Stochastic cosmic ray sources and the TeV break in the all-electron spectrum

arXiv:1809.?????

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TeVPA 2018, Berlin
30 August 2018

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Cosmic ray $e^{\pm}$ and cosmic ray origin

What are the sources of cosmic rays?

No source of **local** cosmic rays has been unambigously identified.

Nuclei

- Far away and old sources can contribute

→ Features from individual sources averaged out

- Use anisotropies?

Difficult!

Electrons and positron

- Only young nearby sources contribute at high energies due to energy losses

→ Can observe features from individual sources

Find sources with high-energy $e^{\pm}$

Green's function

- Solve simplified transport equation for $e^{\pm}$ spectral density ψ

$$\frac{\partial \psi}{\partial t} - \nabla \cdot \kappa \cdot \nabla \psi + \frac{\partial}{\partial p} \left(b(p) \psi \right) = \delta(\boldsymbol{r} - \boldsymbol{r}_0) \delta(t - t_0) Q(p)$$

- For spatially independent κ and $b(p)$, energy becomes pseudo time

$\rightarrow$ Solve heat equation:

$$\psi(\boldsymbol{r}, t, p) = \left(\pi \ell^2(p, t) \right)^{-3/2} \mathrm{e}^{-|\boldsymbol{r} - \boldsymbol{r}_0|^2 / \ell^2(p,t)} \frac{b(p)}{b(p_0(p, t))} Q\left(p_0(p, t) \right)$$

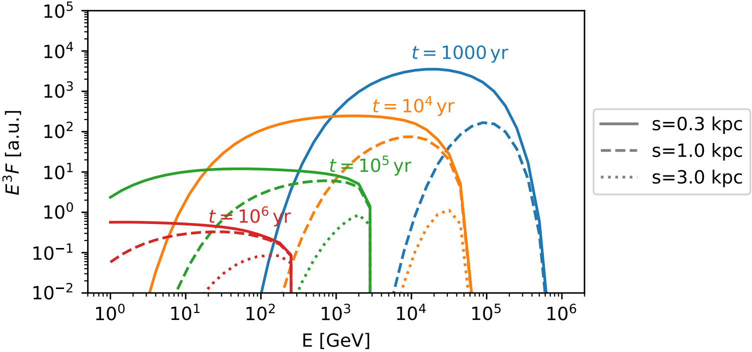
where

$$\ell^2(p, t) = 4 \int_{p_0(p,t)}^{p} \mathrm{d}p' \, \frac{D_{xx}(p')}{b(p')}$$

- Boundary condition in z-direction can be treated by method of mirror sources

Green's function

$$\psi(\boldsymbol{r}, t, p) = \left(\pi \ell^2(p, t)\right)^{-3/2} \mathrm{e}^{-|\boldsymbol{r}-\boldsymbol{r}_0|^2/\ell^2(p,t)} \frac{b(p)}{b(p_0(p,t))} Q\left(p_0(p,t)\right)$$



with $D_{xx} = D_0 p^\delta,\ Q(p_0) \propto p_0^{-\Gamma} \exp[-p_0/p_c]$

Green's function

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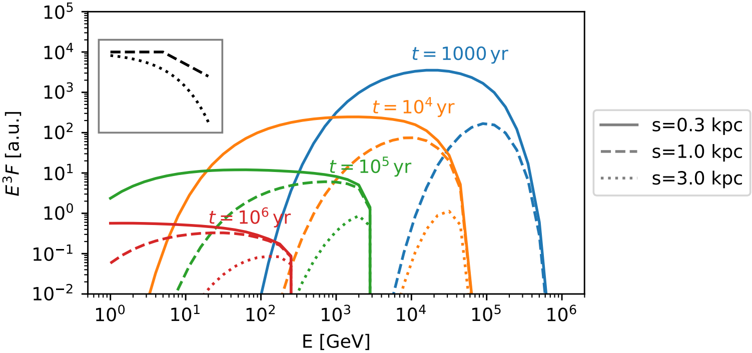

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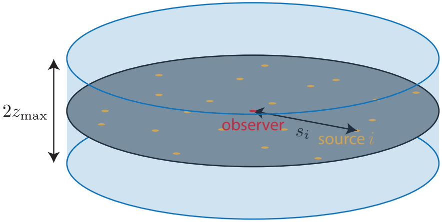
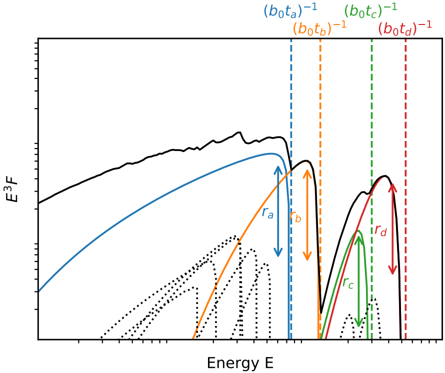
Flux from a population of sources



consider ensemble of sources at distances $\boldsymbol{r}_i$ and with ages t_i

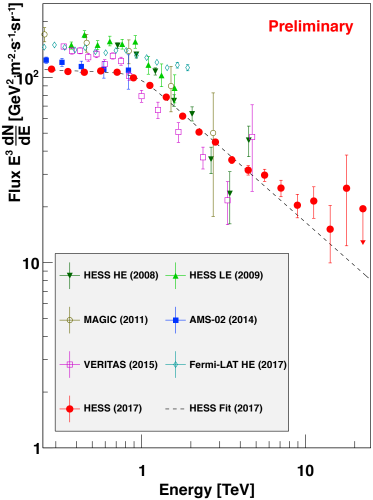


Ignorance of $\boldsymbol{r}_i$ and $t_i \Rightarrow$ cannot predict $e^{\pm}$ spectrum

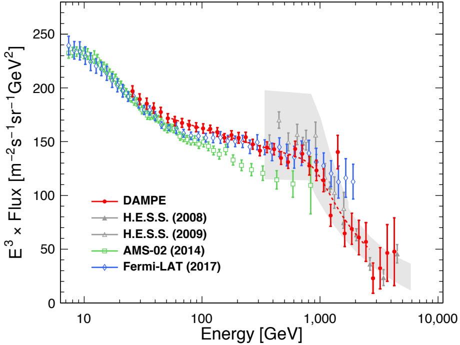
measure $e^{\pm}$ spectrum $\Rightarrow$ learn about $\boldsymbol{r}_i$ and t_i

The TeV break

Kerszberg *et al.*, ICRC 2017

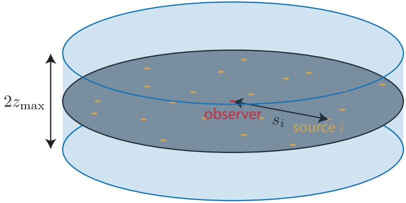


Ambrosi *et al.* (2017)



Is the TeV break compatible with a random ensemble of sources?

A statistical model



- Contribution from source i to ϕ_k depends on distance s_i and age t_i

$\rightarrow$ Spectrum is a random vector: $\boldsymbol{\phi} = \sum_i \boldsymbol{\phi}^i = (\phi_1, \phi_2, \ldots \phi_N)^T$

- Statistically characterised by joint distribution $f(\phi_1, \phi_2, \ldots \phi_N)$

Applications

1. Likelihood of a model: evaluate $f(\hat{\phi}_1, \hat{\phi}_2, \ldots \hat{\phi}_N)$ for measured $\hat{\boldsymbol{\phi}}$
2. Extrapolate to higher energies: $f(\phi_{M+1}, \ldots \phi_N | \phi_1, \phi_2, \ldots \phi_M)$
3. Quickly generate samples from model, e.g. for forecasting

Marginals and copula

What is the joint distribution?

- Non-parametric, e.g. kernel-density estimators?
  $\rightarrow$ curse of dimensionality
- Multi-variate Gaussian?
  $\rightarrow$ would give Gaussian marginals (see below)

Marginals and copula

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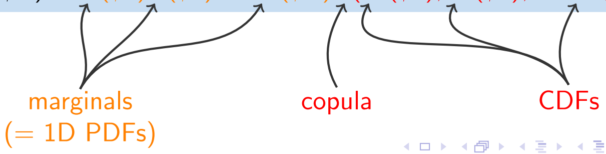
  $\rightarrow$ would give Gaussian marginals (see below)

$\rightarrow$ Use **copulas** to factorise problem:

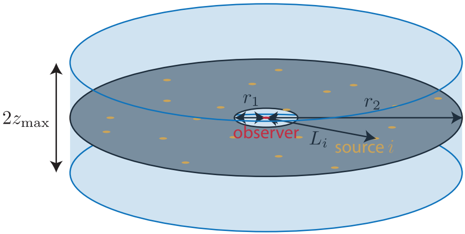
- Multi-variate PDF on unit hypercube
- Have uniform marginals
- Encode correlations

Sklar's theorem

$$f(\phi_1, \phi_2, \ldots \phi_N) = f_1(\phi_1) f_2(\phi_2) \ldots f_N(\phi_N)\, c(F_1(\phi_1), F_2(\phi_2), \ldots F_N(\phi_N))$$



Marginals: analytical approach



- Total flux is sum of fluxes from individual sources

$$J(E) = \sum_{i=1}^{N} J_i(E) = \frac{c}{4\pi} \sum_{i=1}^{N} G(E, t_i, r_i)$$

- r_i and t_i are random variables $\Rightarrow Z_i = G(E, t_i, r_i)$ is a random variable
- What is $f_Z(z)$? Central limit theorem?
- $\langle J \rangle = \frac{c}{4\pi} \langle Z \rangle$ is the flux from smooth distribution of sources.

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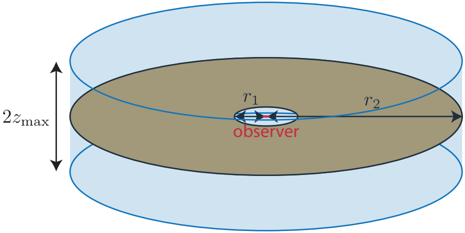

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Diverging variance

Lee (1979), Lagutin & Nikulin (1995), Ptuskin *et al.* (2006), PM (2010), Genolini *et al.* (2016)

- $\langle Z^2 \rangle$ diverges:

$$\langle Z^m \rangle = \frac{1}{t_{\max}} \frac{1}{r_2^2 - r_1^2} \frac{(4D_0)^{1-\frac{3}{2}m}}{(b_0(1-\delta))^{2-\frac{3}{2}m}} \frac{Q_0^m}{m\pi^{\frac{3}{2}m}} E^{-2+\delta+\frac{3}{2}m(1-\delta)-m\Gamma}$$
$$\times \int_0^1 \mathrm{d}\lambda^2 (1-\lambda^2)^{\frac{m(\Gamma-2)+\delta}{1-\delta}} (\lambda^2)^{1-\frac{3}{2}m} \left[\mathrm{e}^{-m\Lambda^2/\lambda^2} \right]_{\rho_2^2}^{\rho_1^2}$$

where

$$\Lambda^2 = \frac{b_0(1-\delta)}{4D_0} E^{1-\delta} L^2 \quad \text{and} \quad \rho_i^2 = \frac{b_0(1-\delta)}{4D_0} E^{1-\delta} r_i^2$$

- Z^m with $m \geq 2$ increases faster with $r \to 0$ than density of sources falls off
- cannot apply central limit theorem
- introduce minimum distance $r_{\min}$?!

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Stable law

PM (2010)

- for $z \to \infty$, $f_Z(z)$ has a power law tail:

$$f_Z(z) \simeq \underbrace{\frac{1}{t_{\mathrm{max}}} \frac{1}{r_{\mathrm{max}}^2} \frac{1}{8\pi^2 D_0} E^{-\delta - \frac{4}{3}\Gamma} Q_0^{4/3}}_{\equiv c^+} z^{-\alpha-1}$$

- Generalised central limit theorem for distributions with power law tail

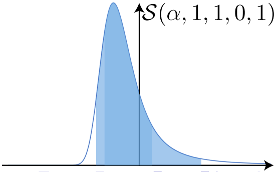
  Gendenko & Kolmogorov (1949)

$$\sum_{i=1}^{N} Z_i \stackrel{N\to\infty}{\longrightarrow} a_N + b_N \mathcal{S}(\alpha, 1, 1, 0, 1)$$

with

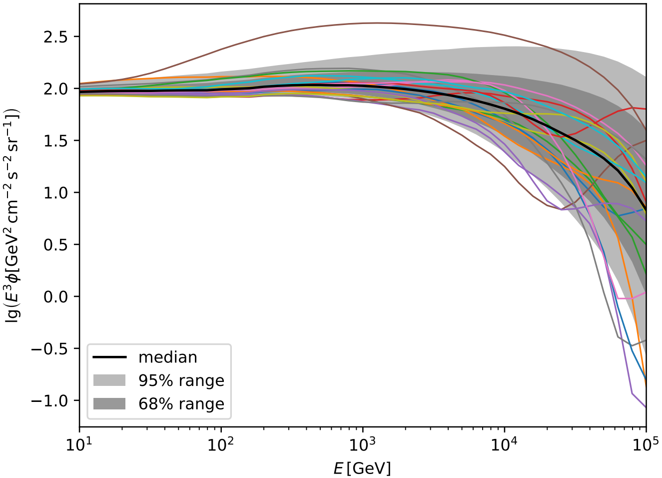
$$a_N = N\langle Z \rangle$$
$$b_N = \left(\frac{\pi c^+}{2\Gamma(1/\alpha)\sin(\pi/2\alpha)} \right)^{\alpha} N^{\alpha}$$

Fluxes distributed as stable law



Numerical result

PM (2018)



SN rate $= 10^{-4}\,\mathrm{yr}^{-1}$, $\Gamma = 2.2$, $E_{\mathrm{cut}} = 10^5\,\mathrm{GeV}$, $z_{\mathrm{max}} = 4\,\mathrm{kpc}$, full KN-losses,
10^4 realisations of source distribution

Semi-analytical model: copula

- Analytical computation of $c(F_1, F_2, \dots F_N)$ seems intractable
- Compute a large ensemble of random samples in a Monte Carlo approach
- Parametrise likelihood by **pair copula construction**

Pair copula construction

Idea: factorise joint PDF into product of (conditional) bi-variate PDFs, e.g.:

$$f(x_1, x_2, x_3) = f_1(x_1) f_2(x_2) f_3(x_3)\, c_{12}(F_1(x_1), F_2(x_2))\, c_{23}(F_2(x_2), F_3(x_3)) \times c_{13|2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2))$$

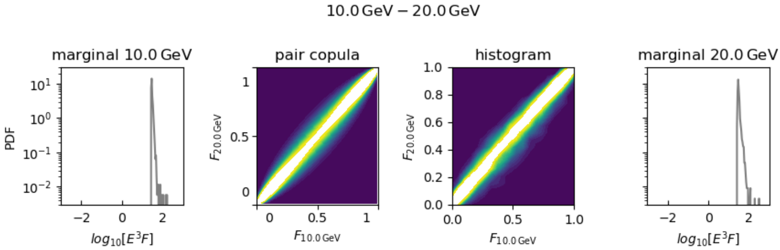
pair copulas

Technical details

- Used regular D-vine
- Tried various copulas, but Normal pair copula fits best
- Determine (conditional) correlation coefficients by fitting

Energy-energy correlations

PM (2018)



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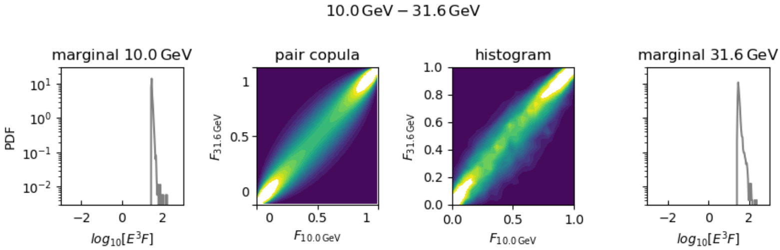

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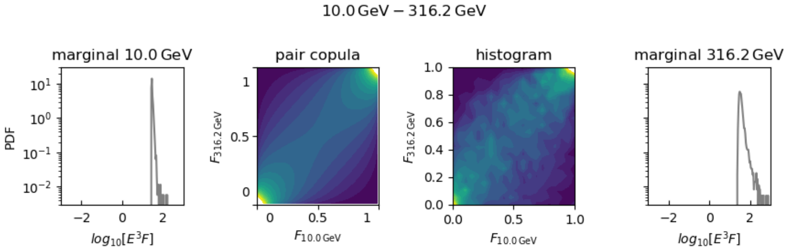

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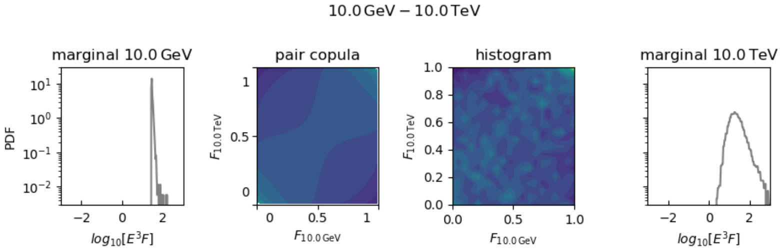

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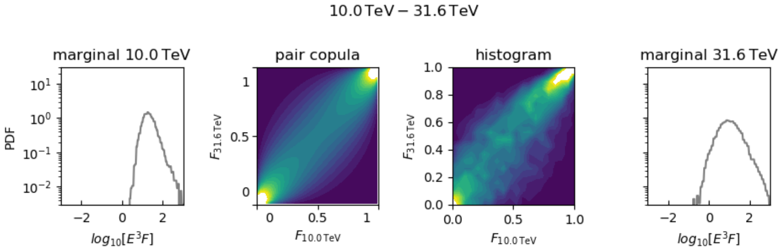

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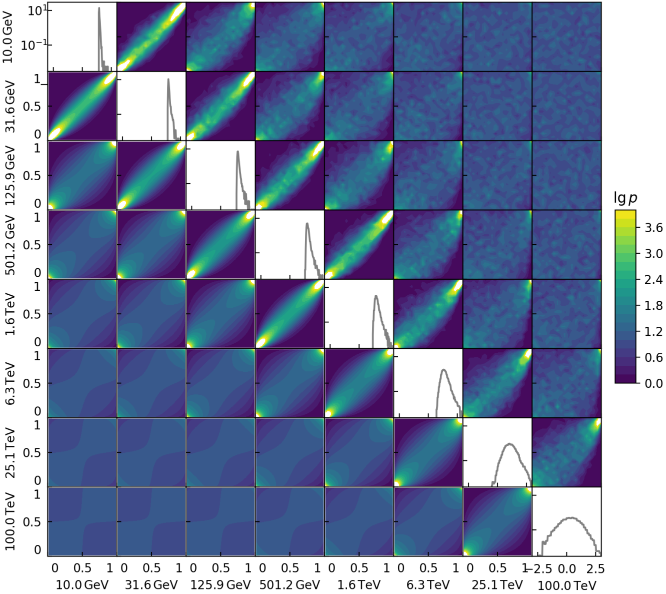


diagonal:
marginals from MC

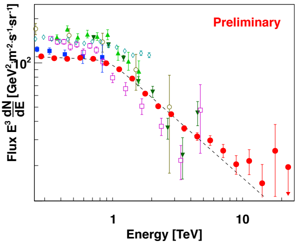
upper triangle:
histograms from MC

lower triangle:
Normal copulas

Goodness of fit

PM (2018)

$$(\hat{\phi}_1, \hat{\phi}_2, \ldots \hat{\phi}_N)^T =$$



Sklar's theorem

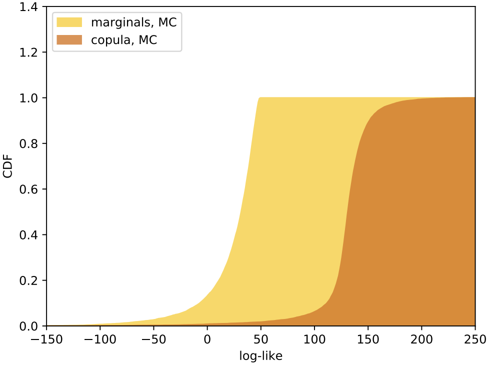
$$f(\phi_1, \phi_2, \ldots \phi_N) = f_1(\phi_1) f_2(\phi_2) \ldots f_N(\phi_N)\, c(F_1(\phi_1), F_2(\phi_2), \ldots F_N(\phi_N))$$

- Compute the log-likelihood for H.E.S.S. broken power-law fit
- Find $\simeq -50$ for marginals, $\simeq 150$ for copula
- Is that …good?

Goodness of fit

PM (2018)

- Distribution of log-likelihoods in MC (SN rate $= 10^4\,\mathrm{Myr}^{-1}$):



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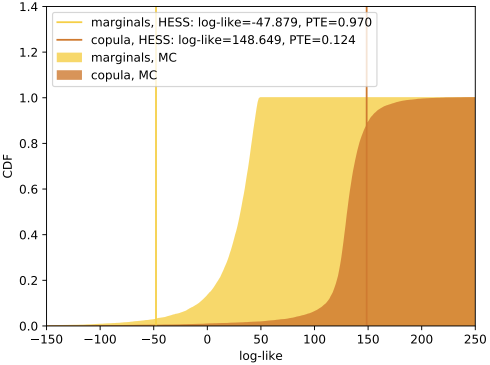


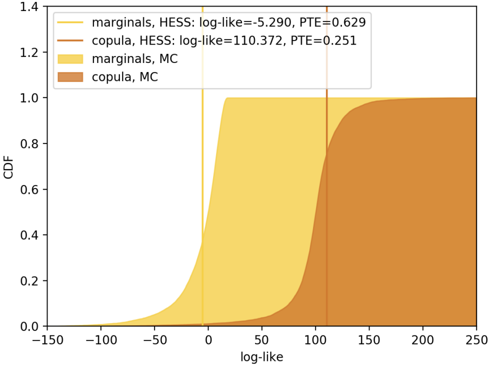
- Compare with log-likelihoods from H.E.S.S. broken power-law:
- Too little fluctuations!

Statistically disfavoured

Goodness of fit

PM (2018)

- Distribution of log-likelihoods in MC (SN rate = $10^3\,\mathrm{Myr}^{-1}$):



- Compare with log-likelihoods from H.E.S.S. broken power-law:

Statistically compatible

Goodness of fit

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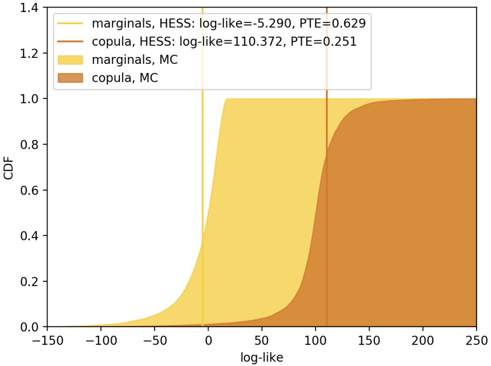


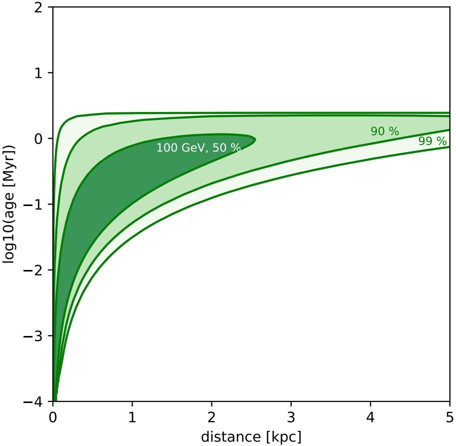
- Compare with log-likelihoods from H.E.S.S. broken power-law:

Statistically compatible

→ Spatial and temporal correlations between SN events?

The problem with catalogues

Contributions from various regions for homogenous source density



→ Effect on flux? PM (2018); also Ahlers, PM, Sarkar (2010)

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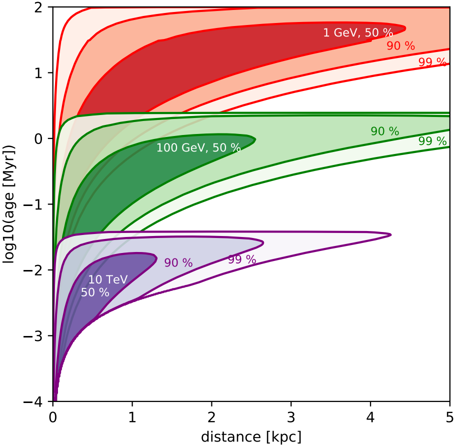

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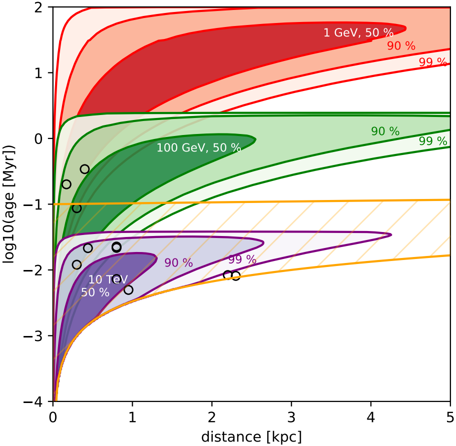

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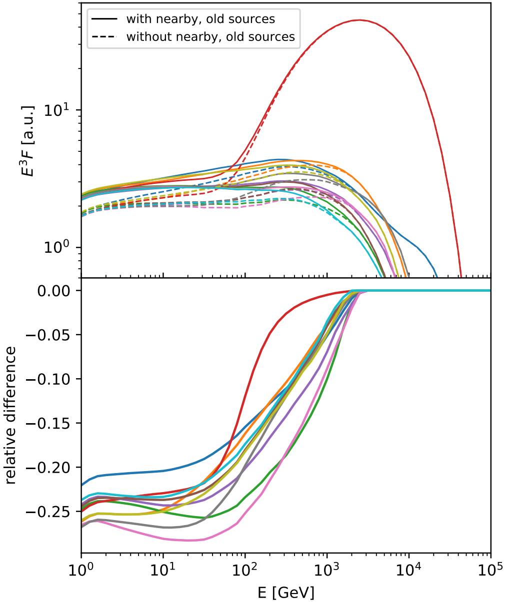
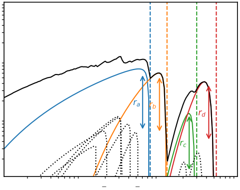


Estimate error due to catalogue incompleteness with MC approach:

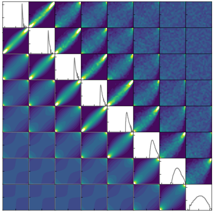
- Homogeneous density in disk
- Constant source rate $10^4\,\mathrm{Myr}^{-1}\mathrm{galaxy}^{-1}$
- Draw samples *with* and *without* nearby ($< 1\,\mathrm{kpc}$), old ($> 0.1\,\mathrm{Myr}$) sources

Underestimates low-energy flux by up to $\sim 25\,\%$! PM (2018)

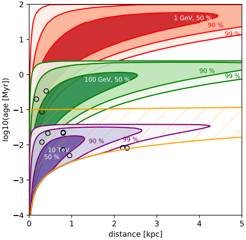
Summary



Green's function approach; features from sources at high energies



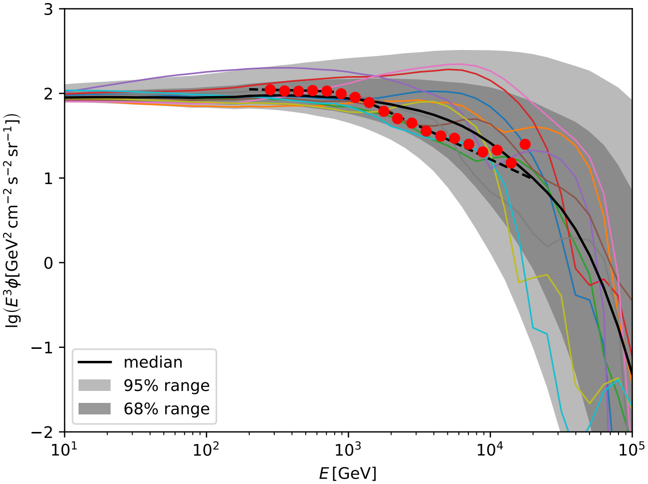
Use pair copula: H.E.S.S. spectrum compatible with correlated sources



Problem with catalogues → use MC approach

Numerical result

PM (2018)



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