State-of-the-art of mixed QCD-EW predictions

Christian Schwinn

— RWTH Aachen —

12.04.2018

QCD corrections

- **NLO** automation:
  One-loop providers (`BlackHat`, `GoSam`, `MadLoop`, `OpenLoops`, `Recola`)
  interfaced with Monte Carlo (`Sherpa`, `MadGraph`, `Munich`, ...),
  matching to Parton showers (`MC@NLO`, `POWHEG`),
  merging of different jet multiplicities(`MINLO`, `MEPS@NLO`, `FxFx`,...)
- **NNLO** for $2 \to 2$ processes VV Vj, Hj, VH, $t\bar{t}$, HH

EW corrections

- **NLO** automation: published results for specific processes

  (⇒ talk by Kuttimalai)

  public release in current/next versions of one-loop frameworks

$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$	$\sigma^{\rm LO}$ [fb]	$\sigma^{\rm NLO}_{\rm EW}$ [fb]	$\Delta\sigma^{\rm LO}$ [σ]	$\Delta\sigma^{\rm LO}$ [‰]	$\Delta\sigma^{\rm NLO}_{\rm EW}$ [σ]	$\Delta\sigma^{\rm NLO}_{\rm EW}$ [‰]
average	448.5414[31]	438.1902[56]				
MUNICH+OPENLOOPS	448.5468[45]	438.1920[75]	+1.6	+0.01	+0.4	+0.00
MOCANLO+RECOLA	448.538[10]	438.193[13]	−0.4	−0.01	+0.2	+0.01
SHERPA+GOSAM/OPENLOOPS/RECOLA	448.5364[46]	438.186[11]	−1.4	−0.01	−0.4	−0.01
MADGRAPH5_AMC@NLO	448.541[40]	438.113[70]	−0.0	−0.00	−1.1	−0.18

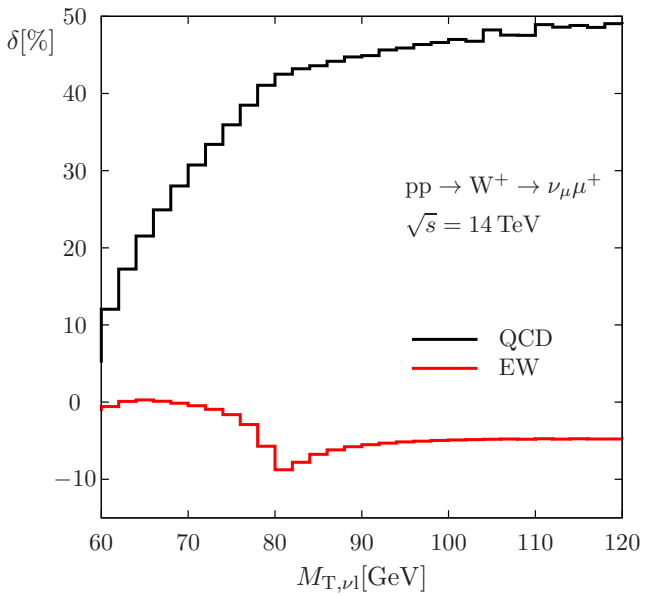
(LesHouches 2017)

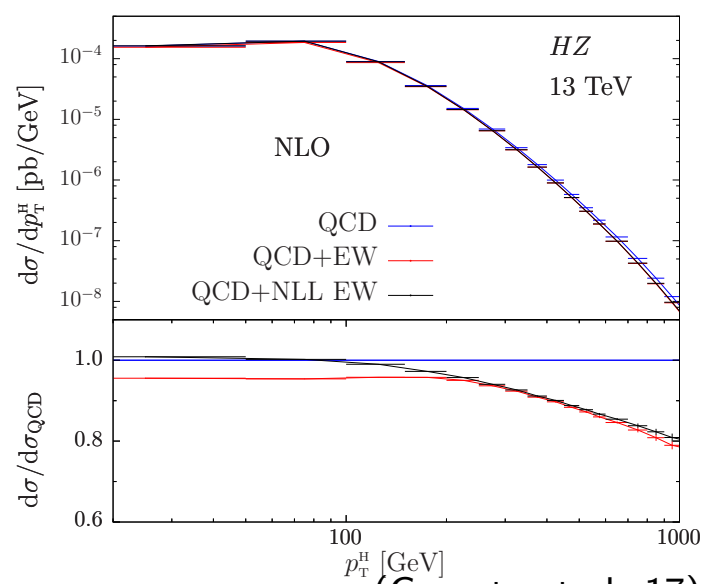
Relevance of EW corrections

- Expectation for generic observables: $\Delta\mathrm{NLO}_{\mathrm{EW}} \sim \Delta\mathrm{NNLO}_{\mathrm{QCD}}$
- Enhanced effects:
  - FSR for EW resonances ($\Rightarrow$ 100 MeV effect on M_W measurement)

    (`Photos` (Golonka/Was 06), γ shower in Pythia, Sherpa)
  - Sudakov logarithms $\alpha \log^{2,1}(Q^2/M_W^2)$ at large Q^2

    (universal, e.g. Denner/Pozzorini 01)





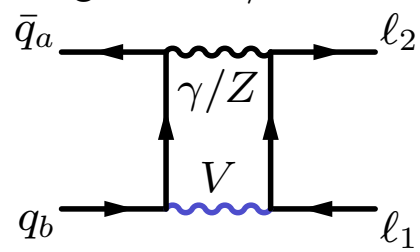
(Granata et al. 17)

Relevance of EW corrections

- Expectation for generic observables: $\Delta\mathrm{NLO}_{\mathrm{EW}} \sim \Delta\mathrm{NNLO}_{\mathrm{QCD}}$
- Enhanced effects:
  - FSR for EW resonances ($\Rightarrow$ $100\,\mathrm{MeV}$ effect on M_W measurement)

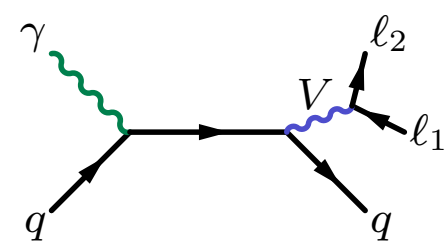
    (`Photos` (Golonka/Was 06), γ shower in Pythia, Sherpa)
  - Sudakov logarithms $\alpha \log^{2,1}(Q^2/M_W^2)$ at large Q^2

    (universal, e.g. Denner/Pozzorini 01)



Features of EW corrections

- connecting initial and final state
- consistent treatment of decay widths necessary (e.g. complex mass scheme)
- reconstruction of "bare" muons
  $\Rightarrow$ logarithmic dependence $\sim \alpha \log\left(m_\mu^2/\hat{s}\right)$



- Photon-induced processes ($\Rightarrow$ LUXQED PDF (Manohar et al. 16))

Importance of mixed $\mathcal{O}(\alpha\alpha_s)$ corrections?

- Expectation for generic observables: $\mathrm{NNLO_{EW/QCD}} \sim \mathrm{N^3LO_{QCD}}$

$\Rightarrow$ relevant for precision physics: DY, $gg \to H$ (Talks by Piccinini, Lindert)

- Dominant effects from FSR/Sudakov expected to factorize; relevant for $t\bar{t}$, VH, $V+$jets... (Talks by Lindert, Pagani)

Impact on DY-type processes:

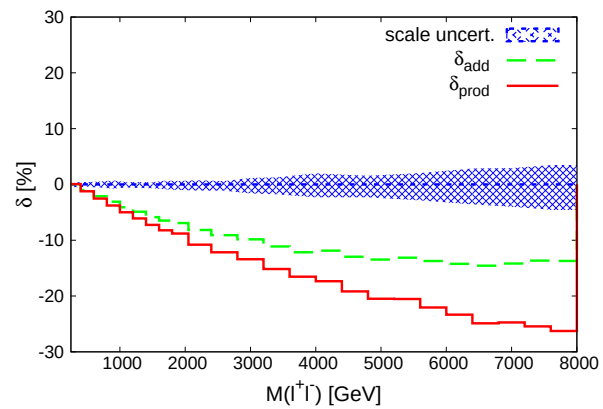
- M_W-measurement: (Dittmaier/Huss/CS 15; Carloni Calame et al. 16)

$$|\Delta M_W^{\mathrm{NNLO}}| \approx 15 \text{ MeV}$$

(approximately included in current analysis through NLO-QCD+Photos)

- Sudakov corrections:
  Estimate of $\mathcal{O}(\alpha\alpha_s)$ corrections larger than NNLO-QCD scale uncertainty for $M_{\ell\ell} > 2\mathrm{TeV}$

  (Campbell/Wackeroth/Zhou 16)



Mixed QCD-EW corrections

Features of mixed EW-QCD corrections

- Classification by orders of α and α_s
- Status of full NNLO calculations

Approaches to approx. $\mathcal{O}(\alpha\alpha_s)$ corrections

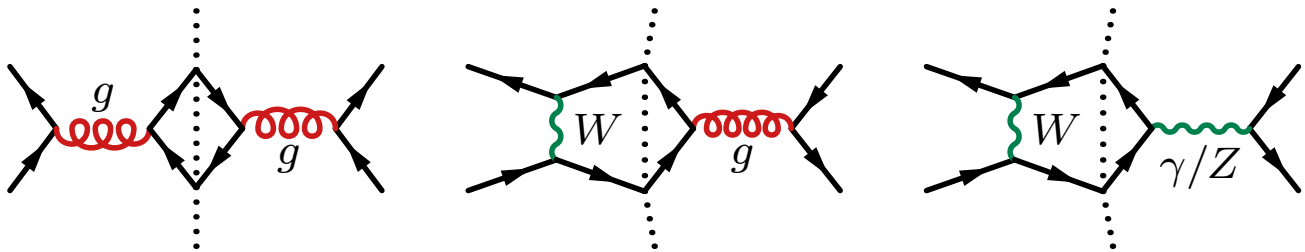
- naive factorization of K-factors
- matching of EW corrections and parton showers
- Pole approximation for resonant processes

Recent examples

- $t\bar{t}$ (additive/multiplicative combination with NNLO QCD: Czakon et al. 17, PS merging: Gütschow et al. 18)
- $V +$ jets (PS merging: Kallweit et al. 15, MC-reweighting: Lindert et al. 17)
- Drell-Yan (Pole approximation: Dittmaier/Huss/CS 14/15; Parton shower matching Carloni Calame et al. 16,. . .)

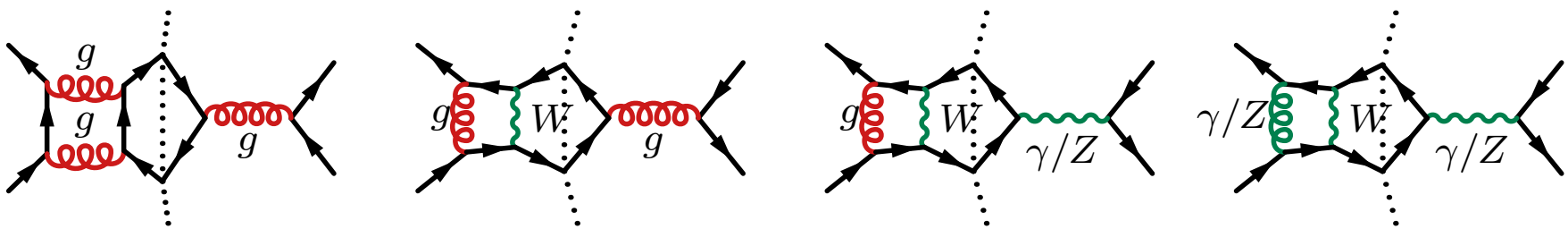
For generic processes both QCD and EW Born contribute:

$\mathcal{O}(\alpha_s^2)$ $\mathcal{O}(\alpha_s\alpha)$ $\mathcal{O}(\alpha^2)$



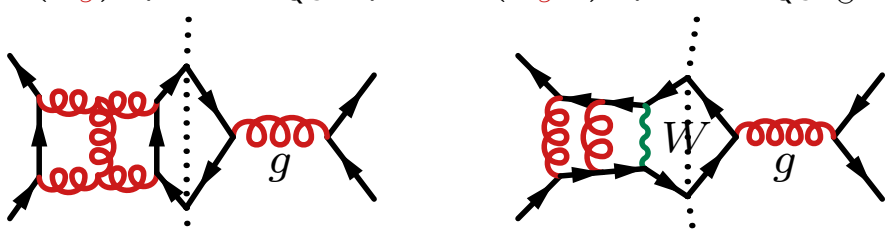
"EW" and "QCD" orders entangled by loop corrections:

$\mathcal{O}(\alpha_s^3)$ ("NLO$_{\text{QCD}}$") $\mathcal{O}(\alpha_s^2\alpha)$ ("NLO$_{\text{EW}}$") $\mathcal{O}(\alpha_s\alpha^2)$ $\mathcal{O}(\alpha^3)$



("subleading" contributions may be enhanced, e.g. in $t\bar{t}W$, $t\bar{t}t\bar{t}$ $\Rightarrow$ talk by Pagani)

$\mathcal{O}(\alpha_s^4)$ ("NNLO$_{\text{QCD}}$") $\mathcal{O}(\alpha_s^3\alpha)$ ("NNLO$_{\text{QCD}\otimes\text{EW}}$") ...

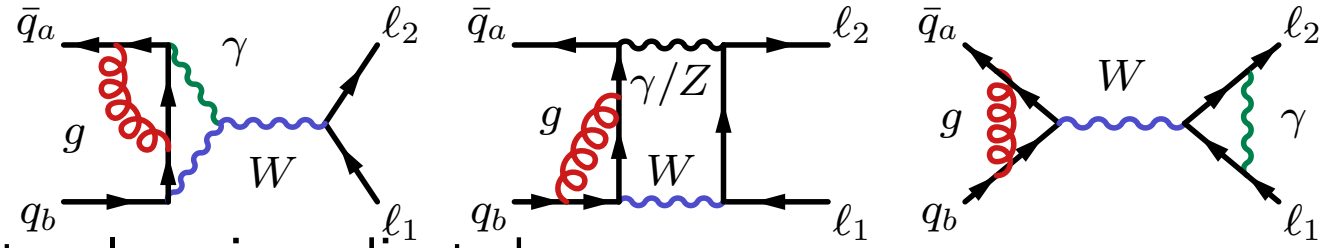


...

state-of-the-art current bottleneck!

Full NNLO QCD⊗EW corrections for DY:

Two-loop diagrams with different mass scales, finite widths:



some two-loop ingredients known

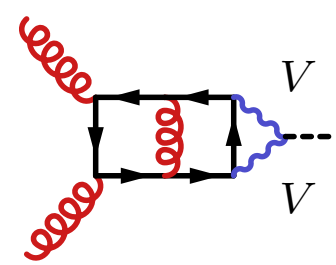
($\mathcal{O}(\alpha_s\alpha)$ corrections to Z/W decay widths: Czarnecki/Kühn 96; Kara 13,

two-loop amplitudes Kotikov/Kühn/Veretin 07; Kilgore/Sturm 12;

Master integrals: Bonciani et al. 16; v.Manteuffel/Schabinger 17)

QCD ⊗ EW corrections to $gg \to H$: (Bonetti et al. 17/18 ⇒ talk by Lindert)

- 3-loop diagrams calculated
- Real corrections in soft approximation



$$\frac{\sigma^{\rm NLO}_{\rm QCD}}{\sigma^{\rm LO}_{\rm QCD}} \approx \frac{\sigma^{\rm NLO}_{\rm EW/QCD}}{\sigma^{\rm LO}_{\rm EW/QCD}} \approx 5.4\%$$

- confirms result from $M_{W/Z} \gg M_H$ limit (Anastasiou et al. 08)

Mixed QCD-EW corrections

Parton-level combinations:

- Additive: $\sigma^{\mathrm{NLO}}_{\mathrm{QCD+EW}} = \sigma_{\mathrm{LO}} + \Delta\sigma^{\mathrm{NLO}}_{\mathrm{QCD}} + \Delta\sigma^{\mathrm{NLO}}_{\mathrm{EW}} = \sigma_{\mathrm{LO}} K^{\mathrm{NLO}}_{\mathrm{QCD}} + \Delta\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$
- Multiplicative: $\sigma^{\mathrm{NLO}}_{\mathrm{QCD\times EW}} = \sigma^{\mathrm{NLO}}_{\mathrm{QCD+EW}} + \frac{\Delta\sigma^{\mathrm{NLO}}_{\mathrm{QCD}}\Delta\sigma^{\mathrm{NLO}}_{\mathrm{EW}}}{\sigma_{\mathrm{LO}}} = \sigma^{\mathrm{NLO}}_{\mathrm{EW}} K^{\mathrm{NLO}}_{\mathrm{QCD}}$

  (schematic, ignores e.g. use of PDFs at different order)
  - expected to capture factorizing soft-QCD/Sudakov-EW effects leading to large corrections
  - misses simultaneous hard photon and gluon emission, non-factorizing virtual effects
  - scale appropriate for $K^{\mathrm{NLO}}_{\mathrm{QCD}}$ might be changed by γ-FSR
- Use $\sigma^{\mathrm{NLO}}_{\mathrm{QCD\times EW}} - \sigma^{\mathrm{NLO}}_{\mathrm{QCD+EW}}$ as error estimate?
  - likely overestimates error in regions dominated by soft-QCD/Sudakov-EW (where corrections are large)
  - appropriate elsewhere (where corrections are small)

Matching of NLO-EW corrections to QCD shower

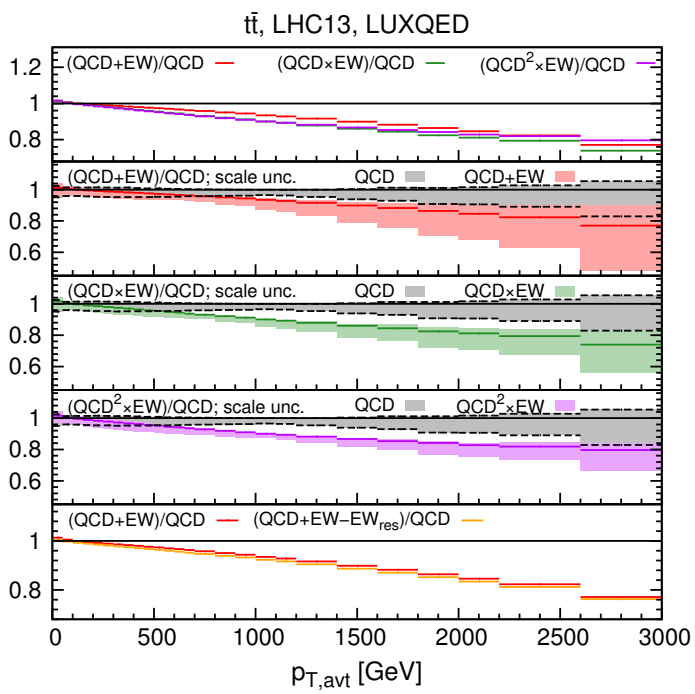
- Modelling of $\mathcal{O}(\alpha_s\alpha)$ effects from collinear gluon emission
  - Drell-Yan (Bernaciak/Wackeroth 12; Barzè et al. 12/13; Carloni Calame et al.; Mück/Oymanns 16)
  - HV, HVj (`OpenLoops+MINLO/POWHEG`: Granata et al. 17)

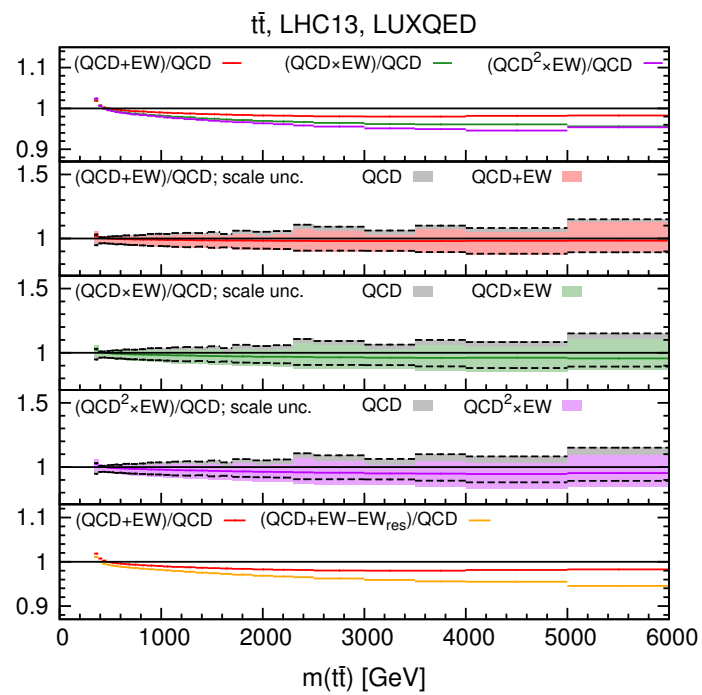
  Needs resonance-aware matching
- **Merging** of EW corrections for different jet multiplicities, includes real-gluon emission of NNLO $\mathcal{O}(\alpha_s\alpha)$ corrections
  - so far only in $\mathrm{EW}_{\text{virt}}$ approximation in `OpenLoops+Sherpa`:
    * $V + 1, 2$ jets (Kallweit et al. 15)
    * $t\bar{t}$ and $t\bar{t}j$ (Gütschow et al. 18)
  - $\mathrm{EW}_{\text{virt}}$ appropriate for Sudakov effects, full NLO-EW needed for precision near resonances

Additive/Multiplicative combination of NLO-EW and NNLO-QCD

(Czakon et al. 17)

- $\mathrm{EW} = \mathrm{LO}_{\mathcal{O}(\alpha_s\alpha)} + \mathrm{NLO}_{\mathcal{O}(\alpha_s^2\alpha)} + \underbrace{\mathrm{LO}_{(\alpha^2)} + \mathrm{NLO}_{\mathcal{O}(\alpha_s\alpha^2)+\mathcal{O}(\alpha^3)}}_{\mathrm{EW}_{\mathrm{res}}}$
- γq initial state included in $\mathrm{LO}_{\mathcal{O}(\alpha_s\alpha)}$
- $\mathrm{EW} \times \mathrm{QCD} = \mathrm{EW} + \mathrm{QCD} + (K_{\mathrm{QCD}}^{\mathrm{NLO}} - 1)\mathrm{NLO}_{\mathcal{O}(\alpha_s^2\alpha)}$
  expected to describe NNLO $\mathrm{EW}_{\mathrm{Sudakov}} \times \mathrm{QCD}_{\mathrm{soft}}$ corrections at $\mathcal{O}(\alpha_s^3\alpha)$.





Additive/Multiplicative combination of NLO-EW and NNLO-QCD

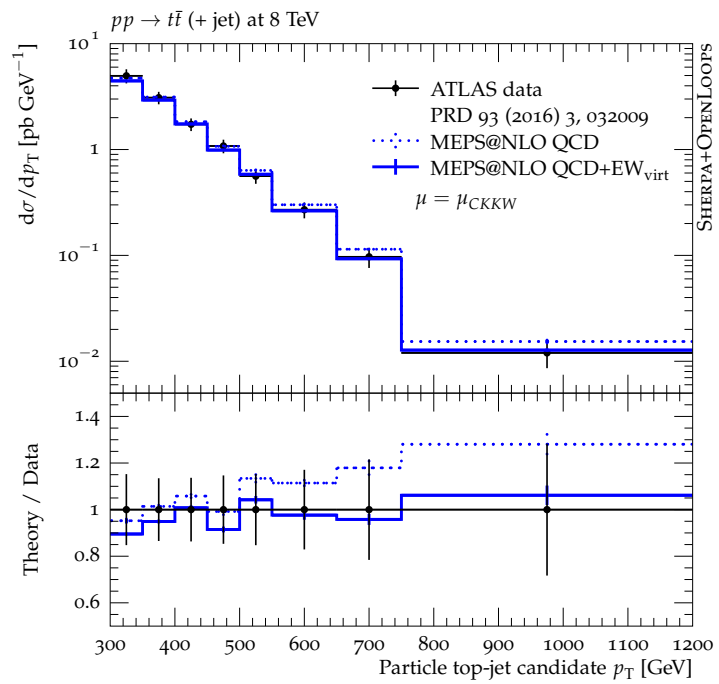
(Czakon et al. 17)

- $\mathrm{EW} = \mathrm{LO}_{\mathcal{O}(\alpha_s\alpha)} + \mathrm{NLO}_{\mathcal{O}(\alpha_s^2\alpha)} + \underbrace{\mathrm{LO}_{(\alpha^2)} + \mathrm{NLO}_{\mathcal{O}(\alpha_s\alpha^2)+\mathcal{O}(\alpha^3)}}_{\mathrm{EW}_{\mathrm{res}}}$
- γq initial state included in $\mathrm{LO}_{\mathcal{O}(\alpha_s\alpha)}$
- $\mathrm{EW} \times \mathrm{QCD} = \mathrm{EW} + \mathrm{QCD} + (K^{\mathrm{NLO}}_{\mathrm{QCD}} - 1)\mathrm{NLO}_{\mathcal{O}(\alpha_s^2\alpha)}$
  expected to describe NNLO $\mathrm{EW}_{\mathrm{Sudakov}} \times \mathrm{QCD}_{\mathrm{soft}}$ corrections at $\mathcal{O}(\alpha_s^3\alpha)$.

Parton-shower merging of $t\bar{t}$ and $t\bar{t}j$ with NLO QCD+EW

(Gütschow, Lindert, Schönherr 18)

- Includes exact virtual EW corrections, real γ in YFS
- merged with $t\bar{t} + 2, 3, 4j$ at LO



Detailed analysis of uncertainties for $V + j$ in context of dark-matter searches (Lindert et al. 17)

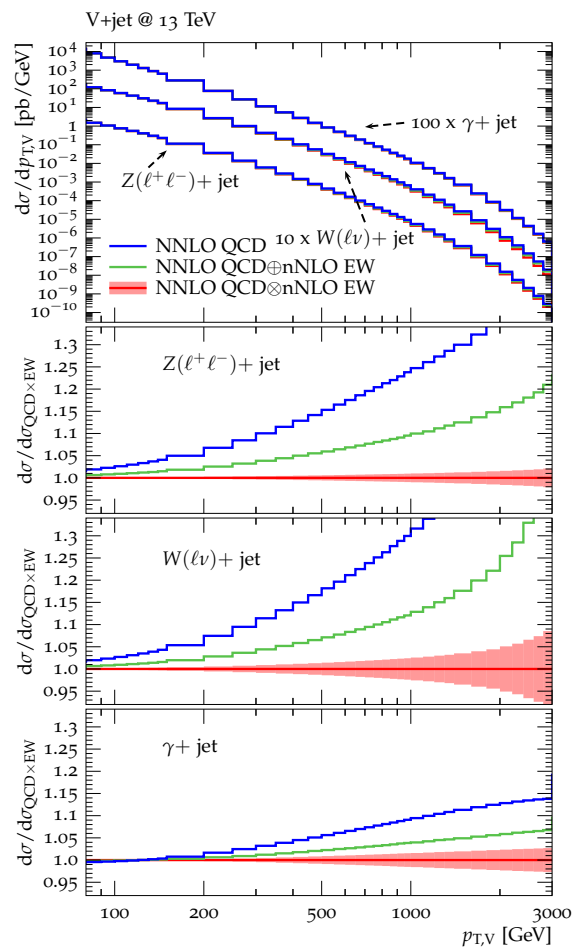
- Higher-order QCD and EW corrections included by MC reweighting

$$\frac{d\sigma_{Vj}}{dp_T^V\,dy} = \frac{d\sigma_{Vj}^{\mathrm{MC}}}{dp_T^V\,dy}\left[\frac{d\sigma_{Vj}^{\mathrm{th}}/dp_T^V}{d\sigma_{Vj}^{\mathrm{MC}}/dp_T^V}\right]$$

- $\mathrm{nNLO_{EW}}$ includes NLL Sudakov logs $\alpha^2 \ln^{4,3}(Q^2/M_V^2)$
- Additive and multiplicative combination of EW and QCD corrections
- Estimate of $\mathcal{O}(\alpha\alpha_s)$ uncertainty

$$\Delta K_{\alpha\alpha_s}^{\mathrm{NNLO}} = \xi(K_{\mathrm{EW}\otimes\mathrm{QCD}} - K_{\mathrm{EW}\oplus\mathrm{QCD}})$$

$\xi = 0.1(Z),\ 0.2(W),\ 0.4(\gamma)$ estimated from NLO EW corrections to $V{+}2j$.



Detailed analysis of uncertainties for $V + j$ in context of dark-matter searches (Lindert et al. 17)

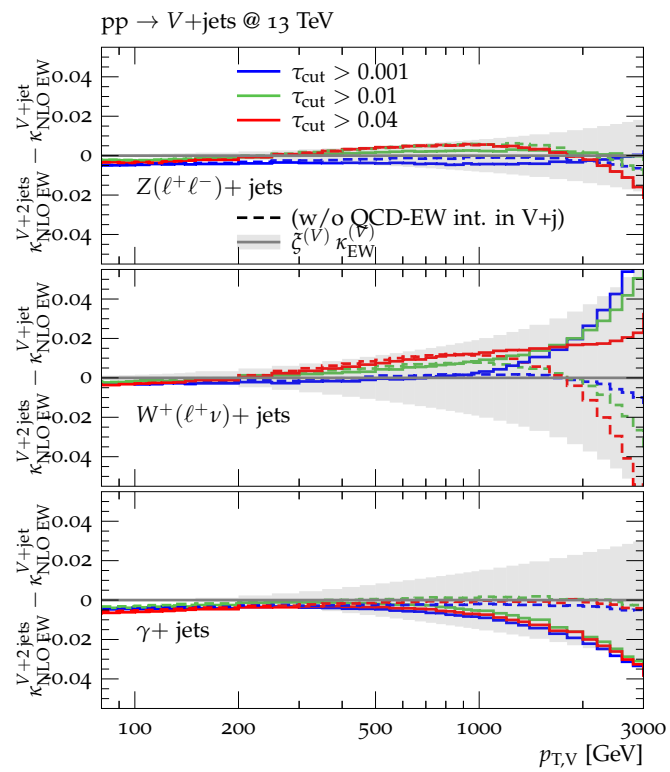
- Higher-order QCD and EW corrections included by MC reweighting

$$\frac{d\sigma_{Vj}}{dp_T^V\,dy} = \frac{d\sigma_{Vj}^{\mathrm{MC}}}{dp_T^V\,dy}\left[\frac{d\sigma_{Vj}^{\mathrm{th}}/dp_T^V}{d\sigma_{Vj}^{\mathrm{MC}}/dp_T^V}\right]$$

- $\mathrm{nNLO_{EW}}$ includes NLL Sudakov logs $\alpha^2 \ln^{4,3}(Q^2/M_V^2)$
- Additive and multiplicative combination of EW and QCD corrections
- Estimate of $\mathcal{O}(\alpha\alpha_s)$ uncertainty

$$\Delta K_{\alpha\alpha_s}^{\mathrm{NNLO}} = \xi(K_{\mathrm{EW}\otimes\mathrm{QCD}} - K_{\mathrm{EW}\oplus\mathrm{QCD}})$$

$\xi = 0.1(Z),\ 0.2(W),\ 0.4(\gamma)$ estimated from NLO EW corrections to $V{+}2j$.

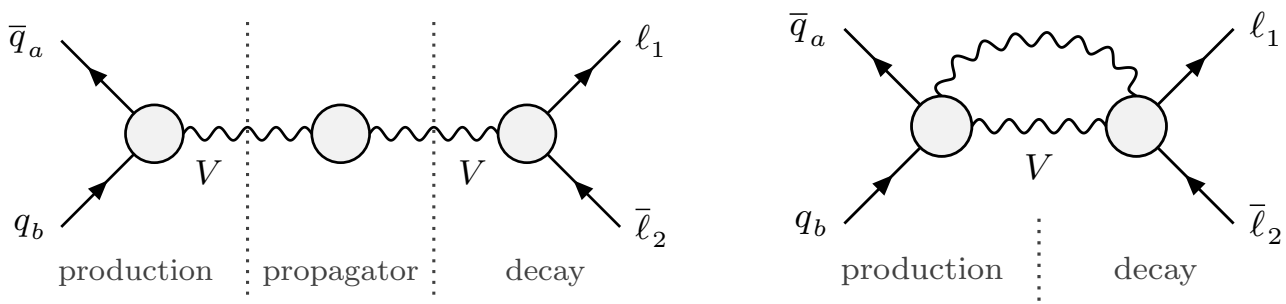


Pole scheme: (Stuart 91; Aeppli/v.Oldenbourgh/Wyler 93)

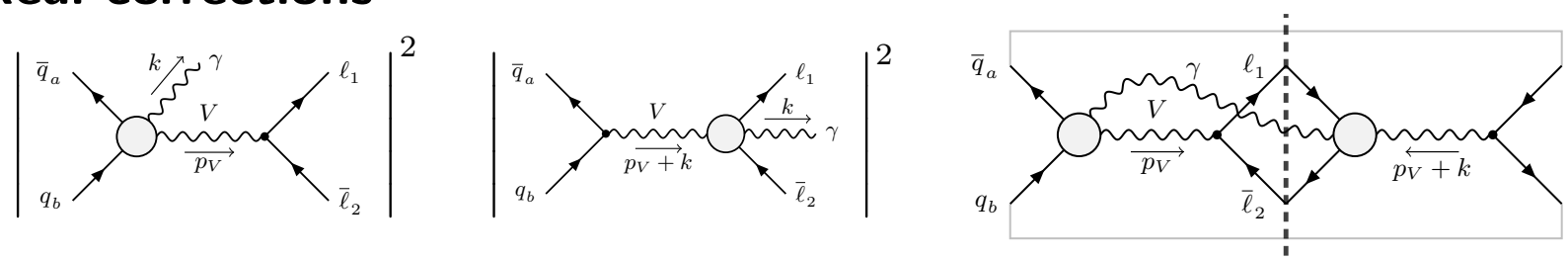
Expand for $p_V^2 \sim \mu_V^2$ with complex pole $\mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V$

- Factorizable corrections to on-shell prod. and decay
- Non-fact. soft-photon corrections

Virtual corrections



Real corrections

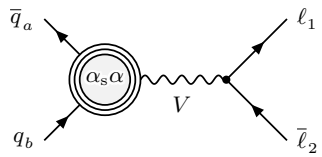


fact.-initial fact.-final non-factorizable

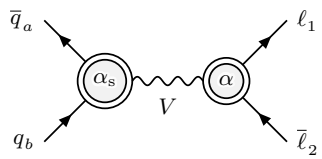
EW/QCD corrections in pole approximation (Dittmaier/Huss/CS 14/15)

(+ corresponding real-virtual and double real)

- Factorizable initial (partial results: Kotikov/Kühn/Veretin 07; Bonciani 11)

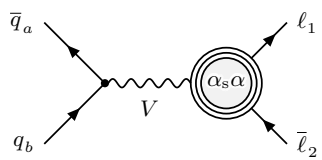


- Factorizable initial×final (expected to be dominant)

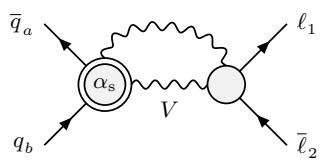


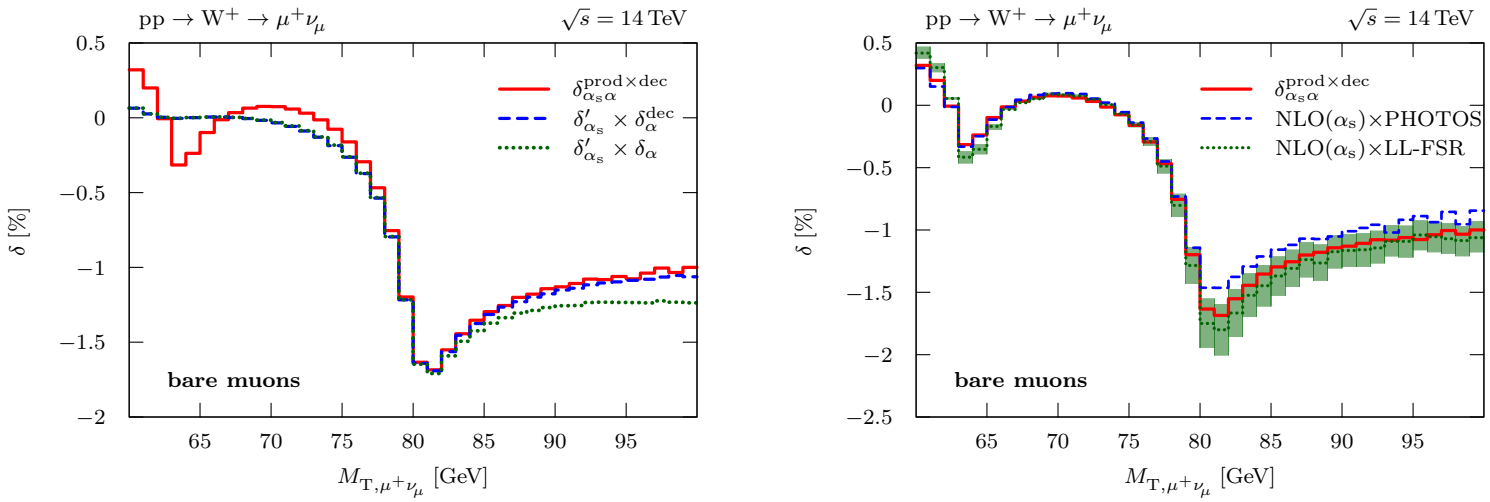
- Factorizable final×final

(finite counterterm from Djouadi/Gambino 93; negligible effect)



- Non-factorizable corrections (numerically negligible)





Comparison of different approximations:

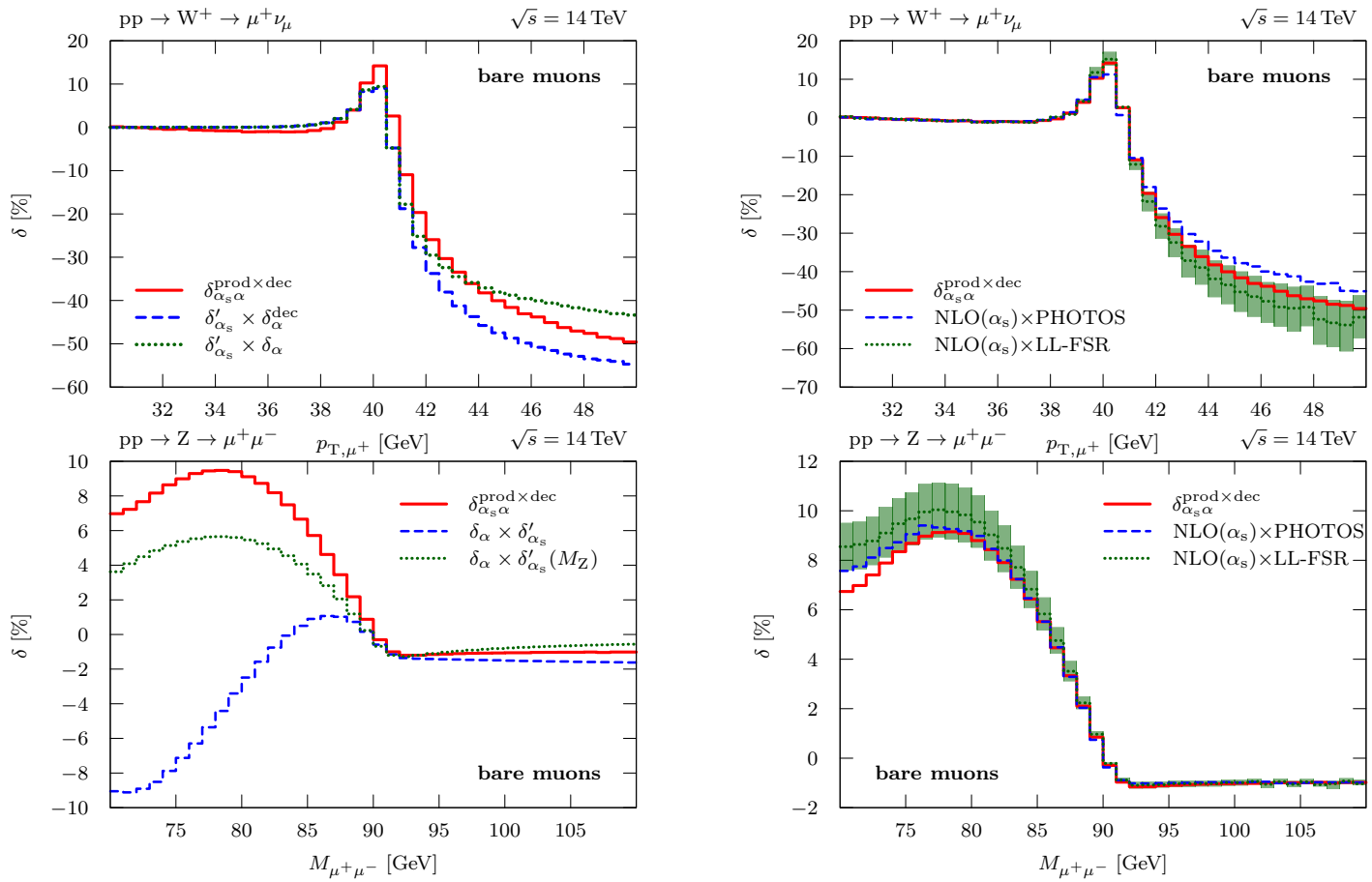
- $\delta^{\text{prod}\times\text{dec}}_{\alpha_s\alpha}$: factorizable initial-final $\mathcal{O}(\alpha_s\alpha)$ corrections
- Naive product of NLO corrections

$$\delta'_{\alpha_s}\delta_\alpha = \left(\frac{\sigma^{\text{NLO}_s}-\sigma^0}{\sigma^{\text{LO}}}\right)\times\frac{\Delta\sigma^{\text{NLO}_{\text{ew}}}}{\sigma^0}$$

where $\sigma^{\text{LO}}/\sigma^0$: LO/NLO PDFs

- $\text{NLO}(\alpha_s)\otimes$ LL[1]FSR: NLO QCD cross section convoluted with LL-FSR structure function
- $\text{NLO}(\alpha_s)\otimes$ PHOTOS: NLO QCD cross section with single photon emission generated with γ-shower (Golonka/Was 06)

Pole app. for EW⊗QCD corrections



- naive product of K-factors only appropriate for observables dominated by resonance and insensitive to ISR
- reasonable agreement of LL-FSR with full result.

(comparison to YFS photon resummation in Sherpa: Huss/Schönherr in Les Houches 15)

Implementation in POWHEG BOX (Carloni Calame et al. 16)

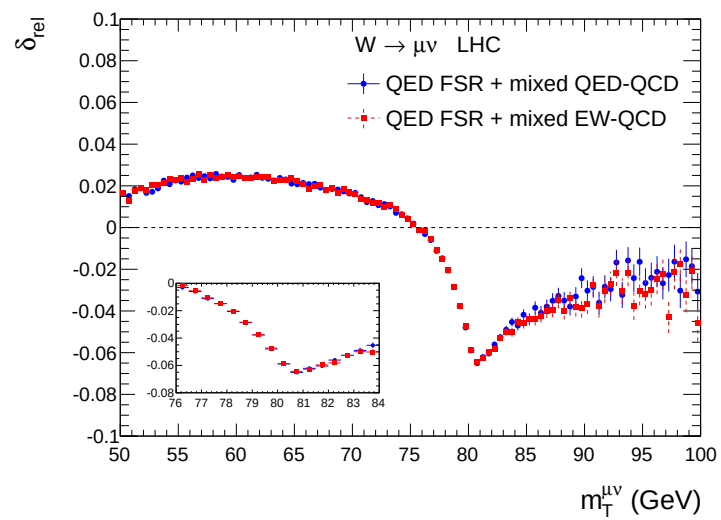
- Full NLO EW and QCD corrections matched to QCD and photon showers (Pythia/Photos)
- $\texttt{POWHEG}_{\texttt{two-rad}}$: generate first photon and gluon emissions with POWHEG (removes spurious $\mathcal{O}(\alpha_s\alpha)$ effect in Barzè et al. 12/13)

  (independent implementation using resonance-improved POWHEG: Mück/Oymanns 16)

$\Rightarrow$ includes approximation to initial-final QCD$\otimes$ EW corrections + additional multi-gluon/photon emission

- $\mathcal{O}(\alpha_s)\otimes$Photos in good agreement with matched NLO-EW
- discrepancies to Pythia photon shower reduced by matching

$\Rightarrow$ matching to NLO-EW for reliable prediction



Summary

NLO-EW corrections entering the age of automation

Prospects of EW precision physics at LHC;

Sudakov corrections increasingly important at 13 TeV

Mixed EW/QCD corrections

- no full calculations available yet (but work in progress for DY, $H \to gg$)
- Approaches in Sudakov regime:
  - factorized approaches expected to be appropriate
  - Multi-jet merging of $\mathrm{EW}_{\mathrm{virt}}$ corrections

    (resonance-aware merging required for full EW corrections)
  - Estimate of uncertainties?
- Approaches for EW precision physics near W/Z resonances:
  - pole expansion: initial-final corrections known.
  - POWHEG matching of NLO EW and QCD shower
  - dominant effects captured by matching NLO QCD+EW to multi-photon radiation in collinear limit.

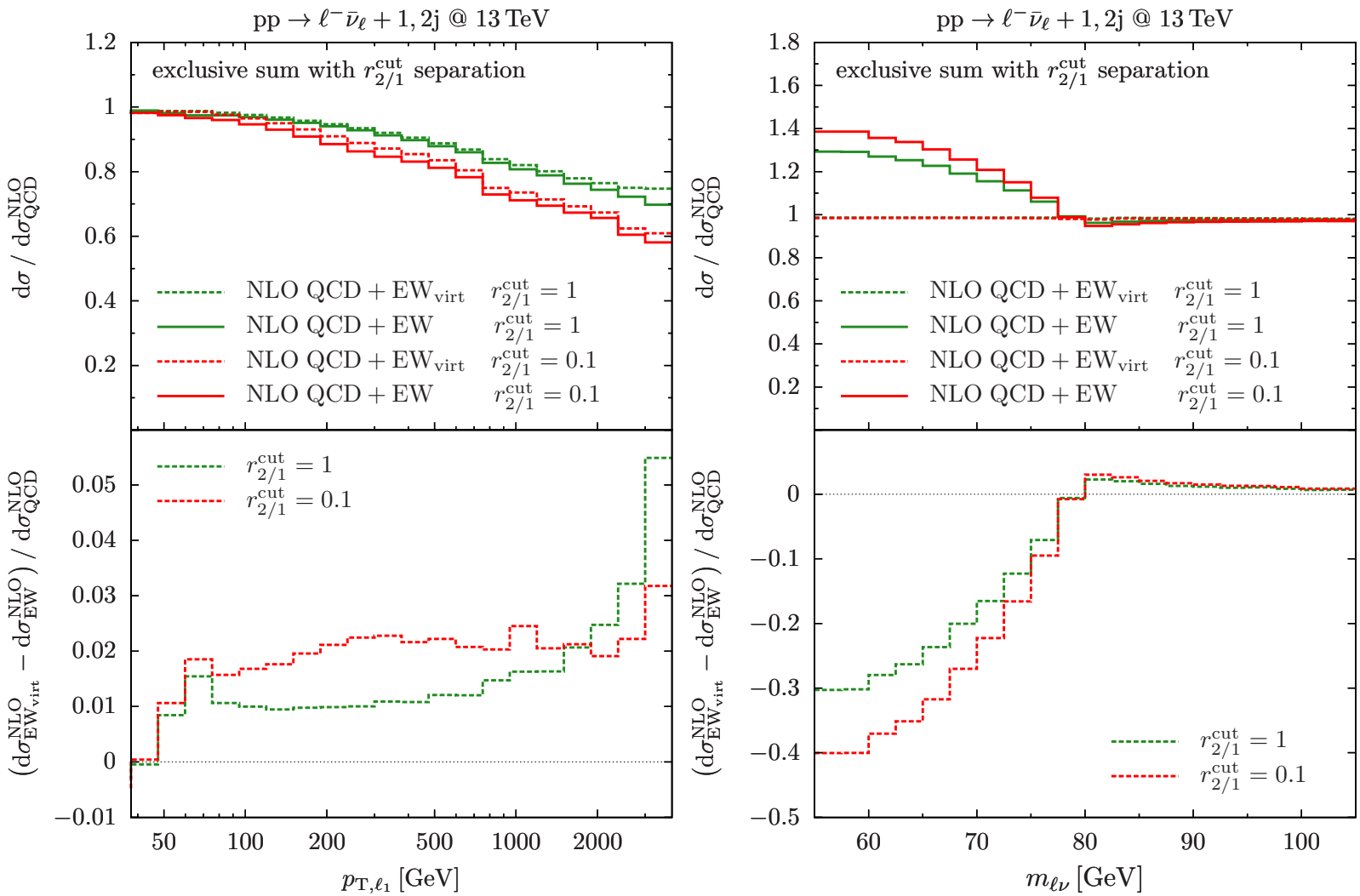
NLO-EW corrections entering the age of automation

Prospects of EW precision physics at LHC;

Sudakov corrections increasingly important at 13 TeV

Mixed EW/QCD corrections

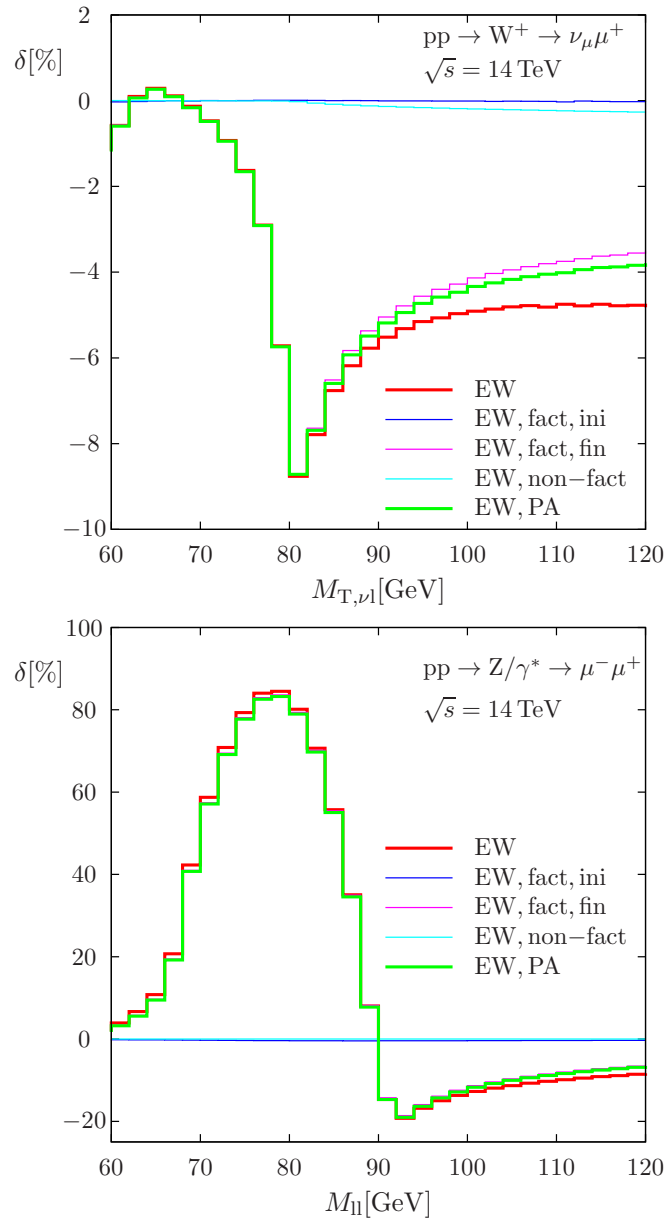
$\mathrm{EW}_{\mathrm{virt}}$ approximation in multi-jet merging in OpenLoops+Sherpa:



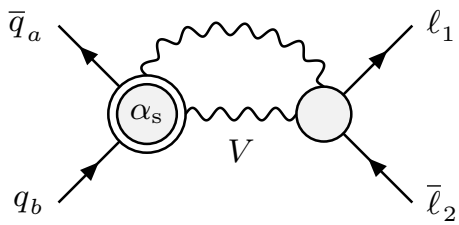
Application of pole approximation to EW corrections at NLO

(Wackeroth/Hollik 96; Baur et al. 98; Dittmaier/Krämer 01; Dittmaier/Huss/CS 14)

- 0.1% accuracy near peak
- final-state factorizable corrections dominant
- initial-state factorizable and soft non-factorizable corrections suppressed



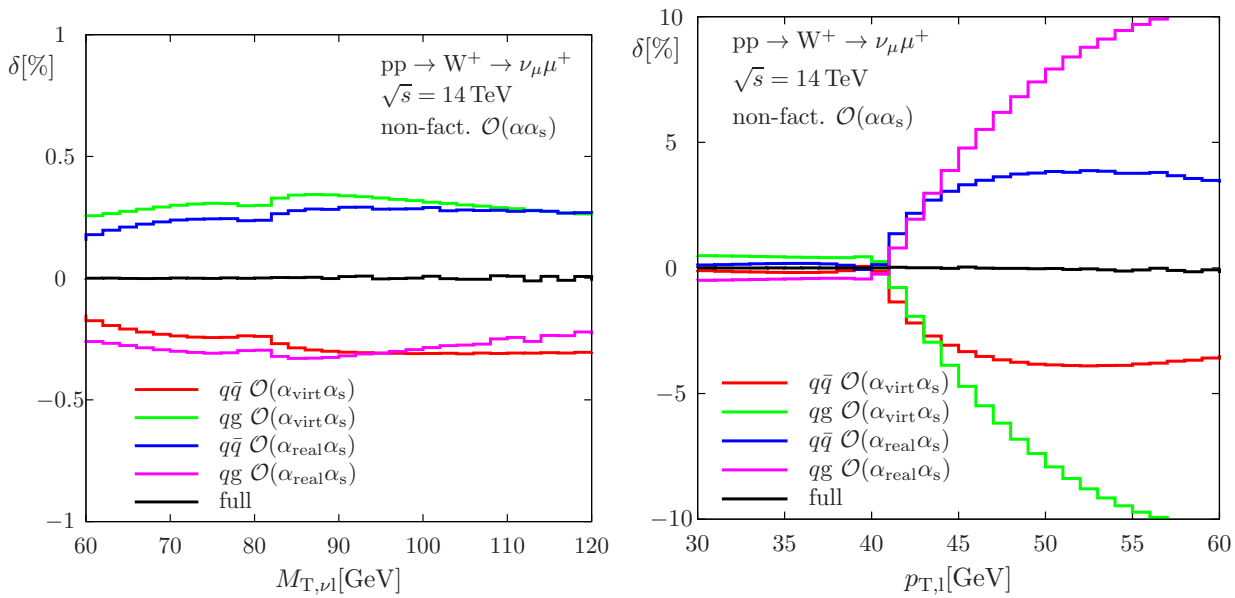
Non-factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections



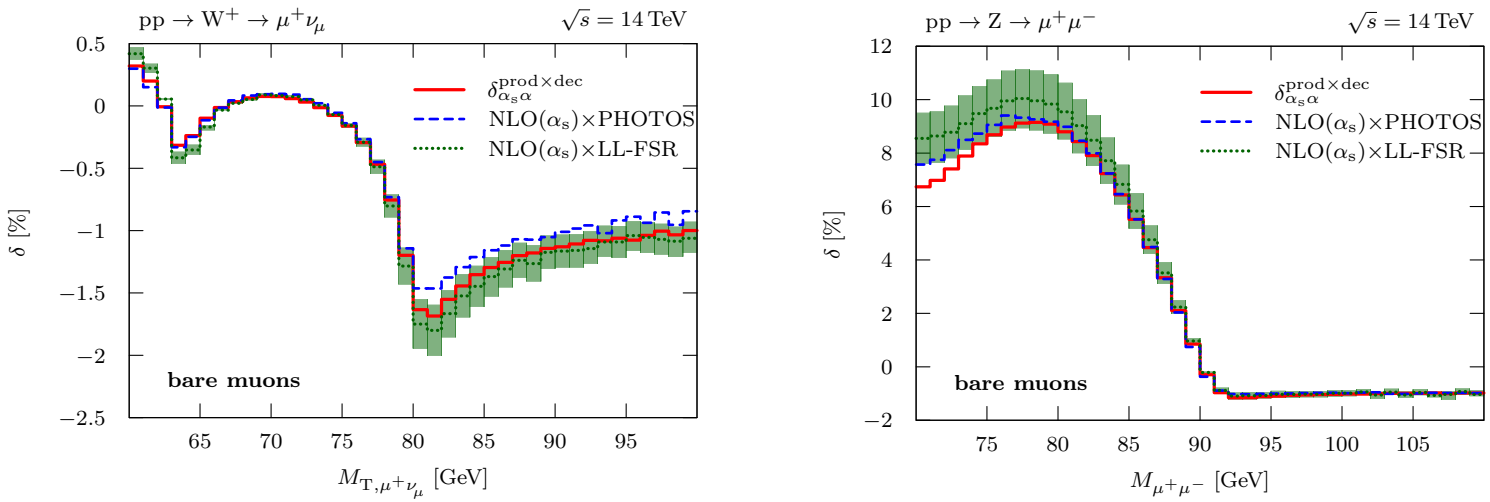
Numerical results:

practically complete cancellation of real and virtual corrections

(defined separately through soft slicing with $\Delta E_\gamma \ll \Gamma_V$ in real corrections)



Comparison of $\mathcal{O}(\alpha_s\alpha)$ corrections in pole-approximation to leading-logarithmic approximation to FSR



- LL1FSR: Convolution of NLO QCD cross section with one-loop structure function

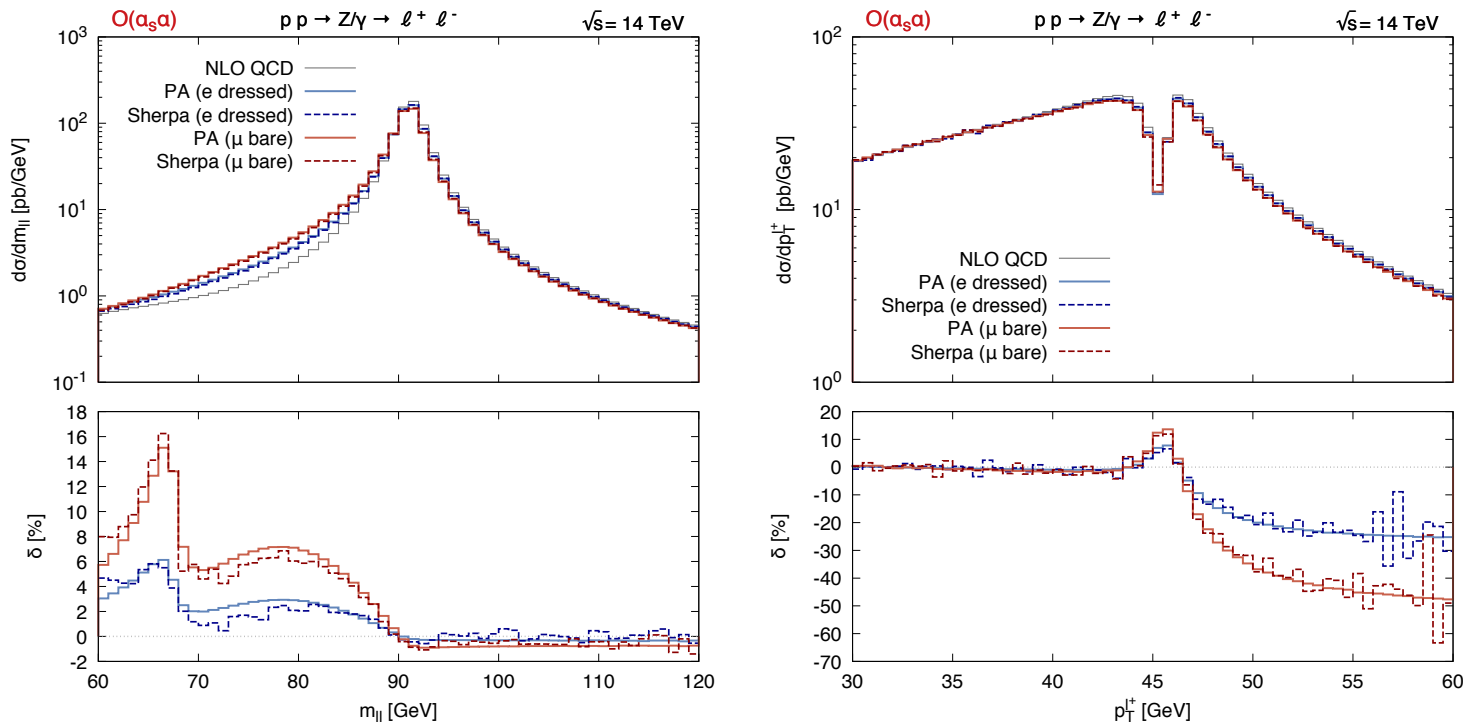
$$\Gamma^{\mathrm{LL},1}_{\ell\ell}(z,Q^2) = \frac{\beta_\ell}{4}\left(\frac{1+z^2}{1-z}\right)_+ \;,\; \beta_\ell = \frac{2\alpha(0)}{\pi}\left[\ln\left(\frac{Q^2}{m_\ell^2}\right)-1\right]$$

- Photos: NLO QCD with γ-shower restricted to single emission

(Golonka/Was 06)

$\Rightarrow$ reasonable agreement of LL approximation with full result.

Comparison of factorizable initial-final $\mathcal{O}(\alpha_s\alpha)$ corrections to YFS photon resummation in Sherpa: (Huss/Schönherr in Les Houches 15)



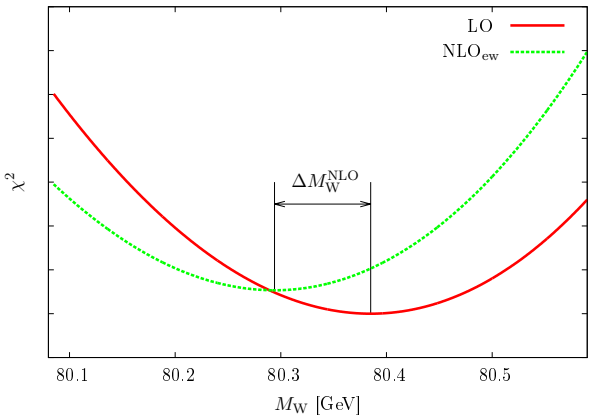
good agreement, although some different effects included:

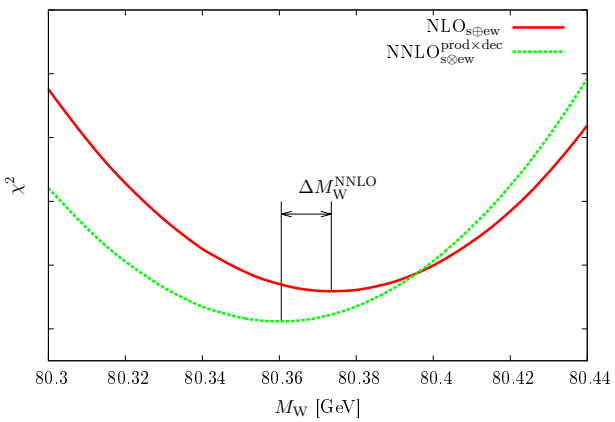
- YFS-Sherpa includes multi-photon emission
- Pole approx. includes finite weak NLO corrections

Effect on M_W measurement

Estimate effect of higher-order corrections on M_W measurement:

- χ^2 fit of $M_{T,\nu\ell}$ distribution in interval
$$M_{T,\nu\ell} = 64.4 - 90.5\,\mathrm{GeV}$$
with $\Delta M_{T,\nu\ell} = 1\mathrm{GeV}$ bins
- "Templates": LO prediction for
$$M_W = \begin{cases} 80.085\ldots80.785\,\mathrm{GeV}, & (\Delta M_W = 10\,\mathrm{MeV}) \\ 80.285\ldots80.485\,\mathrm{GeV}, & (\Delta M_W = 5\,\mathrm{MeV}) \end{cases}$$
- "Data": different theory predictions
(normalized to same σ in $M_{T,\nu\ell}$ interval)
- Shift from LO $\to$ NLO$_{\mathrm{EW}}$:
$$|\Delta M_W^{\mathrm{NLO}}| \approx 90\mathrm{MeV}$$





- Shift from NLO$_{\mathrm{EW+QCD}}$ $\to$ NNLO$^{\mathrm{prod-dec}}$
$$|\Delta M_W^{\mathrm{NNLO}}| \approx 14\ \mathrm{MeV}$$
(partially included in current analysis through NLO-QCD+Photos)