Towards Next-to-Next-to-Leading Order QCD Corrections for Top Quark Pair Production with an Additional Jet at Lepton Colliders

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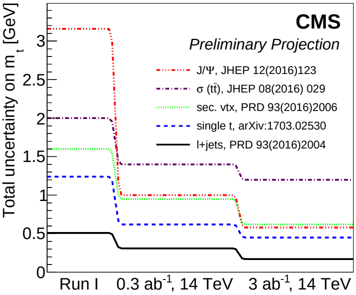
Motivation

Top quark physics

- heaviest quark with mass at electroweak scale
- strongest coupling to Higgs boson
- **top quark mass** important parameter of SM
- decays before hadronization ➔ spin observables
- important background for BSM physics

Experimentally studied: **Tevatron** and **LHC**

- predominantly $t\bar{t}$ production
- $t\bar{t}$ and $t\bar{t}+j$ cross sections measured
- used for top mass determination
- results in good agreement with SM
- precision will increase but is limited by systematic uncertainties



So far: studied at hadron colliders

Lepton Collider

Alternative: e^+e^- collider

- electron and positron are fundamental particles
- initial state energy more precise and small background

➔ **precision studies of SM and QCD**

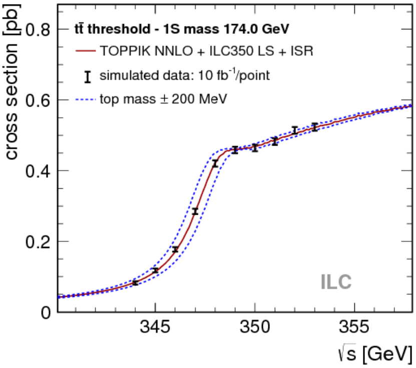
Experiment in the past: LEP with $\sqrt{s} = 209\,\mathrm{GeV}$

➔ **no top quark pairs produced**

In the future: ILC

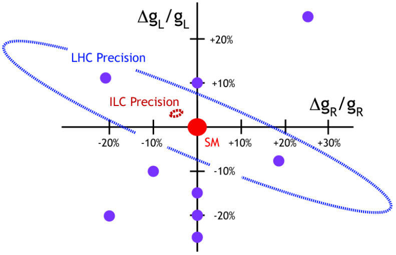
- proposed linear e^+e^- collider with $\sqrt{s} = 500\,\mathrm{GeV} - 1000\,\mathrm{GeV}$
- motivation: Higgs properties, **top quark physics**, BSM searches
- very precise measurements [Fujii et al. '15] e. q.
  - $e^+e^- \to t\bar{t}$ cross section with uncertainty $< 1\%$
  - m_{top} (pole) with $50\,\mathrm{MeV}$ uncertainty

ILC



- measurements at threshold
  ➔ not possible at hadron collider
- **measurement in the continuum as complementary observable**

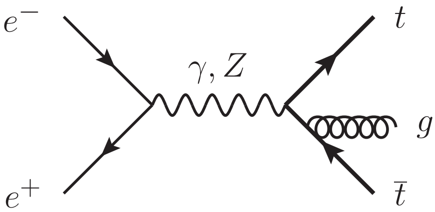
- measurement of (electroweak) couplings with uncertainties below 1%
- Higgs couplings with few percent uncertainty



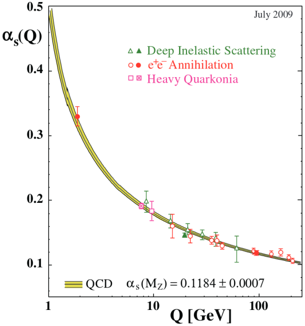
[Fujii et al. '15]

Theoretical predictions must match the experimental precision!

Leading Order: Scale Variation



scale dependence $\sim \alpha_s(\mu)$



[Bethke '09]

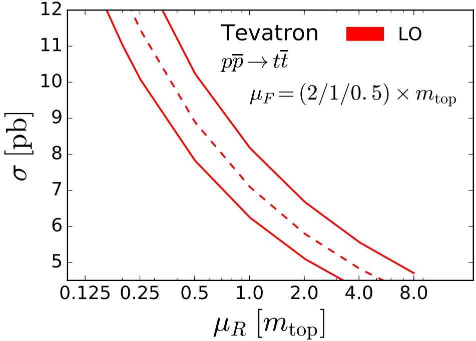
- LO cross section of order α_s
- LO cross section depends on (unphysical) renormalization scale μ
- theoretical uncertainty because of finite order of perturbation theory

Variation of scale μ to estimate uncertainty: $\mathbf{\Delta\sigma \approx 20\%}$ at LO

This is not precise enough!

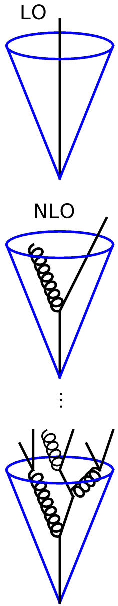
to reduce uncertainty: higher order QCD corrections

Higher Orders in Perturbation Theory



Calculation: HATHOR

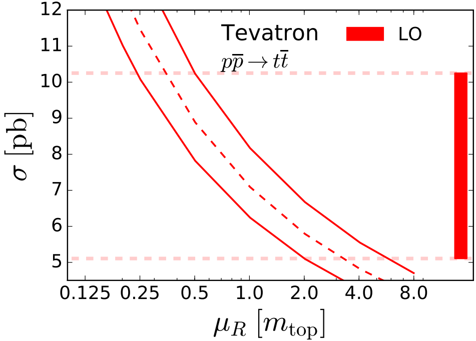
[Aliev et al. '11]



Advantages of higher order QCD corrections:

- smaller and more reliable scale dependence
- check for convergence of perturbation series
- more partons in final state: better matching to jet algorithms

Higher Orders in Perturbation Theory



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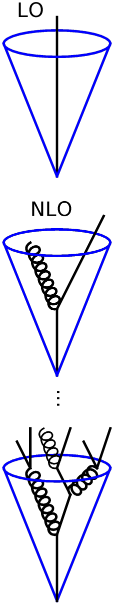

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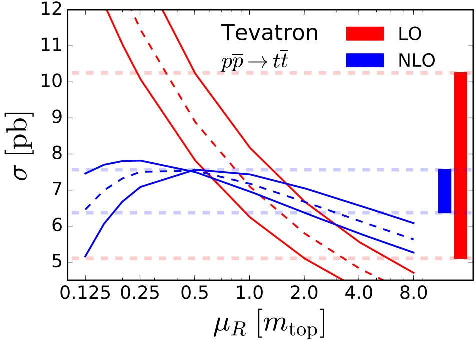

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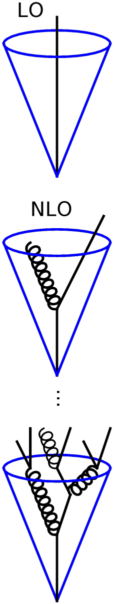

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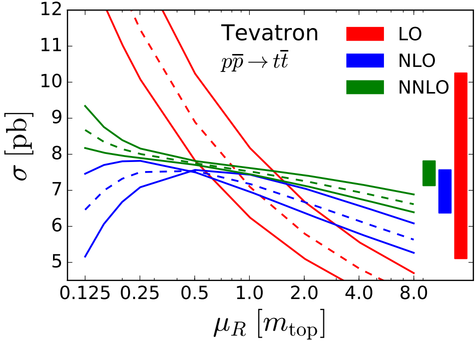
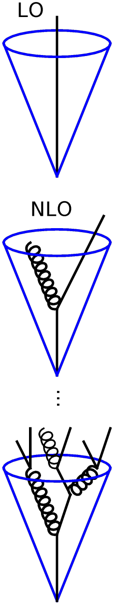


Calculation: HATHOR

[Aliev et al. '11]

NNLO corrections

[Bärnreuther, Czakon, Mitov '12]



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But $e^+e^- \to t\bar{t}j$ is only known at NLO!

Motivation

Why do we need the NNLO corrections for $e^+e^- \to t\bar{t}j$?

1. **important process at ILC**
   - relativly big cross section compared to $t\bar{t}$ (40% of the cross section)
   - NLO: typical uncertainty $\mathcal{O}(10\%)$
   - For $\mathcal{O}(1\%)$ uncertainty one needs NNLO QCD corrections

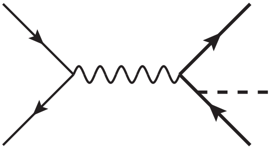
     ➔ **theoretical prediction must match experimental precision**
2. **Development of new and enhanced NNLO techniques**
   - current NNLO techniques limited
   - challenges: processes with many scales, complicated final states

     ➔ use $e^+e^- \to t\bar{t}j$ for development: 'simple' initial state

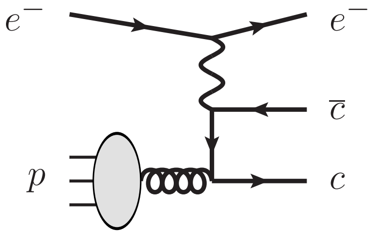
3. **Two loop amplitude for other processes**

   Example: $e^+e^- \to t\bar{t}h$ / $t\bar{t}\gamma$

- similar two loop amplitudes with one more scale
- measurement of top-higgs yukawa coupling



Deep Inelastic Scattering: Open Charm Production



Further application: **open charm production $e^- p \to e^- c\overline{c}$**

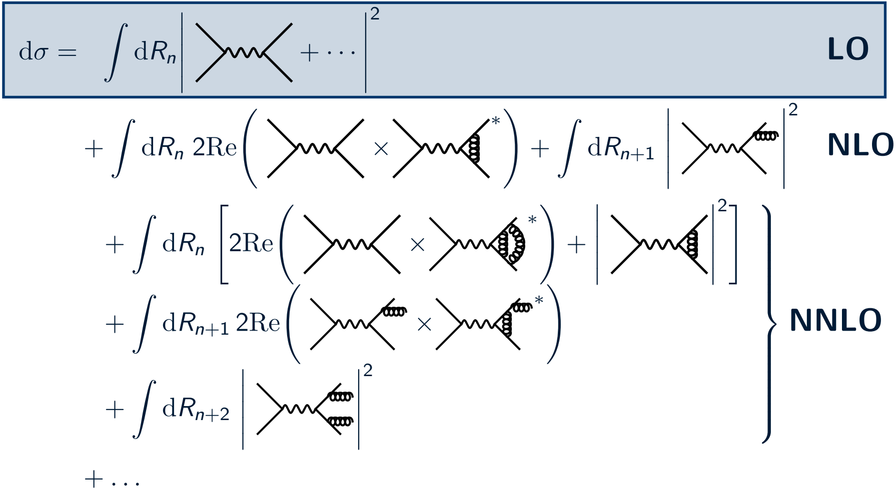
- production of charmed hadrons at deep inelastic scattering
- important process for gluon PDF determination
- here: **charm quark massive** for mass effects in DIS
- 'crossed' $e^+e^- \to t\bar{t}j$ process

experimental data already available

NNLO corrections should be calculated

Structure of the Cross Section

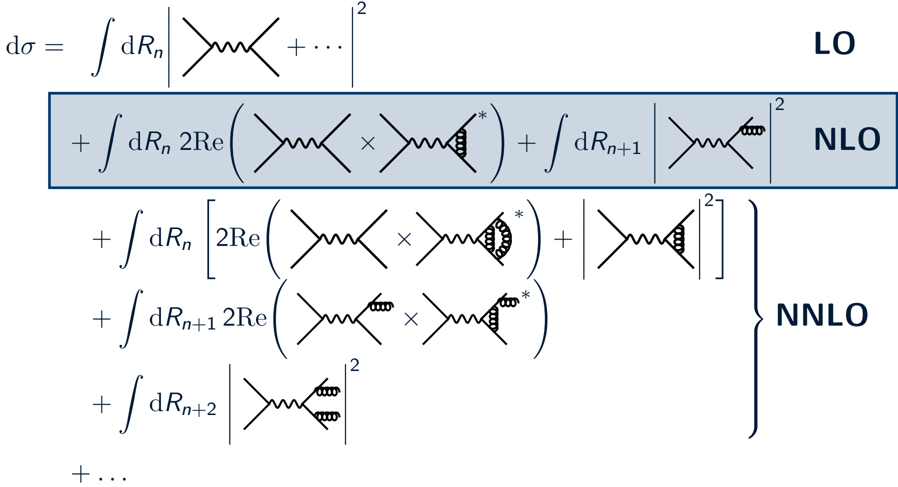
Schematically: (other diagrams and one gluon (particle) not shown)



Born cross section of order α_s

Every new order in perturbation theory: increase one power of α_s

Schematically: (other diagrams and one gluon (particle) not shown)

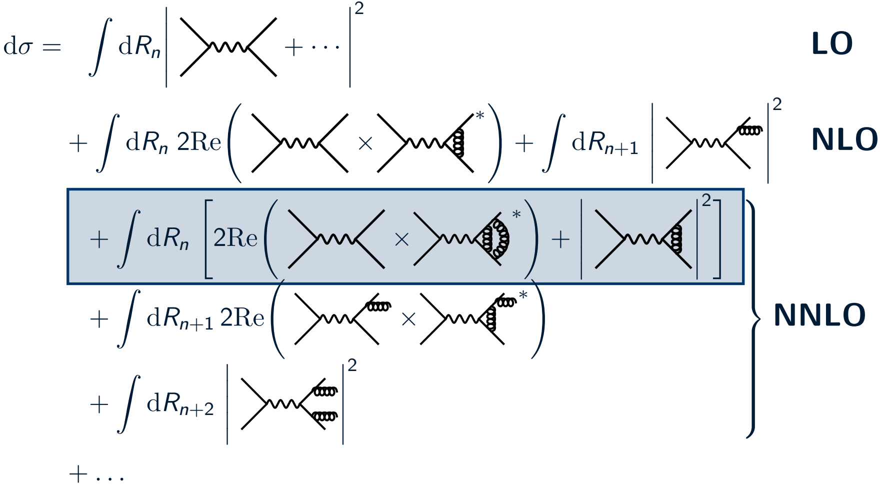


virtual corrections: one loop diagrams

real corrections: one additional (unresolved) particle

Structure of the Cross Section

Schematically: (other diagrams and one gluon (particle) not shown)

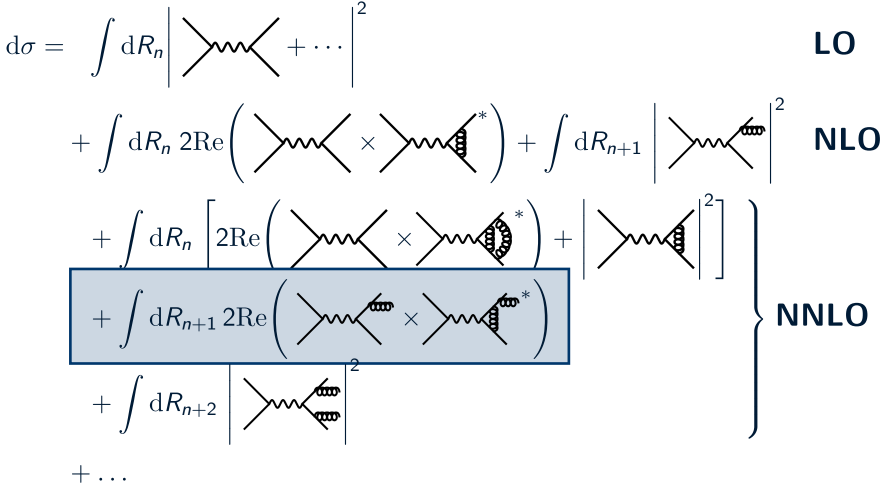


double virtual corrections (VV)

two loop amplitudes and squared one loop amplitudes

Structure of the Cross Section

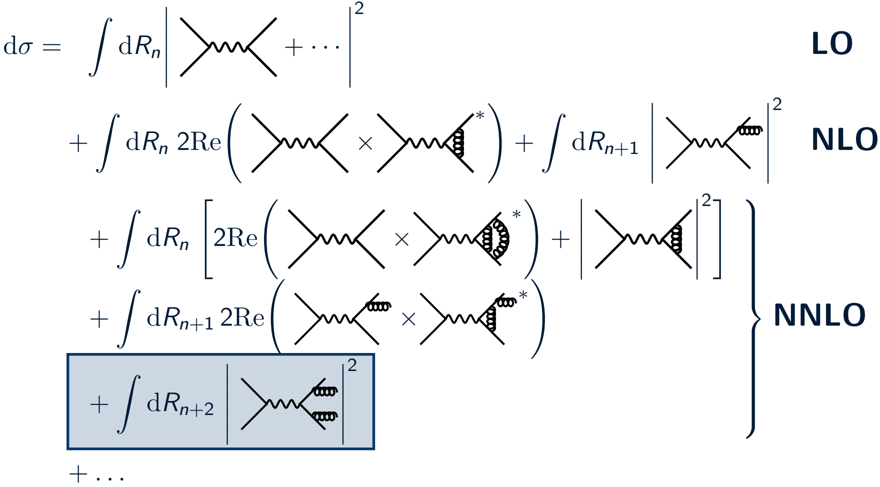
Schematically: (other diagrams and one gluon (particle) not shown)



real virtual corrections (RV)

one loop amplitudes with one additional (unresolved) particle

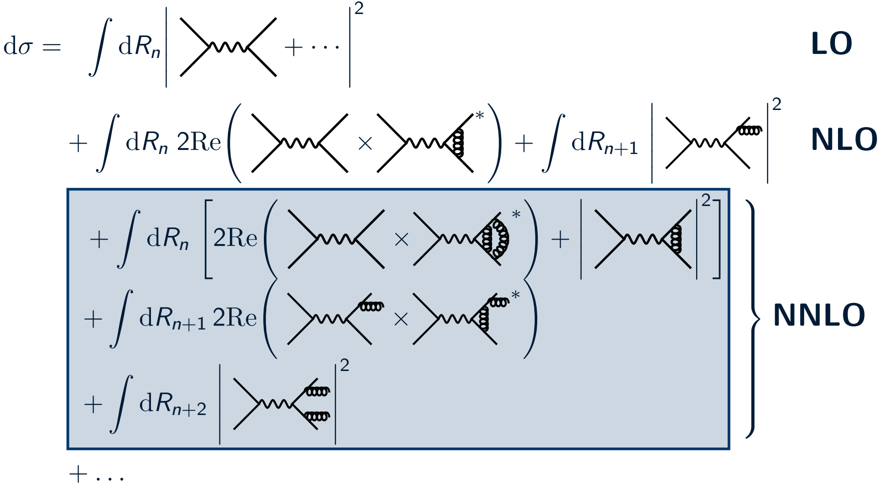
Schematically: (other diagrams and one gluon (particle) not shown)



double real corrections (RR)

tree amplitudes with two additional (unresolved) particles

Schematically: (other diagrams and one gluon (particle) not shown)



Components are seperatly IR divergent

In sum finite because of **KLN-theorem**

Real Virtual and Double Real Corrections

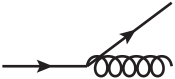
Real Virtual (RV)

$\int \mathrm{d}R_{n+1}\, 2\mathrm{Re}\Big($  $\times$  $^{*}\Big)$

Double Real (RR)

$\int \mathrm{d}R_{n+2}\, \Big|$  $\Big|^2$

singularities arise from phase space integration



p_q

p_g

$\longrightarrow \quad \frac{1}{2p_q p_g} = \frac{1}{2E_q E_g(1-\cos\theta)}$ für $p_q^2 = p_g^2 = 0$

divergent for $E_g \to 0$: **soft singularity**

$\cos\theta \to 1$: **collinear singularity** (mass singularity)

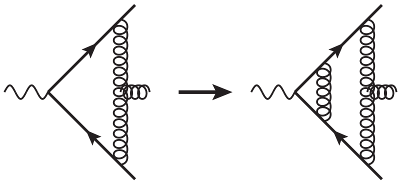
At NNLO: many different soft and collinear configurations

e.q. soft-soft, coll.-soft, coll.-coll., triple coll.

Problem: consistent extraction of singularities

Remaining IR singularities canceled by double virtual corrections

NNLO: Double Virtual Corrections



- two loops: integration over two loop momenta
- QGRAF [Nogueira '93]: around 800 diagrams
  ➔ **many different integrals**

Calculation of two loop amplitudes in two steps:

1. **Reduce tensor/scalar integrals to small set of master integrals**
   e. g. Laporta algorithm with REDUZE 2 [Manteuffel,Studerus '12] or Kira [Maierhoefer,Usovitsch, Uwer '17]
   Note: programs do not give (useable) results for all integrals
   (necessary time/resources of computation, size of results to big)
2. **Calculate master integrals (efficiently)**
   e. g. differential equations, CANONICA [Meyer '17]
   Note: not all master integrals for this process are calculated

Examples not complete or representative

At NNLO: both steps are open problems

Summary and Outlook

Summary:

- LHC: increasing precision but limited by systematics
- e^+e^- collider: high precision measurements
- LO+NLO: not sufficient in the future ➔ NNLO
- NNLO calculation: many challenges
- ➔ NNLO correction for $e^+e^- \to t\bar{t}j$
- Usefull for other processes: $e^-p \to c\bar{c}e^-$ (DIS) and $t\bar{t}h$ production

Status and Outlook:

- still at the beginning of project
- start with one and two loop amplitudes
  - General structure of calculation/setup of tools
  - tensor reduction
- Next: rest of the double virtual corrections

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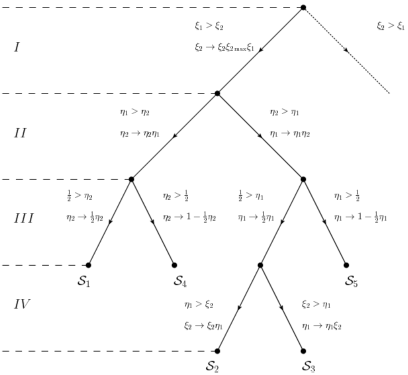
NNLO: Real Virtual and Double Real Corrections

Problem: consistent extraction of singularities

- three particles in final state
- many different soft and collinear configurations
- avoid double counting
- example: decompisition tree for triple collinear phase space



[Czakon, Heymes '14]

Choice and implementation of subtraction scheme

- Slicing techniques: q_t subtraction, N-jettiness
- Subtraction techniques: Antenna, Sector-decomposition+FKS, etc.