Decay Rate of the Electroweak Vacuum in the Standard Model and Beyond
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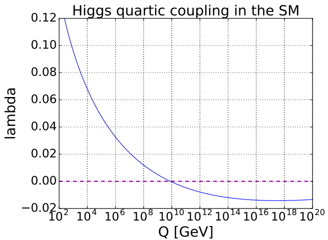
May 24 Planck 2018 @ Bonn

PRL **119** (2017) no.21, 211801 [arXiv:1707.09301]
PRD ??? [arXiv:1803.03902]

Decay of the EW vacuum

Implication of the SM Higgs with $m_h \simeq 125$ GeV:

EW vacuum is NOT absolutely stable in the SM!!



- Many works on the decay rate.
  ['01 Isidori, Ridolfi, and Strumia, etc...]
- Effects of conformal zero-modes were not properly calculated.

Purpose: Complete NLO calculation of the decay rate

Outline

NLO formula of the decay rate

Using the bounce $\bar{\phi}$

- $O(4)$ symmetric solution of EoM
- $\bar{\phi}(\infty) = v$: false vacuum

The decay rate per unit volume @ NLO ['77 Coleman]

$$\gamma = \frac{1}{VT} \,\mathrm{Im}\, \frac{\int_{\text{1-bounce}} \mathcal{D}\Psi \ e^{-S_E}}{\int_{\text{0-bounce}} \mathcal{D}\Psi \ e^{-S_E}} \equiv A e^{-\mathcal{B}} \quad \text{with} \quad \mathcal{B} \equiv S_E[\bar{\phi}] - S_E[v],$$

Expand the action around the bounce

$$S_E[\bar{\phi} + \Psi] = S_E[\bar{\phi}] + \frac{1}{2} \int d^4x \ \Psi \mathcal{M} \Psi + \mathcal{O}(\Psi^3)$$

$$S_E[v + \Psi] = S_E[v] + \frac{1}{2} \int d^4x \ \Psi \widehat{\mathcal{M}} \Psi + \mathcal{O}(\Psi^3)$$

$$\Rightarrow \quad A = \frac{1}{VT} \left| \frac{\mathrm{Det}\ \mathcal{M}}{\mathrm{Det}\ \widehat{\mathcal{M}}} \right|^{-\frac{1}{2}}$$

if $\mathcal{M} f = 0$ for $^\exists f$,
then Det $\mathcal{M} = 0$ and $A \to \infty$

EW vacuum of the standard model

Interested in the scale $\langle\Phi\rangle \sim \mu \gtrsim 10^{10}$ GeV; $V(\Phi) \simeq -|\lambda|(\Phi^\dagger\Phi)^2$

$\langle\Phi\rangle \gg m_{\rm EW}$ approximation

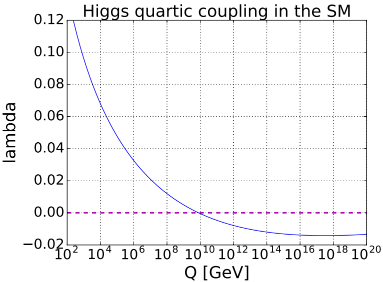
Bounce solution

$$\Phi_{\rm Bounce} = \frac{1}{\sqrt{2}} e^{i\theta^a \sigma^a} \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix} \quad \text{with} \quad \begin{cases} \bar{\phi}(r) \equiv \dfrac{\bar{\phi}_C}{1 + \frac{1}{8}|\lambda|\bar{\phi}_C^2 r^2} \\ \lim_{r\to\infty} \bar{\phi}(r) = 0 : \ \text{FV} \end{cases}$$

Bounce action for the SM

$$\mathcal{B} = \frac{8\pi^2}{3|\lambda|}$$

2 free parameters θ^a, $\bar{\phi}_C$ correspond to zero-modes



The Higgs fluctuation h

Expansion around the bounce

$$\Phi = \frac{1}{\sqrt{2}} e^{i\theta^a \sigma^a} \begin{pmatrix} \phi^1 + i\phi^2 \\ \bar{\phi} + h - i\phi^3 \end{pmatrix}, \quad W^a_\mu = w^a_\mu, \quad B^a_\mu = b^a_\mu$$

Fluctuation operator for the Higgs mode

$$\mathcal{L} \ni \frac{1}{2} h(-\partial^2 - 3|\lambda|\bar{\phi}^2)h \equiv \frac{1}{2} h\mathcal{M}^{(h)} h, \quad \widehat{\mathcal{M}}^{(h)} \equiv -\partial^2$$

Angular momentum expansion of the Higgs mode

$$\mathcal{M}^{(h)}_J = -\left[\partial_r^2 + \frac{3}{r}\partial_r - \frac{4J(J+1)}{r^2} + 3|\lambda|\bar{\phi}^2\right] \quad \text{with} \quad J \in \mathbb{Z}/2$$

- Conformal zero-mode ($J = 0$): $\mathcal{M}^{(h)}_0 \dfrac{\partial\bar{\phi}}{\partial\bar{\phi}_C} = 0$
- Translation zero-mode ($J = 1/2$): $\mathcal{M}^{(h)}_{1/2} \partial_r \bar{\phi} = 0$

Conformal zero-mode

Decomposition using eigenfunctions of $\mathcal{M}_0^{(h)}$ zero-mode: $\mathcal{M}_0^{(h)} h_c = 0$

$$h^{(J=0)} = \sum_{\text{eigen}} h_i^{(J=0)} = \alpha_c \underbrace{\mathcal{N}_c \frac{\partial \bar{\phi}}{\partial \bar{\phi}_C}}_{h_c} + \cdots \quad \text{with} \quad \int d^4 r \; h_c^2 = 2\pi$$

Under $\bar{\phi}_C \to \bar{\phi}_C + \Delta \bar{\phi}_C$

$$\Phi \ni \frac{1}{\sqrt{2}}(\bar{\phi} + h) \to \frac{1}{\sqrt{2}}\left(\bar{\phi} + \frac{\partial \bar{\phi}}{\partial \bar{\phi}_C} \Delta \bar{\phi}_C + \alpha_c \mathcal{N}_c \frac{\partial \bar{\phi}}{\partial \bar{\phi}_C} + \cdots\right)$$

Path integral along the conformal zero-mode direction

$$\int \mathcal{D} h_c \equiv \int d\alpha_c = \int \frac{d\bar{\phi}_C}{\mathcal{N}_c}$$

Higgs $J = 0$ mode contribution

$$\int \mathcal{D}h^{(J=0)} = \left[\mathrm{Det}\mathcal{M}_0^{(h)}\right]^{-1/2} \to \int \frac{d\bar{\phi}_C}{\mathcal{N}_c} \left[\mathrm{Det}'\mathcal{M}_0^{(h)}\right]^{-1/2}$$

Det′ means that zero eigenvalue is omitted from Det

$$\mathrm{Det}'\mathcal{M}_0^{(h)} \equiv \lim_{\nu\to 0} \nu^{-1}\mathrm{Det}(\mathcal{M}_0^{(h)} + \nu)$$

Use “Gelfand-Yaglom theorem”

$$\frac{\mathrm{Det}\mathcal{M}_J}{\mathrm{Det}\widehat{\mathcal{M}}_J} = \lim_{r\to\infty} \frac{f_J(r)}{\widehat{f}_J(r)} \left[\frac{f_J(0)}{\widehat{f}_J(0)}\right]^{-1} \quad \text{with} \quad \begin{cases} \mathcal{M}_J f_J(r) = 0 \\ \widehat{\mathcal{M}}_J \widehat{f}_J(r) = 0 \end{cases}$$

The final result is

['17 Andreassen, Frost, and Schwartz]

['17 SC, Moroi, and Shoji]

$$\left|\frac{\mathrm{Det}\mathcal{M}_0^{(h)}}{\mathrm{Det}\widehat{\mathcal{M}}_0^{(h)}}\right|^{-1/2} \to \int \frac{d\bar{\phi}_C}{\bar{\phi}_C} \left(\frac{16\pi}{|\lambda|}\right)^{1/2}$$

Summary: Decay rate of the EW vacuum

Final decay rate formula

$$\gamma = \int d\ln\bar{\phi}_C \ \left[I^{(h)} I^{(W,Z,\varphi)} I^{(t)} I^{(\mathrm{extra})} e^{-\mathcal{B}} \right]_{\mu\sim\mathcal{O}(\bar{\phi}_C)}$$

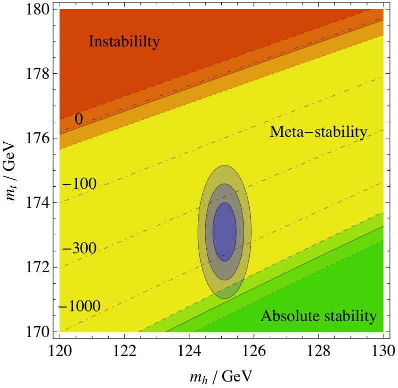
with $I^{(X)}$: contribution of Higgs / gauge and NG / top quark

We provide, Setup: conformal, $1D$ bounce

- analytic expressions (Solve diff. eq., Sum over J)
- public code *ELVAS* : ['18 SC, Moroi, and Shoji]
  application to SM, SM + extra scalars/fermions

Result1: SM

Decay rate $\log_{10}\gamma$ as a function of m_h and m_t



1σ error for α_s

- Dashed lines for $\alpha_s = 0.1192$
- Solid lines for $\alpha_s = 0.1181$
- Dotted lines for $\alpha_s = 0.1170$

Blue: $1, 2, 3\sigma$ contours of m_h and m_t

- $m_h = 125.09 \pm 0.24$ GeV
- $m_t = 173.1 \pm 0.6$ GeV

$$\log_{10}[\gamma \times \text{Gyr Gpc}^3] = -582^{+40}_{-45}{}^{+184}_{-329}{}^{+144}_{-218}{}^{+2}_{-1}$$

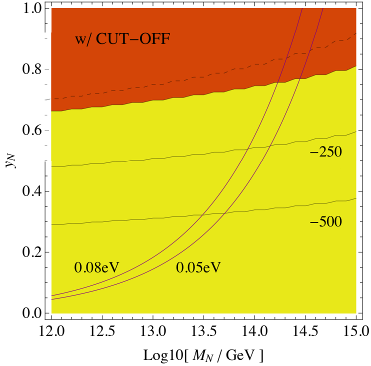
Errors: from m_h, m_t, α_s, μ

Result2: SM + right-handed neutrino

Right-handed neutrino N with Yukawa interaction

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2} M_N \overline{N} N + y_N \Phi^* L \overline{N}$$

Decay rate $\log_{10} \gamma$ as a function of M_N and y_N



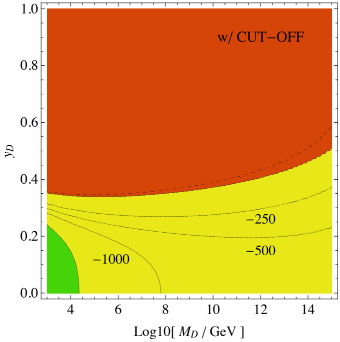
Purple: contours of $m_\nu = 0.08, 0.05$ eV

$y_N \lesssim 0.65 \sim 0.8$ is required

Result3: SM + vector-like quarks/leptons

Vector-like quarks $Q, \overline{Q}, D, \overline{D}$

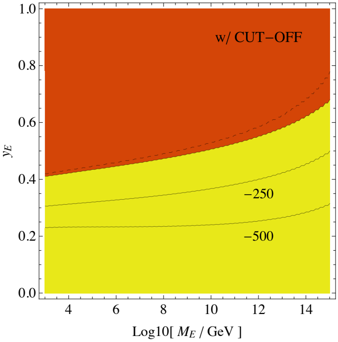
$$\Delta\mathcal{L}_{\text{Yukawa}} = y\Phi^* Q\overline{D} + y\Phi\overline{Q}D$$



$y \lesssim 0.35 \sim 0.5$ is required

Vector-like leptons $L, \overline{L}, E, \overline{E}$

$$\Delta\mathcal{L}_{\text{Yukawa}} = y\Phi^* L\overline{E} + y\Phi\overline{L}E$$



$y \lesssim 0.4 \sim 0.7$ is required

Conclusion

We provide a way to treat conformal zero-mode

This completes NLO calculation of decay rate when Higgs potential is scale invariant

Our works are also characteristic for

- Analytic expressions
- Application is not limited to the SM

Thank you for listening!!

Comparison among several works

$$R \equiv \sqrt{\frac{8}{|\lambda|}}\frac{1}{\bar{\phi}_C}$$

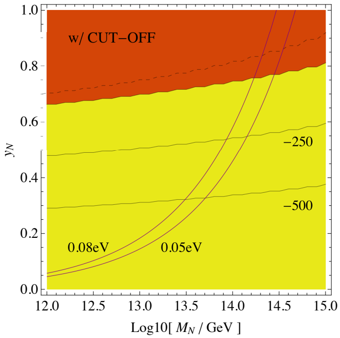
Define the scale R_* of the largest contribution to γ

$$\beta_\lambda(\mu) = 0, \beta'_\lambda(\mu) > 0 \quad \text{at} \quad \mu = R_*^{-1}$$

	'01 Isidori	'17 Andreassen	'17'18 Chigusa
Translation zero mode	○	○	○
Conformal zero mode	Fix $R = R_*$ Dim. analysis	○	○
Gauge zero mode	○	○	○
Gauge fixing	Feynman gauge $R_{\xi=1}$	Fermi gauge $\mathcal{L} = \frac{1}{2\xi}(\partial_\mu A_\mu)^2$	Fermi gauge $\mathcal{L} = \frac{1}{2\xi}(\partial_\mu A_\mu)^2$
Renorm. scale	$\mu = R_*^{-1}$	$\mu = R_*^{-1}$ Resum $\ln^n(\mu R)$	$\mu = R^{-1}$

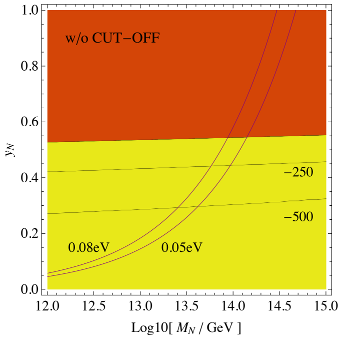
Effect of gravity (SM + right-handed neutrino)

$$\gamma_{\rm pl} \equiv \int^{\bar\phi_C = M_{\rm pl}} d\ln\bar\phi_C[\cdots]$$



- Insensitive to Planck suppressed operators
- Lower limit on γ

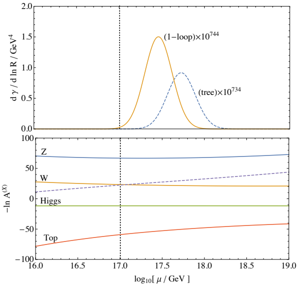
$$\gamma_\infty \equiv \int^{\bar\phi_C = \infty} d\ln\bar\phi_C[\cdots]$$

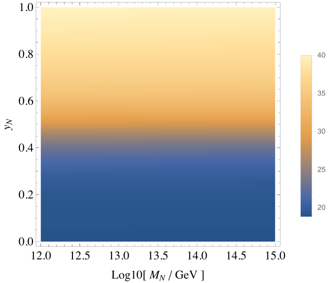


- Modified by Planck suppressed operators
- $\gamma_\infty = \gamma$ if no such operators

Largest contribution to γ

SM

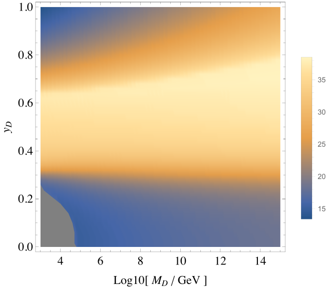


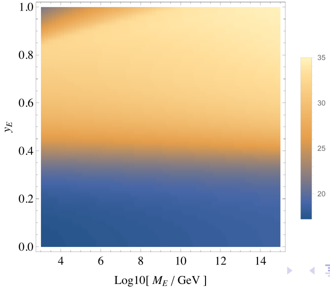


RHN

$\log_{10} \bar{\phi}_C^{\max}$

VLQ





VLL

Renormalization in MSbar scheme

For example, consider the Higgs fluctuation $\ln\mathcal{A}^{(h)}$ determined by

$$\mathcal{M}_J = -\Delta_J + 3\lambda\bar{\phi}^2 \quad \widehat{\mathcal{M}}_J = -\Delta_J$$

UV divergence comes from $\sum_J^\infty$

First regulate it with $\epsilon_h > 0$ as

$$-\frac{1}{2}\sum_J^\infty \frac{(2J+1)^2}{(1+\epsilon_h)^J}\ln\frac{\mathrm{Det}\mathcal{M}_J}{\mathrm{Det}\widehat{\mathcal{M}}_J}$$

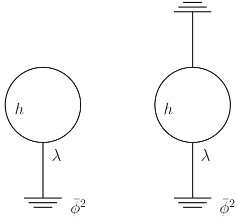
Renormalizable theory:

Only upto $\mathcal{O}(\lambda^2)$ terms diverge

$$-\frac{1}{2}\sum_J^\infty \frac{(2J+1)^2}{(1+\epsilon_h)^J}\ln\left[\frac{\mathrm{Det}\mathcal{M}_J}{\mathrm{Det}\widehat{\mathcal{M}}_J}\right]_{\mathcal{O}(\lambda^2)}$$

Call it $[\ln\mathcal{A}^{(h)}]_{\mathrm{div},\epsilon_h}$

Divergent part can be evaluated in $\overline{\mathrm{MS}}$ scheme by diagramatic approach



Call it $[\ln\mathcal{A}^{(h)}]_{\mathrm{div},\epsilon}$

Renormalized contribution of Higgs:

$$\ln\mathcal{A}^{(h)} - [\ln\mathcal{A}^{(h)}]_{\mathrm{div},\epsilon_h} + [\ln\mathcal{A}^{(h)}]_{\mathrm{div},\epsilon}$$

$$+\,(\text{counter term in } \overline{\mathrm{MS}})$$

Effects of other zero-modes

Also take care of the translation zero-mode ['77 Callan, Coleman]

$$\frac{\mathcal{A}^{(h)}}{VT} \to \int \frac{d\bar{\phi}_C}{\bar{\phi}_C} \left(\frac{16\pi}{|\lambda|}\right)^{\frac{1}{2}} 16\pi^2 \bar{\phi}_C^4 \prod_{J\geq 1} \left[\frac{\mathrm{Det}\mathcal{M}_J^{(h)}}{\mathrm{Det}\widehat{\mathcal{M}}_J^{(h)}}\right]^{-\frac{(2J+1)^2}{2}}$$

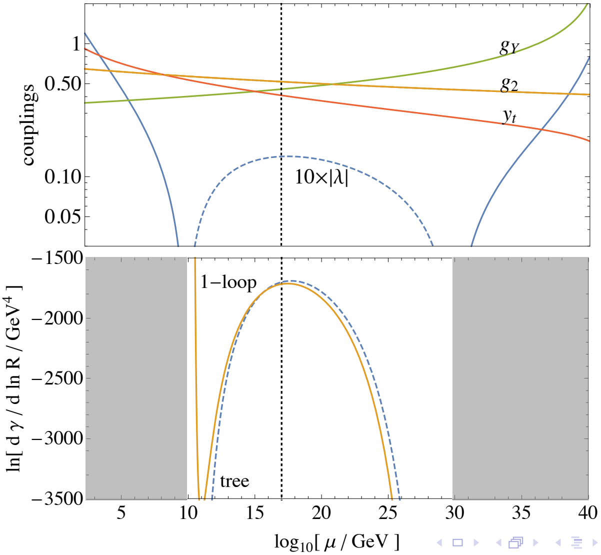
$(2J+1)^2$: degeneracy of the J mode

Gauge $J=0$ mode has a zero-mode related to gauge symmetry

$$\mathcal{A}^{(\mathcal{A}^\mu,\varphi)} = \mathcal{V}_{\mathrm{SU}(2)} \left(\frac{16\pi}{|\lambda|}\right)^{\frac{3}{2}} \prod_{J\geq 1/2} \left[\frac{\mathrm{Det}\mathcal{M}_J^{(A^\mu,\varphi)}}{\mathrm{Det}\widehat{\mathcal{M}}_J^{(A^\mu,\varphi)}}\right]^{-\frac{(2J+1)^2}{2}}$$

with $\mathcal{V}_{\mathrm{SU}(2)} = 2\pi^2$: gauge volume

RG evolution and $d\gamma/d\ln R$ in SM

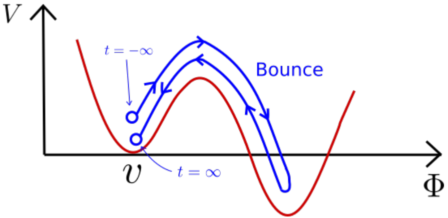


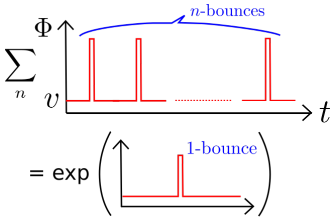
The decay rate and the bounce

The decay rate γ is related to the $4D$ Euclidean action:

$$Z \equiv \langle \mathrm{FV} \,|\, e^{-\mathcal{H}T} \,|\, \mathrm{FV} \rangle = N \int \mathcal{D}\Psi \; e^{-S_E} \propto \exp(i\gamma V T)$$

The path integral is dominated by the bounce $\bar{\phi}$





The bounce $\bar{\phi}$: $O(4)$ symmetric solution of EoM

$$\left[\partial_r^2 \Phi + \frac{3}{r}\partial_r \Phi - \frac{\partial V}{\partial \Phi}\right]_{\Phi \to \bar{\phi}} = 0 \quad \text{with} \quad \bar{\phi}(\infty) = v : \;\; \text{false vacuum}$$

The decay rate per unit volume

$$\gamma = \frac{1}{VT}\ \mathrm{Im}\ \frac{\int_{\text{1-bounce}} \mathcal{D}\Psi\ e^{-S_E}}{\int_{\text{0-bounce}} \mathcal{D}\Psi\ e^{-S_E}} \equiv A e^{-\mathcal{B}}$$

Expand the action around the classical path

$$S_E[\bar{\phi} + \Psi] = S_E[\bar{\phi}] + \frac{1}{2}\int d^4x\ \Psi \mathcal{M} \Psi + \mathcal{O}(\Psi^3)$$

$$S_E[v + \Psi] = S_E[v] + \frac{1}{2}\int d^4x\ \Psi \widehat{\mathcal{M}} \Psi + \mathcal{O}(\Psi^3)$$

$$\Rightarrow \quad \begin{cases} \mathcal{B} = S_E[\bar{\phi}] - S_E[v] \\ A = \dfrac{1}{VT} \left| \dfrac{\text{Det}\ \mathcal{M}}{\text{Det}\ \widehat{\mathcal{M}}} \right|^{-\frac{1}{2}} \end{cases}$$

Longer description of the Higgs mode

Expansion around the bounce

$$\Phi = \frac{1}{\sqrt{2}} e^{i\theta^a \sigma^a} \begin{pmatrix} \phi^1 + i\phi^2 \\ \bar{\phi} + h - i\phi^3 \end{pmatrix}, \quad W_\mu^a = w_\mu^a, \quad B_\mu^a = b_\mu^a$$

Fluctuation operator for the Higgs mode

$$\mathcal{L} \ni \frac{1}{2} h(-\partial^2 - 3|\lambda|\bar{\phi}^2)h \equiv \frac{1}{2} h \mathcal{M}^{(h)} h, \quad \widehat{\mathcal{M}}^{(h)} \equiv -\partial^2$$

Angular momentum expansion in $4D$ using $J \in \mathbb{Z}/2$

$$h(x) = \sum_{n,J,m_A,m_B} \mathcal{R}_{n,J}(r) \mathcal{Y}_{J,m_A,m_B}(\hat{\mathbf{r}})$$

$$\mathcal{M}_J^{(h)} = -\Delta_J - 3|\lambda|\bar{\phi}^2 = -\left[\partial_r^2 + \frac{3}{r}\partial_r - \frac{4J(J+1)}{r^2} + 3|\lambda|\bar{\phi}^2\right]$$

Use [Coleman; Dashen, Hasslacher, and Neveu; Kirsten and McKane, etc...]

$$\frac{\mathrm{Det}\mathcal{M}_J}{\mathrm{Det}\widehat{\mathcal{M}}_J} = \lim_{r\to\infty} \frac{f_J(r)}{\widehat{f}_J(r)} \left[\frac{f_J(0)}{\widehat{f}_J(0)}\right]^{-1} \quad \text{with} \quad \begin{cases} \mathcal{M}_J f_J(r) = 0 \\ \widehat{\mathcal{M}}_J \widehat{f}_J(r) = 0 \end{cases}$$

Higgs mode contribution to $\mathcal{A}$ $(2J+1)^2$: degeneracy of the J-mode

$$\mathcal{A}^{(h)} = \lim_{r\to\infty} \prod_J \left[\frac{f_J^{(h)}(r)}{r^{2J}}\right]^{-\frac{(2J+1)^2}{2}} \quad \text{with} \quad \mathcal{M}_J^{(h)} f_J^{(h)}(r) = 0$$

However, $f_0^{(h)}(r\to\infty) = f_{1/2}^{(h)}(r\to\infty) = 0$

- Conformal zero-mode in $J=0$: $f_0^{(h)} = \dfrac{\partial\bar{\phi}}{\partial\bar{\phi}_C}$
- Translation zero-mode in $J=1/2$: $f_{1/2}^{(h)} = -\dfrac{4}{|\lambda|\bar{\phi}_C}\partial_r\bar{\phi}$

R integral and choice of μ

Simpler example: Decay width of t

$$\Gamma_t = \frac{1}{M_t} \text{Im } \mathcal{M}_{t\to t} = \int \prod_i^L \frac{d^d k_i}{(2\pi)^d} \cdots \delta(\text{on-shell conditions})$$

- Integral just sums up all possible configurations
- Each configuration is "physical"
- Each configuration is μ-independent (up to higher loop)

Vacuum decay rate

$$\gamma = \frac{1}{VT} \text{ Im } \frac{\int_{\text{1-bounce}} \mathcal{D}\Psi \ e^{-S_E}}{\int_{\text{0-bounce}} \mathcal{D}\Psi \ e^{-S_E}} = \int d \ln R \ [\cdots]_{\mu=R}$$

- Integrand should be μ-independent
- Choice of $\mu = R$ is appropriate