Holographic complexity from topology in AdS_3/CFT_2

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Arxiv 1710.01327
Abt, Erdmenger, Hinrichsen, Melby-Thompson, Northe, Meyer

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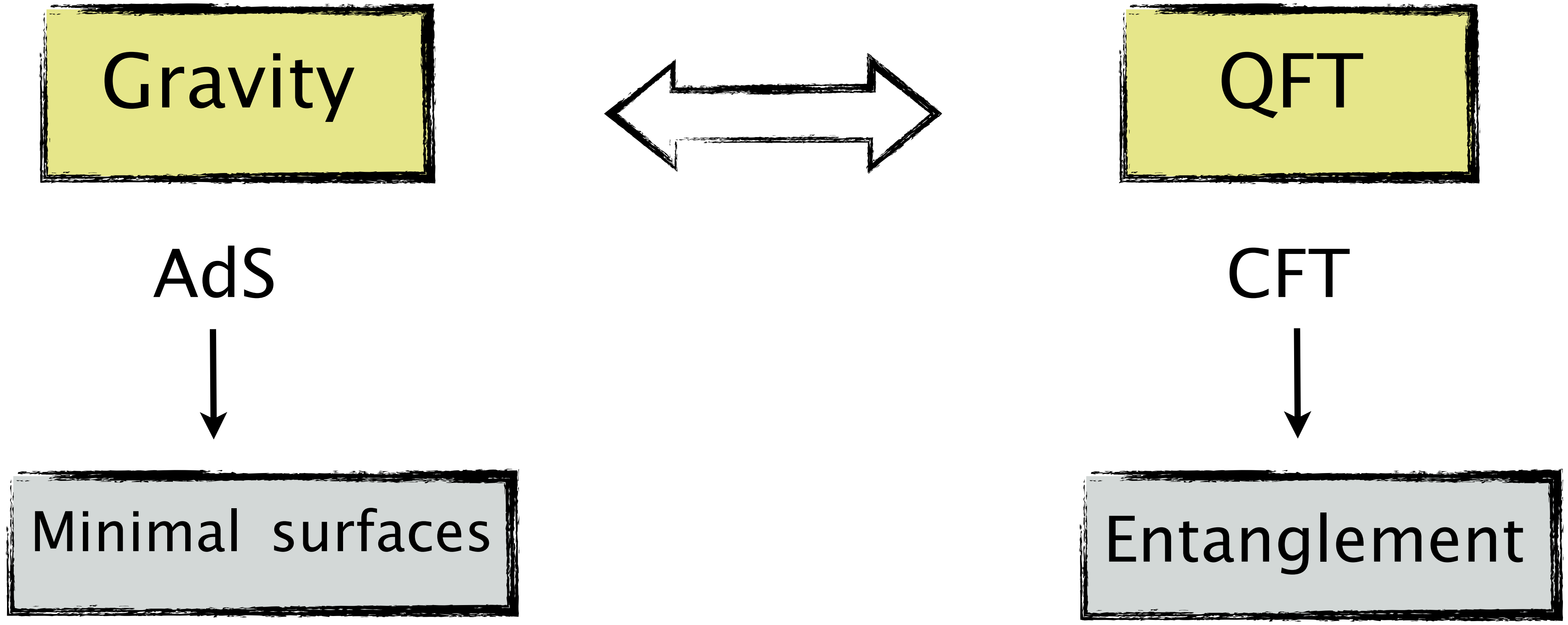
Intro: Holographic subregion complexity

Gravity: Gauss-Bonnet

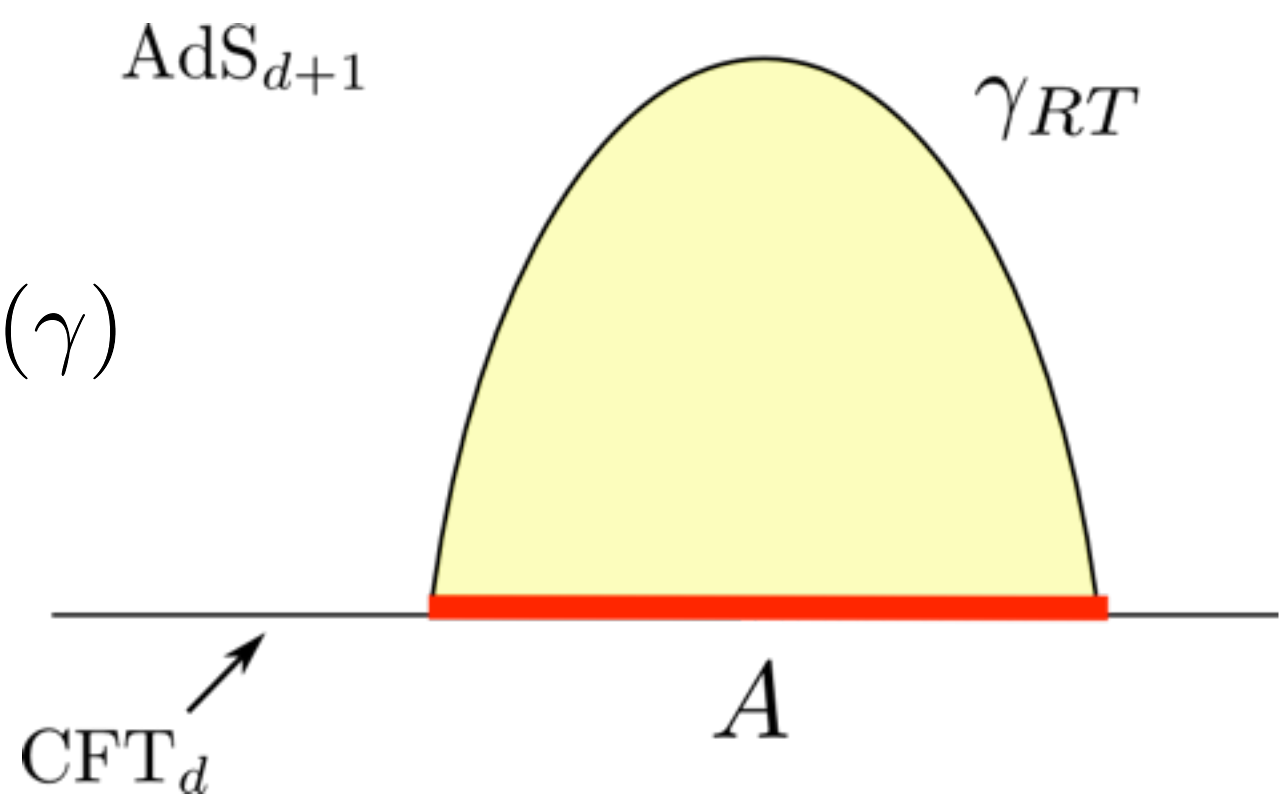
Random Tensor Networks

CFT: Kinematic space



Ryu–Takayanagi '06

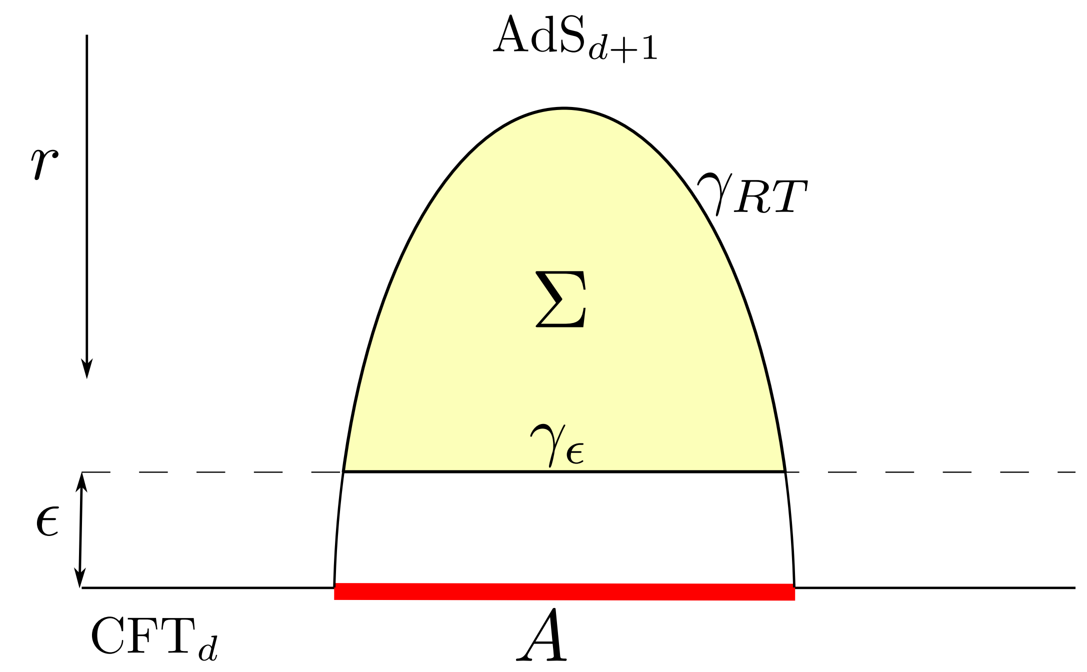
$$S_{EE}(A) = \frac{1}{4G}\min\ \mathrm{Area}(\gamma)$$



Holographic complexity

Alishahiha '15: "Holographic Complexity" for time slice,

$$\mathcal{C}(A) \equiv \frac{\mathrm{Vol}(\Sigma)}{GL}$$



We propose in AdS3/CFT2,

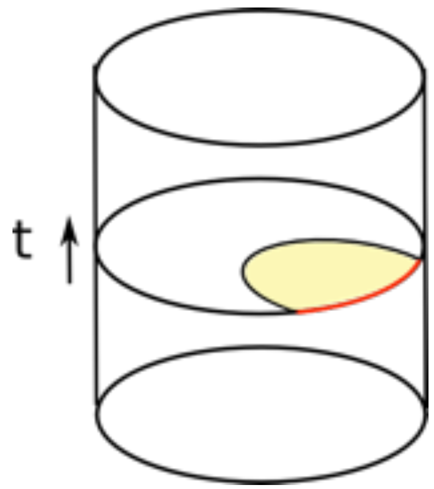
$$\mathcal{C}(A) \equiv -\frac{1}{2}\int_\Sigma R\, d\sigma \quad \sim V(\Sigma) \text{ if } R = \text{const}$$

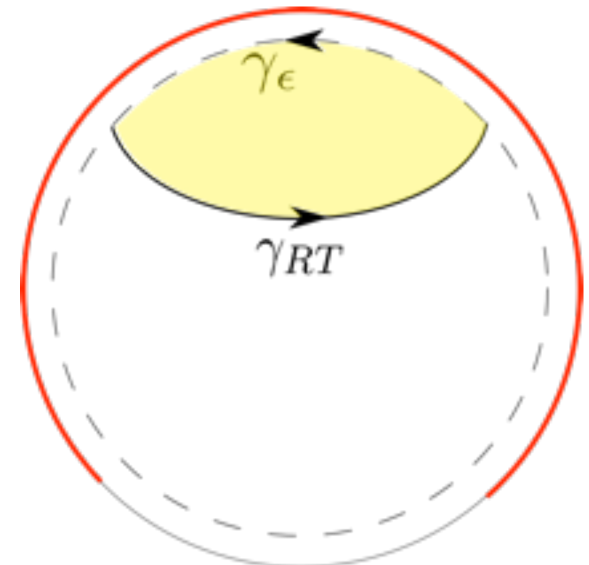
What system?

Gravity

Zero temperature

Constant time slice of AdS3, single interval





Gauss Bonnet $\mathcal{C}(A) = -\frac{1}{2}\int_\Sigma R\, d\sigma = \int_{\partial\Sigma} k_g ds - 2\pi\chi(\Sigma)$

k_g : geodesic curvature, $\chi(\Sigma) = 1$

$$\int_{\partial\Sigma} k_g ds = \int_{\gamma_{RT}} k_g ds + \int_{\gamma_\epsilon} k_g ds + \text{corner angles} = \frac{x}{\epsilon} + \pi$$

$$\boxed{\mathcal{C}(x) = \frac{x}{\epsilon} - \pi}$$

Finite temperature

CFT2 at finite T dual to black hole at same T

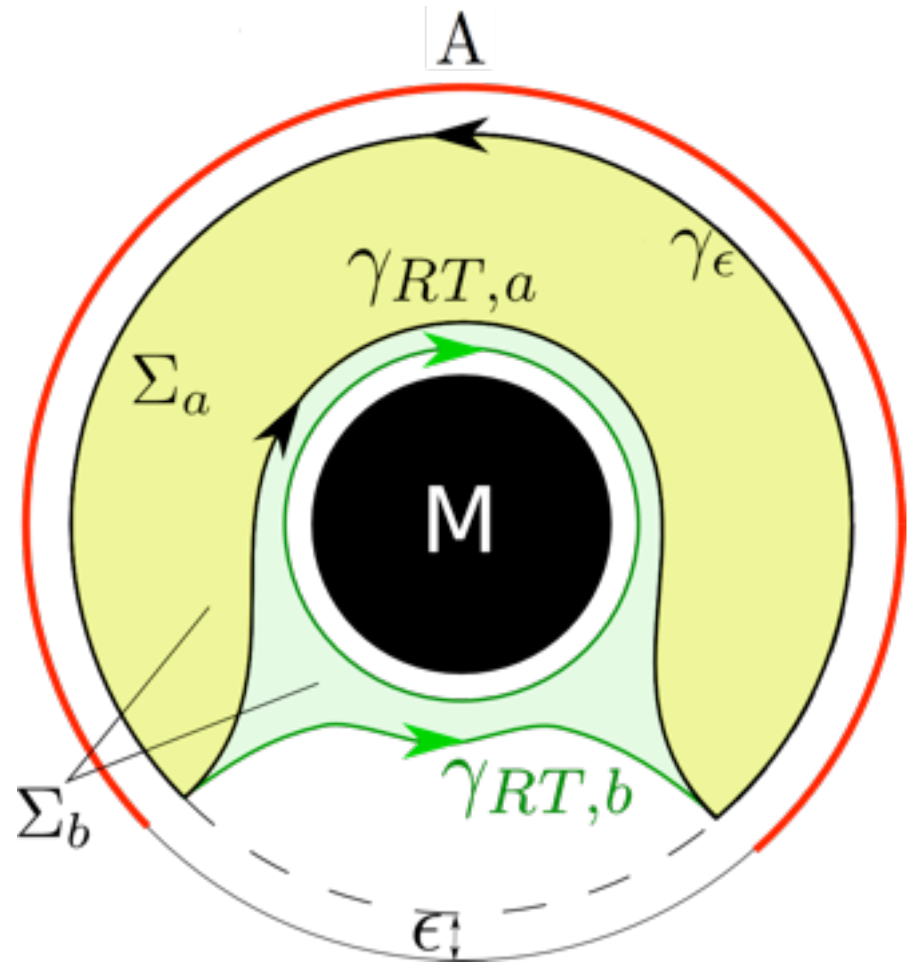
Hubeny et al. '13: two possible phases – γ_{RT} homologous to A

Phase a

identical to AdS

$$\chi(\Sigma_a) = 1$$

$$\Delta\mathcal{C} = 2\pi$$



Phase b

horizon also a geodesic

$$\chi(\Sigma_b) = 0$$

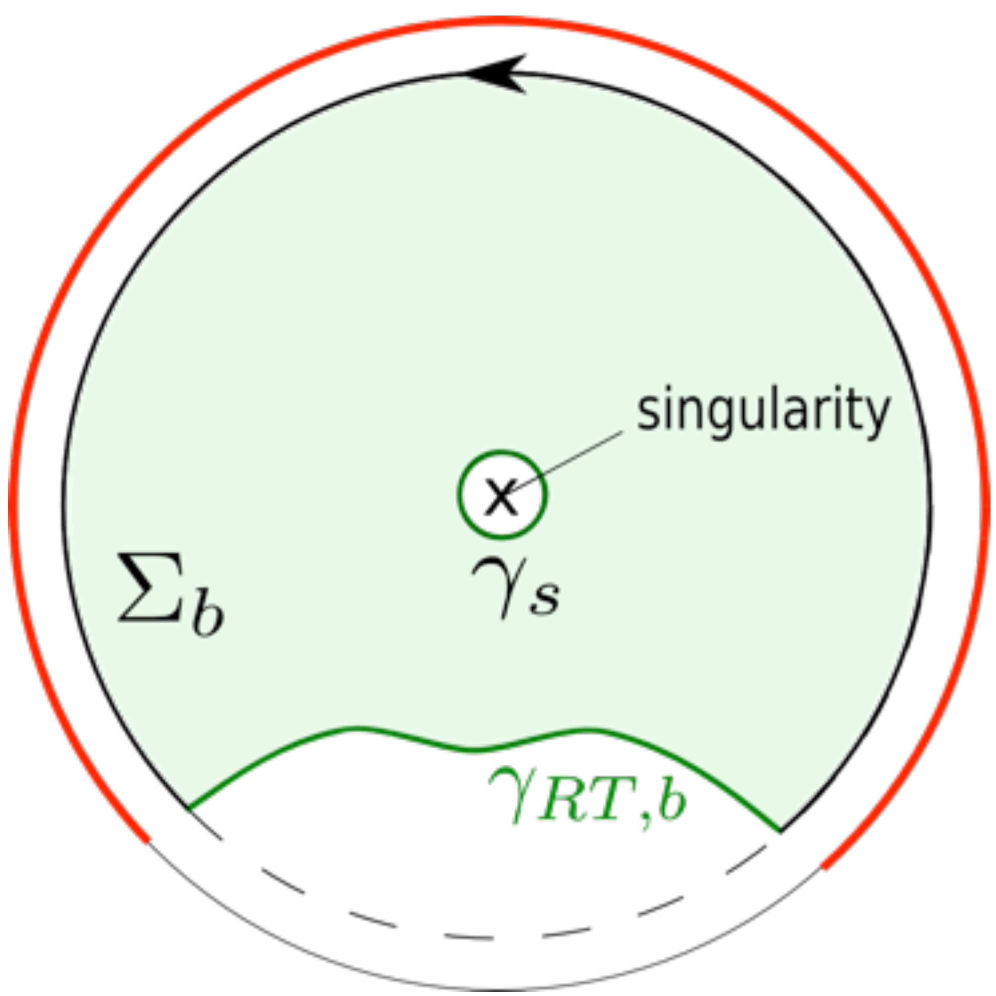
$$\int_{\gamma_{RT}} k_g ds = 0$$

Subregion complexity is temperature independent* in CFT2

Naked singularities

At M=0 horizon disappears

-1<M<0 correspond to conical singularities



Enforce homology condition on RT

↓

cut small ring γ_s around singularity

$$\oint_{\gamma_s} k_g ds = 2\pi\sqrt{f(r)} \xrightarrow[r\to 0]{} 2\pi\sqrt{-M} \quad , \; M<0$$

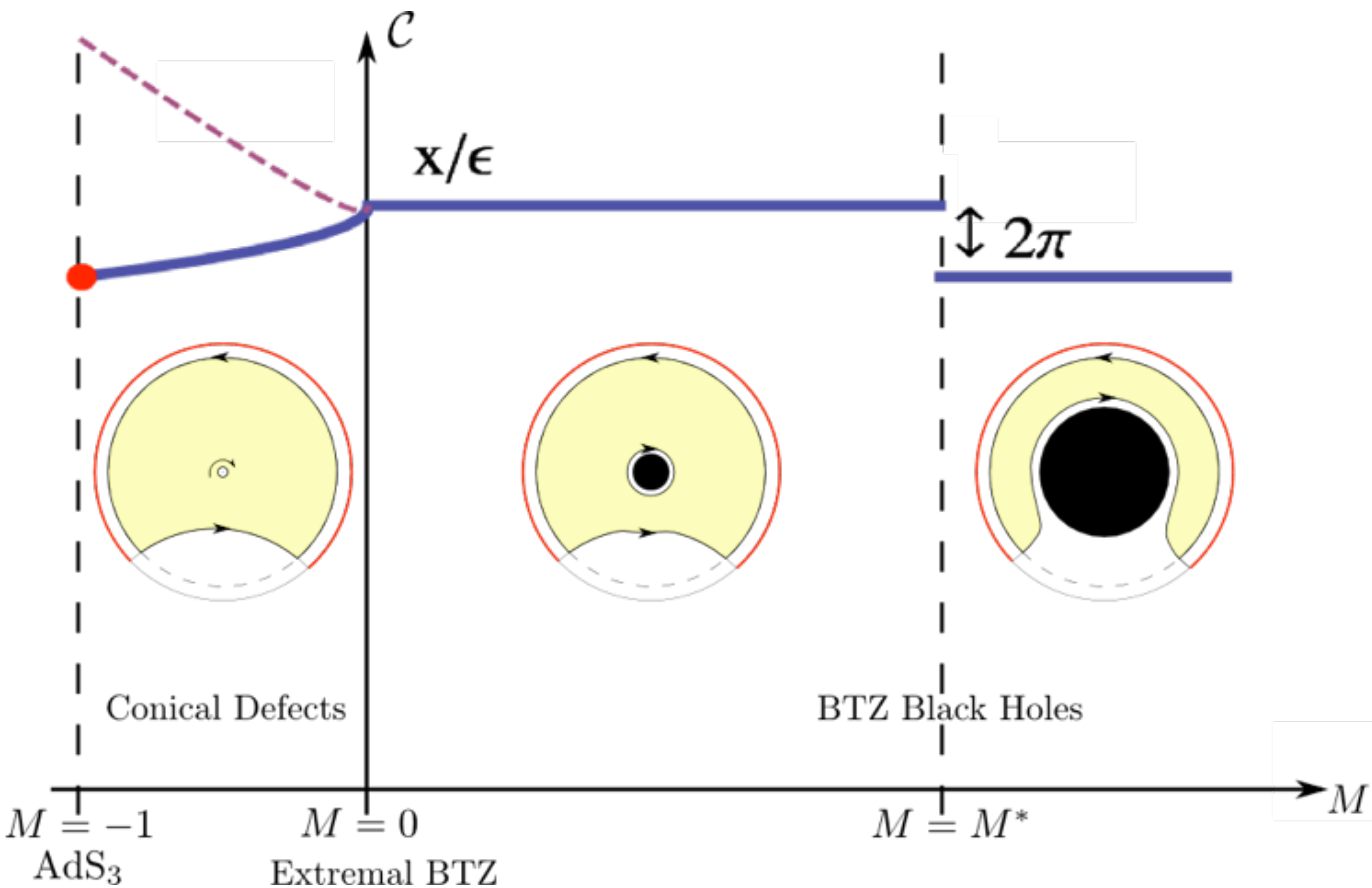
Complexity is M-dependent:

$$\mathcal{C} = \frac{x}{\epsilon} + \pi - 2\pi\sqrt{-M}$$

Homology condition of RT is crucial to get correct answer!

Gravity side

Complexity as function of mass, for fixed entangling region



Tensor Networks

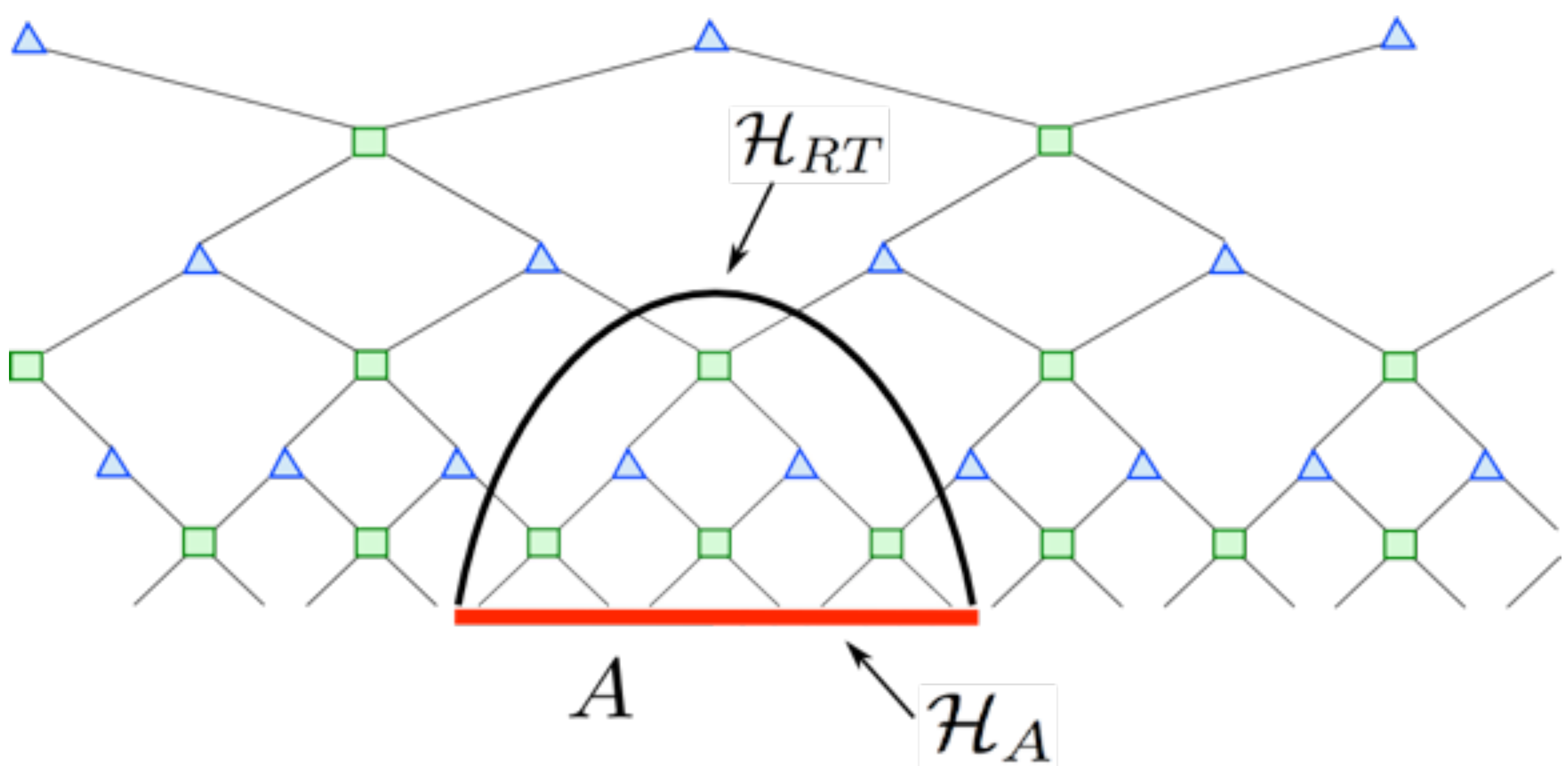
Tensor Networks

AdS/MERA: Swingle '09 MERA → lower bound for EE → RT

Complexity: min. number gates to achieve some task

Holographic complexity $\implies$ what task?

Compression: represent ρ_A in lowest dim. Hilbert space



$$S_E(A) \leq \log(d)|\gamma_A|$$

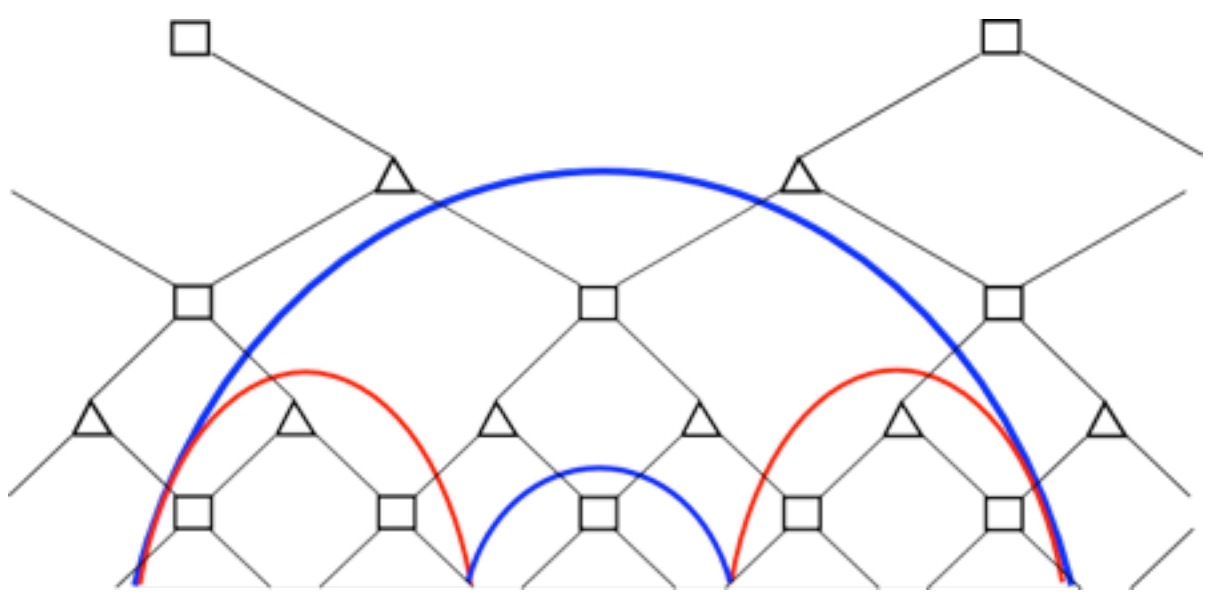
$\Rightarrow$ Complexity of running the renormalisation procedure

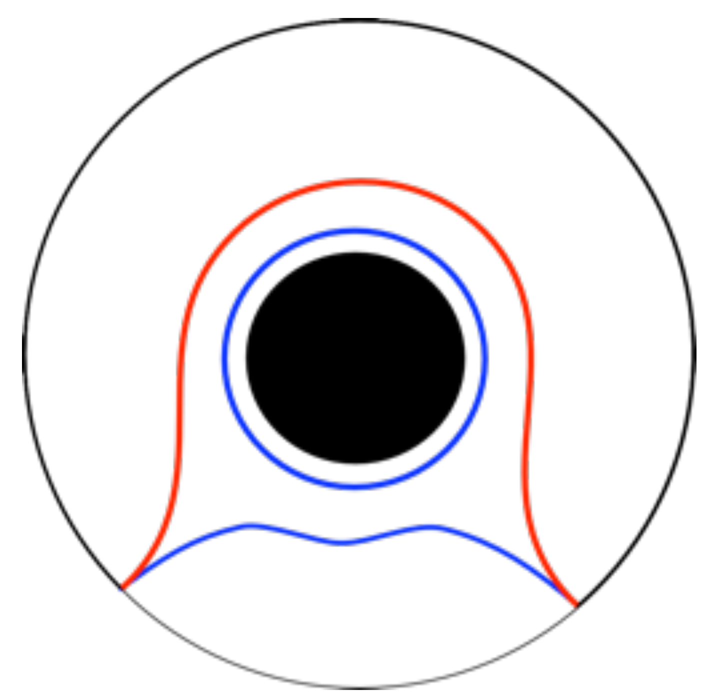
$$\mathcal{C}(A) \equiv -\frac{1}{2}\int_{\Sigma} R\, d\sigma$$

counts number of tensors inside RT surface

RT transition: still compress to same size, but it costs more.

Reordering of information is topologically different.





Random Tensor Networks

Random Tensor Networks [1601.01694] (M. Walter's talk) saturate the RT formula

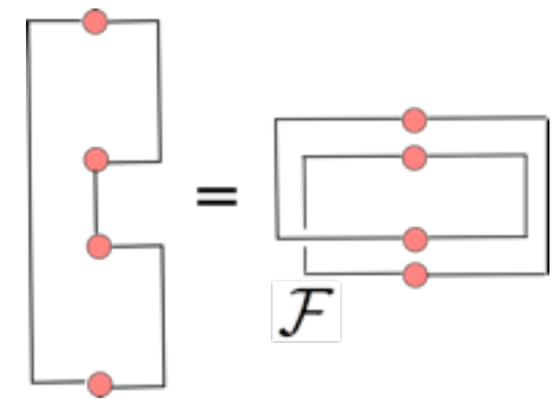
Tessellation of hyperbolic plane

Compute second Rényi entropy

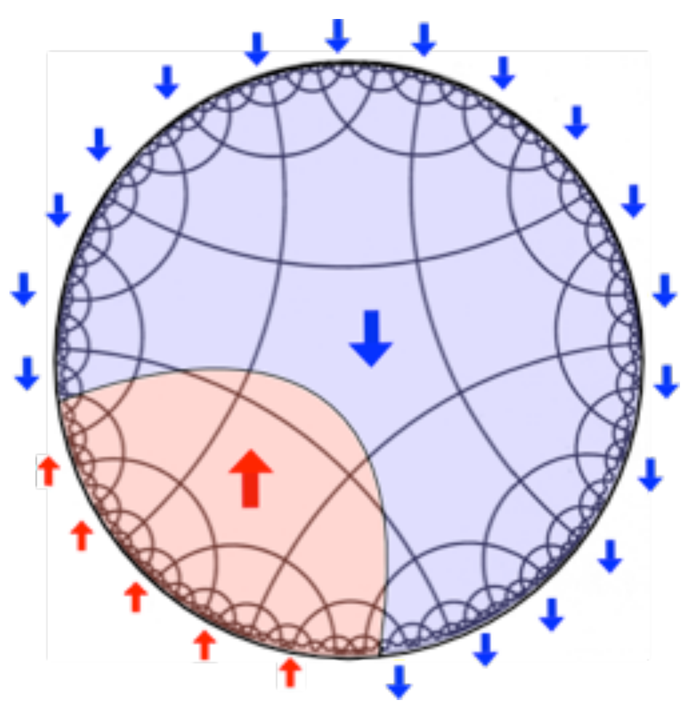
Average over random tensors

$$\overline{|V_x\rangle\langle V_x| \otimes |V_x\rangle\langle V_x|} \sim I_x + \mathcal{F}_x$$

Result: $\overline{S_A^{(2)}} = F_1 - F_0$



'Swap trick'

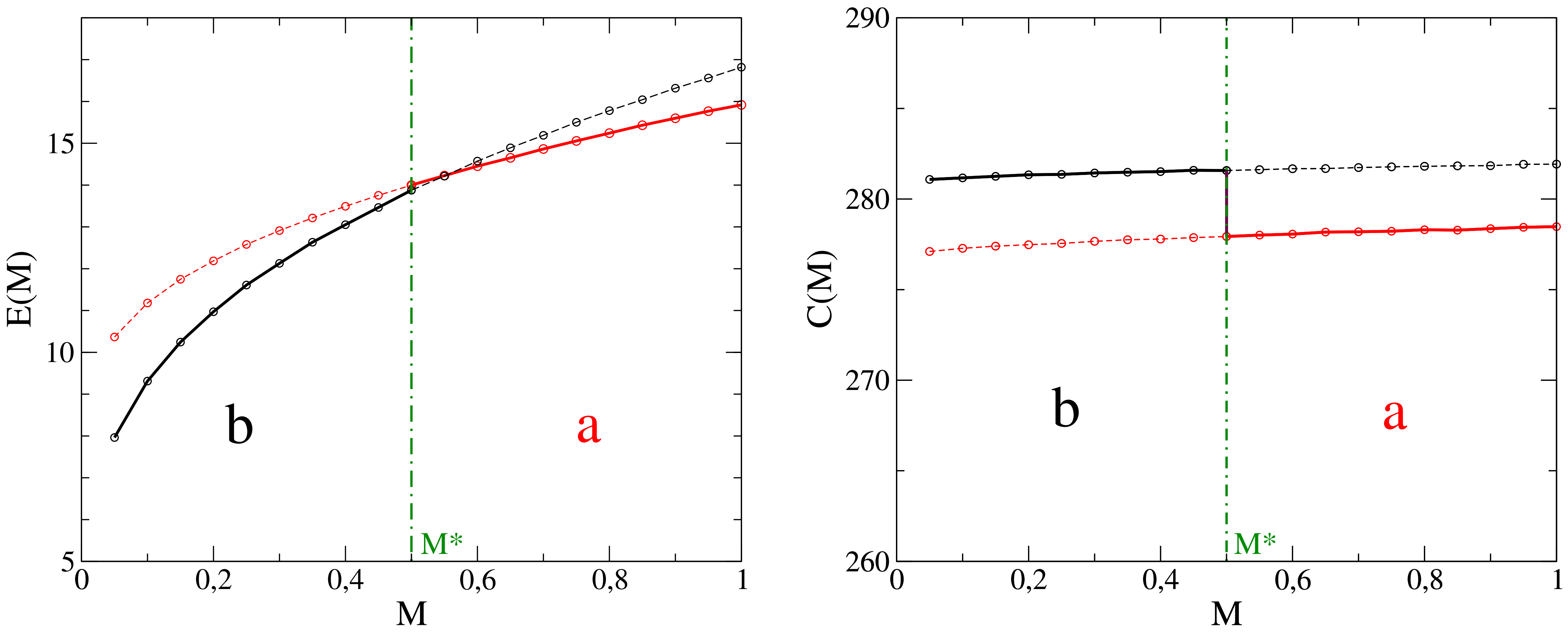


Numerical simulation

To mimic BH metric, we work with variable bond dimension that reproduces the finite temperature EE

Complexity is mapped to magnetization

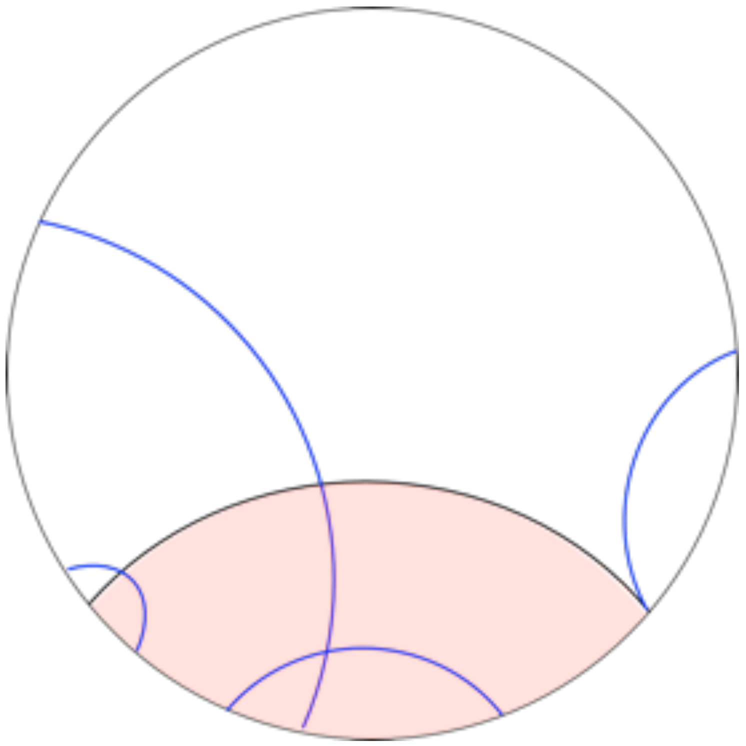
Deviations due to lattice effects



CFT: kinematic space

Czech et al ’15:

Reconstructing the bulk using the EE of boundary



How are entanglement and subregion complexity related?
→ Probe subregion with other subregions

e.g. for the entire circle,

$$\mathcal{C}(\text{circle}) = -\frac{1}{2}\int_0^{\pi} d\alpha\ S(\alpha)\partial_\alpha^2 S(\alpha)$$

Conclusions

Compression complexity in CFT2 is T- independent/‘quantized’

Numerical study → RTN

New CFT construction through kinematic space

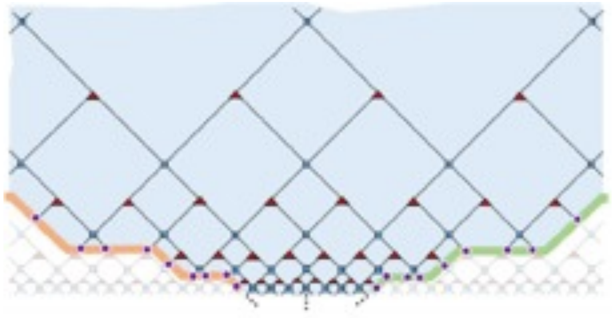
Outlook

Curvature proposal: non-constant examples

Compression algorithm → T-dependence?

How does CFT know about topology?

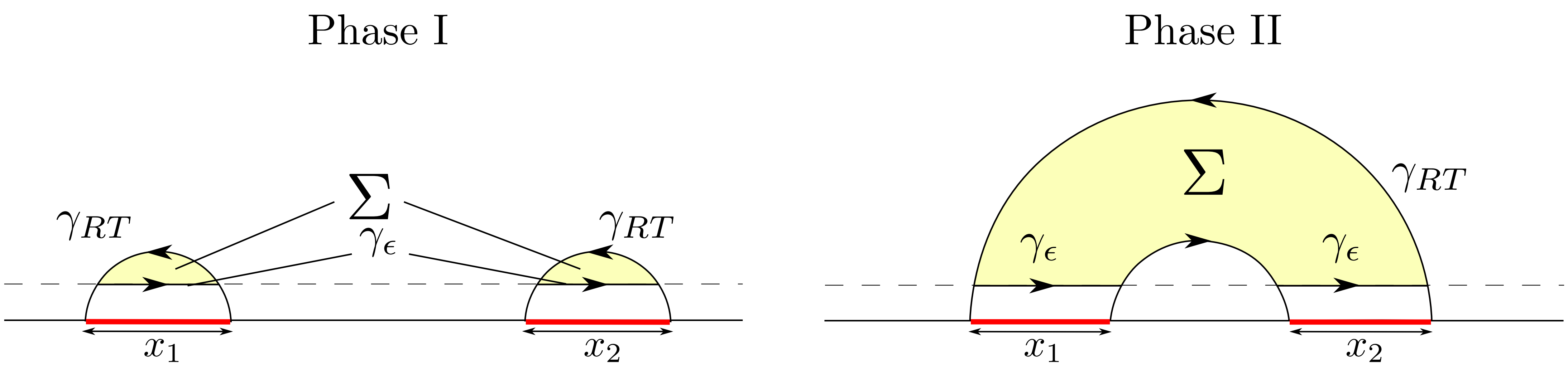
Entanglement ⟺ Subregion complexity



Transitions: zero temperature

The RT surface can have multiple 'phases': the global minimal surface gives the correct answer

Two intervals in AdS



$$\mathcal{C} = \int_{\partial\Sigma} k_g ds - 2\pi\chi(\Sigma)$$

$$\Delta\mathcal{C} = \mathcal{C}_{II} - \mathcal{C}_I = -2\pi\Delta\chi = 2\pi$$