Electroweak Symmetry Breaking by a Neutral Sector : Dynamical Relaxation of the Little Hierarchy Problem

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- The naturalness problem of EW scale and Higgs boson mass has been the most important issue for last four decades.
- The MSSM has been the most promising BSM candidate.
- No evidence of BSM has been observed yet at LHC.

→ Theoretical puzzles raised in the SM still remain UNsolved.

- A barometer of the solution to the naturalness problem is the stop mass .

  The stop mass bound has been already > 1 TeV.

  (The gluino mass bound has exceeded > 2 TeV.)

→ They start threatening the traditional status of SUSY as a solution to the naturalness problem of the EW phase transition.

- ATLAS and CMS have discovered the SM(-like) Higgs with 125-126 GeV mass, which is too heavy as a SUSY Higgs.

- According to the recent analyses, 10-20 TeV stop mass is necessary for the 125 GeV Higgs mass (without a large stop mixing).

$$\Delta m^2_{h_u}|_{1-\text{loop}} \approx \frac{3|y_t|^2}{8\pi^2}\widetilde{m}_t^2 \log\left(\frac{\widetilde{m}_t^2}{\Lambda^2}\right)\left[1+\frac{1}{2}\frac{A_t^2}{\widetilde{m}_t^2}\right],$$

$$\Delta m^2_{H}|_{1-\text{loop}} \approx \frac{3m_t^4}{4\pi^2 v_h^2}\left[\log\left(\frac{\widetilde{m}_t^2}{m_t^2}\right)+\frac{A_t^2}{\widetilde{m}_t^2}\left(1-\frac{1}{12}\frac{A_t^2}{\widetilde{m}_t^2}\right)\right],$$

$$\frac{1}{2}m_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2\tan^2\beta}{\tan^2\beta - 1} - |\mu|^2$$

- ATLAS and CMS have discovered the SM(-like) Higgs with 125-126 GeV mass, which is too heavy as a SUSY Higgs.

- According to the recent analyses, 10-20 TeV stop mass is necessary for the 125 GeV Higgs mass (without a large stop mixing).

A fine-tuning of 10^{-3} – 10^{-4}

seems to be unavoidable !! ??

- Recently some new ideas (without SUSY) have been suggested to relax the gauge hierarchy problem.

- For UV completion, however, embedding them in SUSY also have been discussed.

- Recently some new ideas (without SUSY) have been suggested to relax the gauge hierarchy problem.
- For UV completion, however, embedding them in SUSY also have been discussed.

We will attempt to address
the (little) hierarchy problem
in the SUSY framework.

Little Hierarchy Problem

$$|\mu|^2 + \frac{1}{2}M_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

Why is M_Z^2 [$=(g_2^2+g_Y^2)(v_u^2+v_d^2)/2$] so small compared to the soft masses ?

[$v_u^2 + v_d^2 \equiv \langle |H|^2 \rangle = (174\ \mathrm{GeV})^2$]

Problems in SUSY models

Gravity Mediated SUSY Breaking mech.

μ and $B\mu$ terms are O.K.

But Flavor and CP problems would arise.

Gauge Mediated SUSY Breaking mech.

Flavor and CP problems are absent.

But μ and $B\mu$ problems would be serious.

Model

$$W = (\lambda_1 X + \lambda_2 \phi + \mu)\, h_u h_d + M\, XY + (\kappa/2)\, Y \phi^2$$

Model

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$$W_{\rm UV} \supset \Psi \left(y_1 X_1 + y_2 X_2 \right) Z + y_3 \Psi^c Z \phi$$
$$+ \left(y_4 X_1 + y_5 X_2 \right) h_u h_d + \frac{\left(\Psi^c\right)^2}{M_P} \left(y_6 X_1 + y_7 X_2 \right) Y + \frac{\kappa}{2} Y \phi^2 ,$$

Superfields	$X_{1,2}$	Y	Z	ϕ	Ψ	Ψ^c
$\mathrm{U}(1)_{\rm PQ}$	-1	$4/3$	$5/6$	$-2/3$	$1/6$	$-1/6$

Model

$$W = (\lambda_1 X + \lambda_2 \phi + \mu)\, h_u h_d + M\, XY + (\kappa/2)\, Y\phi^2$$

$$V \supset |H|^2|\lambda_1 X + \lambda_2 \phi + \mu|^2 + |\lambda_1 h_u h_d + MY|^2 + \left|\frac{\kappa}{2}\phi^2 + MX\right|^2$$
$$+|\lambda_2 h_u h_d + \kappa Y \phi|^2 + m_X^2|X|^2 + m_Y^2|Y|^2 + m_\phi^2|\phi|^2$$
$$+\left\{(\lambda_1 a_1 X + \lambda_2 a_2 \phi) h_u h_d + MbXY + \frac{\kappa}{2} aY\phi^2 + \text{h.c.}\right\},$$

where $|H|^2 \equiv |h_u|^2 + |h_d|^2$

Model

W = (λ_1X + $\lambda_2\phi$ +μ) huhd + M XY + (κ/2) Yϕ^2

$$V \supset |H|^2|\lambda_1 X + \lambda_2\phi + \mu|^2 + |\lambda_1 h_u h_d + MY|^2 + \underline{\underline{\left|\frac{\kappa}{2}\phi^2 + MX\right|^2}}$$
$$+|\lambda_2 h_u h_d + \kappa Y\phi|^2 + m_X^2|X|^2 + m_Y^2|Y|^2 + m_\phi^2|\phi|^2$$
$$+\left\{(\lambda_1 a_1 X + \lambda_2 a_2\phi)h_u h_d + MbXY + \frac{\kappa}{2}aY\phi^2 + \text{h.c.}\right\},$$

where $|H|^2 \equiv |h_u|^2 + |h_d|^2$

Model

$$W = (\lambda_1 X + \lambda_2 \phi + \mu)\, h_u h_d + M\, XY + (\kappa/2)\, Y\phi^2$$

$$V \supset |H|^2|\lambda_1 X + \lambda_2\phi + \mu|^2 + |\lambda_1 h_u h_d + MY|^2 + \underline{\left|\frac{\kappa}{2}\phi^2 + MX\right|^2}$$

$+|\lambda_2 h$

where $|H|^2 =$

$$\boxed{(\kappa/2)\phi^2 + MX = 0}$$

FLAT direction ***(= modulus-like)***

in SUSY limit, with $h_u = h_d = Y = 0$

Model

$$W = (\lambda_1 X + \lambda_2 \phi + \mu)\, h_u h_d + M\, XY + (\kappa/2)\, Y\phi^2$$

$$V \supset |H|^2|\lambda_1 X + \lambda_2\phi + \mu|^2 + |\lambda_1 h_u h_d + MY|^2 + \left|\frac{\kappa}{2}\phi^2 + MX\right|^2$$

$+|\lambda$ …

$+$ …

where $|H|^2 \equiv |h_u|^2 + |h_d|^2$

<u>***Suppose***</u>

m_X^2, m_Y^2, m_ϕ^2, a, b, etc.

$\sim O(m^2)$ << MSSM soft para.

Effective mu and Bmu

$$\boxed{\mu_{\text{eff}} = \lambda_1 \langle X \rangle + \lambda_2 \langle \phi \rangle + \mu,}$$

$$\boxed{B\mu_{\text{eff}} = \left(\lambda_1 M^* + \lambda_2 \kappa^* \langle \phi^* \rangle\right) \langle Y^* \rangle + \lambda_1 a_1 \langle X \rangle + \lambda_2 a_2 \langle \phi \rangle + B\mu,}$$

Extreme Conditions

extreme conditions for X, Y, and ϕ

$$\begin{cases} \mathcal{M}_X^2 X + M^* b^* Y^* = -\frac{\kappa}{2} M^* \phi^2 - (\lambda_2 \phi + \mu)\lambda_1^* |H|^2 \\ \qquad\qquad\qquad\qquad -\lambda_1^* a_1^* h_u^* h_d^*, \\ \mathcal{M}_Y^2 Y^* + M b X = -\frac{\kappa}{2} a \phi^2 - (\lambda_1^* M + \lambda_2^* \kappa \phi)\, h_u^* h_d^*, \\ \left(|\kappa Y|^2 + |\lambda_2 H|^2 + m_\phi^2\right)\phi + \left(\frac{\kappa}{2}\phi^2 + M X\right)\kappa^* \phi^* \\ \qquad\qquad +(\lambda_1 X + \mu)\lambda_2^* |H|^2 + \lambda_2^* a_2^* h_u^* h_d^* \\ \qquad\qquad +(\lambda_2 h_u h_d + a^* \phi^*)\kappa^* Y^* = 0. \end{cases}$$

Solutions of Extrm. Condi.

$$X \approx \frac{-\kappa\phi^2}{2\mathcal{M}_X^2} M^* \left[1 - \frac{(a-b)b^*}{\mathcal{M}_Y^2} + \frac{2(\lambda_2\phi + \mu)\lambda_1^* |H|^2}{\kappa\phi^2 M^*} \right],$$

$$Y^* \approx \frac{-\kappa\phi^2}{2\mathcal{M}_Y^2} (a-b) - \frac{(\lambda_1^* M + \lambda_2^* \kappa\phi) h_u^* h_d^*}{\mathcal{M}_Y^2}.$$

where $\quad \mathcal{M}_X^2 \equiv |\lambda_1 H|^2 + m_X^2 + |M|^2 \qquad (\approx |M|^2)$

$$\mathcal{M}_Y^2 \equiv |\kappa\phi|^2 + m_Y^2 + |M|^2$$

Extrm. Condi. for Φ

The extremum condition for $T_\zeta\ (\equiv \kappa\phi/M)$

$$\boxed{\begin{aligned}&\frac{1}{2}|T_\zeta|^2\left(|\lambda_1 H|^2+m_X^2\right)-T_\zeta^*\left(\lambda_2+\frac{\mu}{\phi}\right)\lambda_1^*|H|^2-\frac{1}{2}T_\zeta\lambda_2^*\lambda_1|H|^2\\&+\frac{\mu}{\phi}\lambda_2^*|H|^2+\left(|\lambda_2 H|^2+m_\phi^2\right)\approx\frac{|T_\zeta|^2(|T_\zeta|^2+2)}{4(|T_\zeta|^2+1)^2}\,|a-b|^2,\end{aligned}}$$

Extrm. Condi. for Φ

The extremum condition for $\underline{T_\zeta\ (\equiv \kappa\phi/M)}$

$$\frac{1}{2}|T_\zeta|^2\left(|\lambda_1 H|^2 + \underline{m_X^2}\right) - T_\zeta^*\left(\lambda_2 + \frac{\mu}{\phi}\right)\lambda_1^*|H|^2 - \frac{1}{2}T_\zeta\lambda_2^*\lambda_1|H|^2$$

$$\text{or}\quad |H|^2 \approx \frac{-m_\phi^2 - \frac{1}{2}\left(m_X^2 - |a-b|^2\right)|T_\zeta|^2}{\left(\lambda_2 - \frac{1}{2}T_\zeta\lambda_1 + \frac{\mu}{\phi}\right)\left(\lambda_2^* - T_\zeta^*\lambda_1^*\right)},$$

Extrm. Condi. for Φ

The extrem… for $T_\zeta\ (= \kappa\phi/M)$

$$|H|^2 \sim -m_{\phi,X}^2 / |\lambda_{2,1}|^2$$

$$\text{or} \quad |H|^2 \approx \frac{-m_\phi^2 - \frac{1}{2}\left(m_X^2 - |a-b|^2\right)|T_\zeta|^2}{\left(\lambda_2 - \frac{1}{2}T_\zeta\lambda_1 + \frac{\mu}{\phi}\right)\left(\lambda_2^* - T_\zeta^*\lambda_1^*\right)},$$

Dynamical Relaxation

$$\boxed{\mu_{\rm eff} = \lambda_1\langle X\rangle + \lambda_2\langle\phi\rangle + \mu,} \quad \boxed{\approx \frac{MT_\zeta}{\kappa}\left(\lambda_2 - \frac{1}{2}\lambda_1 T_\zeta\right) + \mu,}$$

$\mu_{\rm eff}$

$$\boxed{|\mu|^2 + \frac{1}{2}M_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2\tan^2\beta}{\tan^2\beta - 1}} \quad (\approx -m_{h_u}^2)$$

satisfying the conditions for EW symmetry breaking,

$B\mu_{\rm eff}$

$$2B\mu \;<\; (m_{h_u}^2 + |\mu|^2) + (m_{h_d}^2 + |\mu|^2)$$

$$(B\mu)^2 \;>\; (m_{h_u}^2 + |\mu|^2)(m_{h_d}^2 + |\mu|^2)$$

Dynamical Relaxation

$$\boxed{\mu_{\rm eff} = \lambda_1\langle X\rangle + \lambda_2\langle\phi\rangle + \mu,} \quad \boxed{\approx \frac{MT_\zeta}{\kappa}\left(\lambda_2 - \frac{1}{2}\lambda_1 T_\zeta\right) + \mu,}$$

$\mu_{\rm eff}$

$$\boxed{|\mu|^2 + \frac{1}{2}M_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2\tan^2\beta}{\tan^2\beta - 1}} \quad (\approx -m_{h_u}^2)$$

sati g.

$B\mu_{\rm eff}$

$(B$

For $|\lambda_2 M/\kappa|^2 >> -m_{hu}^2$

$|T_\zeta(1-T_\zeta\lambda_1/2\lambda_2)| << 1$

Little Hierarchy Problem

$$|\mu|^2 + \frac{1}{2}M_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

Why is M_Z^2 [$=(g_2^2+g_Y^2)(v_u^2+v_d^2)/2$] so small compared to the soft masses ?

[$v_u^2 + v_d^2 \equiv \langle |H|^2 \rangle = (174\ \text{GeV})^2$]

Little Hierarchy Problem

It is because m_ϕ, m_χ are so small compared to the MSSM soft masses.

Why is M_Z^2 $[=(g_2^2+g_Y^2)(v_u^2+v_d^2)/2]$ so small compared to the soft masses ?

$[\, v_u^2 + v_d^2 \equiv \langle |H|^2 \rangle = (174\ \mathrm{GeV})^2 \,]$

For small enough $m_{\phi,X}^2$

Introduce Gauge Med. SUSY Breaking
as well as Gravity Med. SUSY breaking

Gauge Med. $\longrightarrow$ Heavy MSSM soft masses
avoiding exp. Bounds and SUSY flavor and CP problems

Gravity Med. $\longrightarrow$ Small MSSM singlet masses and $B\mu$ term

For small enough $m_{\phi,X}^2$

Introduce Gauge Med. SUSY Breaking
as well as Gravity Med. SUSY breaking

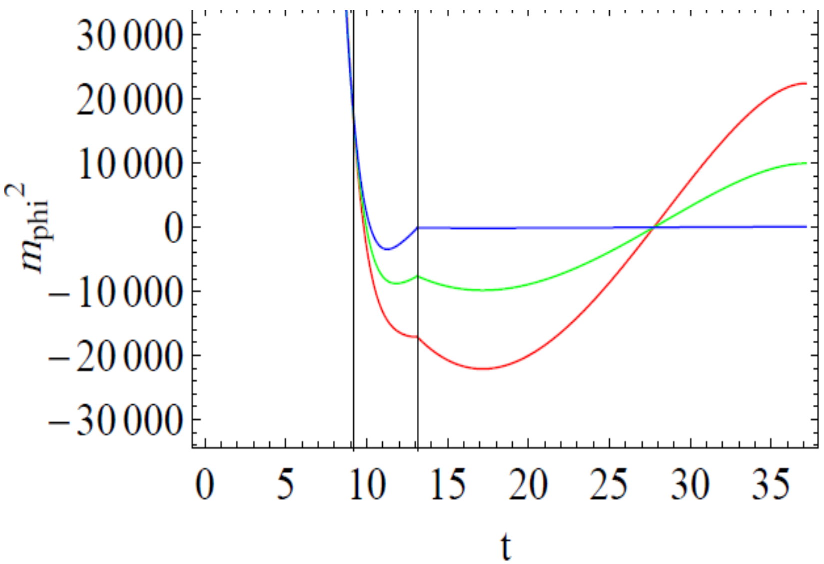
ms

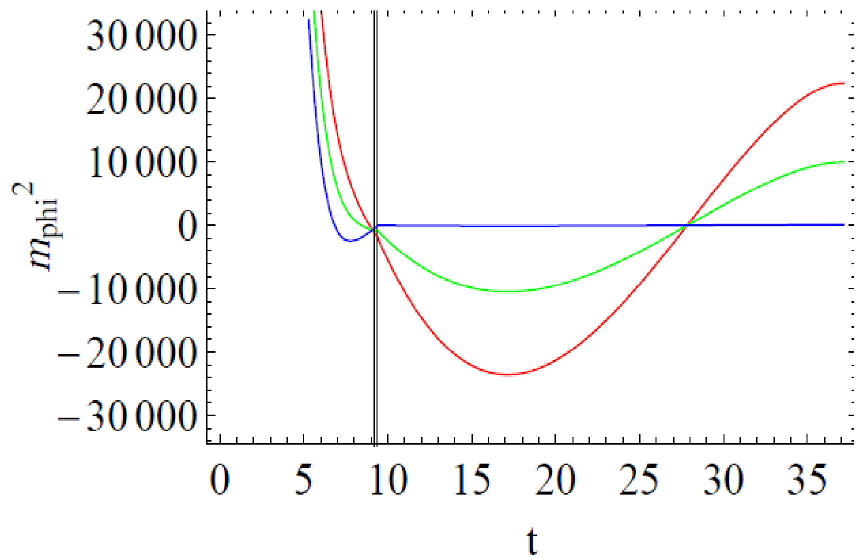
- *λ_2 and/or λ_1 should be* ***small enough.***
- ***Messenger Scale*** *of the* ***Gauge Med.***

needs to be ***LOW enough.***

Focus Point

(λ_1=0.7 ≫ λ_2=0.02)





RG evolutions of $m_\phi{}^2$ under various trial $m_0{}^2$s.

the messenger scale = 500 TeV (L) and 12 TeV (R).

In both cases, the stop mass scales = 10 TeV.

Focus Point

Case I	$\tan\beta = 5$	Case II	$\tan\beta = 15$
$\lambda_2 = 0.03$	$\widetilde{m}_t = 10\,\mathrm{TeV}$	$\lambda_2 = 0.07$	$\widetilde{m}_t = 20\,\mathrm{TeV}$
$\lambda_1 = 0.7$	$\Lambda_{\mathrm{mess}} = 15\,\mathrm{TeV}$	$\lambda_1 = 0.7$	$\Lambda_{\mathrm{mess}} = 25\,\mathrm{TeV}$
$\boldsymbol{\Delta}_{\mathbf{m_0^2}}$	-4.1	$\boldsymbol{\Delta}_{\mathbf{m_0^2}}$	-10.6
$\boldsymbol{\Delta}_{\mathrm{M_U}}$	47.4	$\boldsymbol{\Delta}_{\mathrm{M_U}}$	128.3
$\boldsymbol{\Delta}_{\lambda_2^2}$	29.7	$\boldsymbol{\Delta}_{\lambda_2^2}$	61.9
$\boldsymbol{\Delta}_{\mathrm{GM}}$	10.3	$\boldsymbol{\Delta}_{\mathrm{GM}}$	23.2
$\boldsymbol{\Delta}_{\mathrm{mess}}$	-15.0	$\boldsymbol{\Delta}_{\mathrm{mess}}$	49.1

Focus Point

Case I	$\tan\beta = 5$	Case II	$\tan\beta = 15$
$\lambda_2 = 0.03$	$\widetilde{m}_t = 10\,\mathrm{TeV}$	$\lambda_2 = 0.07$	$\widetilde{m}_t = 20\,\mathrm{TeV}$
$\lambda_1 = 0.7$	$\Lambda_{\mathrm{mess}} = 15\,\mathrm{TeV}$	$\lambda_1 = 0.7$	$\Lambda_{\mathrm{mess}} = 25\,\mathrm{TeV}$
$\boldsymbol{\Delta}_{\mathbf{m_0^2}}$	-4.1	$\boldsymbol{\Delta}_{\mathbf{m_0^2}}$	-10.6
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$\boldsymbol{\Delta}_{\lambda_2^2}$	29.7	$\boldsymbol{\Delta}_{\lambda_2^2}$	61.9

$(M_3, M_2, M_1) \approx (12, 5, 3)$ TeV (I) , $(21, 9, 5)$ TeV (II)

$m_{hu}^2 = -(2.5\ \mathrm{TeV})^2$ (L) and $-(1.8\ \mathrm{TeV})^2$ TeV (II).

SUSY particles' masses of the 1st, 2nd generations are much heavier.

Mass Matrix

$$\begin{pmatrix} m_H^2 & \lambda_2 H \mu_{\text{eff}}^* & \lambda_1 H \mu_{\text{eff}}^* \\ \lambda_2^* H^* \mu_{\text{eff}} & m_\phi^2 + |\lambda_2 H|^2 + |\kappa\phi|^2 & \lambda_2^* \lambda_1 |H|^2 + \kappa^* \phi^* M \\ \lambda_1^* H^* \mu_{\text{eff}} & \lambda_1^* \lambda_2 |H|^2 + \kappa\phi M^* & m_X^2 + |\lambda_1 H|^2 + |M|^2 \end{pmatrix}$$

in the basis of $\{H, \phi, X\}$

Mixing Constraints

$$\mathcal{O}_3^T \cdot \text{diag.}(m_1^2, m_2^2, M_3^2) \cdot \mathcal{O}_3$$

yields a symmetric matrix $\mathcal{M}^2_{ij}$ $(= \mathcal{M}^2_{ji})$ with the following elements:

$$\begin{aligned}
\mathcal{M}^2_{11} &\approx M_3^2\ \varepsilon_2^2 + m_2^2\ \varepsilon_1^2 + m_1^2, \\
\mathcal{M}^2_{12} &\approx M_3^2\ \varepsilon_2\ \sin\theta + m_2^2\ \varepsilon_1\ \cos\theta - m_1^2\ \epsilon_1, \\
\mathcal{M}^2_{13} &\approx M_3^2\ \varepsilon_2\ \cos\theta - m_2^2\ \varepsilon_1\ \sin\theta - m_1^2\ \epsilon_2, \\
\mathcal{M}^2_{23} &\approx \Delta M^2_{32}\left(1 - \vec{\epsilon}^{\,2}\right)\ \sin\theta\ \cos\theta, \\
\mathcal{M}^2_{22} &\approx \Delta M^2_{32}\ \sin^2\theta + m_2^2 - \left\{\Delta M^2_{32}\ \sin^2\theta + \Delta m^2_{21}\right\}\epsilon_1^2, \\
\mathcal{M}^2_{33} &\approx \Delta M^2_{32}\ \cos^2\theta + m_2^2 - \left\{\Delta M^2_{32}\ \cos^2\theta + \Delta m^2_{21}\right\}\epsilon_2^2 \\
&\qquad -\Delta M^2_{32}\ \sin 2\theta\ \epsilon_1\epsilon_2,
\end{aligned}$$

Mixing Constraints

$$\mathcal{O}_3^T \cdot \mathrm{diag.}(m_1^2, m_2^2, M_3^2) \cdot \mathcal{O}_3$$

yields a sym

$$\mathcal{O}_3 = \begin{pmatrix} c_1 c_2 & -s_1 & -c_1 s_2 \\ c_2 c_3 s_1 - s_2 s_3 & c_1 c_3 & -c_3 s_1 s_2 - c_2 s_3 \\ c_3 s_2 + c_2 s_1 s_3 & c_1 s_3 & c_2 c_3 - s_1 s_2 s_3 \end{pmatrix}$$

$$\mathcal{M}_{11}^2 \approx$$

$$\mathcal{M}_{12}^2 \approx$$

$$\mathcal{M}_{13}^2 \approx M_3^2\ \varepsilon_2\ \cos\theta - m_2^2\ \varepsilon_1\ \sin\theta - m_1^2\ \epsilon_2,$$

$$\mathcal{M}_{23}^2 \approx \Delta M_{32}^2\,(1 - \bar{\epsilon}^2)\ \sin\theta\ \cos\theta,$$

$$\mathcal{M}_{22}^2 \approx \Delta M_{32}^2\ \sin^2\theta + m_2^2 - \{\Delta M_{32}^2\ \sin^2\theta + \Delta m_{21}^2\}\ \epsilon_1^2,$$

$$\mathcal{M}_{33}^2 \approx \Delta M_{32}^2\ \cos^2\theta + m_2^2 - \{\Delta M_{32}^2\ \cos^2\theta + \Delta m_{21}^2\}\ \epsilon_2^2$$
$$-\Delta M_{32}^2\ \sin2\theta\ \epsilon_1\epsilon_2,$$

Mixing Constraints

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yields a symmetric matrix $\mathcal{M}^2_{ij}$ $(= \mathcal{M}^2_{ji})$ with the following elements:

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$$\mathcal{M}^2_{12} \approx M_3^2\ \varepsilon_2\ \sin\theta + m_2^2\ \varepsilon_1\ \cos\theta - m_1^2\ \epsilon_1,$$
$\mathcal{M}^2_{13}$

$\mathcal{M}^2_{23}$

$\mathcal{M}^2_{22}$

$\mathcal{M}^2_{33}$

$$\varepsilon_{2,1} \equiv \epsilon_{2,1}\ \cos\theta \pm \epsilon_{1,2}\ \sin\theta,$$
$$\bar{\epsilon}^2 \equiv \frac{1}{2}\left(\epsilon_1^2 + \epsilon_2^2\right) + \tan\theta\ \epsilon_1\epsilon_2 + \frac{2(m_2^2 - m_1^2)\ \epsilon_1\epsilon_2}{(M_3^2 - m_2^2)\ \sin 2\theta},$$
$$\Delta M_{32}^2 \equiv M_3^2 - m_2^2\ , \quad \text{and} \quad \Delta m_{21}^2 \equiv m_2^2 - m_1^2.$$

Mixing Constraints

(1,1) :

$$m_H^2 \approx M_Z^2 \cos^2 2\beta + \left| \frac{\kappa \phi^2 |\lambda_1|^2}{|M|^2} \right|^2 |H|^2 + \Delta m_H^2$$

$$\left(\boxed{\left| \frac{\kappa \phi^2 |\lambda_1|^2}{|M|^2} \right|^2 |H|^2 \approx M_3^2 \varepsilon_2^2 + M_2^2 \varepsilon_1^2} \right)$$

(1,2), **(1,3)** :

$$\lambda_2 H \mu_{\text{eff}} \approx M_3^2 \; \varepsilon_2 \; \sin\theta + M_2^2 \; \varepsilon_1 \; \cos\theta,$$

$$\lambda_1 H \mu_{\text{eff}} \approx M_3^2 \; \varepsilon_2 \; \cos\theta - M_2^2 \; \varepsilon_1 \; \sin\theta,$$

$$\boxed{\frac{\varepsilon_1}{\varepsilon_2} \equiv \frac{\epsilon_1 \cos\theta - \epsilon_2 \sin\theta}{\epsilon_2 \cos\theta + \epsilon_1 \sin\theta} = \frac{M_3^2}{M_2^2} \, \frac{\frac{\lambda_2}{\lambda_1} - \tan\theta}{1 + \frac{\lambda_2}{\lambda_1} \tan\theta},}$$

Mixing Constraints

(2,3) : $\kappa\phi M = M_\Sigma^2 \sin\theta\ \cos\theta, \quad \lambda_1\lambda_2|H|^2 = \delta M^2 \sin\theta\ \cos\theta,$

$$M_\Sigma^2 + \delta M^2 \approx \Delta M_{32}^2(1 - \bar{\epsilon}^2),$$

Mixing Constraints

<u>**(2,3)**</u> **:** $\kappa\phi M = M_\Sigma^2 \sin\theta \, \cos\theta, \quad \lambda_1\lambda_2|H|^2 = \delta M^2 \sin\theta \, \cos\theta,$

$$\boxed{M_\Sigma^2 + \delta M^2 \approx \Delta M_{32}^2(1-\bar{\epsilon}^2),}$$

$$\boxed{\frac{\kappa\phi}{M} \equiv \tan\zeta, \quad \frac{\lambda_2}{\lambda_1} \equiv \tan\xi,}$$

T_ζ

Mixing Constraints

$$
\begin{aligned}
m_\phi^2 \approx\ & m_2^2 + \Delta M_{32}^2\left(\bar{\epsilon}^2 - \epsilon_1^2\right)\sin^2\theta - \Delta m_{21}^2\epsilon_1^2 \\
& + M_\Sigma^2 \sin\theta\ \cos\theta\left(\tan\theta - \tan\zeta\right) \\
& + \delta M^2 \sin\theta\ \cos\theta\left(\tan\theta - \tan\xi\right), \\
m_X^2 \approx\ & m_2^2 + \Delta M_{32}^2\left(\bar{\epsilon}^2 - \epsilon_2^2\right)\cos^2\theta - \Delta m_{21}^2\epsilon_2^2 \\
& - \Delta M_{32}^2\ \sin 2\theta\ \epsilon_1\epsilon_2 + M_\Sigma^2 \sin\theta\ \cos\theta\left(\cot\theta - \cot\zeta\right) \\
& + \delta M^2 \sin\theta\ \cos\theta\left(\cot\theta - \cot\xi\right)
\end{aligned}
$$

Around $\theta = \frac{\pi}{2} + \zeta$, i.e. when $\theta = \frac{\pi}{2} + \delta\theta$ $(|\delta\theta|, |\zeta| \ll 1)$,

(2,2), $$m_\phi^2 \approx M_3^2 - M_2^2\left[\frac{\epsilon_1^2 + \epsilon_2^2}{2} + (\epsilon_1\epsilon_2 + \zeta - \delta\theta)\,\delta\theta\right],$$

(3,3) : $$m_X^2 \approx M_2^2\left[1 - 2\epsilon_1\epsilon_2\delta\theta - \epsilon_2^2 - \frac{\delta\theta}{\zeta} - \mathcal{O}(\delta\theta^2)\right],$$

Mixing Constraints

$$m_\phi^2 \approx m_2^2 + \Delta M_{32}^2\left(\bar{\epsilon}^2 - \epsilon_1^2\right)\sin^2\theta - \Delta m_{21}^2\epsilon_1^2$$
$$+ M_\Sigma^2 \sin\theta\,\cos\theta\,(\tan\theta - \tan\zeta)$$
$$+ \delta M^2 \sin\theta\,\cos\theta\,(\tan\theta - \tan\xi)\,,$$

m … $\mathrm{ot}\zeta)$

$$\boxed{\delta\theta \approx \frac{\zeta\left(1-\epsilon_2^2\right)}{1+2\epsilon_1\epsilon_2\zeta}\,, \qquad M_3^2 \approx M_2^2\left[\frac{\epsilon_1^2+\epsilon_2^2}{2} + \epsilon_1\epsilon_2\zeta\right]}$$

$$+ \delta M^2 \sin\theta\,\cos\theta\,(\cot\theta - \cot\xi)$$

Around $\theta = \frac{\pi}{2} + \zeta$, i.e. when $\theta = \frac{\pi}{2} + \delta\theta$ $(|\delta\theta|, |\zeta| \ll 1)$,

(2,2), $$m_\phi^2 \approx M_3^2 - M_2^2\left[\frac{\epsilon_1^2+\epsilon_2^2}{2} + (\epsilon_1\epsilon_2 + \zeta - \delta\theta)\,\delta\theta\right],$$

(3,3) : $$m_X^2 \approx M_2^2\left[1 - 2\epsilon_1\epsilon_2\delta\theta - \epsilon_2^2 - \frac{\delta\theta}{\zeta} - \mathcal{O}(\delta\theta^2)\right],$$

Mixing Constraints

We can fulfill the constraints e.g. with

- $\lambda_1 \approx 0.7$, $|\lambda_2 / \lambda_1| \approx 0.03$, $\tan\zeta < 10^{-1}$

- $M_3 \sim 5$ TeV, $m_2 \sim 500$ GeV

$\varepsilon_{1,2} \sim 10^{-1} - 10^{-2}$, $|\tan\theta| > 10^{+1}$

(almost maximal mixing btw ϕ and X)

- $F_{GG} / \langle S \rangle \sim 6$ TeV

Conclusion

- The MSSM μ term is dynamically adjusted by singlets such that the min. cond. of the Higgs is fulfilled .
- The large VEV of singlets (μ term) are efficiently controlled by a Higgs VEV of order 100 GeV.
- A relatively small soft mass of a singlet is responsible for the small <H> (or small M_Z).
- SUSY particles' masses are well-above the exp. bounds, and FCNC would adequately be suppressed.
- Mixings btw the Higgs and singlets constrain the allowed parameter space.