Lepton Masses and Mixing in the *2HDM*

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Introduction

Within the framework of the Standard Model (SM) a discrete symmetry [1, 2] for Yukawa couplings gives the relations for mass matrices of charged leptons (M_l) and neutrinos (M_ν):

$$A_L^{i\dagger}\left(M_l M_l^\dagger\right)A_L^i = \left(M_l M_l^\dagger\right), \quad (1)$$
$$A_L^{i\dagger}\left(M_\nu M_\nu^\dagger\right)A_L^i = \left(M_\nu M_\nu^\dagger\right), \quad (2)$$

where $A_L^i = A_L(g_i),\ i = 1, 2, \dots, N$ are 3D *rep* matrices for the LH lepton doublets for some N-order flavour symmetry ($\mathcal{FS}$) group $\mathcal{G}$. Then the I-st Schur's lemma implies that $M_l M_l^\dagger$ and $M_\nu M_\nu^\dagger$ are proportional to the identity matrices, which obviously entails the trivial lepton mixing matrix (U_{PMNS}). Usually $\mathcal{FS}$ is broken spontaneously by scalar, singlet Higgs field (flavons) or by introducing more Higgs multiplets.

Objectives

We have studied $\mathcal{FS}$ in the context of 2HDM [3] (type III) for groups up to order 1025 (each of our groups must have at least one faithful, 3D *irr rep* [4]) avoiding the consequences of Schur's lemma.

Dirac Neutrinos Case

$\mathcal{FS}$ of our theory means, that after transformation of fields occurring in the 2HDM Lagrangian by the 3 dimensional (A_L, A_l^R, A_ν^R) and 2 dimensional (A_Φ) representations of a flavour group $\mathcal{G}$:

$$L_{\alpha L} \to L'_{\alpha L} = (A_L)_{\alpha,\chi} L_{\chi L},$$
$$l_{\beta R} \to l'_{\beta R} = \left(A_l^R\right)_{\beta,\delta} l_{\delta R}$$
$$\nu_{\beta R} \to \nu'_{\beta R} = \left(A_\nu^R\right)_{\beta,\delta} \nu_{\delta R},$$
$$\Phi_i \to \Phi'_i = (A_\Phi)_{ik}\Phi_k,$$

the full 2HDM Lagrangian does not change.

In order to find symmetric Yukawa matrices $h_i^{(l)}$, $h_i^{(\nu)}$, $i = 1, 2$, one can express the symmetry conditions as the eigenequation for direct product of unitary group *rep* to the eigenvalue 1:

$$\left((A_\Phi)^\dagger \otimes (A_L)^\dagger \otimes (A_l^R)^T\right)(h^l) = (h^l),$$
$$\left((A_\Phi)^T \otimes (A_L)^\dagger \otimes (A_\nu^R)^T\right)(h^\nu) = (h^\nu).$$

Both relations need to be satisfied for any group's element $g \in \mathcal{G}$. It is however sufficient, that they are fulfilled only for the group generators.

The invariance equations for the mass matrices are not trivial.

For symmetric Higgs potential:

$$A_L\, M^{l(\nu)}\left(A^R_{l(\nu)}\right)^\dagger = \quad (3)$$
$$= \frac{1}{\sqrt{2}}\sum_{i,k=1}^{2} h_i^{l(\nu)}\left(A_\Phi\right)_{i,k} v_k \neq M^{l(\nu)},$$

Eq.(1-2) are not satisfied and we avoid consequences of the Schur's Lemma.

Majorana Neutrino Case

Since for Dirac case, lepton and neutrino mass matrices were defined as follows:

$$M^l = -\frac{1}{\sqrt{2}}\left(v_1^* h_1^{(l)} + v_2^* h_2^{(l)}\right), \quad (4)$$
$$M^\nu = \frac{1}{\sqrt{2}}\left(v_1 h_1^{(\nu)} + v_2 h_2^{(\nu)}\right), \quad (5)$$

GAP investigations

Using the **GAP** [5] system for computational discrete algebra, with the included **Small Groups Library** [6] and **REPSN** [7] packages, we have found in total **10862** groups with at least one 2 dimensional and at least one 3 dimensional irreducible representations but only **413** of these groups are subgroups of the $U(3)$ group. Either a group has at least one faithful 3 dimensional irreducible representation (there are **173** such groups) or it has at least one faithful 1+2 reducible representation (there are **240** such groups). Some groups are also subgroups of the $U(2)$ group.

An example of examined groups

Table 1: Here: "[o , i]" the i-th group of the order o in the Small Groups Library catalogue, "StructureDescription" a short string which provides some insight into the structure of the group under consideration, "2-D" the number of 2-dimensional irreducible representations, "3-D" the number of 3-dimensional irreducible representations, "$U(2)$" an indicator why the group is classified as a subgroup of the $U(2)$ group (at least one 2-dimensional irreducible is faithful), "$U(3)$" an indicator why the group is classified as a subgroup of the $U(3)$ group (either at least one 3-dimensional irreducible or one 1+2 reducible representation is faithful), "L" the number of different combinations of representations for charged leptons, "DN" the number of different combinations of representations for Dirac neutrinos, "MN" the number of different combinations of representations for Majorana neutrinos, "L+DN" the number of pairs of different combinations of representations for charged leptons and Dirac neutrinos, "L+MN" the number of pairs of different combinations of representations for charged leptons and Majorana neutrinos. Note that the "L" and the "DN" are always equal and that the "L+DN" is twice that number.

[o , i]	StructureDescription	2-D	3-D	$U(2)$	$U(3)$	L	DN	MN	L+DN	L+MN
[24 , 3]	SL(2,3)	3	1	2	1+2			3		
[24 , 12]	S4	1	2		3	4	4	2	8	4
[48 , 28]	C2.S4=SL(2,3).C2	3	2	2	1+2	4	4	6	8	4
[48 , 29]	GL(2,3)	3	2	2	1+2	4	4	6	8	4
[48 , 30]	A4:C4	2	4		3	16	16	8	32	16
[48 , 32]	C2xSL(2,3)	6	2		1+2			12		
[48 , 33]	SL(2,3):C2	6	2	2	1+2					
[48 , 48]	C2xS4	2	4		3	16	16	8	32	16
[54 , 8]	((C3xC3):C3):C2	4	4		3	32	32		64	
[72 , 3]	Q8:C9	9	3	2	1+2			9		
[72 , 25]	C3xSL(2,3)	9	3	2	1+2			9		
[72 , 42]	C3xS4	3	6		3	36	36	6	72	12
[96 , 64]	((C4xC4):C3):C2	1	6		3	12	12	2	24	4
[96 , 65]	A4:C8	4	8		3	64	64	16	128	32
[96 , 66]	SL(2,3):C4	6	4		1+2	16	16	24	32	16
[96 , 67]	SL(2,3):C4	6	4	2	1+2	16	16	8	32	16
[96 , 69]	C4xSL(2,3)	12	4		1+2			24		
[96 , 74]	((C8xC2):C2):C3	12	4	2	1+2					
...	...	...	...	...	...	...	...	...	...	...

for Majorana case, neutrino mass matrix is equal:

$$M^\nu_{\alpha,\beta} = \frac{g}{M}\sum_{i,k=1}^{2} v_i v_k h^{(i,k)}_{\alpha,\beta}. \quad (6)$$

Similarly as in the previous case, from the requirement of $\mathcal{FS}$ for the Yukawa Lagrangian the neutrino Yukawa matrices must satisfy the equation:

$$\left((A_\Phi)^T \otimes (A_\Phi)^T \otimes \right. \quad (7)$$
$$\left. \otimes \quad (A_L)^\dagger \otimes (A_L)^\dagger\right)(h^\nu) = h^\nu.$$

Results

- Among the groups that we considered, there exist 267 groups that gave in total 748672 different combinations of 2 and 3 dimensional irreducible representations that give 1 dimensional degeneration subspace for all generators, which is the solution of the equations in Eq.(3).
- Among the groups that we considered, there exist 195 groups that gave in total 20888 solutions. All found symmetries, as a solution of Eq.(7).

Conclusions

1. We have avoided having to break the family symmetry and introduce flavon fields.
2. None of studied group can reproduce the current experimental data.
3. The set of symmetric Yukawa matrices for charged leptons is independent from the nature of the neutrinos. In the models with two Higgs particles, regardless of the adopted neutrino sector and the set of groups that we are considering, we will not find a symmetry that gives real masses of charged leptons, even approximately.

Main References

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