XXVIII International Conference on Neutrino Physics and Astrophysics (NEUTRINO 2018), 4-9 June 2018, Heidelberg, Germany

Neutrino flavour, spin and spin-flavour oscillations and consistent account for a constant magnetic field

Artem Popov[a]**, Alexander Studenikin**[a,b]

[a]Department of Theoretical Physics, Moscow State University, 119992 Moscow, Russia;

[b]Joint Institute for Nuclear Research, Dubna 141980, Moscow Region, Russia

E-mail addresses: ar.popov@physics.msu.ru, studenik@srd.sinp.msu.ru

Introduction

Massive neutrinos have nontrivial electromagnetic properties (see [2] for a review, the update can be found in [3]). And for many years since [4], it is known that at least the magnetic moment is not zero ($\mu_i \neq 0$ are magnetic moments of the mass states of neutrino). The best terrestrial upper bounds on the level of $\mu_\nu < 2.9 \div 2.8 \times 10^{-11}\mu_B$ on neutrino magnetic moments is obtained by the GEMMA reactor neutrino experiment [5] and recently by the Borexino collaboration [6] from solar neutrino fluxes. An order of magnitude more strict astrophysical bound on the neutrino magnetic moment is provided by the observed properties of globular cluster stars [7, 8, 9].

The neutrino magnetic moment precession in the transversal magnetic field $\mathbf{B}_\perp$ was first considered in [10], then spin-flavor precession in vacuum was discussed in [11], the importance of the matter effect was emphasized in [12]. The effect of resonant amplification of neutrino spin oscillations in $\mathbf{B}_\perp$ in the presence of matter was proposed in [13, 14], the magnetic field critical strength the presence of which makes spin oscillations significant was introduced [15], the impact of the longitudinal magnetic field $\mathbf{B}_\parallel$ was discussed in [16] and just recently in [17]. In a series of papers [18, 19, 20, 21] the solution of the solar neutrino problem was discussed on the basis of neutrino oscillations with subdominant effect from the neutrino transition magnetic moments conversion in the solar magnetic field (the spin-flavour precession).

Following to the general idea first implemented in [22, 23], we further develop a new approach to description of the relativistic neutrino flavour $\nu_e^L \leftrightarrow \nu_\mu^L$, spin $\nu_e^L \leftrightarrow \nu_e^R$ and spin-flavour $\nu_e^L \leftrightarrow \nu_\mu^R$ oscillations in the presence of an arbitrary constant magnetic field. Our approach is based on the use of the exact stationary states in the magnetic field for classification of neutrino spin states, contrary to the customary approach when the neutrino helicity states are used for this purpose.

Within this customary approach the helicity operator is used for classification of a neutrino spin states in a magnetic field. The helicity operator does not commute with the neutrino evolution Hamiltonian in an arbitrary constant magnetic field and the helicity states are not stationary in this case. This resembles situation of the flavour neutrino oscillations in the presence of matter when the neutrino mass states are also not stationary. In the presence of matter the neutrino flavour states are considered as superpositions of stationary states in matter. These stationary states are characterized by "masses" $\widetilde{m}_i(n_{eff})$ that are dependent on the matter density n_{eff} and the effective neutrino mixing angle $\tilde{\theta}_{eff}$ is also a function of the matter density.

The proposed alternative approach to neutrino oscillations in a magnetic field is based on the use of the exact solutions of the corresponding Dirac equation for a massive neutrino wave function in the presence of a magnetic field that stipulates the description of the neutrino spin states with the corresponding spin operator that commutes with the neutrino dynamic Hamiltonian in the magnetic field. In what follows, we also account for the complete set of conversions between four neutrino states.

Massive neutrino in a magnetic field

Consider two flavour neutrinos with two chiralities accounting for mixing

$$\nu_e^{L(R)} = \nu_1^{L(R)}\cos\theta + \nu_2^{L(R)}\sin\theta, \nu_\mu^{L(R)} = -\nu_1^{L(R)}\sin\theta + \nu_2^{L(R)}\cos\theta,$$

where $\nu_i^{L(R)}$ are the chiral neutrino mass states, $i = 1, 2$. For the relativistic neutrinos the chiral states approximately coincide with the helicity states $\nu_i^{L(R)} \approx \nu_i^{h^-(h^+)}$. Note that the helicity mass states $\nu_i^{h^-(h^+)}$ are not stationary states in the presence of a magnetic field. In our further evaluations we shall expand $\nu_i^{h^-(h^+)}$ over the neutrino stationary states $\nu_i^{-(+)}$ in the presence of a magnetic field.

The wave function ν_i^s $(s = \pm 1)$ of a massive neutrino that propagates along $\mathbf{n_z}$ direction in the presence of a constant and homogeneous arbitrary orientated magnetic field can be found as the solution of the Dirac equation

$$(\gamma p - m_i - \mu_i \mathbf{\Sigma B})\nu_i^s(p) = 0, \tag{1}$$

where μ_i is the neutrino magnetic moment and the magnetic field is given by $\mathbf{B} = (B_\perp, 0, B_\parallel)$. In the discussed two-neutrino case the possibility for nonzero neutrino transition moment μ_{ij} $(i \neq j)$ is not considered and two equations for two neutrinos states ν_i^s are decoupled. The equation (1) can be re-written in the equivalent form

$$\hat{H}_i\nu_i^s = E\nu_i^s, \tag{2}$$

where the Hamiltonian is

$$\hat{H}_i = m_i\gamma_0 + \gamma_0\boldsymbol{\gamma p} + \mu_i\gamma_0\mathbf{\Sigma}\boldsymbol{B}. \tag{3}$$

The spin operator that commutes with the Hamiltonian (3) can be chosen in the form [24]

$$\hat{S}_i = \frac{m_i}{\sqrt{m_i^2\boldsymbol{B}^2 + \boldsymbol{p}^2 B_\perp^2}}\left[\mathbf{\Sigma}\boldsymbol{B} - \frac{i}{m_i}\gamma_0\gamma_5[\mathbf{\Sigma}\times\boldsymbol{p}]\boldsymbol{B}\right]. \tag{4}$$

For the neutrino energy spectrum one obtains

$$E_i^s = \sqrt{m_i^2 + p^2 + \mu_i{}^2\mathbf{B}^2 + 2\mu_i s\sqrt{m_i^2\mathbf{B}^2 + p^2 B_\perp^2}}, \tag{5}$$

where $s = \pm 1$ correspond to two different eigenvalues of the Hamiltonian (3) and $p = |\boldsymbol{p}|$. Hence, we specify the neutrino spin states as the stationary states for the Hamiltonian in the presence of the magnetic field, contrary to the customary approach to the description of neutrino oscillations when the helicity states are used.

The spin operator $\hat{S}_i$ commutes with the Hamiltonian $\hat{H}_i$, and for the neutrino stationary states we have

$$\hat{S}_i\left|\nu_i^s\right\rangle = s\left|\nu_i^s\right\rangle, s = \pm 1, \tag{6}$$

and

$$\langle\nu_i^s|\nu_k^{s'}\rangle = \delta_{ik}\delta_{ss'}. \tag{7}$$

Following this line, the corresponding projector operators can be introduced

$$\hat{P}_i^\pm = \frac{1 \pm \hat{S}_i}{2}. \tag{8}$$

It is clear that projectors act on the stationary states as follows

$$\langle\nu_k^{s'}|\hat{P}_i^s|\nu_i^s\rangle = \delta_{ik}\delta_{ss'}. \tag{9}$$

Now in order to solve the problem of the neutrino flavour $\nu_e^L \leftrightarrow \nu_\mu^L$, spin $\nu_e^L \leftrightarrow \nu_e^R$ and spin-flavour $\nu_e^L \leftrightarrow \nu_\mu^R$ oscillations in the magnetic field we expand the neutrino chiral states over the neutrino stationary states

$$\nu_i^L(t) = c_i^+\nu_i^+(t) + c_i^-\nu_i^-(t), \tag{10}$$

$$\nu_i^R(t) = d_i^+\nu_i^+(t) + d_i^-\nu_i^-(t), \tag{11}$$

where $c_i^\pm$ and $d_i^\pm$ are independent on time.

The quadratic combinations of the coefficients $c_i^{+(-)}$ and $d_i^{+(-)}$ are given by matrix elements of the projector operators (8)

$$|c_i^\pm|^2 = \langle\nu_i^L|\hat{P}_i^\pm|\nu_i^L\rangle, \tag{12}$$

$$|d_i^\pm|^2 = \langle\nu_i^R|\hat{P}_i^\pm|\nu_i^R\rangle, \tag{13}$$

$$(d_i^\pm)^*c_i^\pm = \langle\nu_i^R|P_i^\pm|\nu_i^L\rangle. \tag{14}$$

Since $|c_i^\pm|^2$, $|d_i^\pm|^2$ and $(d_i^\pm)^*c_i^\pm$ are time independent, they can be determined from the initial conditions. Now let's take into account the fact that only chiral states can participate in weak interaction and, consequently, in processes of neutrino creation and detection. It means, that the spinor structure of the neutrino initial and final states is determined by

$$\nu^L = \frac{1}{\sqrt{2}L^{\frac{3}{2}}}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}, \quad \nu^R = \frac{1}{\sqrt{2}L^{\frac{3}{2}}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}, \tag{15}$$

where L is the normalization length. Thus, for the quadratic combinations of the coefficients we get

$$|c_i^\pm|^2 = \langle\nu^L|\hat{P}_i^\pm|\nu^L\rangle = \frac{1}{2}\left(1 \pm \frac{m_i B_\parallel}{\sqrt{m_i^2B^2 + p^2B_\perp^2}}\right), \tag{16}$$

$$|d_i^\pm|^2 = \langle\nu^R|\hat{P}_i^\pm|\nu^R\rangle = \frac{1}{2}\left(1 \mp \frac{m_i B_\parallel}{\sqrt{m_i^2B^2 + p^2B_\perp^2}}\right), \tag{17}$$

$$(d_i^+)^*c_i^+ = \langle\nu^R|P_i^+|\nu^L\rangle = -\frac{1}{2}\frac{p(B_1 - iB_2)}{\sqrt{m_i^2B^2 + p^2B_\perp^2}}, \tag{18}$$

$$(d_i^-)^*c_i^- = \langle\nu^R|P_i^-|\nu^L\rangle = \frac{1}{2}\frac{p(B_1 - iB_2)}{\sqrt{m_i^2B^2 + p^2B_\perp^2}}. \tag{19}$$

In the case $B_\perp = 0$ the helicity states are stationary and $(d_i^+)^*c_i^+ = (d_i^-)^*c_i^- = |c_i^-|^2 = |d_i^+|^2 = 0$, $|c_i^+|^2 = |d_i^-|^2 = 1$.

Using eqs. (10), (11) and accounting for the fact that stationary states' propagation law has the form $\nu_i^s(t) = e^{-iE_i^st}\nu_i^s(0)$, we get that the evolution in time (space) of the relativistic neutrino flavour state ν_e^L is given by

$$\nu_e^L(t) = \left(c_1^+e^{-iE_1^+t}\nu_1^+ + c_1^-e^{-iE_1^-t}\nu_1^-\right)\cos\theta + \left(c_2^+e^{-iE_2^+t}\nu_2^+ + c_2^-e^{-iE_2^-t}\nu_2^-\right)\sin\theta, \tag{20}$$

where $\nu_i^s \equiv \nu_i^s(0)$. In exactly the same way we can write out the decomposition of the wave function of a muon neutrino. The probability of the flavour neutrino oscillations $\nu_e^L \leftrightarrow \nu_\mu^L$ is just

$$P_{\nu_e^L\to\nu_\mu^L}(t) = \left|\langle\nu_\mu^L|\nu_e^L(t)\rangle\right|^2.$$

The probability of oscillations $\nu_e^L \leftrightarrow \nu_\mu^L$ is simplified if one accounts for the relativistic neutrino energies ($p \gg m$) and also for realistic values of the neutrino magnetic moments and strengths of magnetic fields ($p \gg \mu B$). In this case we have

$$E_i^s \approx p + \frac{m_i^2}{2p} + \frac{\mu_i^2B^2}{2p} + \mu_i sB_\perp. \tag{21}$$

It is reasonably to suppose that $\mu B << m$, then the contribution $\frac{\mu_i^2B^2}{2p}$ can be neglected in (21). In the considered case we also have

$$|c_i^s|^2|c_k^{s'}|^2 \approx \frac{1}{4}. \tag{22}$$

Finally, for the probability of flavour oscillations $\nu_e^L \leftrightarrow \nu_\mu^L$ we get

$$P_{\nu_e^L\to\nu_\mu^L}(t) = \sin^2 2\theta\Big\{\cos(\mu_1B_\perp t)\cos(\mu_2B_\perp t)\sin^2\frac{\Delta m^2}{4p}t + \\ +\sin^2\left(\mu_+B_\perp t\right)\sin^2(\mu_-B_\perp t)\Big\}, \tag{23}$$

where $\mu_\pm = \frac{1}{2}(\mu_1 \pm \mu_2)$.

From the obtained expression (23) a new phenomenon in the neutrino flavour oscillation in a magnetic field can be seen. It follows that the neutrino flavour oscillations in general can be modified by the neutrino magnetic moment interactions with the transversal magnetic field $B_\perp$. In the case of zeroth magnetic moment and/or vanishing magnetic field eq.(23) reduces to the well known probability of the flavour neutrino oscillations in vacuum.

In quite similar evaluations we also obtain probabilities of neutrino spin $\nu_e^L \leftrightarrow \nu_e^R$ and spin-flavour $\nu_e^L \leftrightarrow \nu_\mu^R$ oscillations. In particular, for of neutrino spin $\nu_e^L \leftrightarrow \nu_e^R$ oscillations we get

$$P_{\nu_e^L\to\nu_e^R} = \Big\{\sin\left(\mu_+B_\perp t\right)\cos\left(\mu_-B_\perp t\right) + \cos 2\theta\sin\left(\mu_-B_\perp t\right)\cos\left(\mu_+B_\perp t\right)\Big\}^2 - \\ - \sin^2 2\theta\sin(\mu_1B_\perp t)\sin(\mu_2B_\perp t)\sin^2\frac{\Delta m^2}{4p}t. \tag{24}$$

For the probability of the neutrino spin-flavour oscillations $\nu_e^L \leftrightarrow \nu_\mu^R$ we get

$$P_{\nu_e^L\to\nu_\mu^R}(t) = \sin^2 2\theta\Big\{\sin^2\mu_-B_\perp t\cos^2\left(\mu_+B_\perp t\right) + \\ +\sin(\mu_1B_\perp t)\sin(\mu_2B_\perp t)\sin^2\frac{\Delta m^2}{4p}t\Big\}. \tag{25}$$

For completeness, we also calculate within our approach the neutrino survival probability $\nu_e^L \leftrightarrow \nu_e^L$ and get

$$P_{\nu_e^L\to\nu_e^L}(t) = \Big\{\cos\left(\mu_+B_\perp t\right)\cos\left(\mu_-B_\perp t\right) - \cos 2\theta\sin\left(\mu_+B_\perp t\right)\sin\left(\mu_-B_\perp t\right)\Big\}^2 - \\ - \sin^2 2\theta\cos(\mu_1B_\perp t)\cos(\mu_2B_\perp t)\sin^2\frac{\Delta m^2}{4p}t. \tag{26}$$

It is just straightforward that the sum of the obtained four probabilities (23), (24), (25) and (26) is

$$P_{\nu_e^L\to\nu_\mu^L} + P_{\nu_e^L\to\nu_e^R} + P_{\nu_e^L\to\nu_\mu^R} + P_{\nu_e^L\to\nu_e^L} = 1. \tag{27}$$

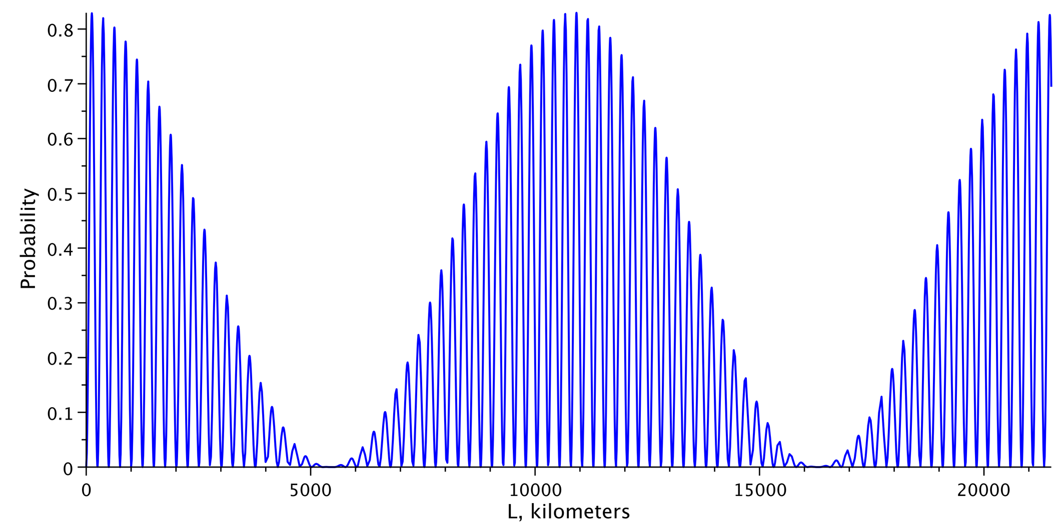

Figure 1: The probability of the neutrino flavour oscillations $\nu_e^L \to \nu_\mu^L$ in the transversal magnetic field $B_\perp = 10^8\ G$ for the neutrino energy $p = 1\ MeV$, $\Delta m^2 = 7\times 10^{-5}\ eV^2$ and magnetic moments $\mu_1 = \mu_2 = 10^{-12}\mu_B$.

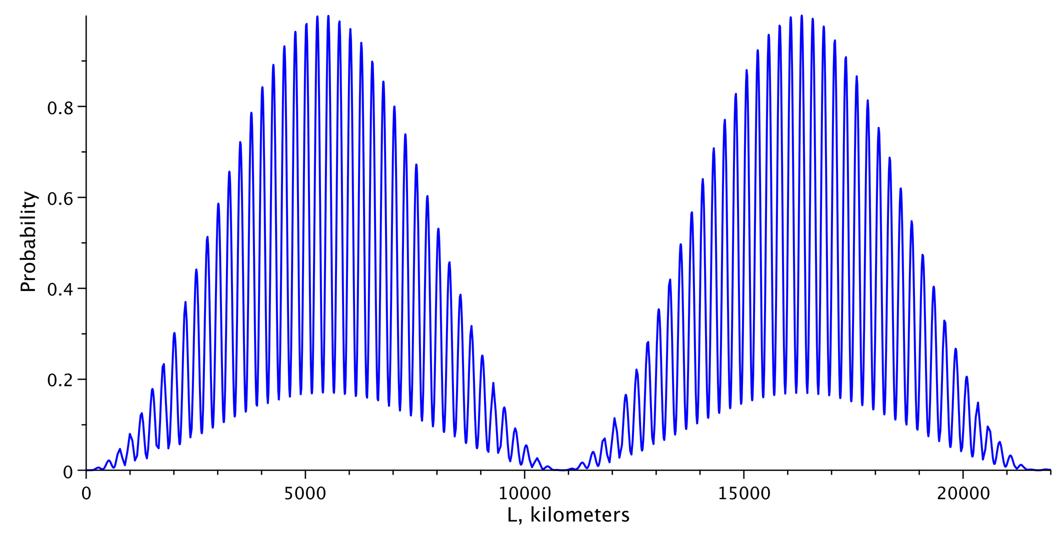

Figure 2: The probability of the neutrino spin oscillations $\nu_e^L \to \nu_e^R$ in the transversal magnetic field $B_\perp = 10^8\ G$ for the neutrino energy $p = 1\ MeV$, $\Delta m^2 = 7\times 10^{-5}\ eV^2$ and magnetic moments $\mu_1 = \mu_2 = 10^{-12}\mu_B$.

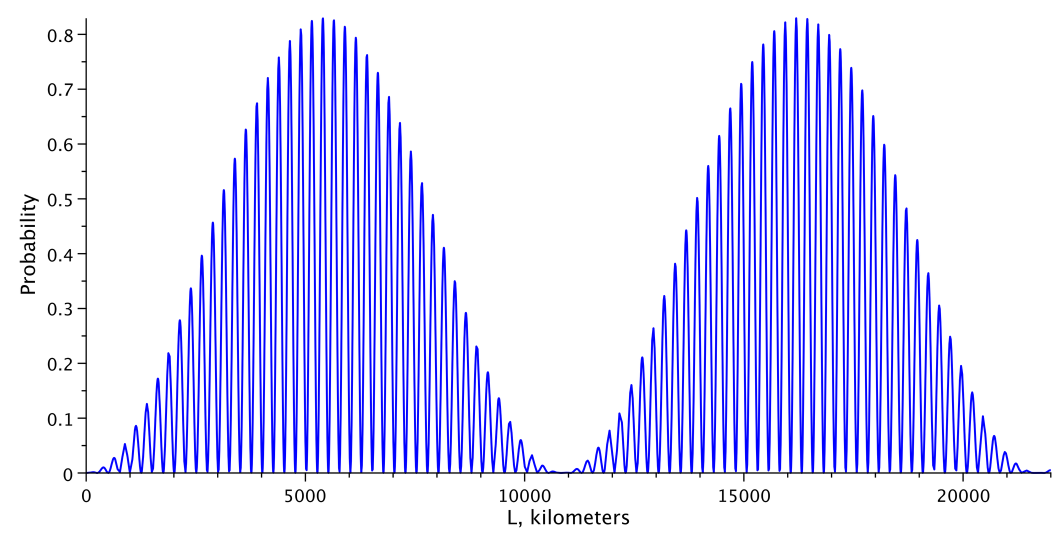

Figure 3: The probability of the neutrino spin flavour oscillations $\nu_e^L \to \nu_\mu^R$ in the transversal magnetic field $B_\perp = 10^8\ G$ for the neutrino energy $p = 1\ MeV$, $\Delta m^2 = 7\times 10^{-5}\ eV^2$ and magnetic moments $\mu_1 = \mu_2 = 10^{-12}\mu_B$.

Finally, the obtained closed expressions (23), (24), (25) and (26) show that the neutrino oscillation $P_{\nu_e^L\to\nu_\mu^L}(t)$, $P_{\nu_e^L\to\nu_e^R}(t)$, $P_{\nu_e^L\to\nu_\mu^R}(t)$ and also survival $P_{\nu_e^L\to\nu_e^L}(t)$ probabilities exhibits quiet complicated interplay of the harmonic functions that are dependent on six different frequencies. On this basis we predict modifications of the neutrino oscillation patterns that might provide new important phenomenological consequences in case of neutrinos propagation in extreme astrophysical environments where magnetic fields are present.

References

[1] A. Popov and A. Studenikin, arXiv:1803.05755 [hep-ph].

[2] C.Giunti and A.Studenikin, Rev. Mod. Phys. **87**, 531 (2015)

[3] A. Studenikin, PoS EPS **-HEP2017** (2017) 137 doi:10.22323/1.314.0137 [arXiv:1801.08887 [hep-ph]].

[4] K.Fujikawa and R.Shrock, Phys. Rev. Lett. **45**, 963 (1980)

[5] A.Beda, V.Brudanin, V.Egorov, D.Medvedev et al, Adv. High Energy Phys. **2012**, 350150 (2012)

[6] M. Agostini *et al.* [Borexino Collaboration], arXiv:1707.09355 [hep-ex].

[7] G.Raffelt, Phys. Rev. Lett. **64**, 2856 (1990)

[8] N. Viaux, M. Catelan, P. B. Stetson, G. Raffelt, J. Redondo, A. A. R. Valcarce and A. Weiss, Astron. Astrophys. **558** (2013) A12 doi:10.1051/0004-6361/201322004 [arXiv:1308.4627 [astro-ph.SR]].

[9] S. Arceo-Daz, K.-P. Schrder, K. Zuber and D. Jack, Astropart. Phys. **70** (2015) 1. doi:10.1016/j.astropartphys.2015.03.006

[10] A.Cisneros, Astrophys. Space Sci. **10**, 87 (1971)

[11] J.Schechter and J.W.F.Valle Phys. Rev. D **24**. 1883 (1981)

[12] L. Okun, M.Voloshin and M.Vysotsky, Sov. Phys. JETP **64**, 446 (1986)

[13] E. Akhmedov, Phys. Lett. B **213**, 64 (1988)

[14] C.-S.Lim and W. Marciano, Phys. Rev. D **37**, 1368 (1988)

[15] G. G. Likhachev and A. I. Studenikin, J. Exp. Theor. Phys. **81** (1995) 419 [Zh. Eksp. Teor. Fiz. **108** (1995) 769].

[16] E.Akhmedov and M.Khlopov, Mod. Phys. Lett.A bf 3, 451 (1988)

[17] R. Fabbricatore, A. Grigoriev and A. Studenikin, J. Phys. Conf. Ser. **718** (2016) no.6, 062058 doi:10.1088/1742-6596/718/6/062058 [arXiv:1604.01245 [hep-ph]].

[18] J. Pulido and E. K. Akhmedov, Astropart. Phys. **13** (2000) 227 doi:10.1016/S0927-6505(99)00121-8 [hep-ph/9907399].

[19] E. K. Akhmedov and J. Pulido, Phys. Lett. B **485** (2000) 178 doi:10.1016/S0370-2693(00)00695-X [hep-ph/0005173].

[20] E. K. Akhmedov and J. Pulido, Phys. Lett. B **529** (2002) 193 doi:10.1016/S0370-2693(02)01254-6 [hep-ph/0201089].

[21] E. K. Akhmedov and J. Pulido, Phys. Lett. B **553** (2003) 7 doi:10.1016/S0370-2693(02)03182-9 [hep-ph/0209192].

[22] A. Dmitriev, R. Fabbricatore and A. Studenikin, PoS CORFU **2014**, 050 (2015) [arXiv:1506.05311 [hep-ph]]

[23] A. Studenikin, EPJ Web Conf. **125** (2016) 04018 doi:10.1051/epjconf/201612504018 [arXiv:1705.05944 [hep-ph]].

[24] I. M. Ternov, V. G. Bagrov and A. M. Khapaev, JETP **21**, 613 (1965).

[25] P. Kurashvili, K. A. Kouzakov, L. Chotorlishvili and A. I. Studenikin, Phys. Rev. D **96** (2017) no.10, 103017 doi:10.1103/PhysRevD.96.103017 [arXiv:1711.04303 [hep-ph]].