid computation?

Involve classical or dynamical gauge fields

Tensor networks

Philosophy: parallel approach

Examples: High-Tc superconductors & quantum magnetism, gauge theories

Quantum simulation

Topological Materials, quantum memories,….

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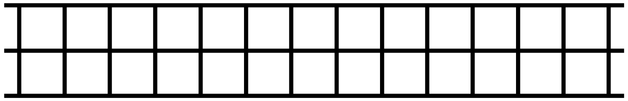
New observables, New vision

This talk: quasi-1D approach

Here: from synthetic magnetic fields to lattice gauge theories with plaquette interactions

Minimal step: ladder/strip



"Easy" as 1D to simulate
-classically, e.g. DMRG
-quantum, e.g. synthetic lattices
encoding Gauss law

Capture some 2D properties
e.g. topological behavior

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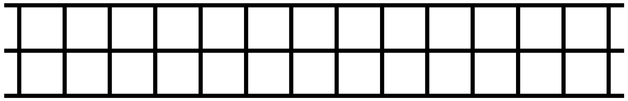

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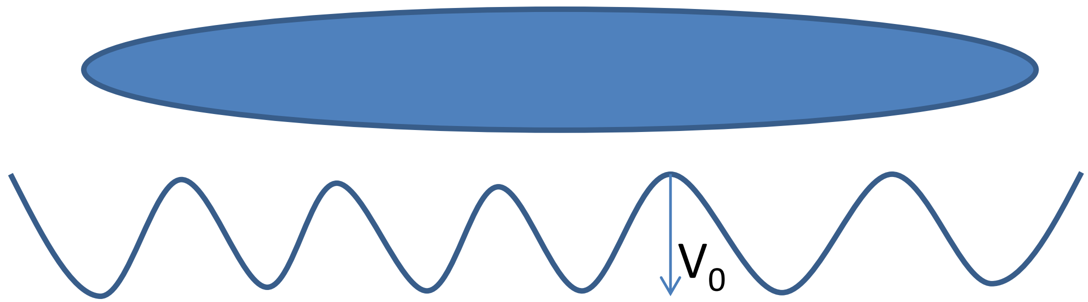
Capture some 2D properties
e.g. topological behavior

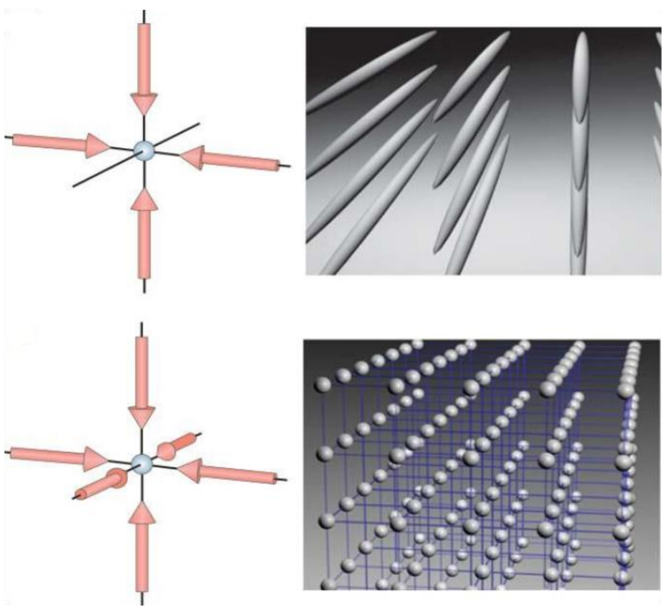
General question: when is the crossover between 1D and 2D?
how is it affecting the different properties?

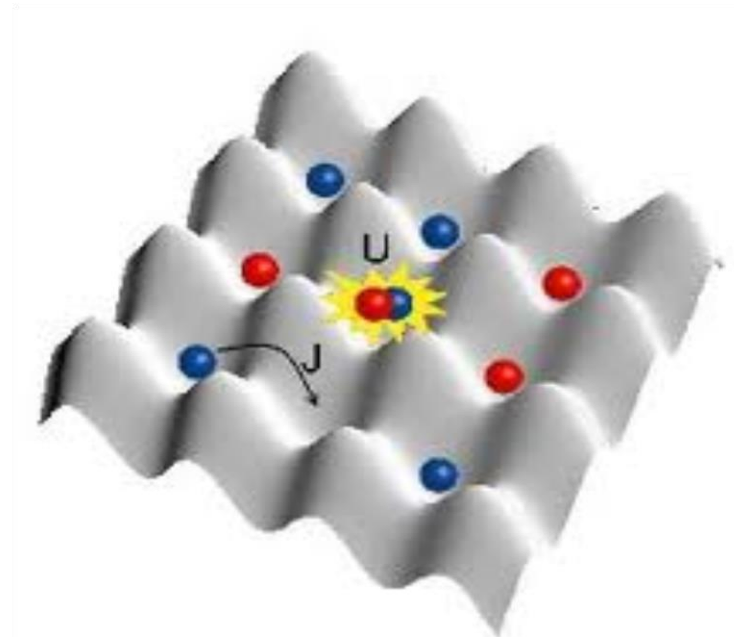
Plan

- Synthetic gauge fields and synthetic lattices (Extradimension)
- Synthetic strips as minimal quantum Hall systems
  Edge states in narrow strips
  Topological response in narrow strips
- Dimerized interacting ladder
  Meissner/Vortex phase (in analogy to type II superconductors)
  Effect of the dimerization:
  Reverse of chiral current (single particle)
  Commensurate-Incommensurate transition (strong interactions)
- Interacting flux ladder from ion chains
- U(1) Lattice gauge theory on the ladder (very very preliminary)
- Prospects

Ultracold atoms in optical lattices as ideal electrons in metals







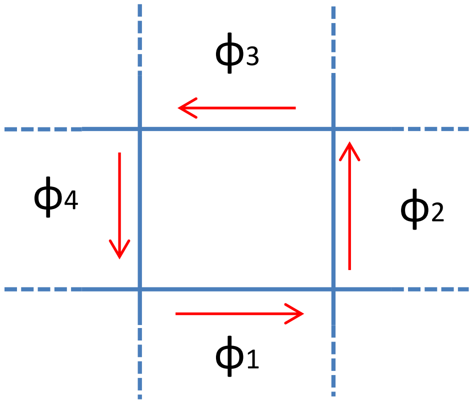
Mott-superfluid phase transition (predicted 1998- observed 2002)

Ultracold atoms in optical lattices as charged electrons

Increasing complexity optical lattice simulator:

- Modify Hopping & connectivity
- Tune interaction (intensity, range)
- Distinguish internal states of atoms

Hopping with phases → Synthetic magnetic field for neutral atoms



Synthetic Aharonov-Bohm effect

$\phi = \Sigma_i \, \phi_i$ = magnetic flux

Ultracold atoms in optical lattices as charged electrons

Many ways of generating magnetic fluxes on the lattice

- Rotation (= constant magnetic field)
- Shaking (also non-Abelian *PRL 109 145301 (2012)…*)

Exp. collaboration with Hamburg: search for
Spin liquid phases in frustrated antiferromagnets with *Bosons*

Ultracold atoms in optical lattices as charged electrons

Many ways of generating magnetic fluxes on the lattice

- Rotation (= constant magnetic field)
- Shaking
- Raman laser + 2D superlattice

Theory : Jaksch & Zoller *NJP* ***5*** *56* (2003)

.....

Experiments: *PRL 107 255301 (2012), PRL 111 185301 (2013)* (I.Bloch group)
PRL 111 185302 (2013) (W.Ketterle group)

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- Raman laser + *"Extradimension"*

Simulating an extra dimension

[Boada,AC,Latorre,Lewenstein, PRL 108, 133001 (2012)]

In optical lattices 1D-3D Hubbard model by tuning optical potential

And > 3D? In a lattice Dimensionality ≡ Connectivity

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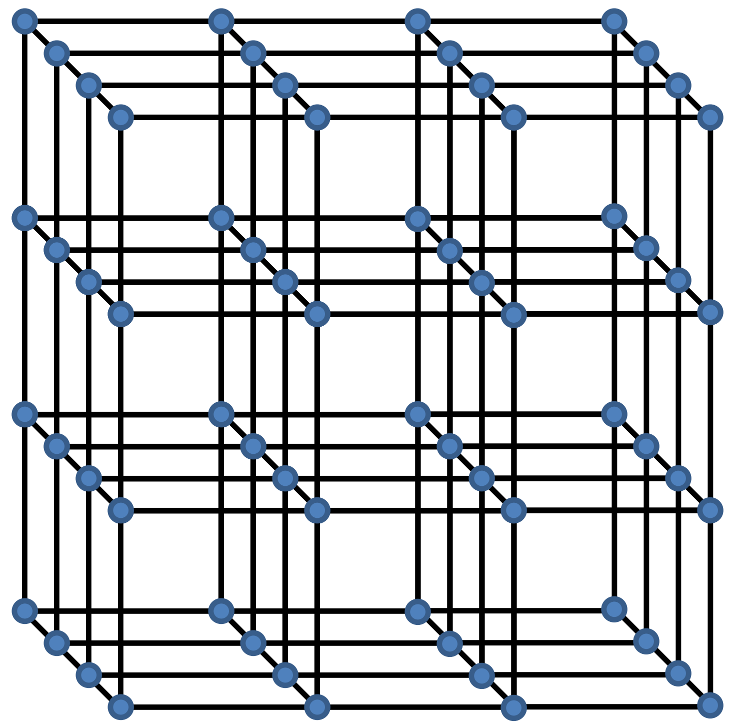
Hopping in *D*+1 hypercubic lattice as



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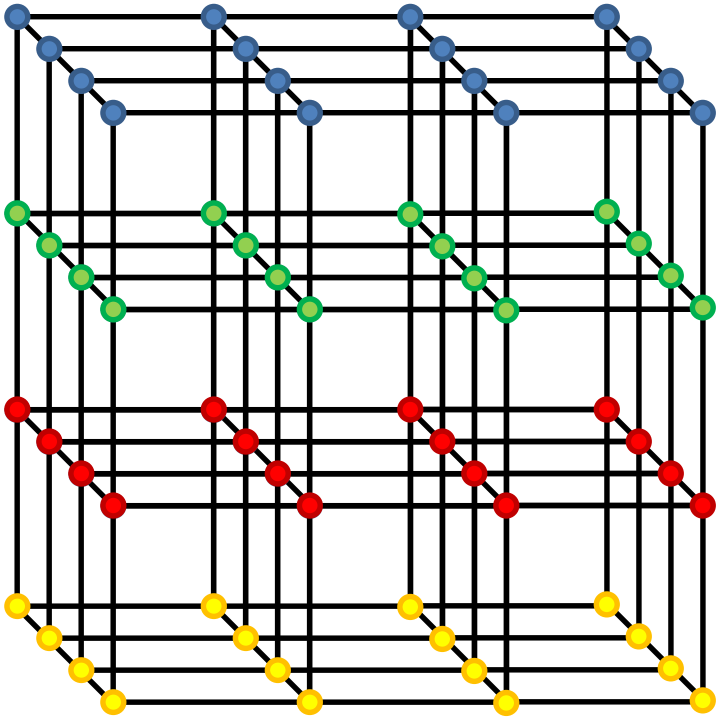

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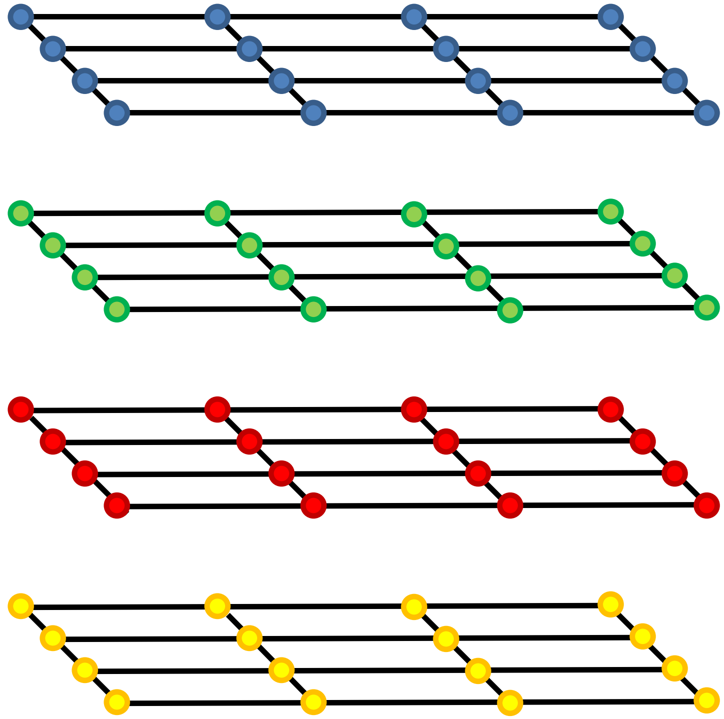

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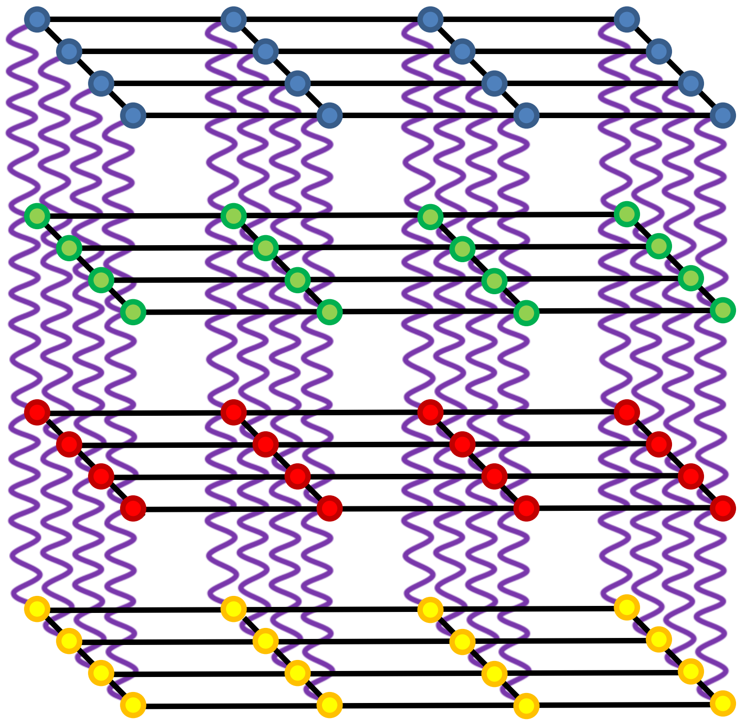
Coupled atomic states hopping in *D*-lattices



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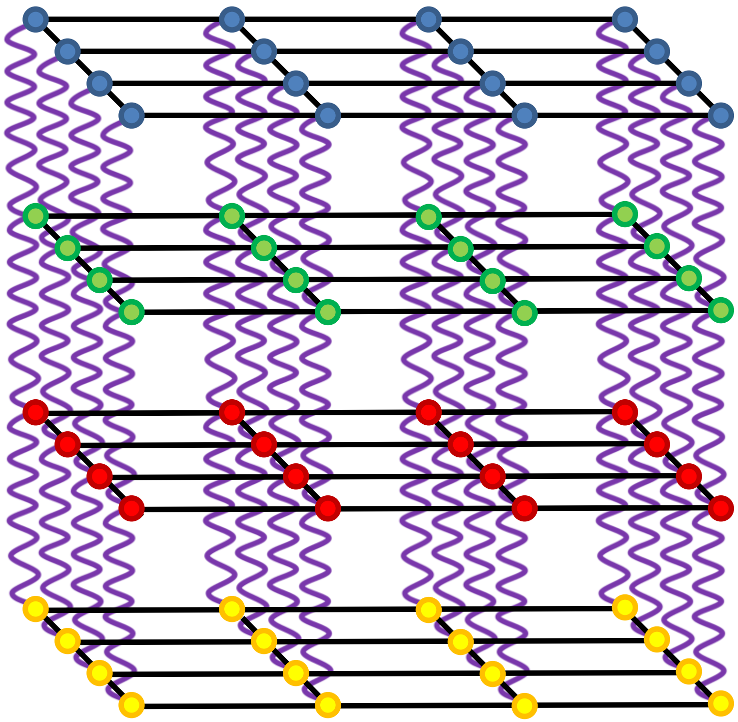

Not only spin states
Momentum states
Trap modes…

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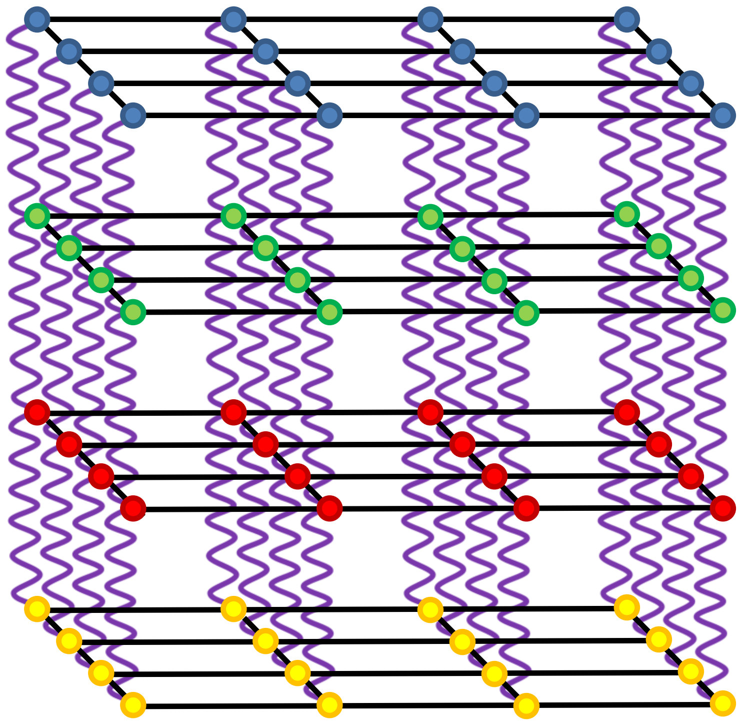
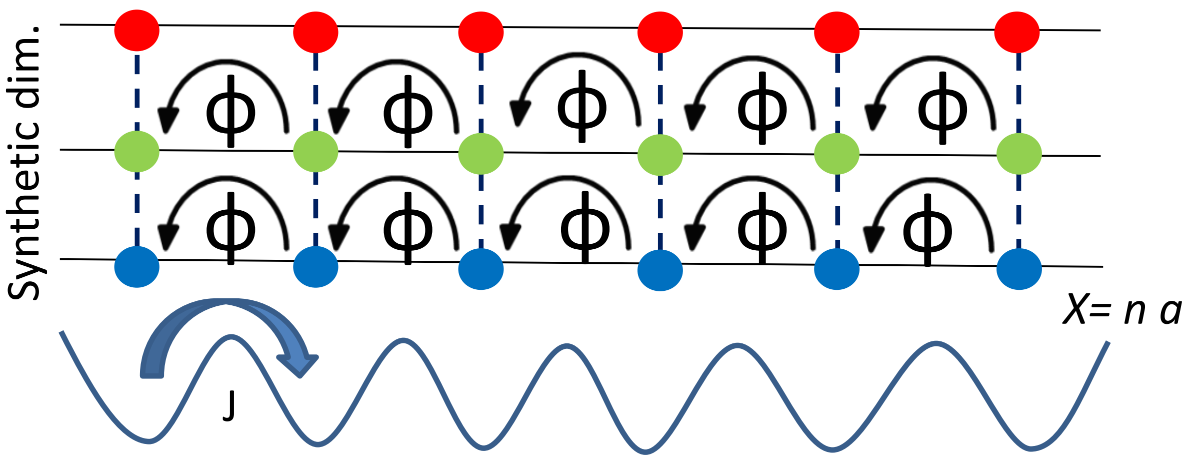


Not only spin states
Momentum states
Trap modes…

Not only atoms
Cold molecules,
Photonic crystal,
Ring resonators…

Synthetic gauge fields in synthetic dimension

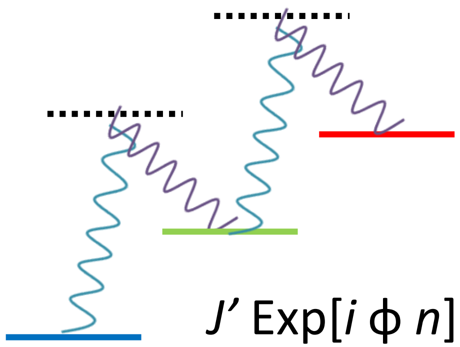
[AC *et al* PRL 112 , 043001 (2014)]



1d-lattice loaded e.g. with ^{87}Rb (F=1, m=-1,0,1)

+

Raman dressing

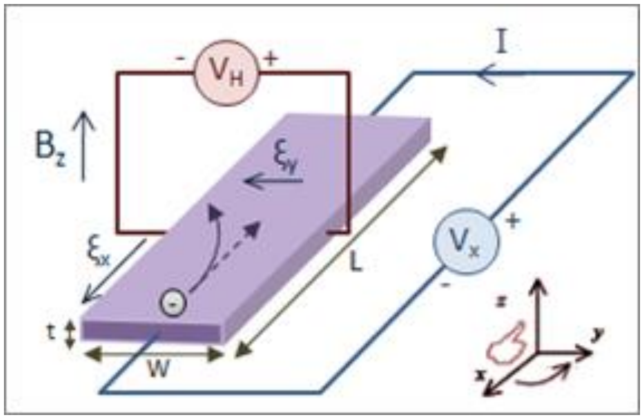


Constant magnetic flux ϕ!

Minimal instance of a quantum Hall system!

Quantum Hall effect

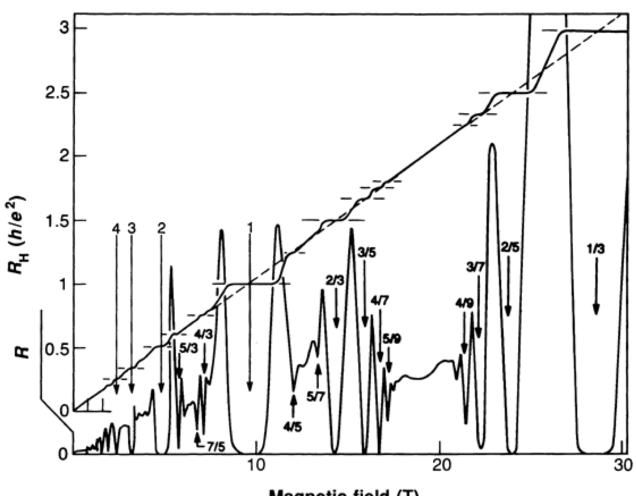
1879: Classical Hall effect (consequence of Lorentz force)



E. Hall

1980: Quantum Hall effect: Electric conductivity quantized

$$\sigma = \frac{I_{\text{channel}}}{V_{\text{Hall}}} = \nu \frac{e^2}{h}$$



K. Von Klitzing

Integer Quantum Hall effect in a lattice

IQH explained in terms of single particle physics (Landau level filling)

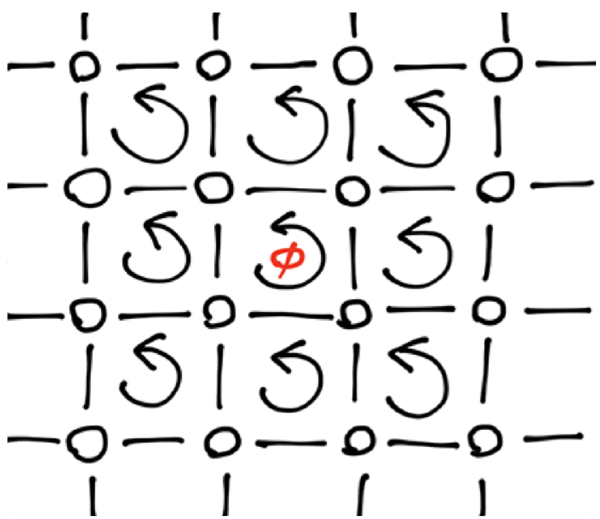
$$H = -\sum_{n,m}(J a^{\dagger}_{n+1,m} + J' e^{i\Phi n} a^{\dagger}_{n,m+1}) a_{n,m} + h.c.$$

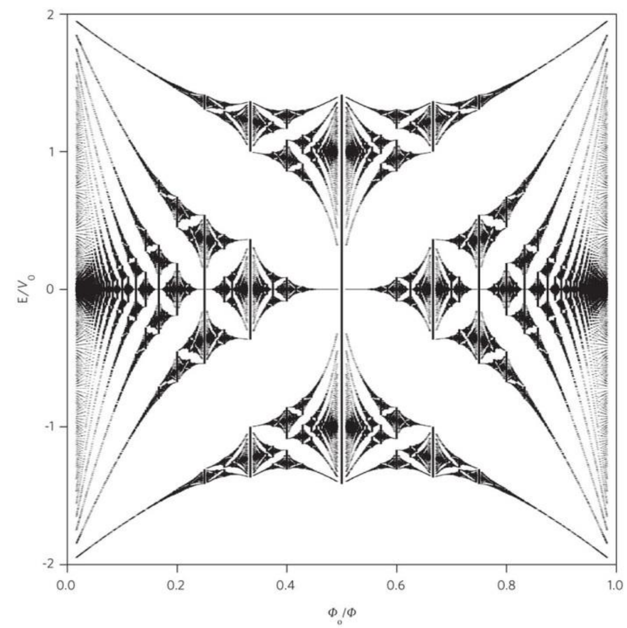
Quantization determined by topology of filled bands (1-Chern number)

Bulk-boundary correspondence (Topological insulator prototype)

Edge states determined by spectrum of periodic system

On square lattice simple formulation: Hofstadter model





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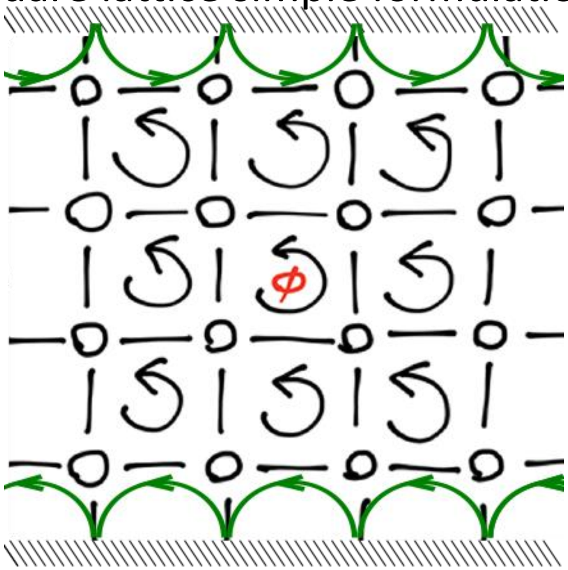
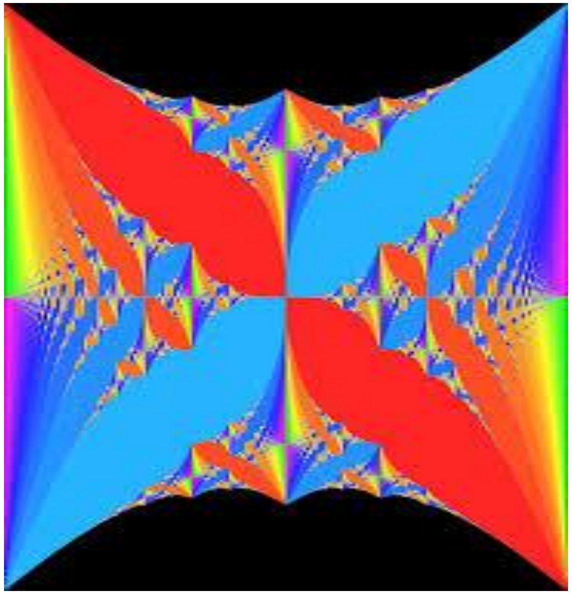
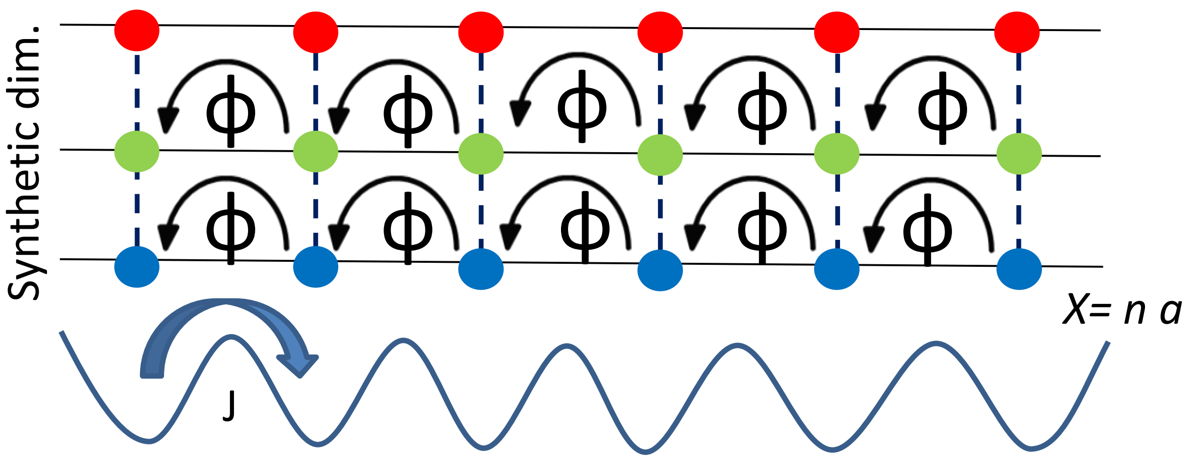




Synthetic gauge fields in synthetic dimension

[AC *et al* PRL 112 , 043001 (2014)]



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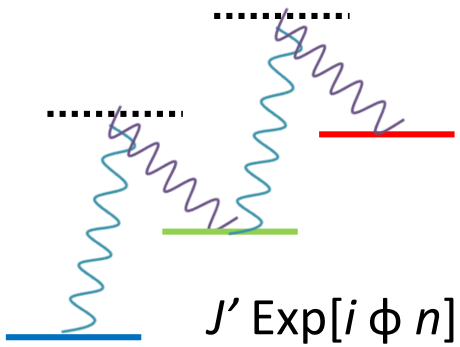

Constant magnetic flux ϕ!

Sharp Boundaries ➡ Edge currents (hard to get in real 2d lattice)

signal of Topological nature of quantum Hall (bulk-boundary correspondence)

Synthetic gauge fields in synthetic dimension

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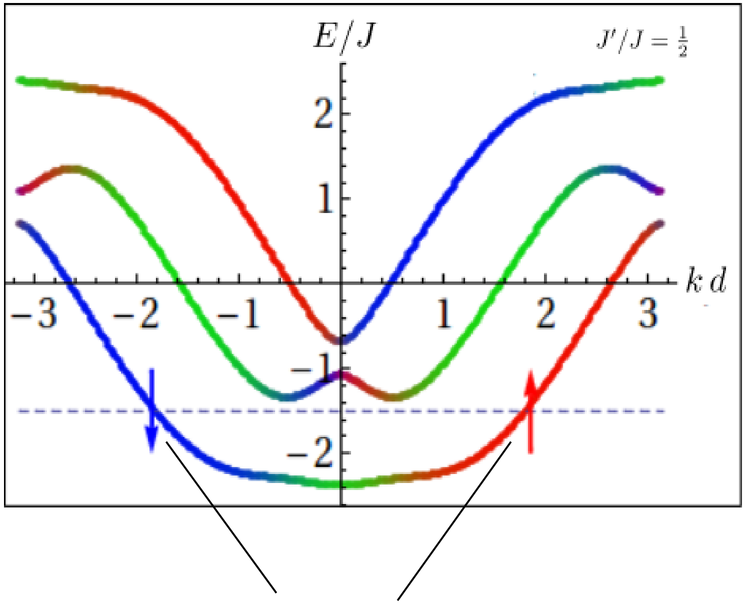
Spectrum



"Genuine" Edge states for small *J'/J*:

-live in the gap,

-have linear dispersion

-have well defined spin

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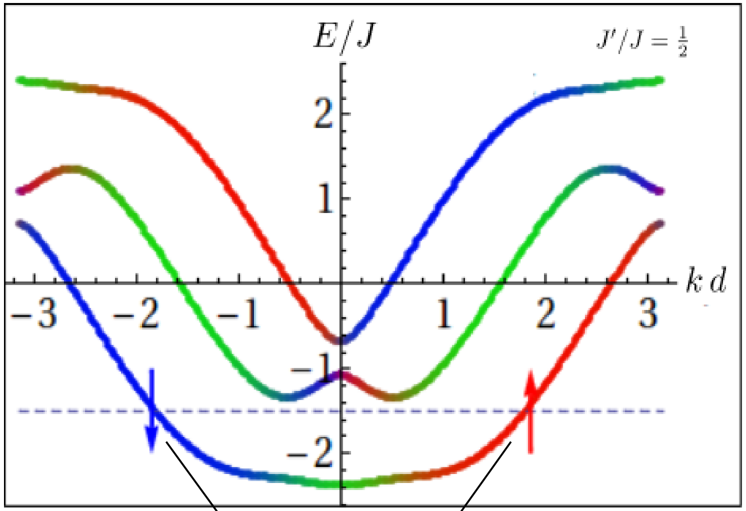

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Experimental Realizations:

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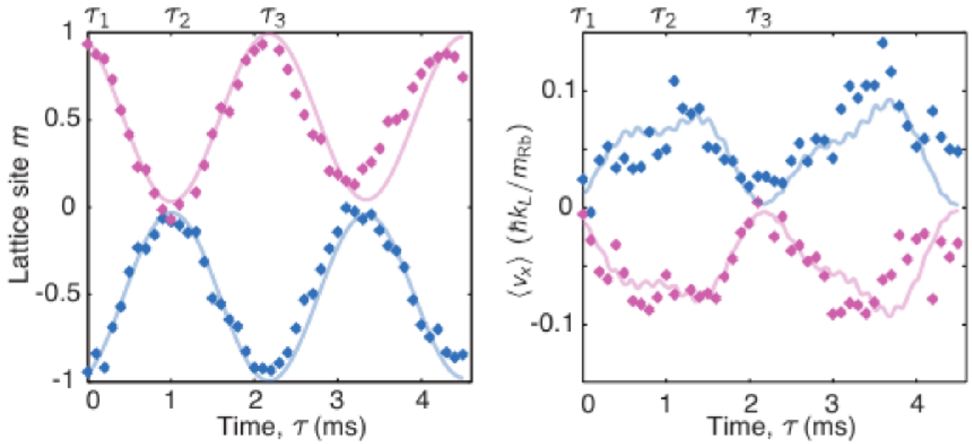

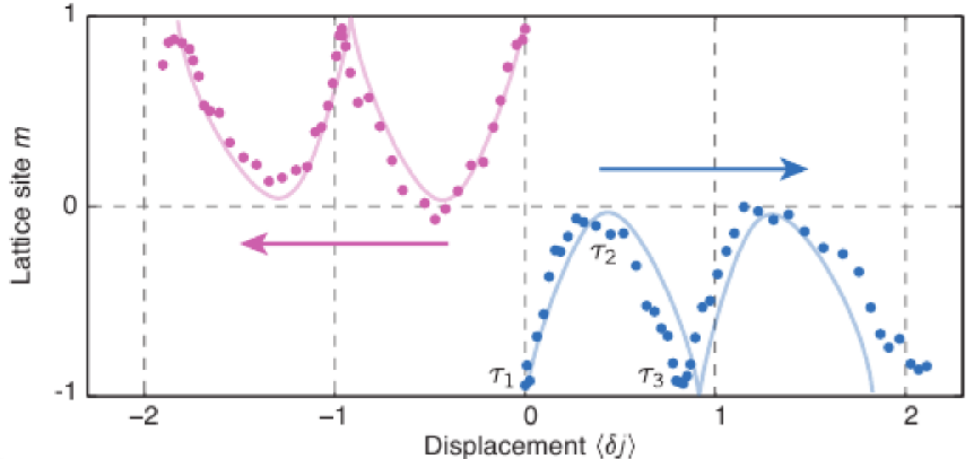

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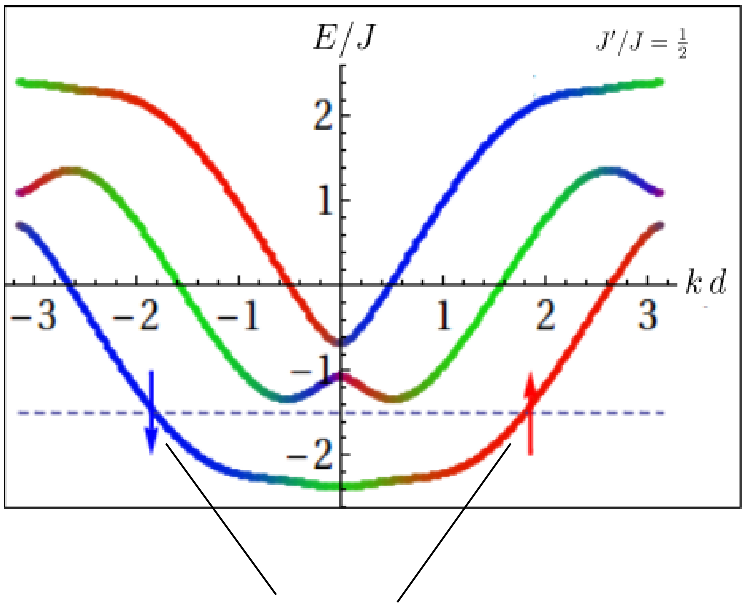

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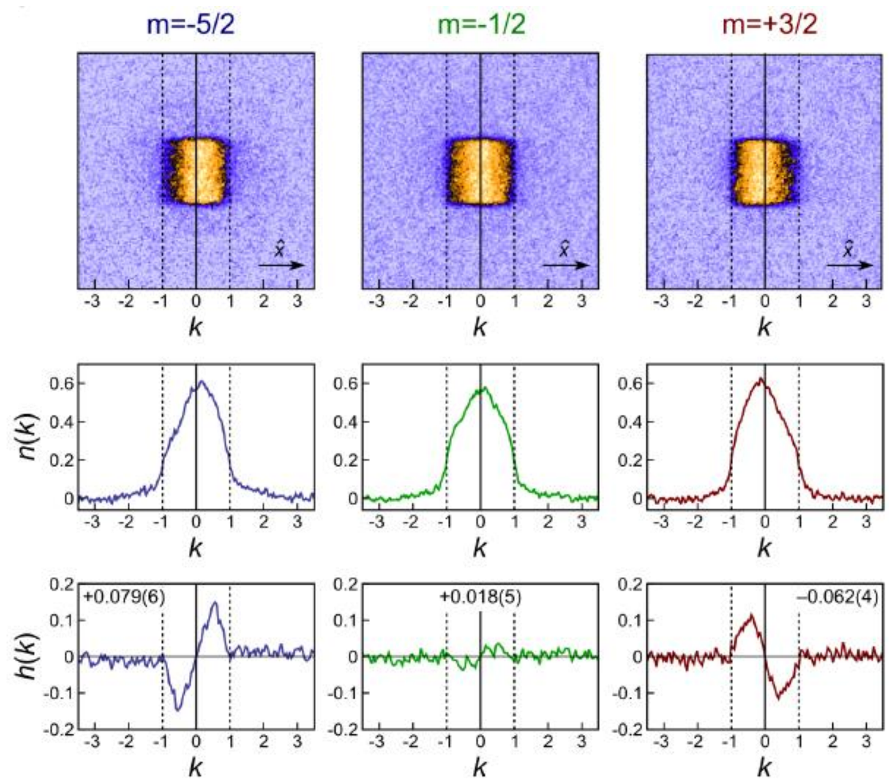

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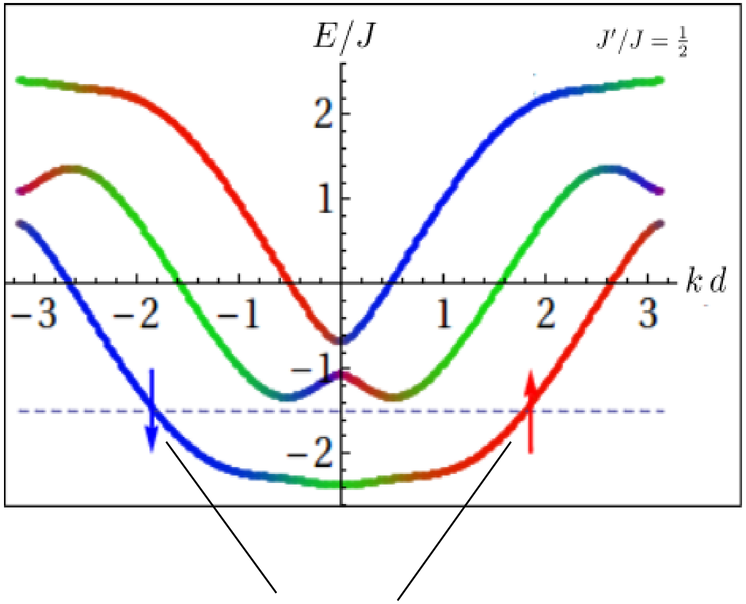

"Genuine" Edge states for small *J′/J*:

-live in the gap,

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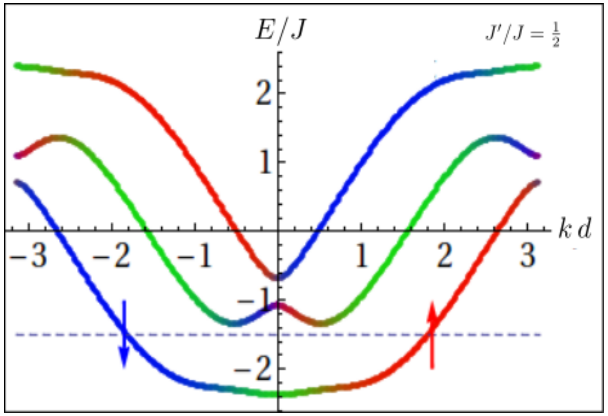
II) Fermions: LENS Fallani group ^{173}Yb [*Science* (2015)]

Also with clock states (ladder)
LENS: Livi et al. PRL 117, 220401 (2016)
JILA: Kolkowitz et al. Nature 542 66 (2017)

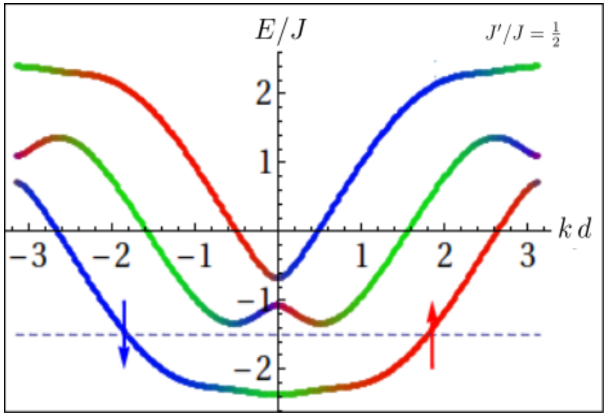
Topology in narrow strips

Narrow Hofstadter strips have edge states



What about the “bulk”?

Topology in narrow strips



Narrow Hofstadter strips have edge states

What about the “bulk”?

How big should it be to display topological properties?

Is there some reminiscence of open/closed boundary correspondence?

Is Chern number defined? / Can we measure it?

Measuring Chern numbers in (narrow) Hofstadter strips, S.Mugel,….AC, SciPost 3, 012, (2017)

Pragmatic approach: measure transverse displacement to a force after a Bloch oscillation, Laughlin pump argument

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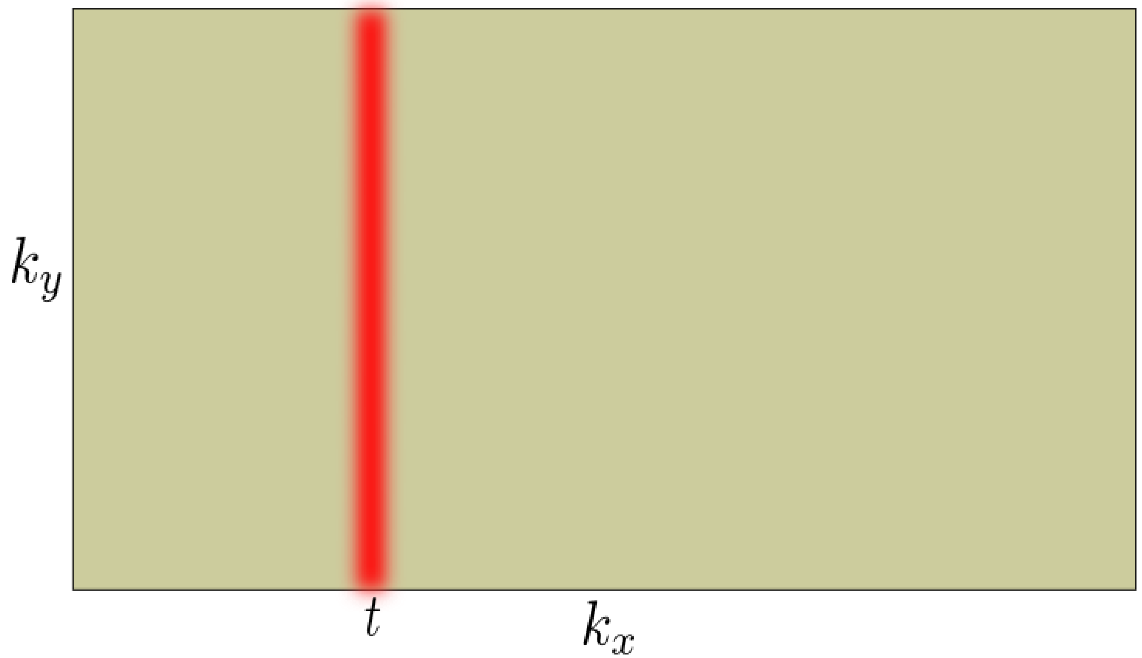
Pragmatic approach: measure transverse displacement to a force after a Bloch oscillation, Laughlin pump argument

- Large (periodic) system, a lowest band state well localized in *y* and spread in *x*
- Apply a force along *x*

Brillouin sketch



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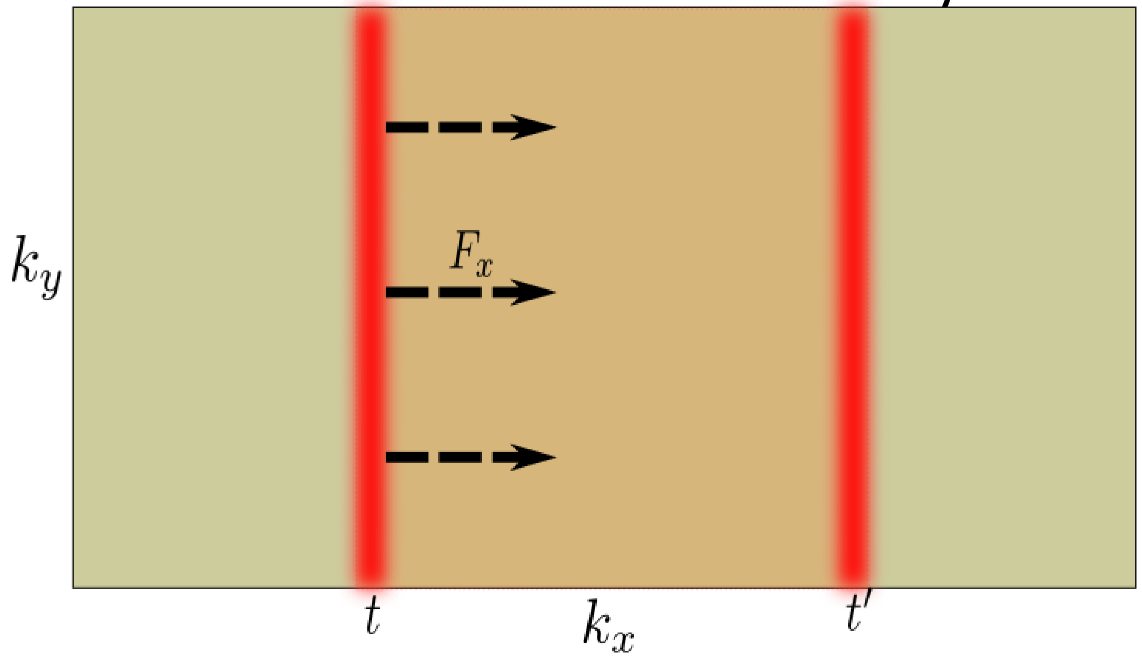
Brillouin sketch of semiclassical dynamics



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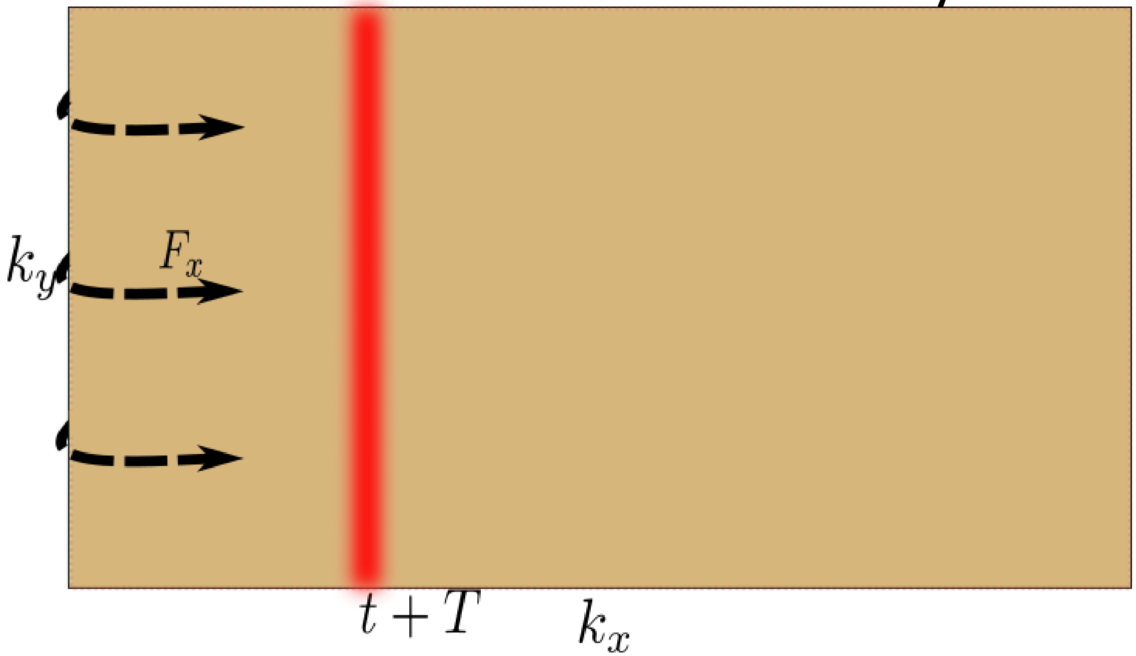
Pragmatic approach: measure transverse displacement to a force after a Bloch oscillation, Laughlin pump argument

- Large (periodic) system, a lowest band state well localized in *y* and spread in *x*
- Apply a force along *x*
- After a Bloch oscillation observe the displacement

Brillouin sketch of semiclassical dynamics



Displacement in *y* due to anomalous velocity!

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In formulae: semiclassical approach

$$\mathbf{k}(t) = \mathbf{k}_0 + \frac{t}{\hbar d} F_x \, \mathbf{e}_x \longrightarrow \mathbf{v}(\mathbf{k}) = \frac{1}{\hbar} \partial_{\mathbf{k}} E(\mathbf{k}) + \frac{F_x}{\hbar d} \mathcal{F}(\mathbf{k}) \mathbf{e}_y$$

Wave packet:

$$|\psi(\mathbf{k})|^2 \sim \tfrac{1}{A_{BZ}} \delta(k_x - k_x(t))$$

$$\langle \mathbf{r}(T) - \mathbf{r}(0) \rangle = \int_0^T \langle \mathbf{v}(t) \rangle = \tfrac{\hbar d}{|F_x|} \int_{BZ} \mathbf{v}(\mathbf{k}) = \mathrm{sgn}(F_x) \mathcal{C} \, d \, \mathbf{e}_y$$

State easy to prepare if the coupling $J_y \ll J_x$

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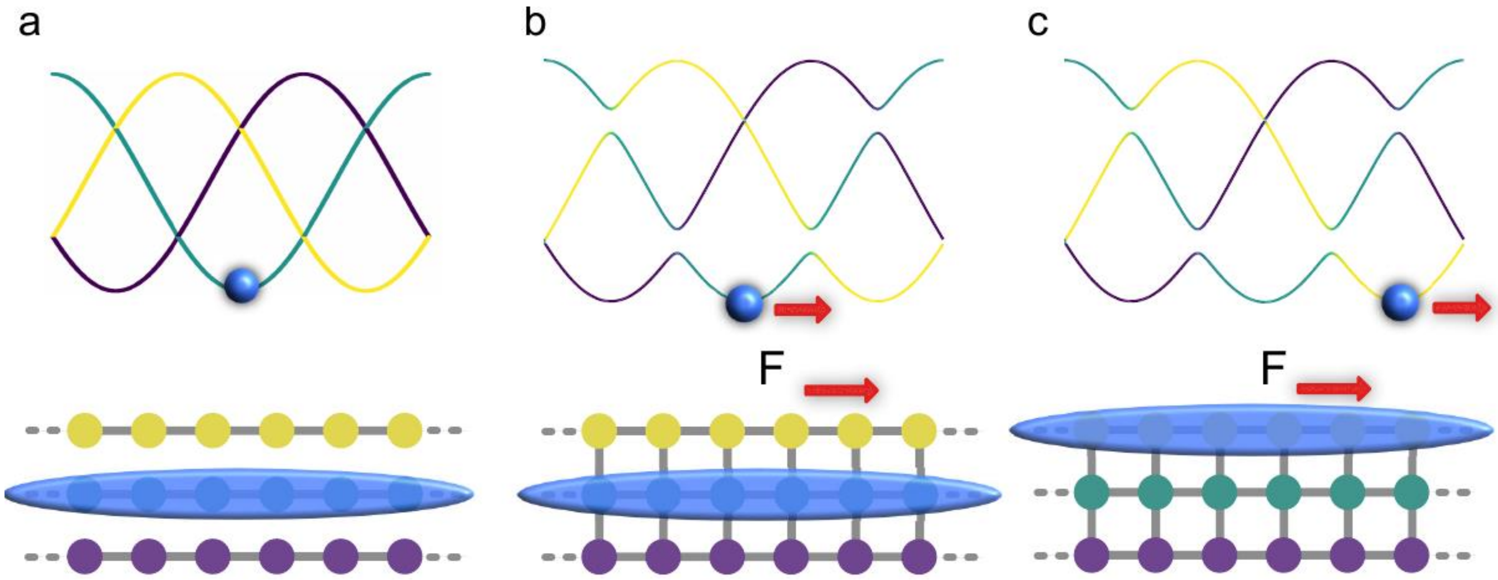
State easy to prepare if the coupling $J_y \ll J_x$

Applicable also to strips until we don't reach the boundary…

Measuring Chern numbers in (narrow) Hofstadter strips, S.Mugel,….AC, SciPost 3, 012, (2017)

Scheme:



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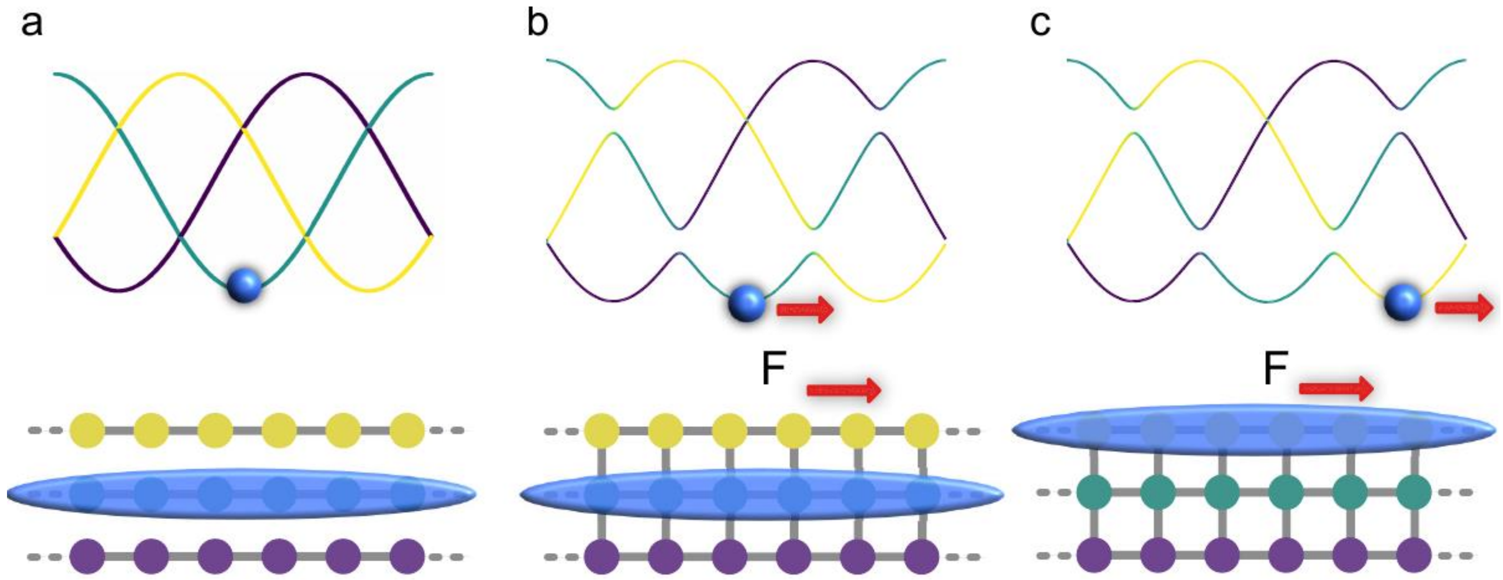
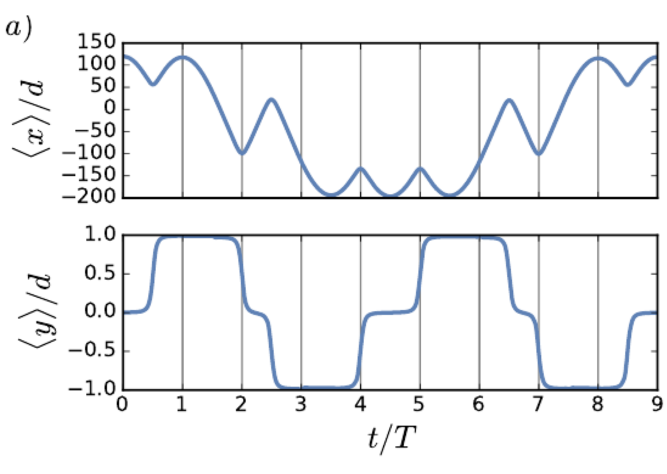
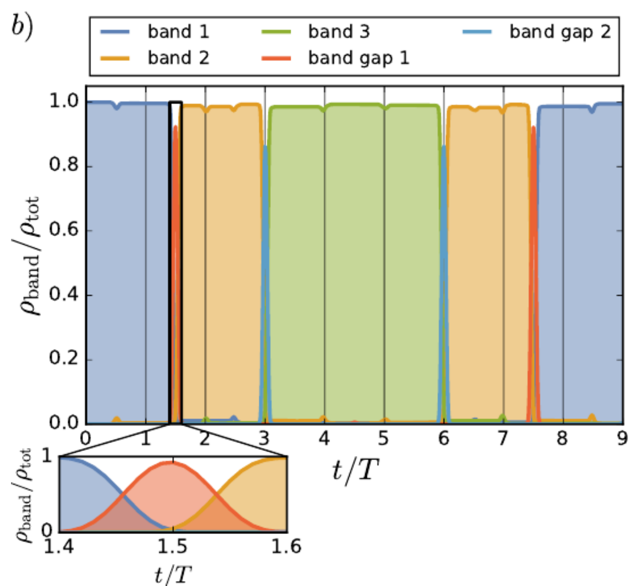


Results: $J_y = \frac{1}{5} J_x, \quad \Phi = \frac{2\pi}{3} \quad N_y = 3$





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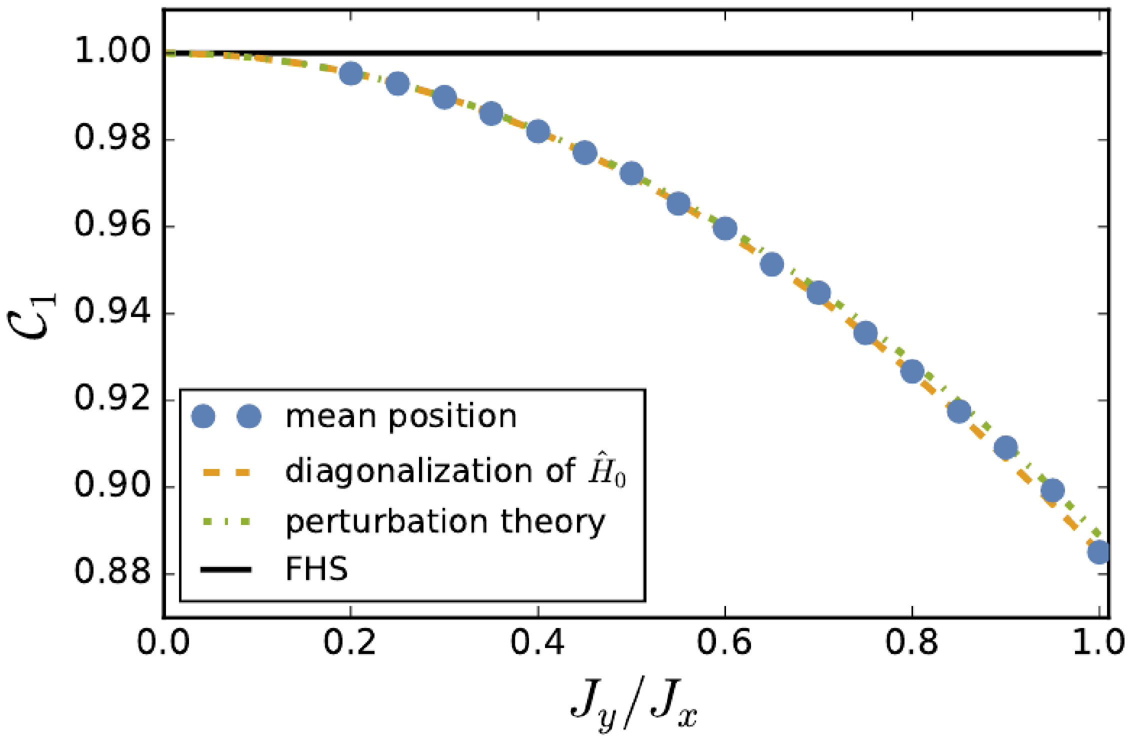
Why does it work? Perturbative argument also for edge states:

- Gap linear in J_y/J_x
- Hybridization spin states (spreading in y) quadratic in J_y/J_x



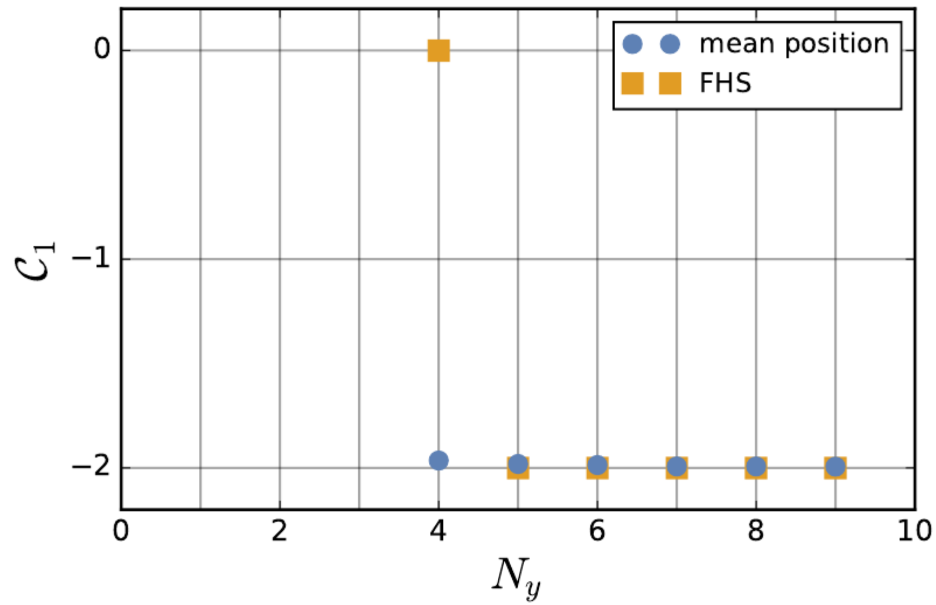
Quadratic degradation of the measurement

Measuring Chern numbers in (narrow) Hofstadter strips, S.Mugel,....AC, SciPost 3, 012, (2017)

Higher $\mathcal{C}$ possible for $N_y \geq \mathcal{C} + 2$

Ex: $\Phi = \frac{4\pi}{5} \rightarrow \mathcal{C}_1 = -2$

"Better" than Fukui-Hatsugai-Suzuki algorithm J. Phys. Soc. Jpn. (2005)



Robust to disorder (< gap) and typical harmonic confinement

Interactions? Gap small (although may hold thought adiabatic argument, see later)

Synthetic lattices in interaction

- Interesting route to interaction -> Fractional QH effect?!
  -> Manybody localization?!

  No heating expected

- Peculiarity: Interactions are naturally long range in the synthetic dimension

- Quasi 1D approach to 2D interesting both theoretically & practically

Synthetic lattices in interaction

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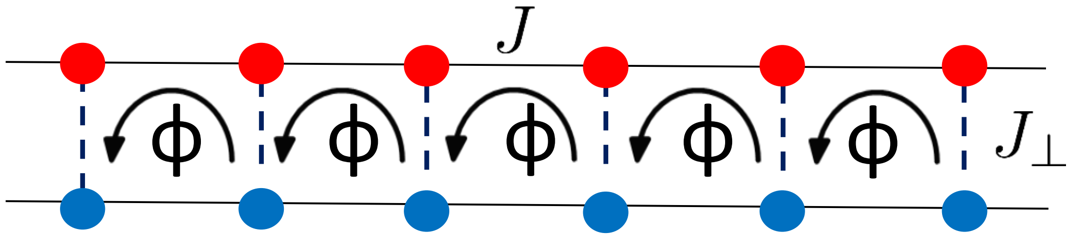
- Quasi 1D approach to 2D interesting both theoretically & practically

Many studies: Meissner-vortex and commensurable incommensurable transitions, Fractional pumping, Laughlin like states, pseudo Majorana…

Here: effect of dimerization on synthetic Hofstadter ladder

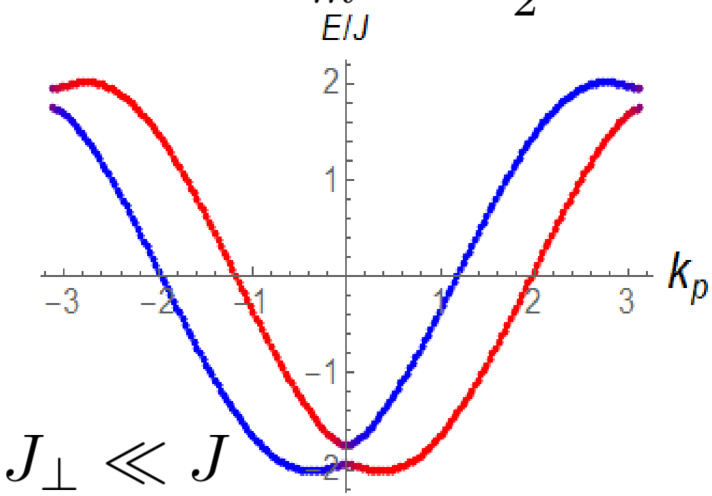
Meissner/Vortex phase in flux ladder



No interactions: real = synthetic ladder

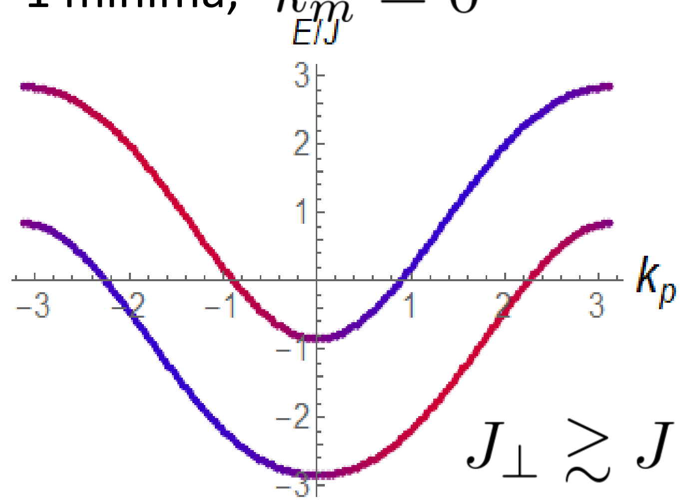
Weak interleg (Raman) coupling:

2 minima, $k_m \sim \pm\frac{\phi}{2}$



Strong interleg (Raman) coupling:

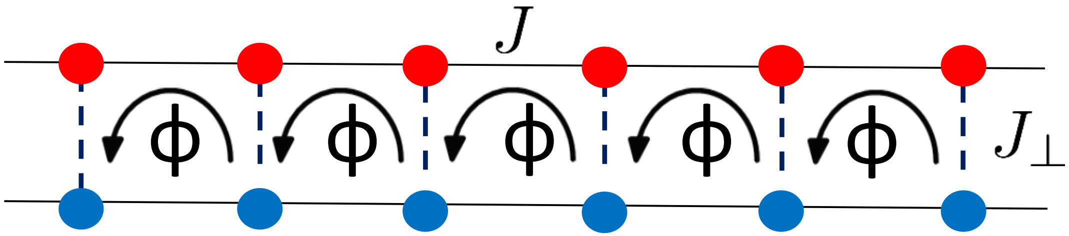
1 minima, $k_m = 0$



[Orignac,Giamarchi, PRB 2001] Analogous to type II, also in presence interactions

Real ladder experiment [Atala et, Nature Phys. 2014]

Meissner/Vortex phase in flux ladder



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Observables

$$J_c(j,m) = i \langle \hat{a}^\dagger_{j+1,m} \hat{a}_{j,m} \rangle + H.c.$$

$$J_\perp(j) = i \langle \hat{a}^\dagger_{j,1/2} \hat{a}_{j,-1/2} \rangle + H.c.$$

$J_\perp \ll J$

$J_\perp \gtrsim J$

[Orignac,Giamarchi, PRB 2001] Analogous to type II, also in presence interactions

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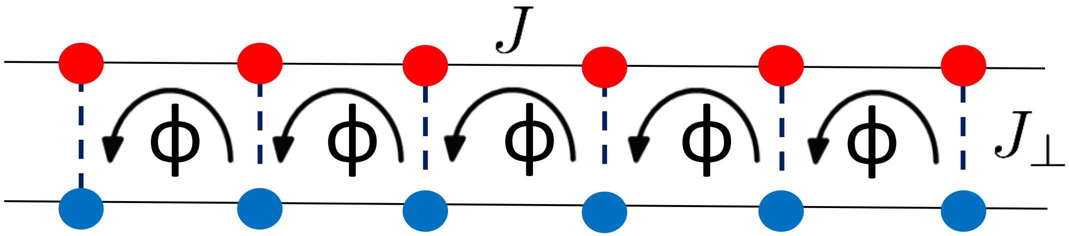

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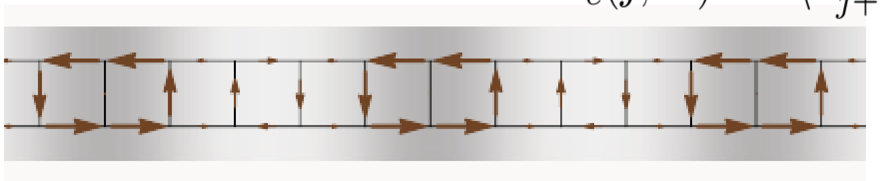

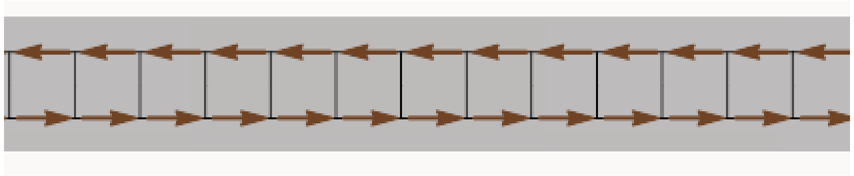

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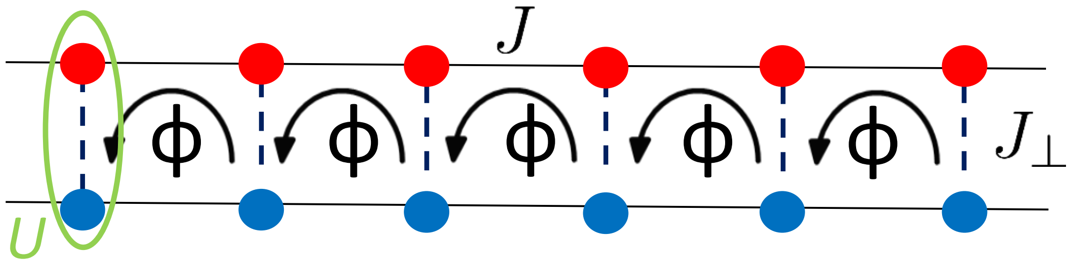


Effect of interactions: suppress vortex phase

Real ladder: vortex phase survives in the hard-core limit for ϕ large

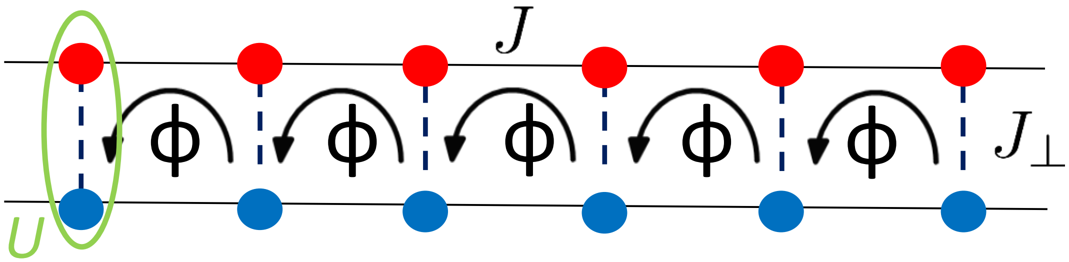
more phases at $U \neq \infty$

see [Petrescu, Le Hur,PRL 2013]
[Piraud et al, PRB 2015]

Synthetic ladder: vortex phase disappears in the hard-core limit

more phases at $U \neq \infty$ ....

Meissner/Vortex phase in flux ladder



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see [Petrescu, Le Hur, PRL 2013]
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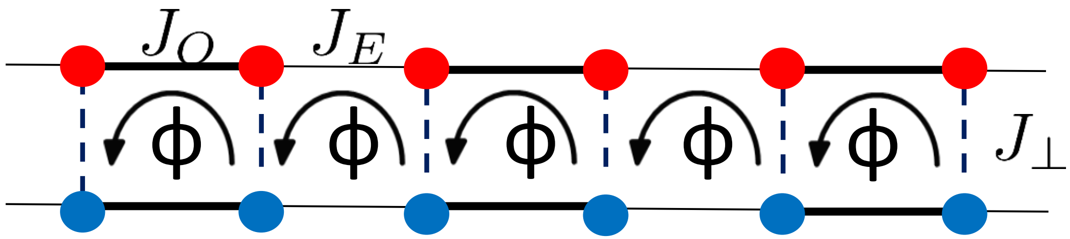
Synthetic ladder: vortex phase disappears in the hard-core limit

more phases at $U \neq \infty$ ....

Idea: nucleate vortices by dimerizing the lattice ("easy" exp. handle)

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$$J = \frac{J_O + J_E}{2}$$



Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

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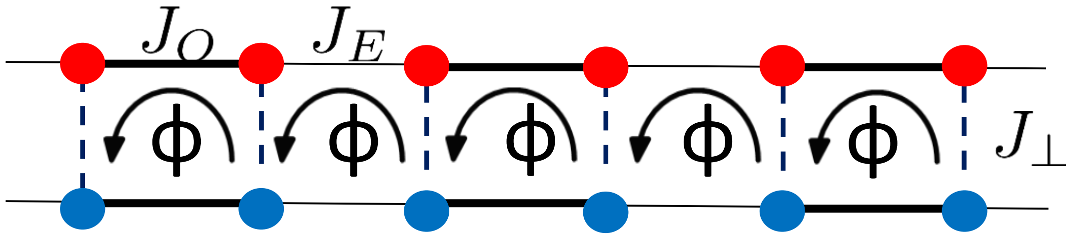
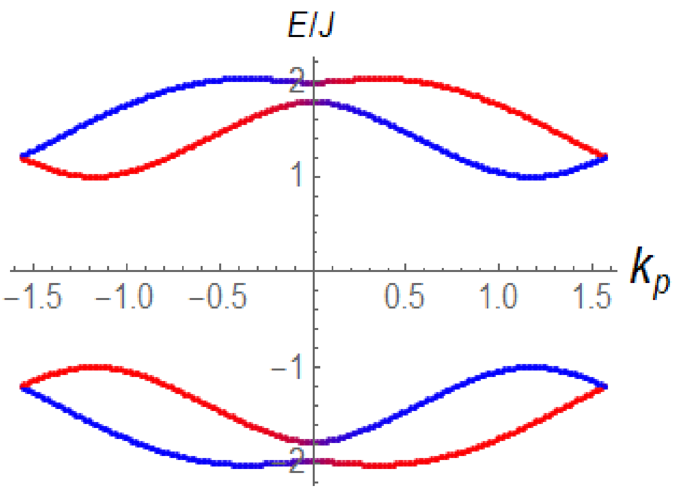


Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

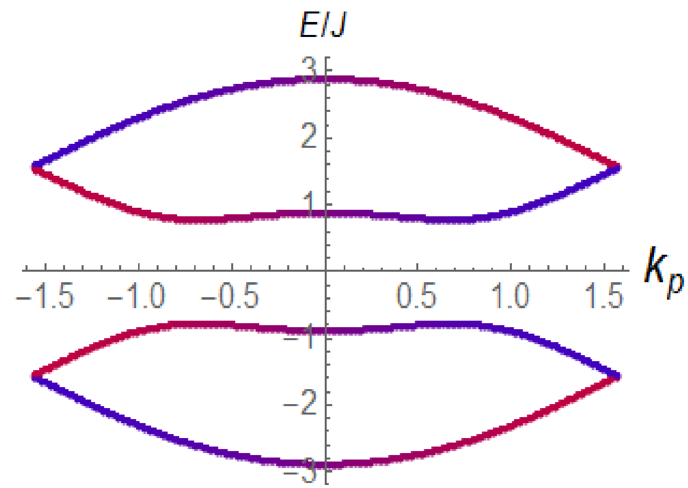
No interactions: 4 bands



$J_\perp \ll J$

$\Delta = 0.5$

Bands deform & mix

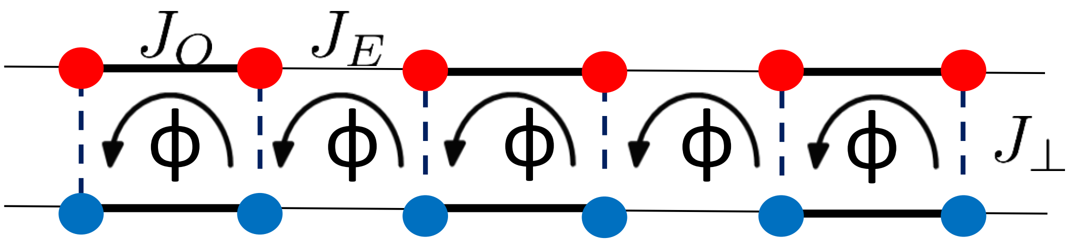


$J_\perp \gtrsim J$

Minima separate: dimerization enhances vortex phase!

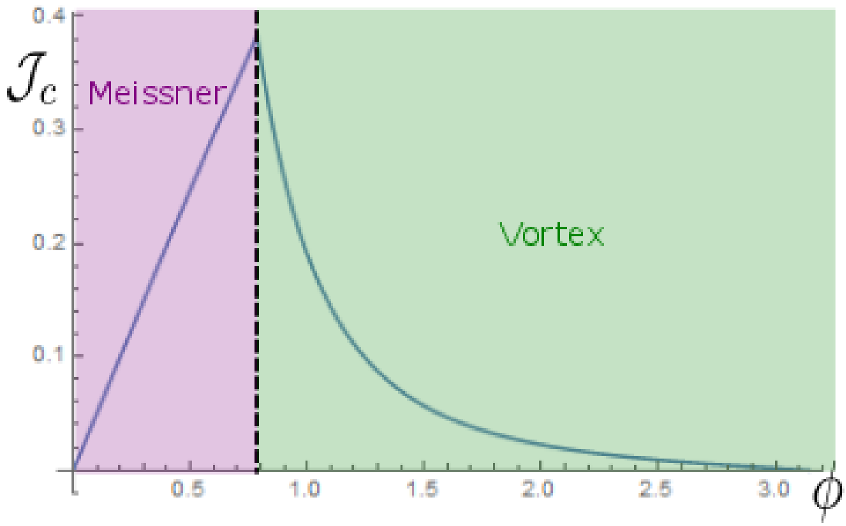
Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J = \frac{J_O + J_E}{2}$

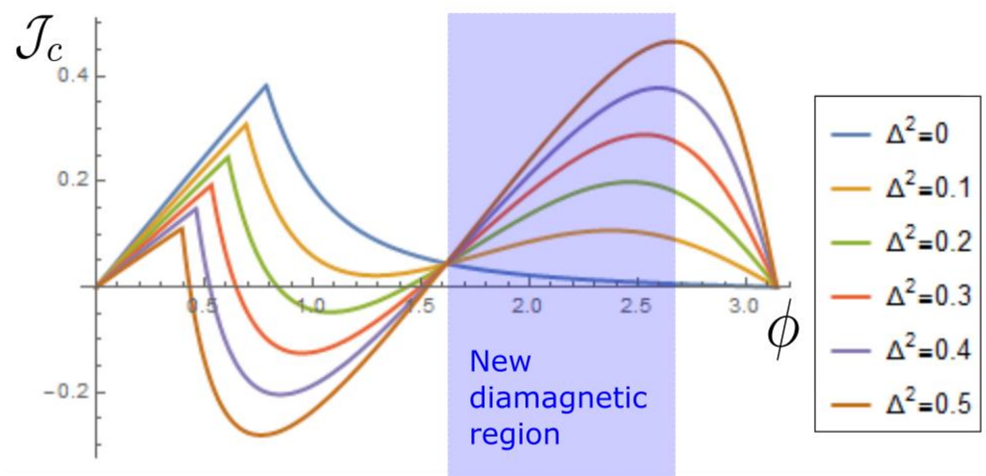


Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

No interactions: Reverse of chiral current



$\Delta = 0$



$\Delta \neq 0$

Current behavior confirms vortex enhancement!

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

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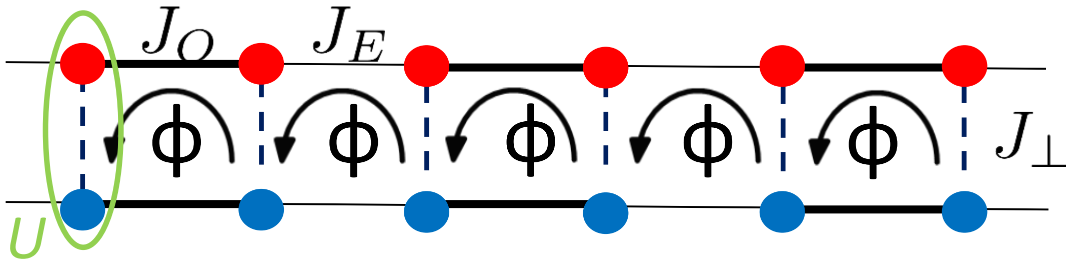

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Interactions: $U \to \infty$ 3 states per rung

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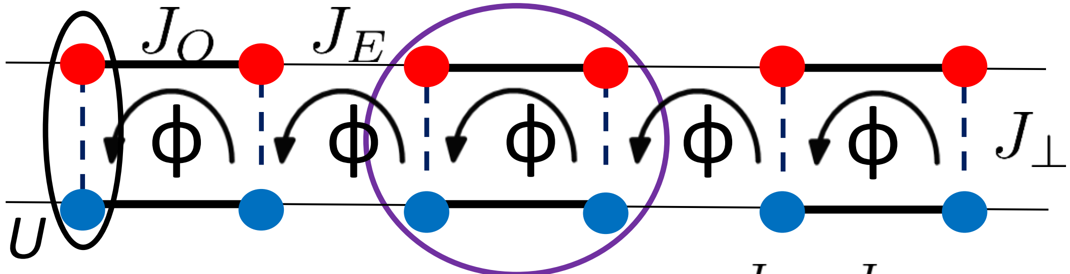

Effect of dimerization: new handle $\Delta = \frac{J_O - J_E}{J_O + J_E}$

Interactions: $U \to \infty$ 3 states per rung

$J_E \ll J_O$ 9 states per plaquette

	1 *n=0,*	4 *n=1,*	4 *n=2*
Spectrum plaquette	0	$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 - 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	$\pm 2J_\perp$
		$\pm J_\perp \sqrt{1 + \left(\frac{J_O}{J_\perp}\right)^2 + 2\cos\frac{\phi}{2}\left(\frac{J_O}{J_\perp}\right)}$	± 0

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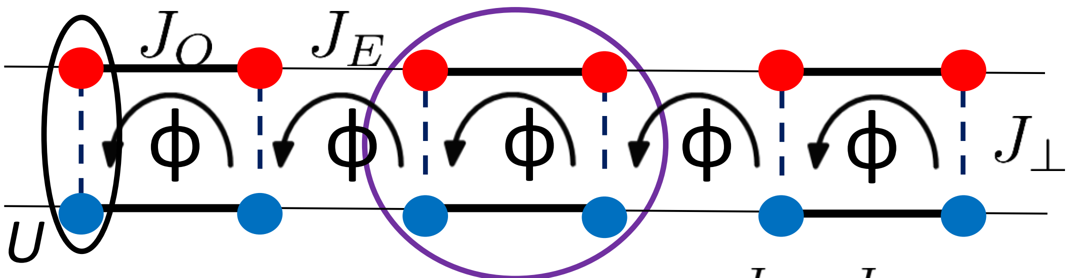

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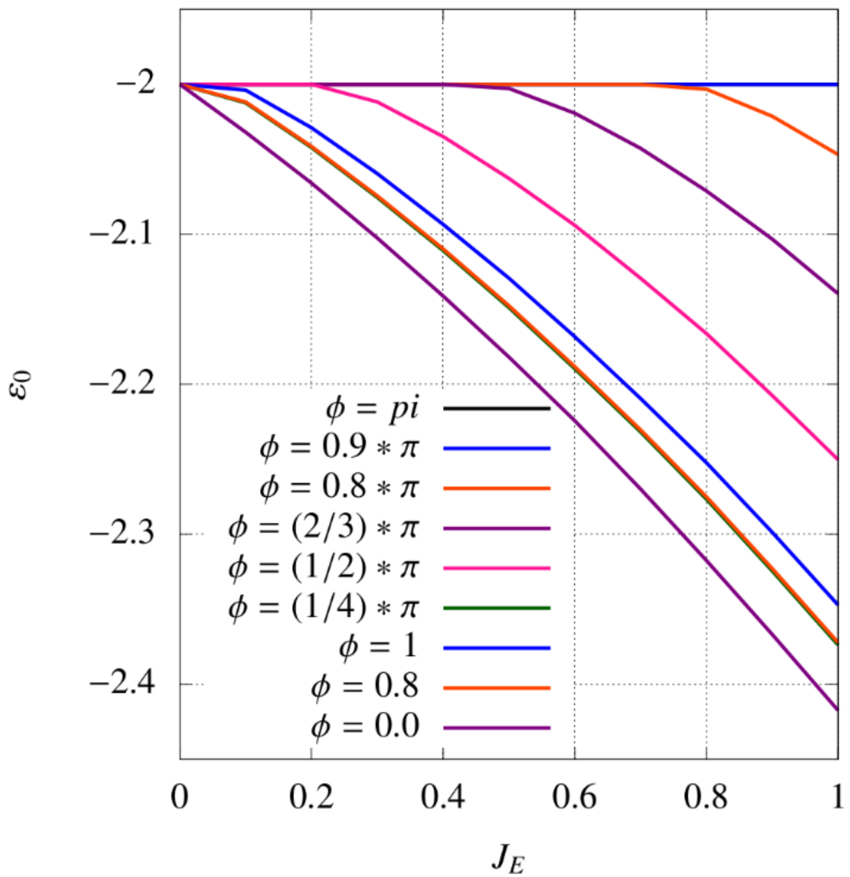
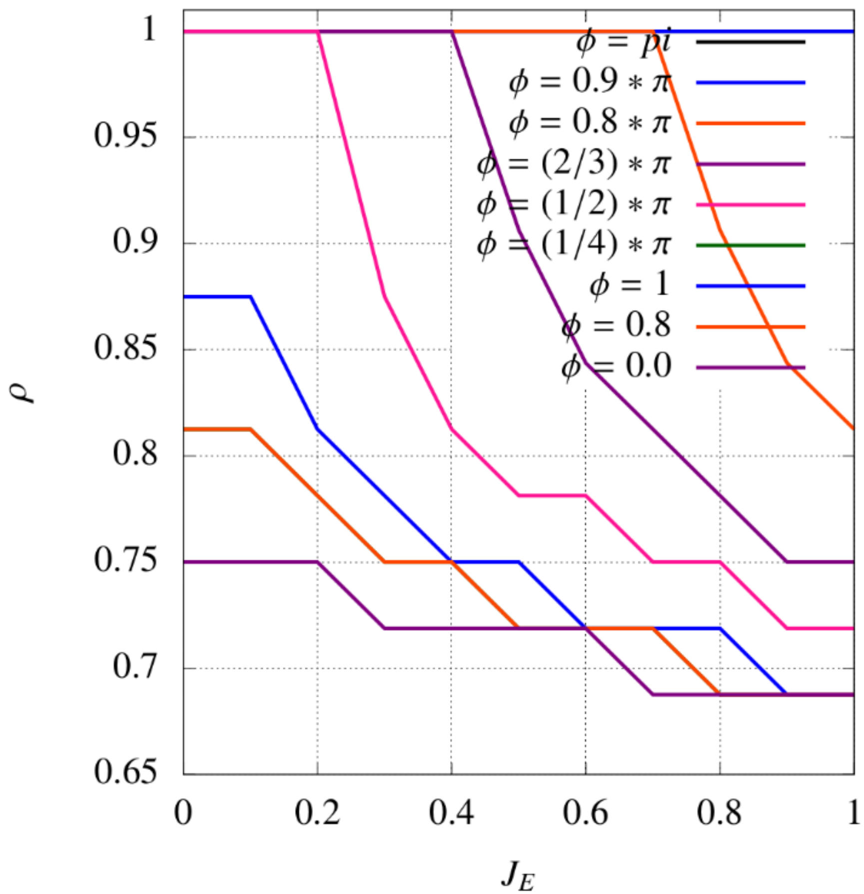
$J_\perp \gtrsim J_O$ Plaquette in *n=2* → Band insulator

$J_\perp < J_O$ Plaquette in *n=1* → Imprinted vortex

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

DMRG calculations confirm perturbative expectations Ex. $J_{\perp} = J_O = 1$

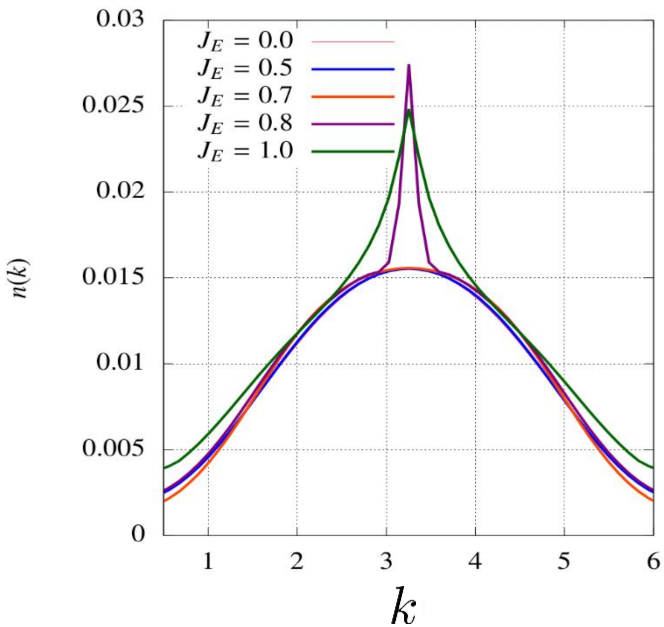


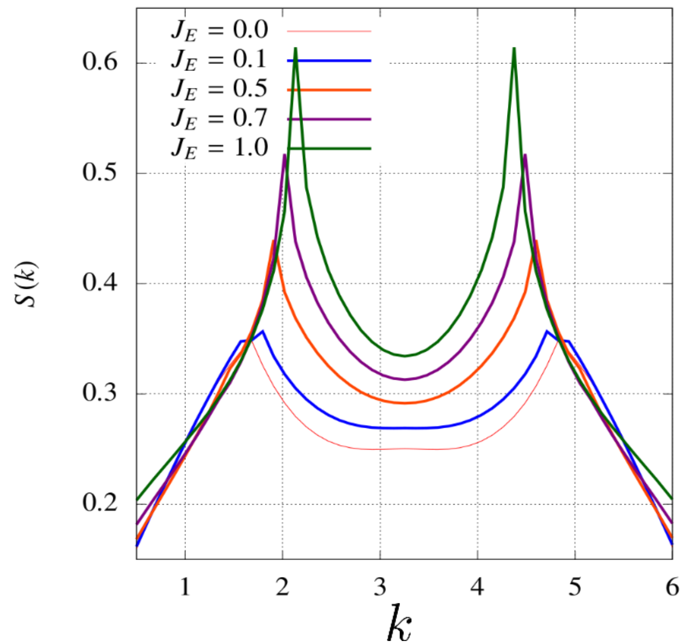


Phase diagram through calculation of currents and structure factors

Vortex Nesting and Melting in synthetic ladders, E. Tirrito, R. Citro, M.Lewenstein, AC *in progress*

$J_O/J_\perp \sim 2.81$ Ex. $\phi = 2\pi/3$





0 → J_E → J_O

Melted vortex → Meissner charge density wave → Meissner

Similar to attractive interleg interaction, [Orignac et al, PRB 96, 014518 (2017)]

Evidence of commensurate-incommensurate transition

Further steps

- No hard-core boson limit: bosons different fermions

- Study the accessible experimental parameters

- Search for “visible” Laughlin-like states in such regimes

  cf. [Calvanese et al, PRX 7, 021033 (2017)],[Petrescu et al, PRB 96, 014524 (2017)]

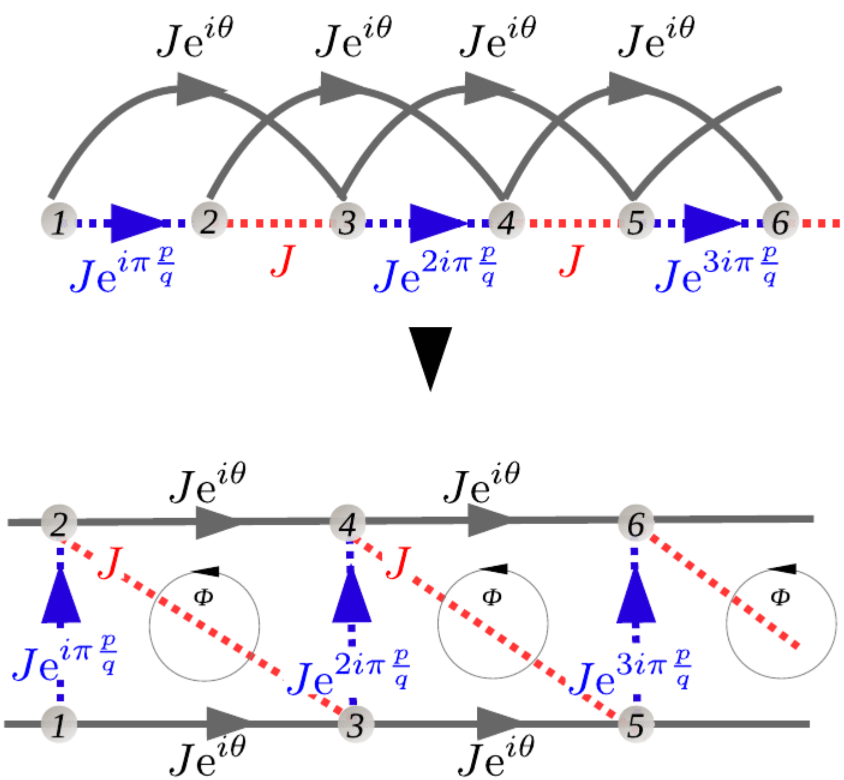
- *…Toy model for many-body localization?*

Alternative route to synthetic interacting ladders… long range interactions!

PHYSICAL REVIEW A **91**, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,[1] Christine Muschik,[1,2,3] Alessio Celi,[1] Ravindra W. Chhajlany,[1,4] and Maciej Lewenstein[1,5]
[1]*ICFO-Institut de Ciències Fotòniques, Av. Carl Friedrich Gauss 3, 08860 Barcelona, Spain*



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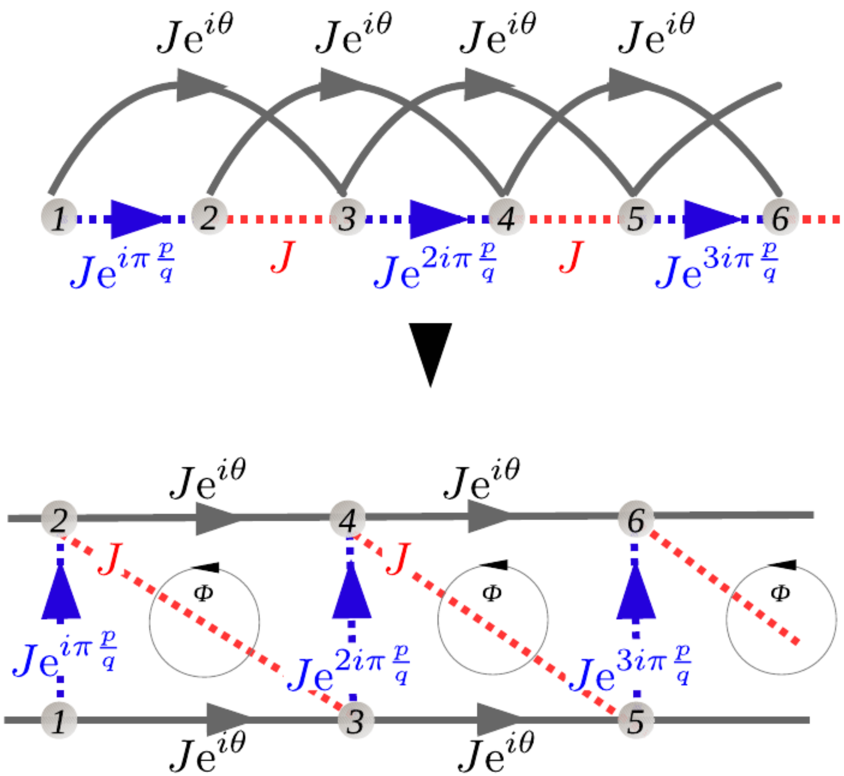
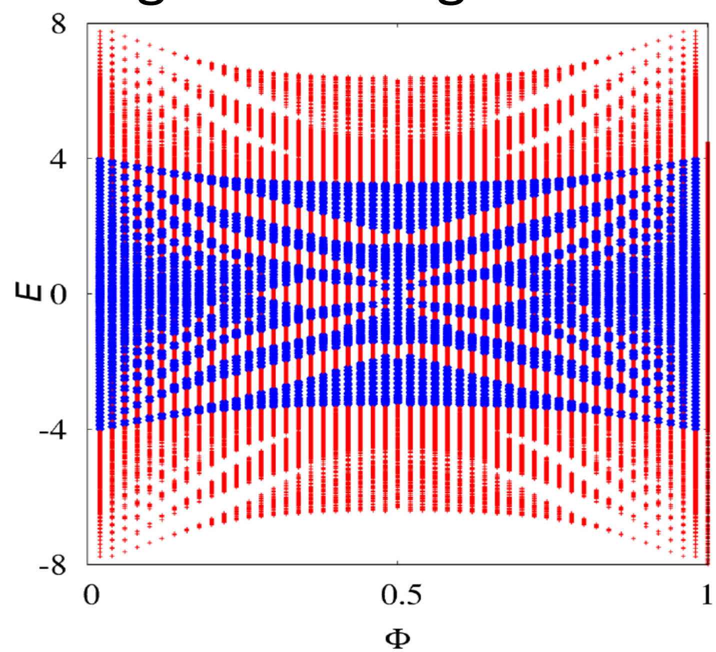


How? E.g. ion analogue simulation

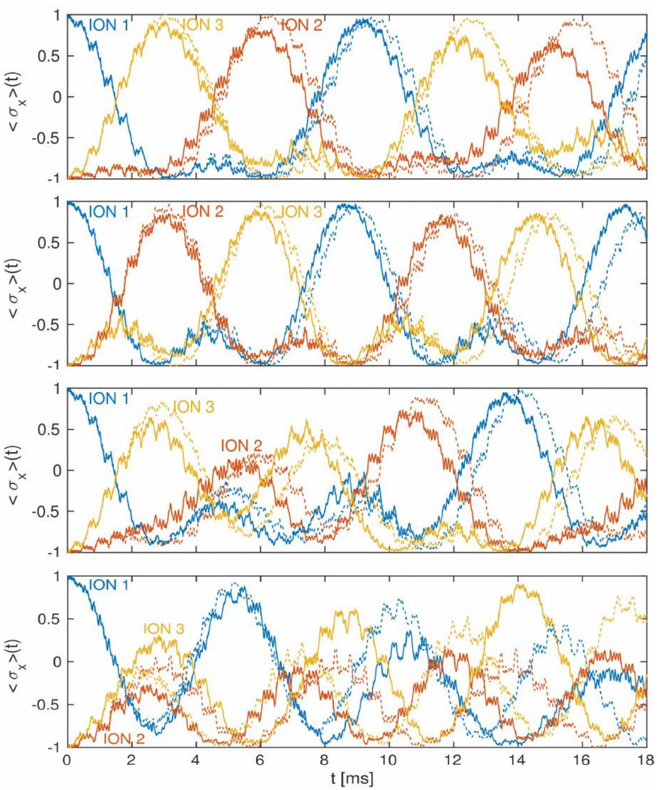


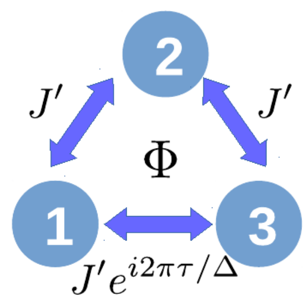
Alternative route to synthetic interacting ladders… long range interactions!

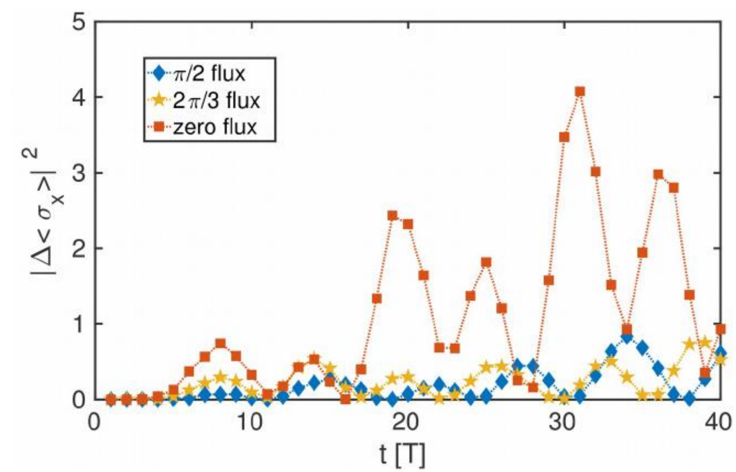
Driven Dicke model with “fluxes” seems robust to photon heating!

$$H_0(t) = \sum_m \hbar\omega_m a_m^\dagger a_m + \sum_{i,m} \hbar\Omega_i \eta_{i,m}(a_m + a_m^\dagger)\sigma_i^x \sin(\omega t) + \sum_i B_i(t)\sigma_i^z$$

Special instance: Triangle! [T. Grass,AC,G. Pagano,M. Lewenstein, arXiv:1708.01882]







Deviation from the effective spin model are suppressed also for strong driving in presence of fluxes! *Topological protection?*

From synthetic to dynamical gauge fields in ultracold atoms

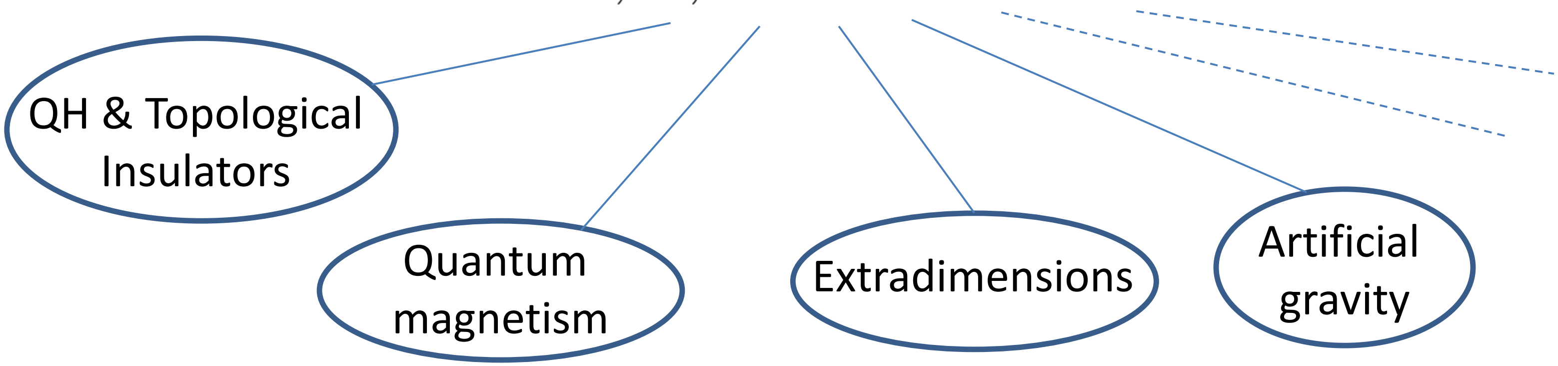
A classical non-dynamical gauge field conf. = fixed Unitary matrix U

Ex.: constant magnetic field (Landau gauge)

$$H = -\sum_{n,m}\left(J a^{\dagger}_{n+1,m} + J' e^{i\Phi n} a^{\dagger}_{n,m+1}\right) a_{n,m} + H.c. + \ldots$$

Simulable with synthetic gauge field in optical lattices

Quantum simulator $J, \Phi, \ldots$ controllable parameters

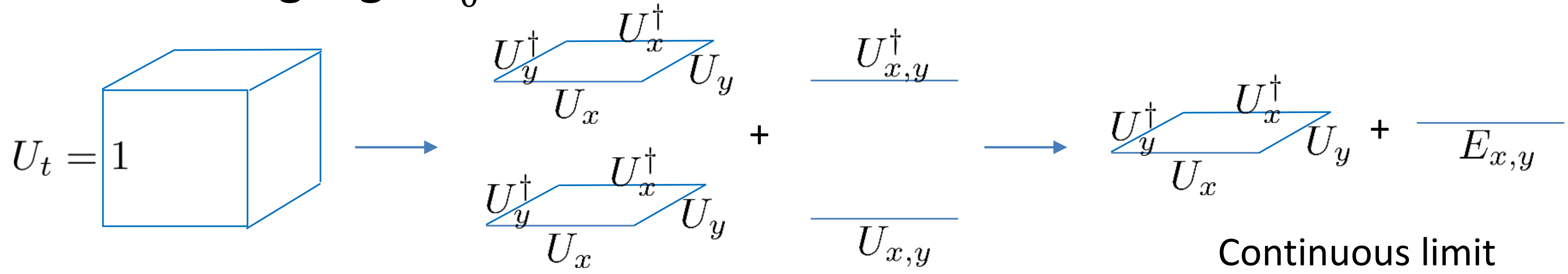


How to get synthetic gauge field dynamical?

Hamiltonian framework *natural* for atoms

Hamiltonian Formulation

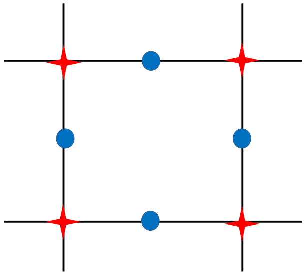
Convenient gauge $A_0=0$ + $\Delta t \rightarrow 0$ [Kogut-Susskind 75, Creutz 77]



Continuous limit

$B^2 + E^2$

Path Integral $\longrightarrow$ Operator approach



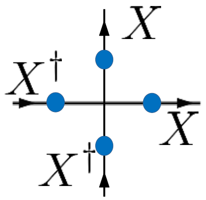
Matter d.o.f. on sites

Gauge d.o.f. on sites

E operators act on links

U operators act on links and matter

Gauge invariance $\longrightarrow$ Gauss law $(\nabla \cdot E = 4\,\pi\,\rho)$



$$X^\dagger X^\dagger X\, X\, |\, l_1\rangle |\, l_2\rangle |\, l_3\rangle |\, l_4\rangle = |\, l_1\rangle |\, l_2\rangle |\, l_3\rangle |\, l_4\rangle$$

Hamiltonian LGT simulation

Recent boom in quantum simulation:

Gauge magnets (link model) ≈ spin formulation of LGT [Horn,Orland,Wiese]

MPQ- Tel Aviv

Zohar… PRL 107 275301 (2011)
109 125302 (2012)
110 055302 (2013)
110 125303 (2013)
….

Innsbruck-Bern

PRL 109 125302 (2012)
110 125304 (2013)
111 110504 (2013)
112 120406 (2014)
PRX 3 041018 (2013)
…..

Platforms:
Rydberg, Earth-alkali atoms
Superconducting qubits
Trapped ions, …

ICFO

Tagliacozzo, AC,… Ann.Phys 330 160 (2013) (Abelian)
Nature Comm. 4 (2013) (Non-Abelian)
….

….

U(1) Gauge magnets = emergent LGT in spin-ice

Hamiltonian LGT simulation

Parallel progresses in classical simulation with tensor networks

Tagliacozzo, Vidal,
PRB 83 115127 (2011)

MPQ-Berlin
Bañuls *et al*
1d Schwinger model
Competitive results
with montecarlo

Gent
PRL 111 091601 (2014)
PRX 5 011024 (2015)
....

Ulm(-Innsbruck)
PRL 112 201601 (2014)
NJP **16** 103015 (2014)
PRB 90 125154 (2014)
....

Osborne
(*webproject)*

......

ICFO

"Tensor Networks for Lattice Gauge Theories with continuous groups",
L. Tagliacozzo, AC, and M. Lewenstein, Phys. Rev. X 4, 041024 (2014)

Quantum Simulation of LGT

Challenge: 4-body interaction at distance not natural for atoms

Two strategies
- Analogue: engineering Hamiltonian
- Digital: engineering time evolution

Gauss Law
- by symmetry
  - Angular momentum cons. [Cirac-Reznik]
  - SU(N) inv. collision [Wiese-Zoller]
- Energy penalty (& dissipation)
  - ICFO….

+

Dynamics
- Electric term easy
- Plaquette hard
  - Analogue: Perturbative (like superexchange)
  - Digital: Trotter decomposition

Quantum Simulation of *LadderGT*

Special cases: 1D encoding (gauge field eliminated) no Gauss law

no plaquette

Ladder encoding (1 link per plaquette) no Gauss law

plaquette

Quantum Simulation of *LadderGT*

Special cases: 1D encoding (gauge field eliminated) no Gauss law 

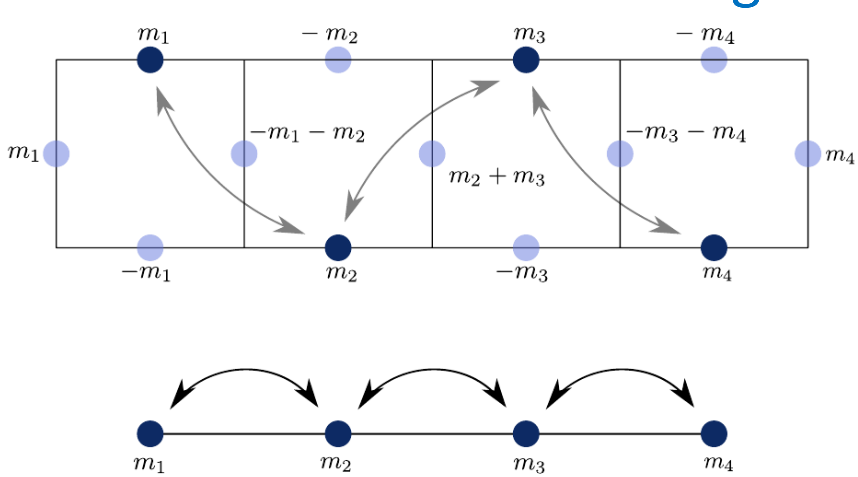
no plaquette 

Ladder encoding (1 link per plaquette) no Gauss law 

plaquette 

= nearest-neighbor interaction chain!

U(1)



$$UUU^\dagger U^\dagger + H.c. \to U + U^\dagger$$

$$\sum_I E_I^2 \to \sum_i E_i E_{i+1}$$

Quantum Simulation of *LadderGT* *(running)*

Simplest experiment: static charges, measure of string tension σ

$$\hat{H} = 2g^2 \sum_{i=1}^{r_1-1} \hat{E}_i^2 + 2g^2 \sum_{i=r_2}^{L} \hat{E}_i^2 + 2g^2 \left(\hat{E}_{r_1} - \frac{q}{2}\right)^2 + 2g^2 \sum_{i=r_1+1}^{r_2-1} \left(\hat{E}_i - \frac{3q}{4}\right)^2 + g^2 \sum_{i=1}^{L-1} \hat{E}_i \hat{E}_{i+1} - \frac{1}{2g^2} \sum_{i=1}^{L} \left(\hat{U}_i + \hat{U}_i^\dagger\right) + g^2 q^2 \frac{7-R}{8}$$

Behavior under truncation?

Weak coupling limit? (still $\sigma \approx \exp[-1/g^2]$?)

Gap and σ related also at strong coupling?

Quantum Simulation of *LadderGT* *(running)*

Simplest experiment: static charges, measure of string tension σ

$$\hat{H} = 2g^2 \sum_{i=1}^{r_1-1} \hat{E}_i^2 + 2g^2 \sum_{i=r_2}^{L} \hat{E}_i^2 + 2g^2 \left(\hat{E}_{r_1} - \frac{q}{2}\right)^2 + 2g^2 \sum_{i=r_1+1}^{r_2-1} \left(\hat{E}_i - \frac{3q}{4}\right)^2 + g^2 \sum_{i=1}^{L-1} \hat{E}_i \hat{E}_{i+1} - \frac{1}{2g^2} \sum_{i=1}^{L} \left(\hat{U}_i + \hat{U}_i^\dagger\right) + g^2 q^2 \frac{7-R}{8}$$

Behavior under truncation?

Weak coupling limit? (still $\sigma \approx \exp[-1/g^2]$?)

DMRG and analytics

Gap and σ related also at strong coupling?

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$$\hat{H} = 2g^2 \sum_{i=1}^{r_1-1} \hat{E}_i^2 + 2g^2 \sum_{i=r_2}^{L} \hat{E}_i^2 + 2g^2 \left(\hat{E}_{r_1} - \frac{q}{2}\right)^2 + 2g^2 \sum_{i=r_1+1}^{r_2-1} \left(\hat{E}_i - \frac{3q}{4}\right)^2$$
$$+ g^2 \sum_{i=1}^{L-1} \hat{E}_i \hat{E}_{i+1} - \frac{1}{2g^2} \sum_{i=1}^{L} \left(\hat{U}_i + \hat{U}_i^\dagger\right) + g^2 q^2 \frac{7-R}{8}$$

Implementation: usual angular momentum -> Schwinger boson rep.

$$\hat{E}_i = \lim_{l\to\infty} \hat{L}_i^z, \quad \hat{U}_i = \lim_{l\to\infty} \frac{\hat{L}_i^+}{\sqrt{l(l+1)}}, \quad \hat{U}_i^\dagger = \lim_{l\to\infty} \frac{\hat{L}_i^-}{\sqrt{l(l+1)}}$$

$$\hat{L}_i^z = \frac{1}{2}\left(\hat{a}_i^\dagger \hat{a}_i - \hat{b}_i^\dagger \hat{b}_i\right), \quad \hat{L}_i^+ = \hat{a}_i^\dagger \hat{b}_i, \quad \hat{L}_i^- = \hat{b}_i^\dagger \hat{a}_i, \quad l = \left(\hat{a}_i^\dagger \hat{a}_i + \hat{b}_i^\dagger \hat{b}_i\right)$$

On-site and nearest-neighbor interactions!

Summary

- Synthetic edge state in synthetic Hofstadter strips
- “Bulk topology” in synthetic Hofstadter strips
- Effect of dimerization in synthetic Hofstadter ladder w/o interactions
- Synthetic fluxes in driven ion chain
- Encoding plaquettes in local terms in ladders…

"Extradimensional" collaborators


J.I. Latorre


O. Boada


M. Lewenstein

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O. Boada

M. Lewenstein

T. Grass

“Extradimensional” collaborators


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O. Boada


M. Lewenstein


T. Grass


G. Juzeliunas


P. Massignan


J. Ruseckas


N. Goldman


I.B. Spielman

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R.W. Chhajlany

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S. Mugel


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A. Dauphin


L. Tarruell

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J. Asboth


C. Lobo


A. Dauphin


L. Tarruell


E. Tirrito


R. Citro

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M. Lewenstein


T. Grass

Atomic implementation


G. Juzeliunas


P. Massignan


J. Ruseckas


N. Goldman


I.B. Spielman


J. Rodriguez-Laguna


C. Muschik


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S. Mugel


J. Asboth


C. Lobo


A. Dauphin


L. Tarruell


E. Tirrito


R. Citro

....

Quantum and TN LGT simulation

M. Lewenstein

L. Tagliacozzo

P. Orland

A. Zamora

M. Mitchell

"LadderGT" collaborators

D. González-Cuadra

E. Tirrito

“LadderGT” collaborators

D. González-Cuadra

E. Tirrito

Atomic implementation

“Quantum guitar” (ions)

C. Muschik