

RG IMPROVEMENT OF MULTI-FIELD EFFECTIVE POTENTIALS

Bogumiła Świeżewska
Utrecht University

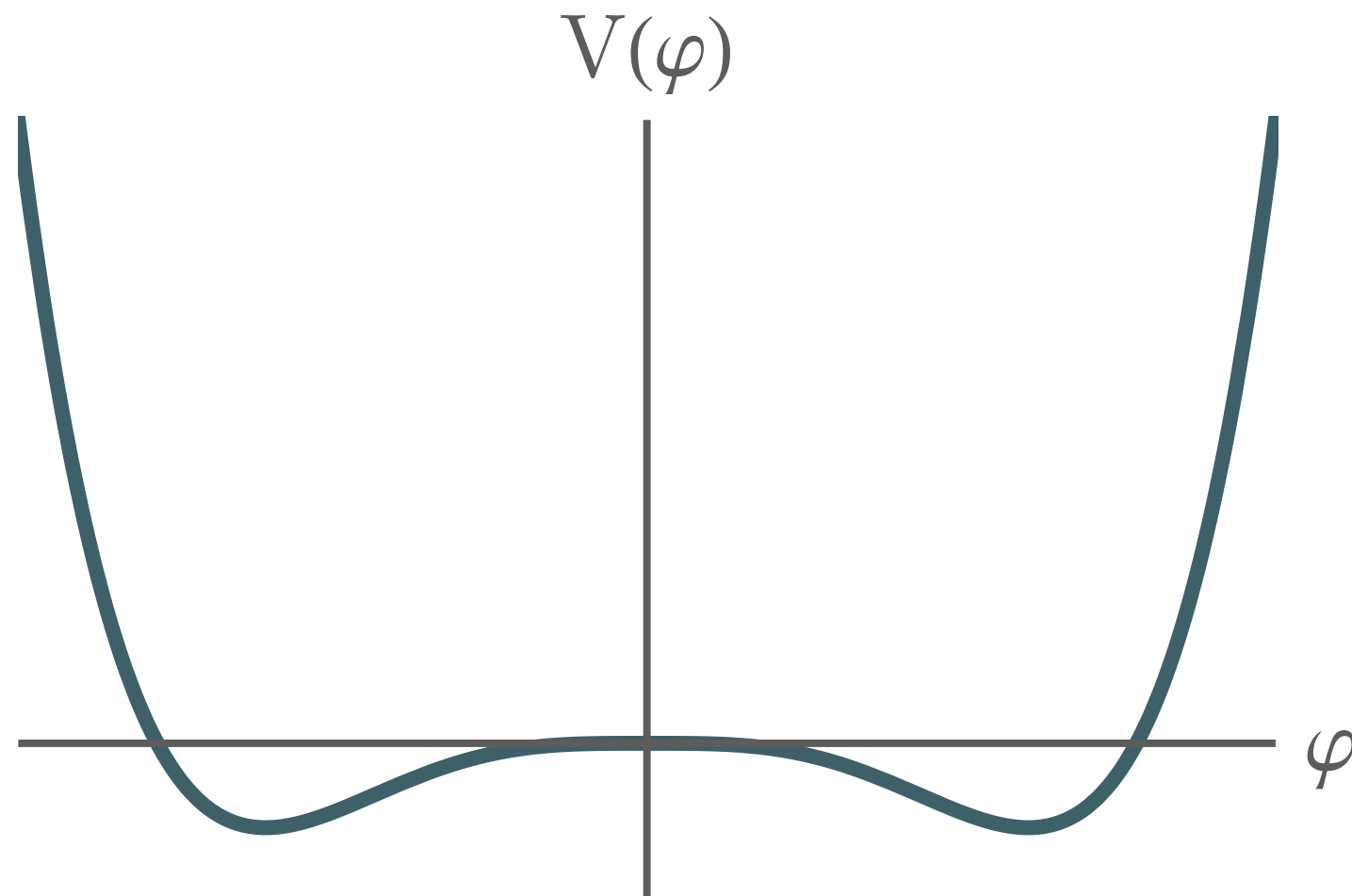
*Based on JHEP 1803 (2018) 014 and arXiv:1805.09292,
in collaboration with
Tomislav Prokopec, Leonardo Chataignier, Michael G. Schmidt*

Planck 2018, 24.05.2018

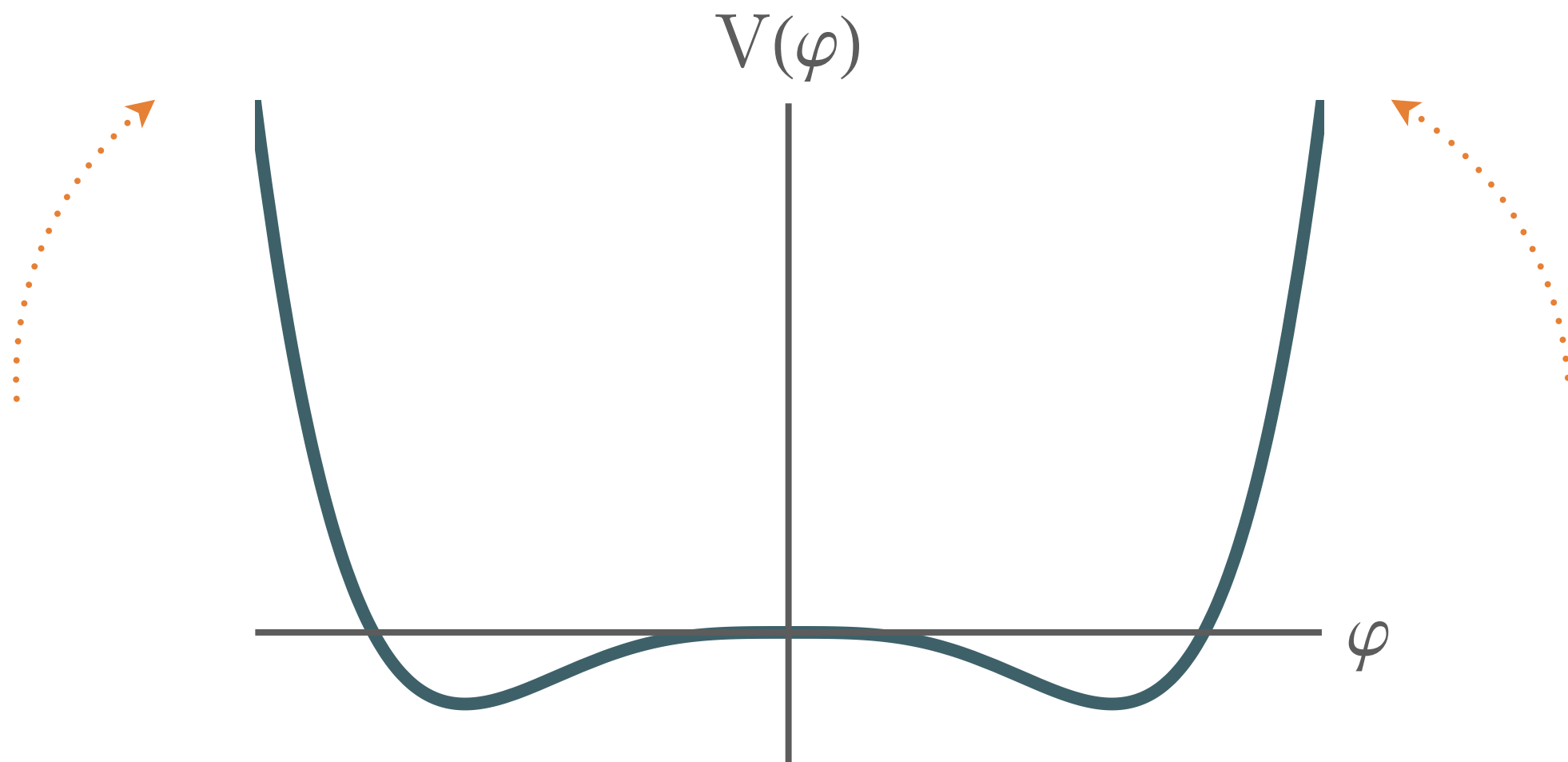
INTRODUCTION



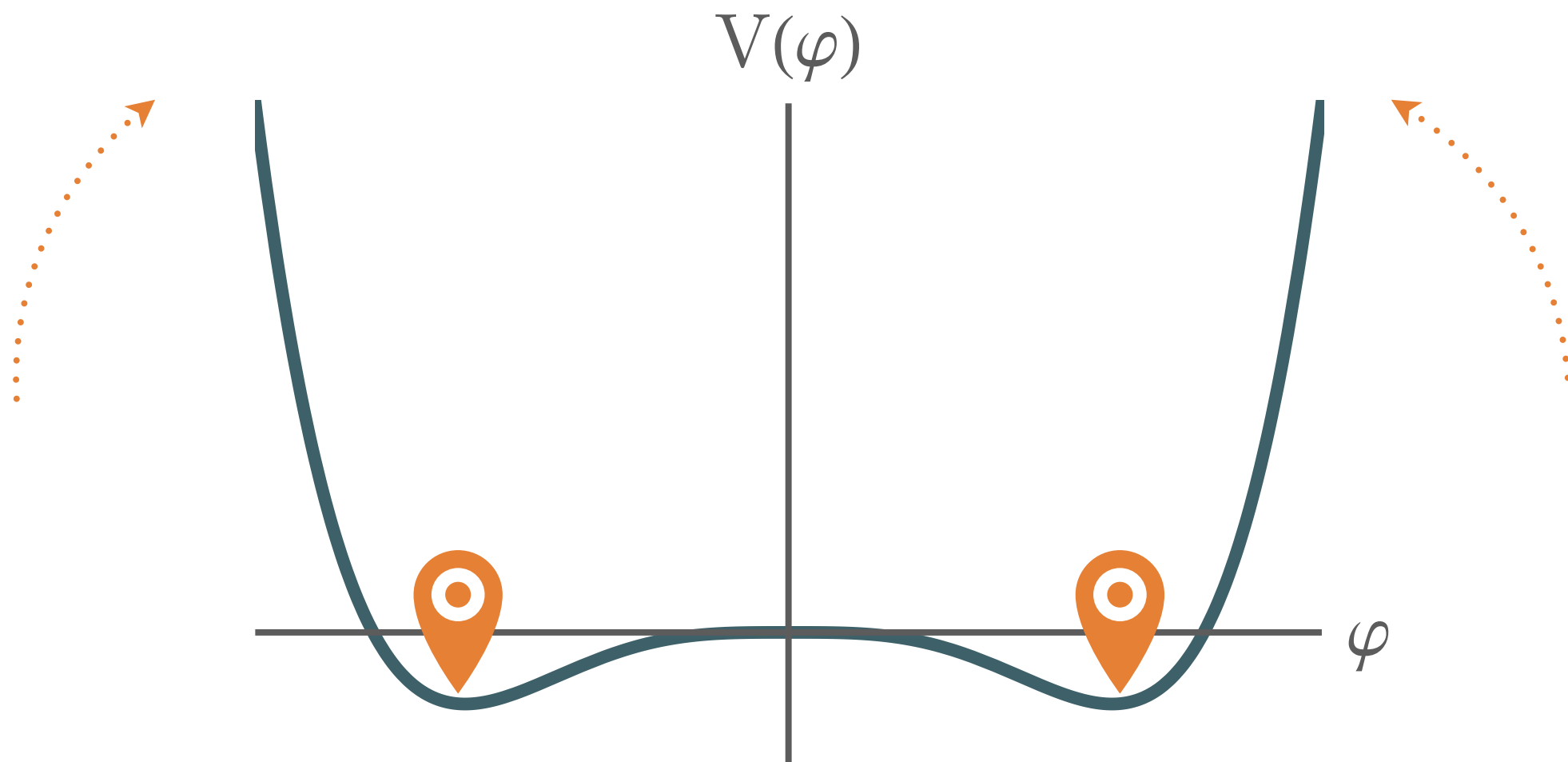
WHY EFFECTIVE POTENTIAL?



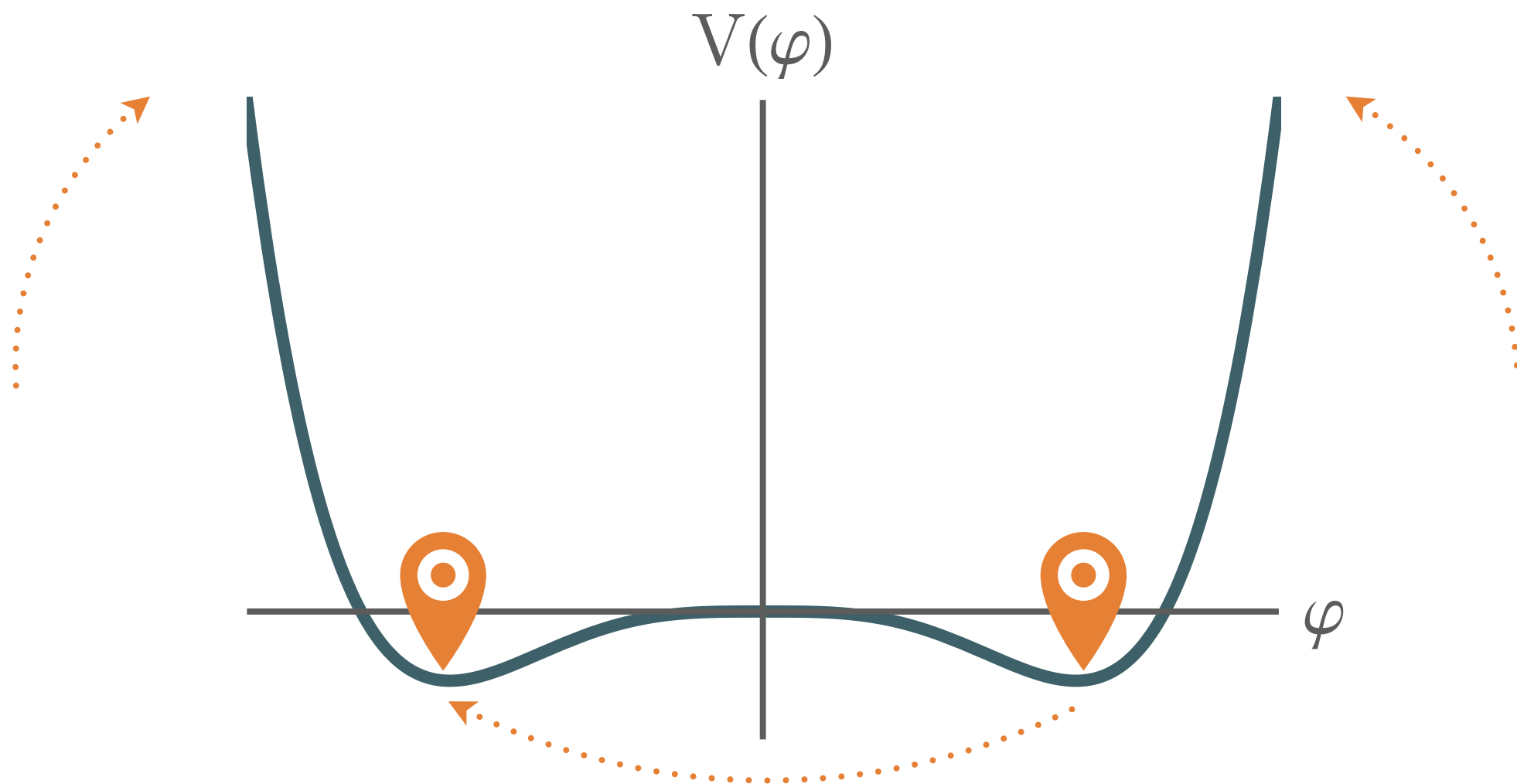
WHY EFFECTIVE POTENTIAL?



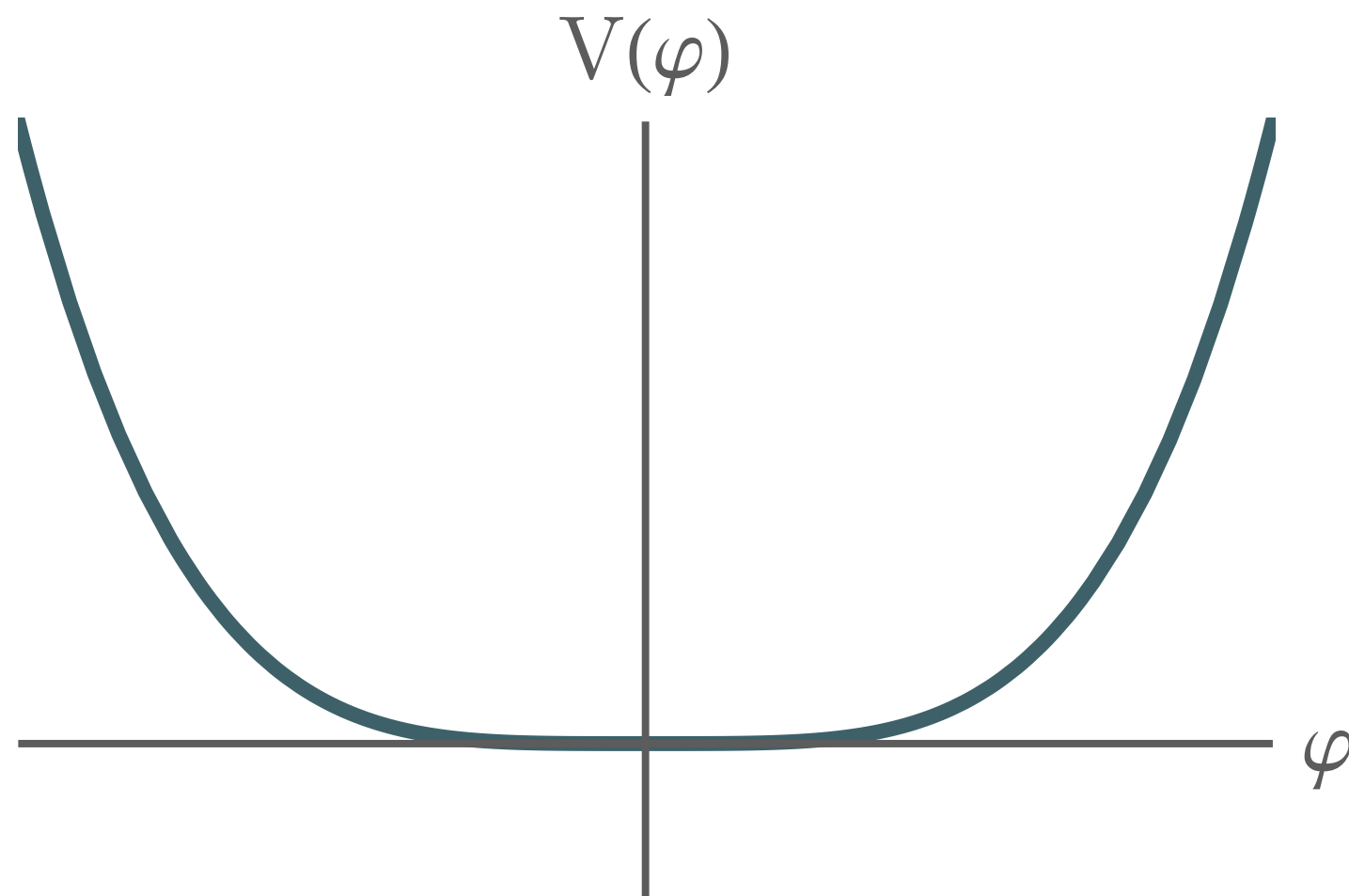
WHY EFFECTIVE POTENTIAL?



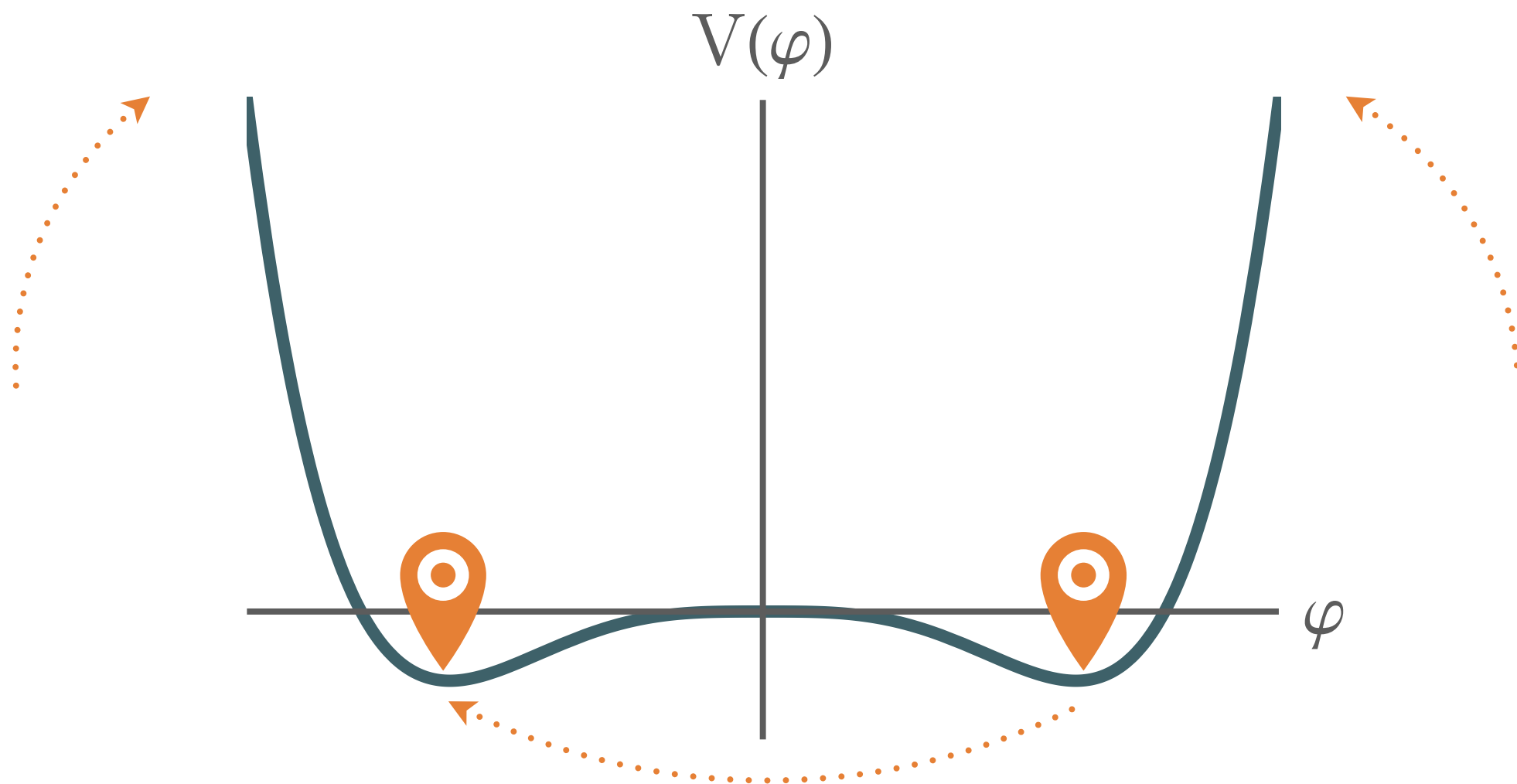
WHY EFFECTIVE POTENTIAL?



WHY EFFECTIVE POTENTIAL?



WHY EFFECTIVE POTENTIAL?



WHY RGE?

Loop expansion parameters: $\lambda, \lambda \log \frac{\phi^2}{\mu^2}$

RGE



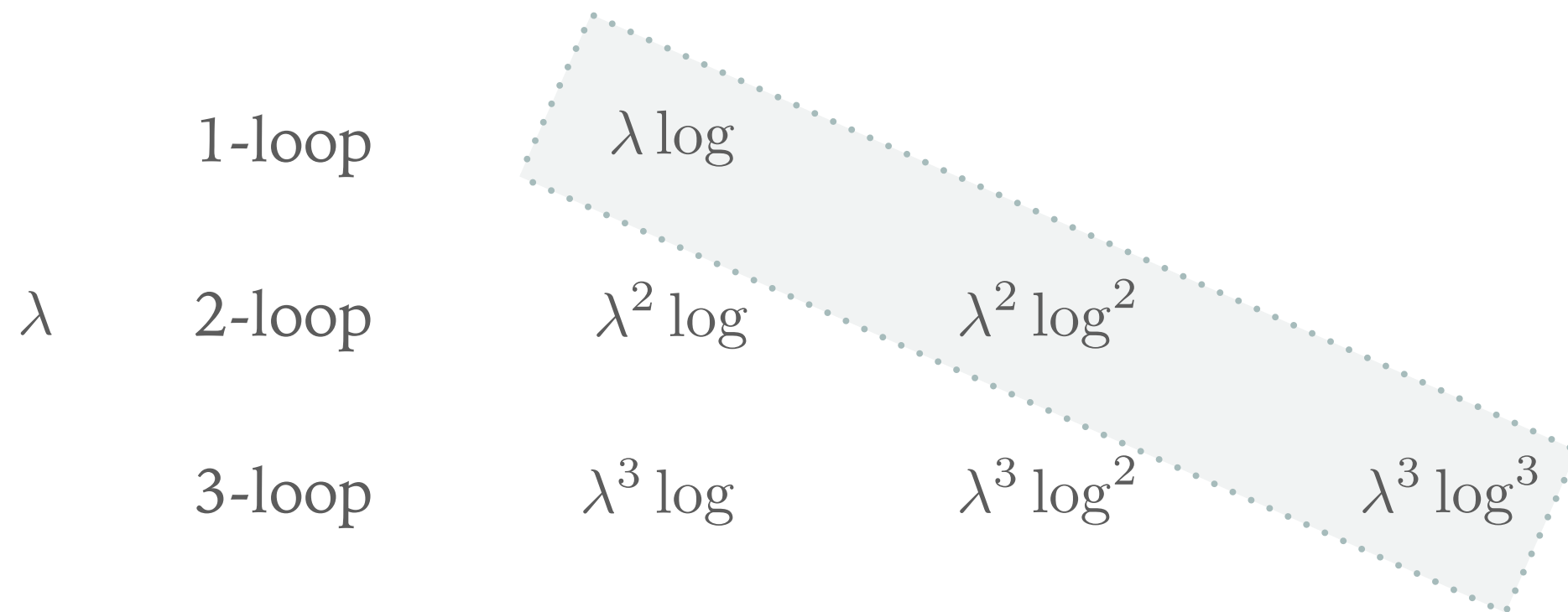
Scale independence of
the effective potential

RG improved potential expansion parameter: $\bar{\lambda} \left(\log \frac{\phi^2}{\mu^2} \right)$

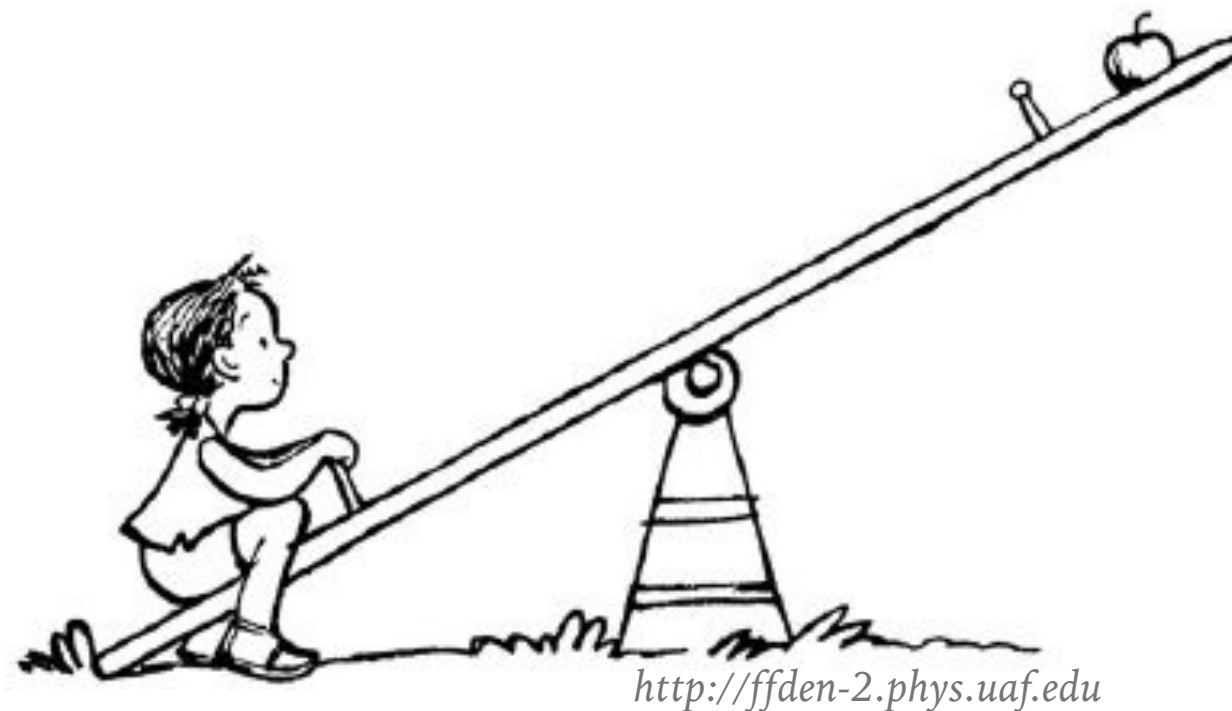
LEADING LOGS

	1-loop	$\lambda \log$		
λ	2-loop	$\lambda^2 \log$	$\lambda^2 \log^2$	
	3-loop	$\lambda^3 \log$	$\lambda^3 \log^2$	$\lambda^3 \log^3$

LEADING LOGS



WHY IS THIS PROBLEMATIC?



[See also: M.B. Einhorn, D.R.T. Jones, Nucl. Phys. B 230 (1984) 261, C. Ford, D.R.T. Jones, P.W. Stephenson, M.B. Einhorn, Nucl. Phys. B 395 (1993) 17, C. Ford, Phys. Rev. D 50 (1994) 7531, C. Ford and C. Wiesendanger, Phys. Lett. B 398 (1997) 342, M. Bando, T. Kugo, N. Maekawa and H. Nakano, Prog Theor Phys (1993) 90, J.A. Casas, V. Di Clemente, M. Quirós, Nucl.Phys. B553 (1999) 511]

THE METHOD



METHOD OF CHARACTERISTICS

$$\mu \frac{dV}{d\mu}(\mu; \lambda, \phi) = \left(\mu \frac{\partial}{\partial \mu} + \sum_{i=1}^{N_\lambda} \beta_i \frac{\partial}{\partial \lambda_i} - \frac{1}{2} \sum_{a=1}^{N_\phi} \gamma_a \phi_a \frac{\partial}{\partial \phi_a} \right) V(\mu; \lambda, \phi) = 0$$

METHOD OF CHARACTERISTICS

$$\mu \frac{dV}{d\mu}(\mu; \lambda, \phi) = \left(\mu \frac{\partial}{\partial \mu} + \sum_{i=1}^{N_\lambda} \beta_i \frac{\partial}{\partial \lambda_i} - \frac{1}{2} \sum_{a=1}^{N_\phi} \gamma_a \phi_a \frac{\partial}{\partial \phi_a} \right) V(\mu; \lambda, \phi) = 0$$

Characteristic curves = RG flow

$$\frac{d}{dt} \bar{\mu}(t) = \bar{\mu}(t),$$

$$\frac{d}{dt} \bar{\lambda}_i(t) = \beta_i(\bar{\lambda}),$$

$$\frac{d}{dt} \bar{\phi}_a(t) = -\frac{1}{2} \gamma_a(\bar{\lambda}) \bar{\phi}_a(t),$$

$$\frac{d}{dt} V(t) = 0$$

METHOD OF CHARACTERISTICS

$$\mu \frac{dV}{d\mu}(\mu; \lambda, \phi) = \left(\mu \frac{\partial}{\partial \mu} + \sum_{i=1}^{N_\lambda} \beta_i \frac{\partial}{\partial \lambda_i} - \frac{1}{2} \sum_{a=1}^{N_\phi} \gamma_a \phi_a \frac{\partial}{\partial \phi_a} \right) V(\mu; \lambda, \phi) = 0$$

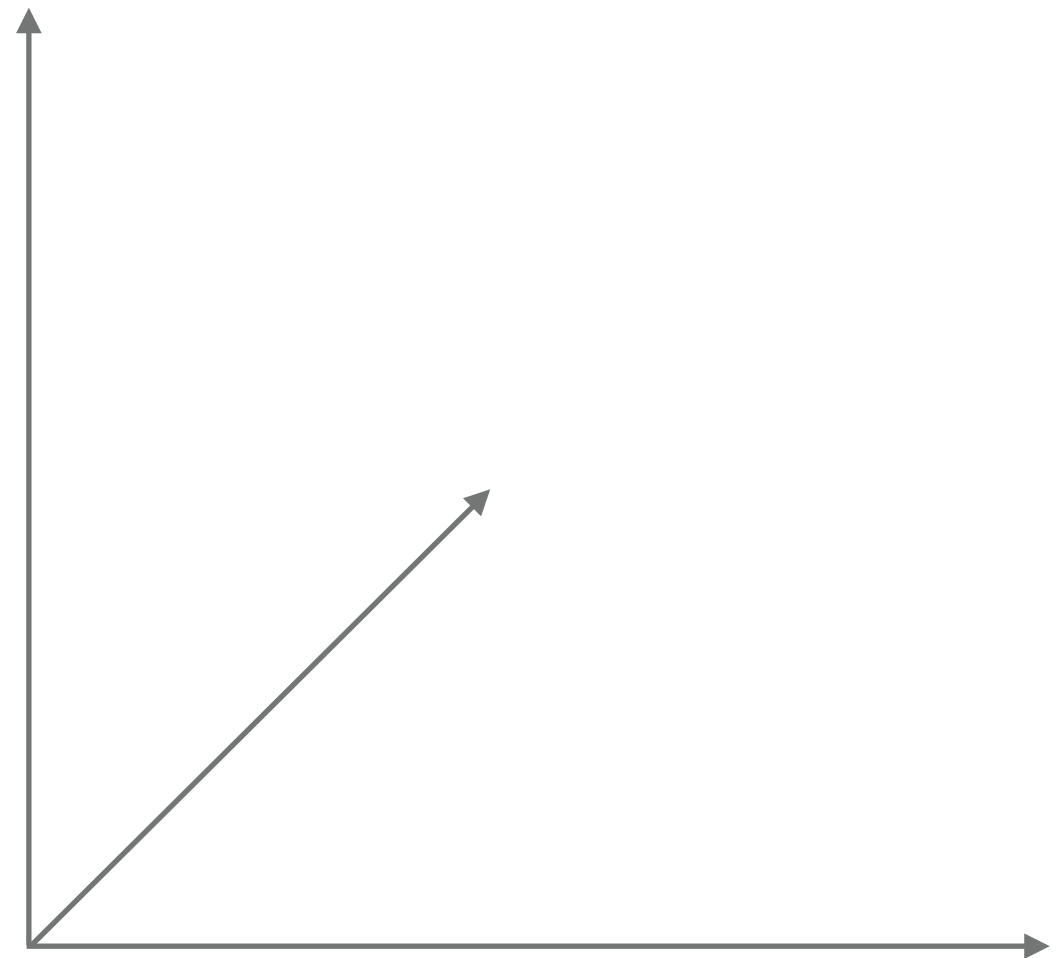
Characteristic curves = RG flow

$$\frac{d}{dt} \bar{\mu}(t) = \bar{\mu}(t),$$

$$\frac{d}{dt} \bar{\lambda}_i(t) = \beta_i(\bar{\lambda}),$$

$$\frac{d}{dt} \bar{\phi}_a(t) = -\frac{1}{2} \gamma_a(\bar{\lambda}) \bar{\phi}_a(t),$$

$$\frac{d}{dt} V(t) = 0$$



METHOD OF CHARACTERISTICS

$$\mu \frac{dV}{d\mu}(\mu; \lambda, \phi) = \left(\mu \frac{\partial}{\partial \mu} + \sum_{i=1}^{N_\lambda} \beta_i \frac{\partial}{\partial \lambda_i} - \frac{1}{2} \sum_{a=1}^{N_\phi} \gamma_a \phi_a \frac{\partial}{\partial \phi_a} \right) V(\mu; \lambda, \phi) = 0$$

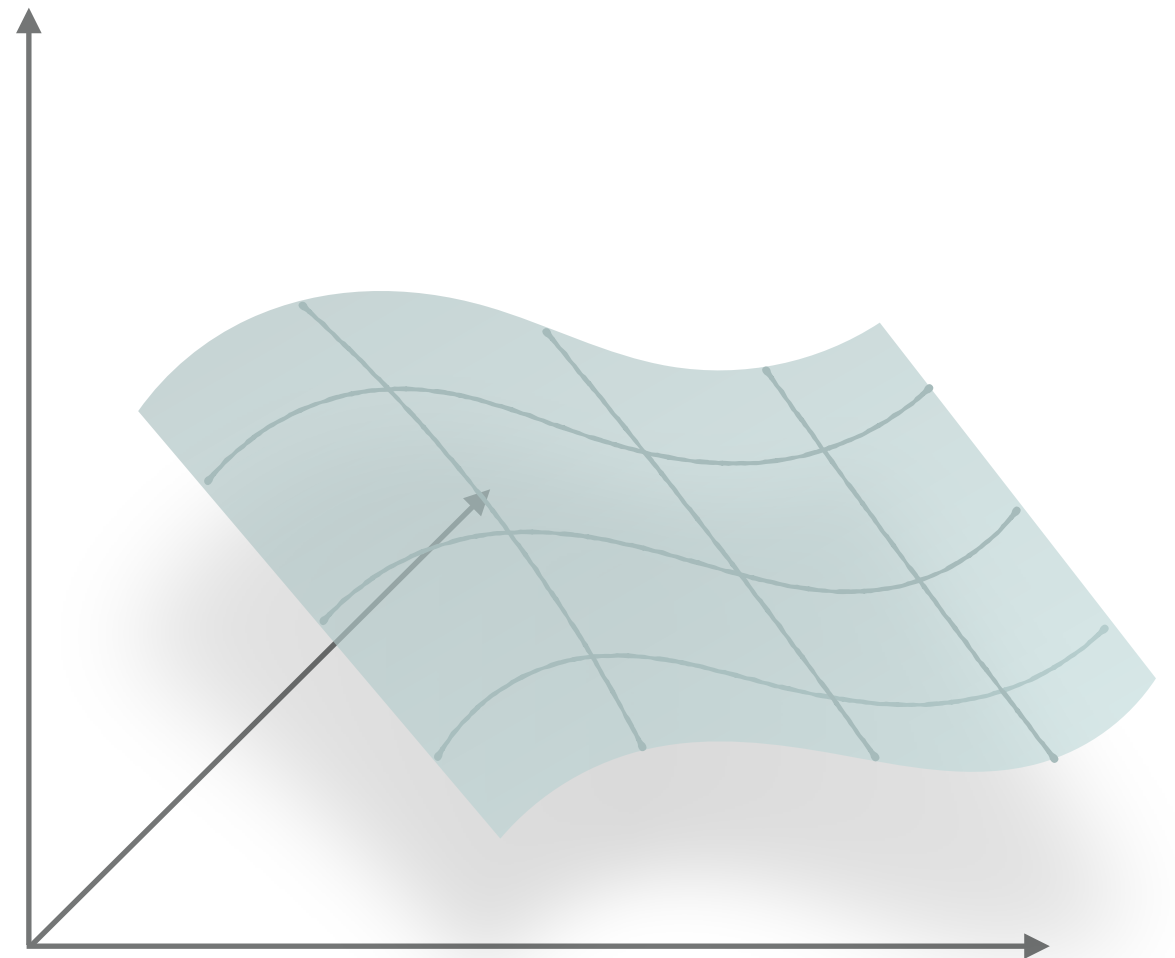
Characteristic curves = RG flow

$$\frac{d}{dt} \bar{\mu}(t) = \bar{\mu}(t),$$

$$\frac{d}{dt} \bar{\lambda}_i(t) = \beta_i(\bar{\lambda}),$$

$$\frac{d}{dt} \bar{\phi}_a(t) = -\frac{1}{2} \gamma_a(\bar{\lambda}) \bar{\phi}_a(t),$$

$$\frac{d}{dt} V(t) = 0$$



METHOD OF CHARACTERISTICS

$$\mu \frac{dV}{d\mu}(\mu; \lambda, \phi) = \left(\mu \frac{\partial}{\partial \mu} + \sum_{i=1}^{N_\lambda} \beta_i \frac{\partial}{\partial \lambda_i} - \frac{1}{2} \sum_{a=1}^{N_\phi} \gamma_a \phi_a \frac{\partial}{\partial \phi_a} \right) V(\mu; \lambda, \phi) = 0$$

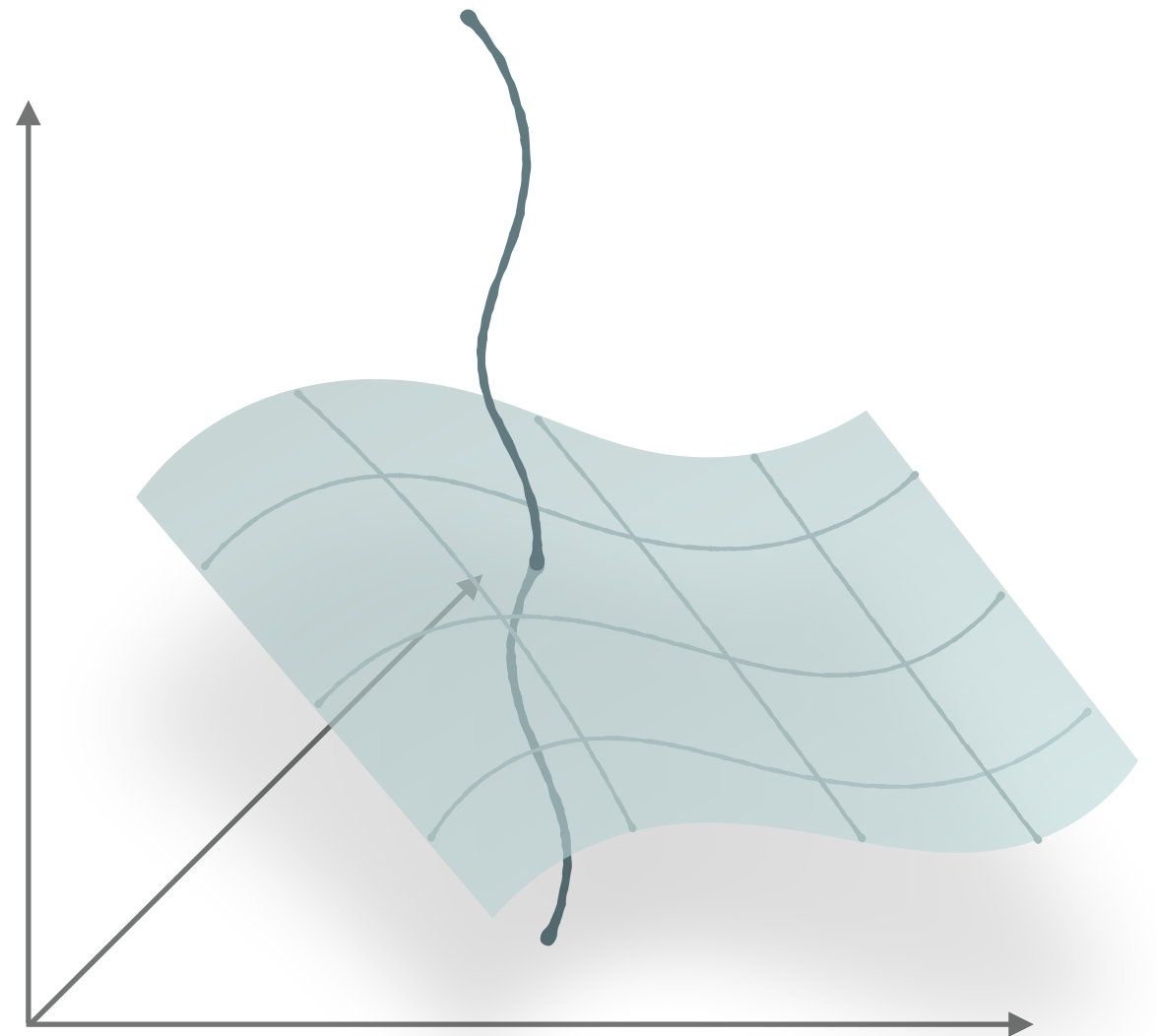
Characteristic curves = RG flow

$$\frac{d}{dt} \bar{\mu}(t) = \bar{\mu}(t),$$

$$\frac{d}{dt} \bar{\lambda}_i(t) = \beta_i(\bar{\lambda}),$$

$$\frac{d}{dt} \bar{\phi}_a(t) = -\frac{1}{2} \gamma_a(\bar{\lambda}) \bar{\phi}_a(t),$$

$$\frac{d}{dt} V(t) = 0$$



METHOD OF CHARACTERISTICS

$$\mu \frac{dV}{d\mu}(\mu; \lambda, \phi) = \left(\mu \frac{\partial}{\partial \mu} + \sum_{i=1}^{N_\lambda} \beta_i \frac{\partial}{\partial \lambda_i} - \frac{1}{2} \sum_{a=1}^{N_\phi} \gamma_a \phi_a \frac{\partial}{\partial \phi_a} \right) V(\mu; \lambda, \phi) = 0$$

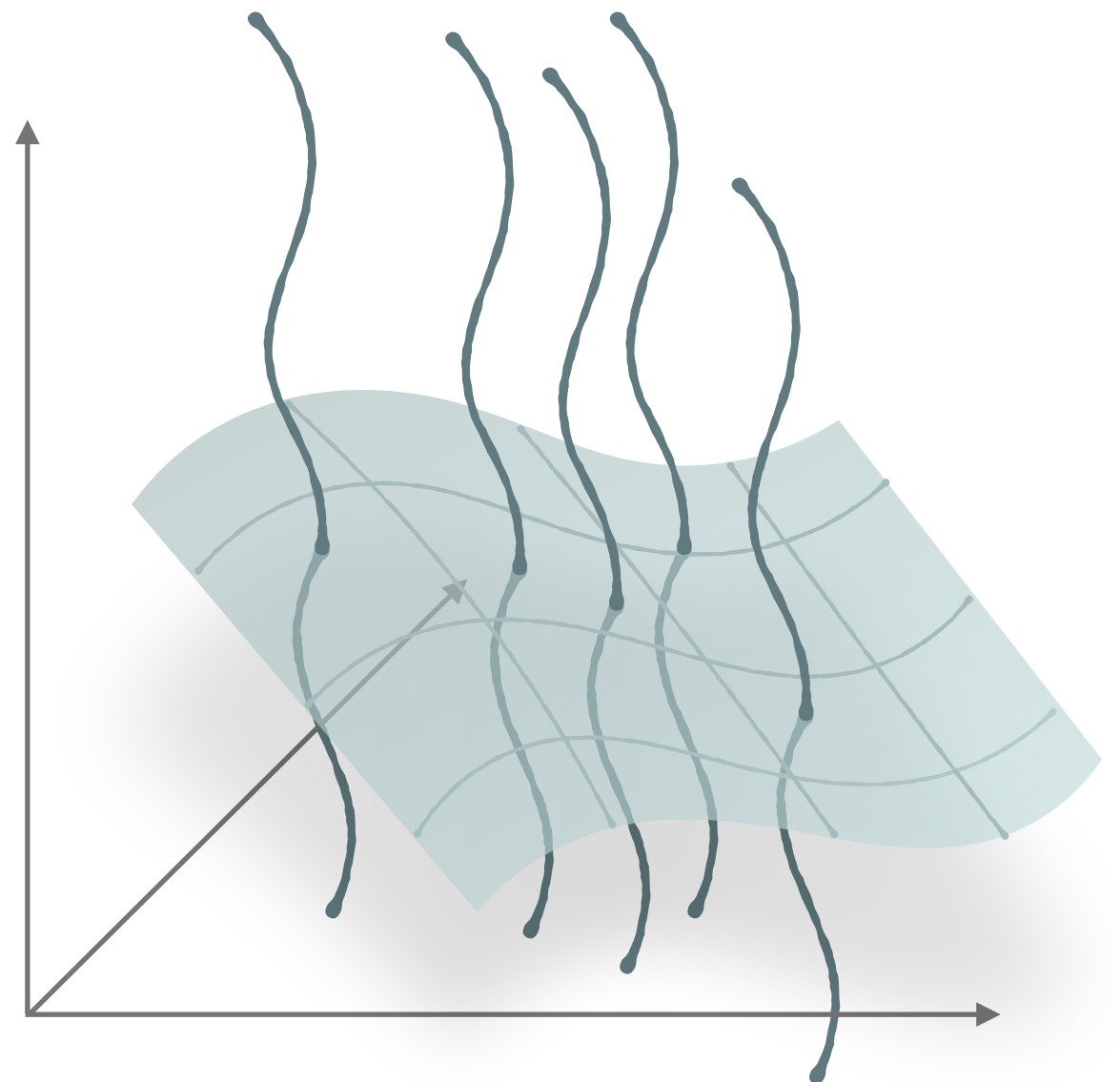
Characteristic curves = RG flow

$$\frac{d}{dt} \bar{\mu}(t) = \bar{\mu}(t),$$

$$\frac{d}{dt} \bar{\lambda}_i(t) = \beta_i(\bar{\lambda}),$$

$$\frac{d}{dt} \bar{\phi}_a(t) = -\frac{1}{2} \gamma_a(\bar{\lambda}) \bar{\phi}_a(t),$$

$$\frac{d}{dt} V(t) = 0$$



METHOD OF CHARACTERISTICS

$$\mu \frac{dV}{d\mu}(\mu; \lambda, \phi) = \left(\mu \frac{\partial}{\partial \mu} + \sum_{i=1}^{N_\lambda} \beta_i \frac{\partial}{\partial \lambda_i} - \frac{1}{2} \sum_{a=1}^{N_\phi} \gamma_a \phi_a \frac{\partial}{\partial \phi_a} \right) V(\mu; \lambda, \phi) = 0$$

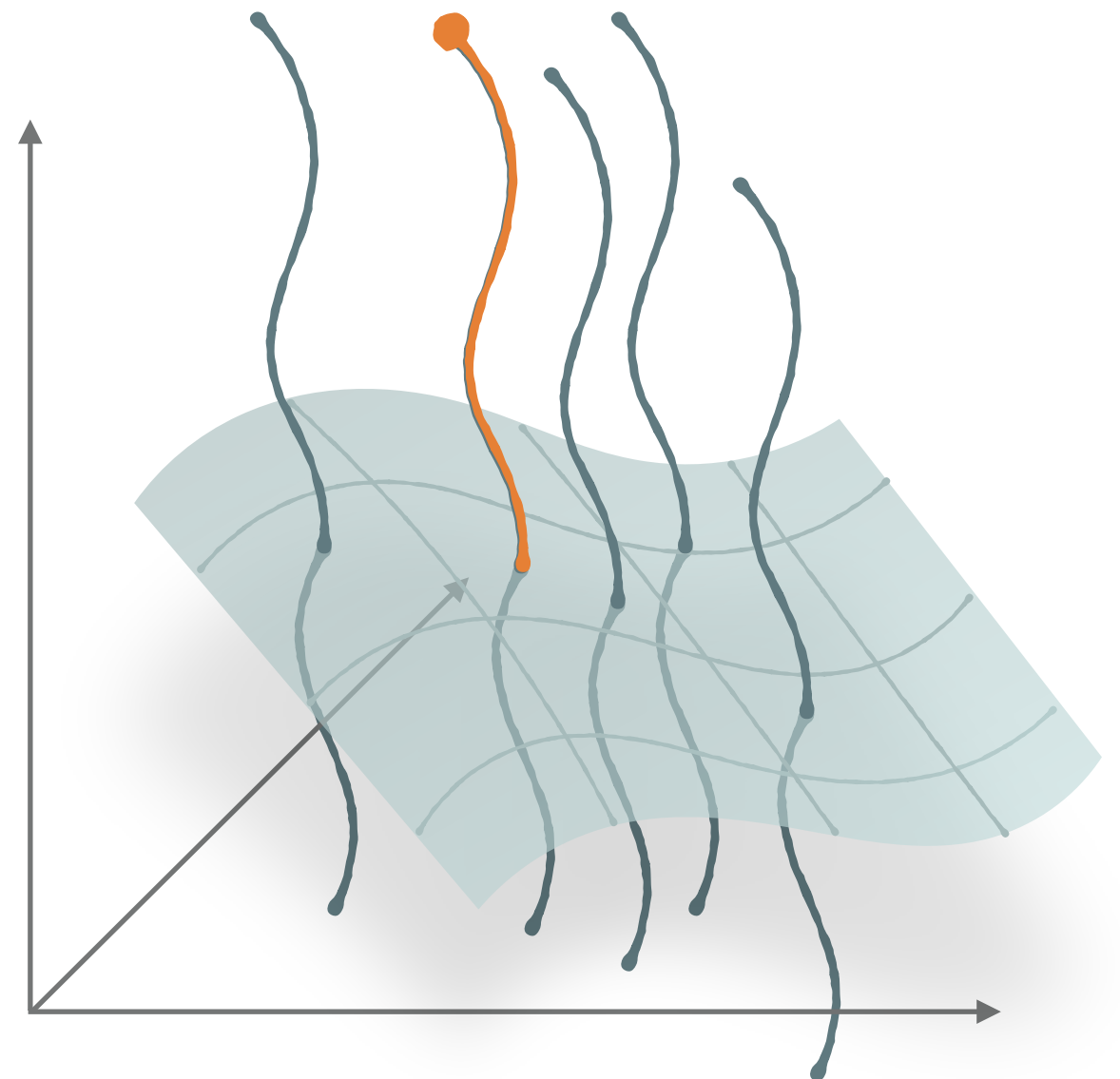
Characteristic curves = RG flow

$$\frac{d}{dt} \bar{\mu}(t) = \bar{\mu}(t),$$

$$\frac{d}{dt} \bar{\lambda}_i(t) = \beta_i(\bar{\lambda}),$$

$$\frac{d}{dt} \bar{\phi}_a(t) = -\frac{1}{2} \gamma_a(\bar{\lambda}) \bar{\phi}_a(t),$$

$$\frac{d}{dt} V(t) = 0$$



MULTI-FIELD MODELS

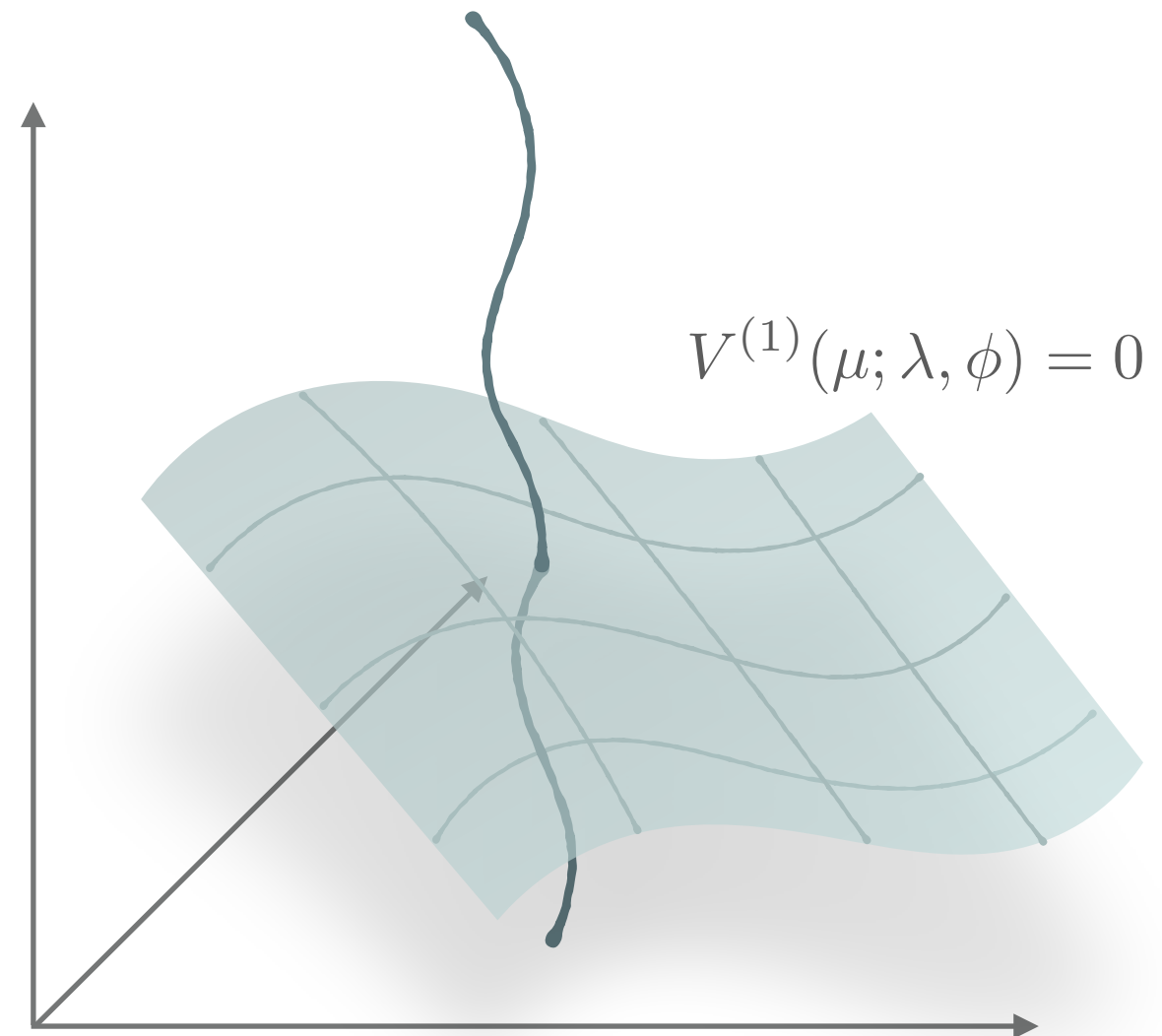
$$V^{(1)}(\mu, \lambda, \phi) = \frac{1}{64\pi^2} \sum_a n_a m_a^4(\lambda, \phi) \left[\log \frac{m_a^2(\lambda, \phi)}{\mu^2} - \chi_a \right]$$

To compute $V(\mu, \lambda, \phi)$

t_* $\left\{ \begin{array}{l} \text{Run to the tree-} \\ \text{level surface} \end{array} \right.$

Evaluate

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$



MULTI-FIELD MODELS

$$V^{(1)}(\mu, \lambda, \phi) =$$

M. Sher, Electroweak Higgs potentials and vacuum stability
 The form of this solution is not surprising; the only question is : what is t ?

$$\left[1 + \frac{m_a^2(\lambda, \phi)}{\mu^2} - \chi_a \right]$$

To compute $V(\mu, \lambda, \phi)$

t_*

Run to the tree-level surface

Evaluate

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

$$V^{(1)}(\mu; \lambda, \phi) = 0$$

MULTI-FIELD MODELS

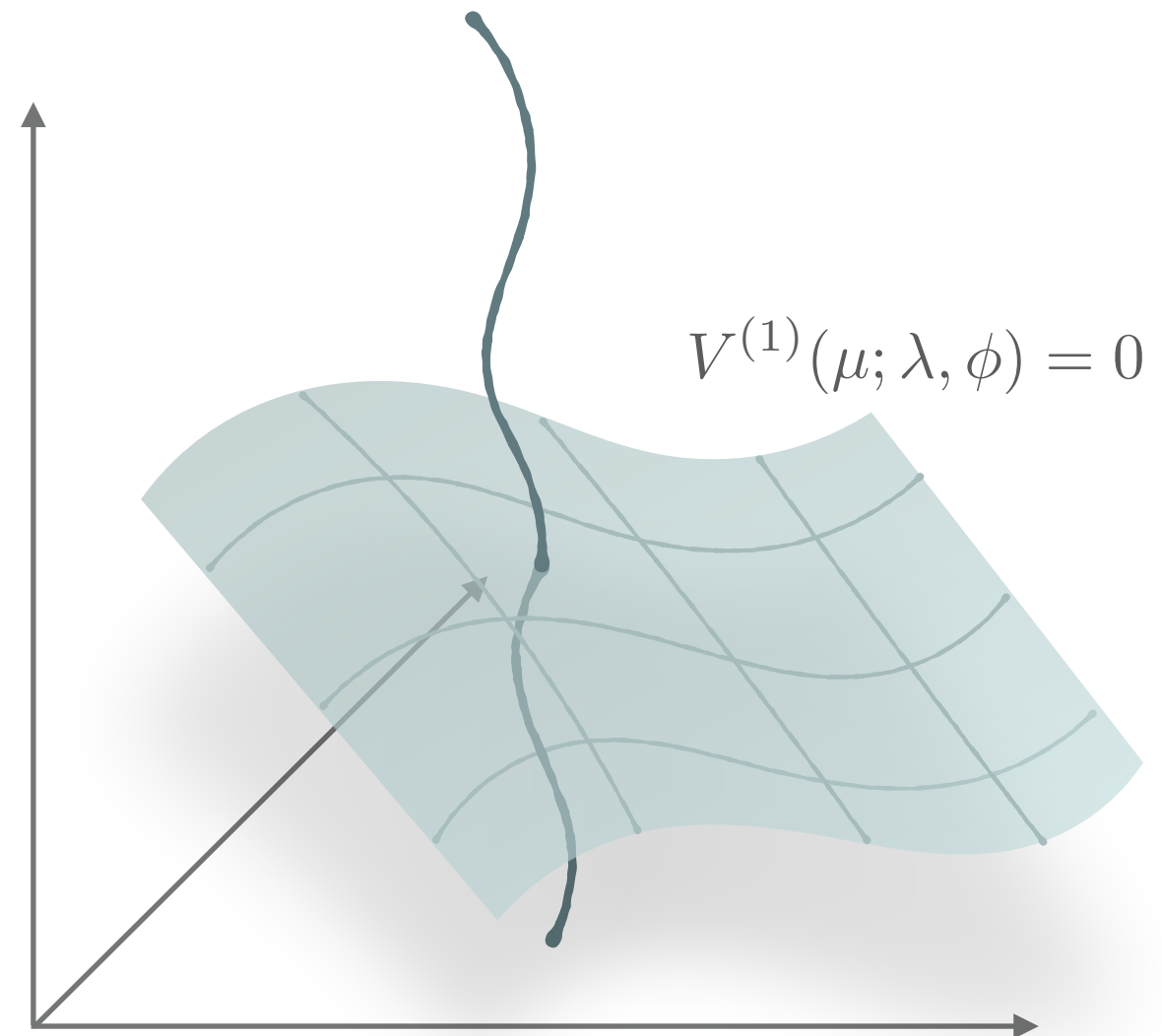
$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

What is t_* ?

$$V^{(1)}(\bar{\mu}(t_*); \bar{\lambda}(t_*), \bar{\phi}(t_*)) = 0$$

To leading
order in $\hbar$

$$t_*^{(0)} = \frac{V^{(1)}(\mu, \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)}$$



*SUBTLETIES

- Properties of the tree-level hypersurface
- Existence and uniqueness of t_*
- Resummation

APPLICATIONS

VACUUM STABILITY

$$\lim_{\phi \rightarrow \infty} V(\phi) = ?$$

One-loop potential unsuitable for this issue  need of improvement

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

VACUUM STABILITY

$$\lim_{\phi \rightarrow \infty} V(\phi) = ?$$

One-loop potential unsuitable for this issue  need of improvement

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$



Enough to consider tree-level conditions evaluated at large scale.

SU(2)CSM

cSM:
 $SU(2)_L \times U(1)_Y$
 Φ

Hidden sector:
 $SU(2)_X$
 Ψ

$$V^{(0)}(\Phi, \Psi) = \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 (\Phi^\dagger \Phi) (\Psi^\dagger \Psi) + \lambda_3 (\Psi^\dagger \Psi)^2,$$

[See also: T.Hambye, A.Strumia, PRD88 (2013) 055022, C.D.Carone, R.Ramos, PRD88 (2013) 055020, V.V.Khoze, C.McCabe, G.Ro, JHEP 08 (2014) 026]

SU(2)CSM

cSM:

$$SU(2)_L \times U(1)_Y$$
$$\Phi$$

Hidden sector:

$$SU(2)_X$$
$$\Psi$$

$$V^{(0)}(\Phi, \Psi) = \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 (\Phi^\dagger \Phi) (\Psi^\dagger \Psi) + \lambda_3 (\Psi^\dagger \Psi)^2,$$

Perturbative, stable
and self-consistent up
to the Planck scale

DM candidate

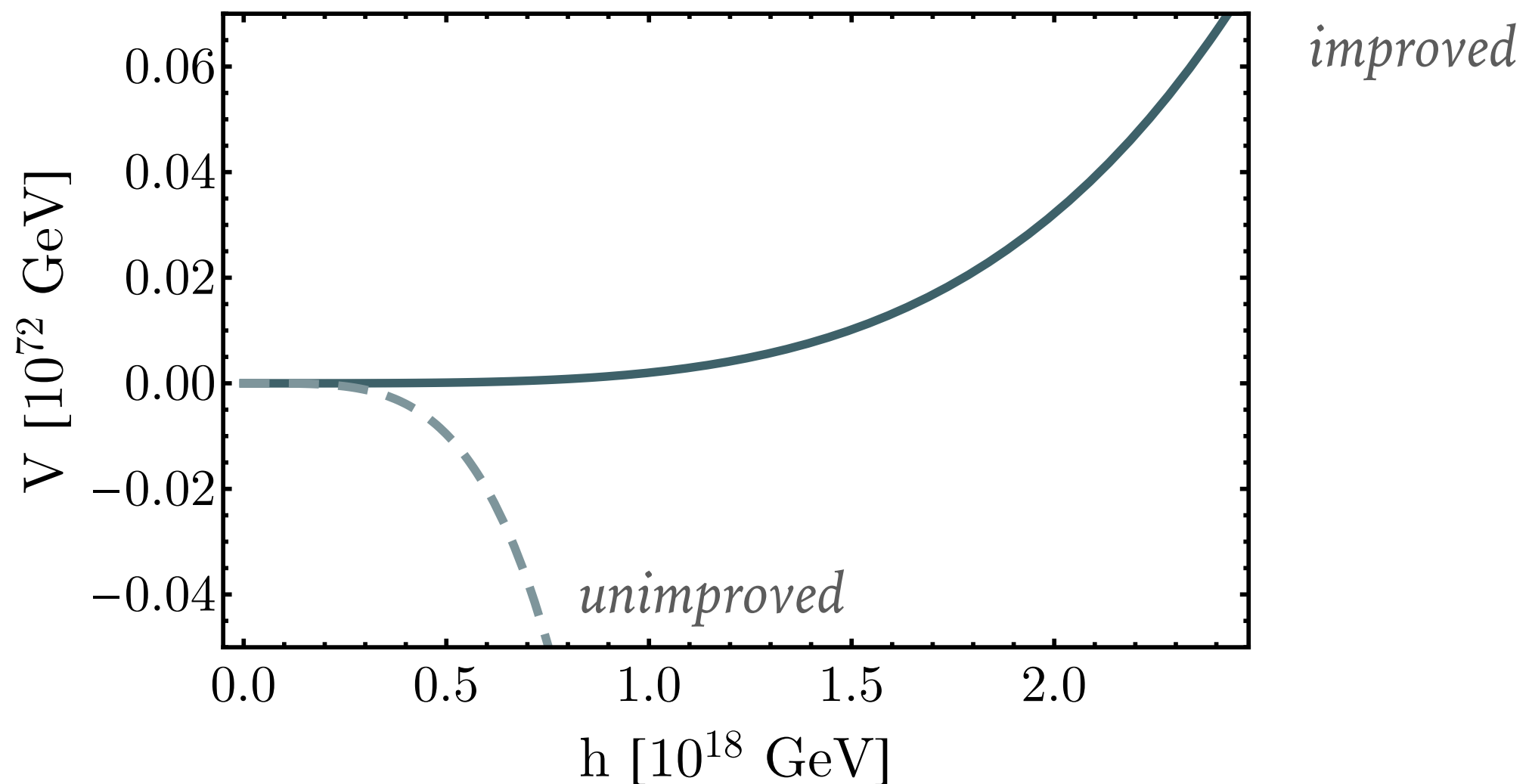
All masses generated radiatively

Potential for baryogenesis

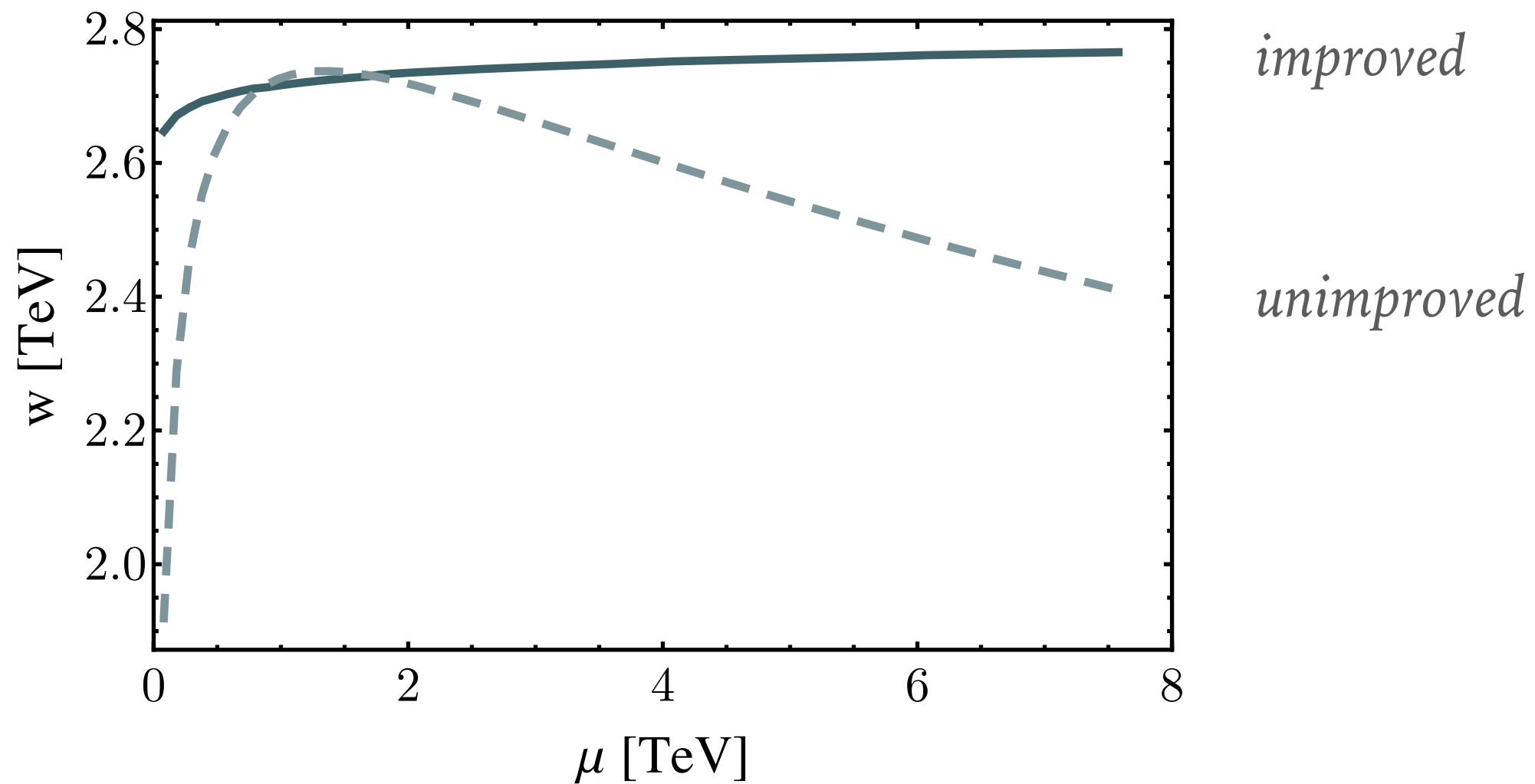
[See also: T.Hambye, A.Strumia, PRD88 (2013) 055022, C.D.Carone, R.Ramos, PRD88 (2013) 055020, V.V.Khoze, C.McCabe, G.Ro, JHEP 08 (2014) 026]

VACUUM STABILITY IN SU(2)CSM

Tree-level vacuum stability conditions satisfied at the Planck scale



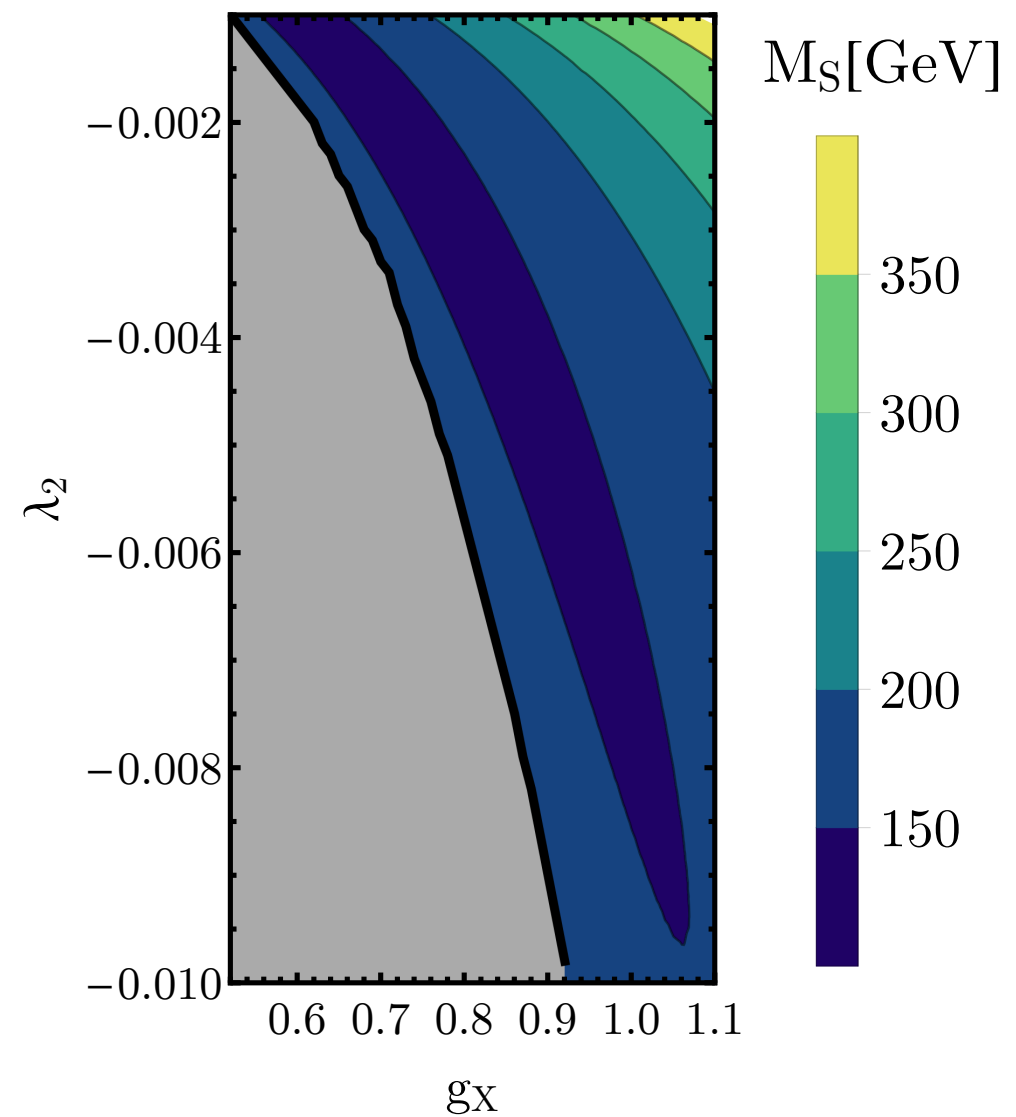
RUNNING VEVs



RADIATIVE SYMMETRY BREAKING IN $SU(2)$ CSM

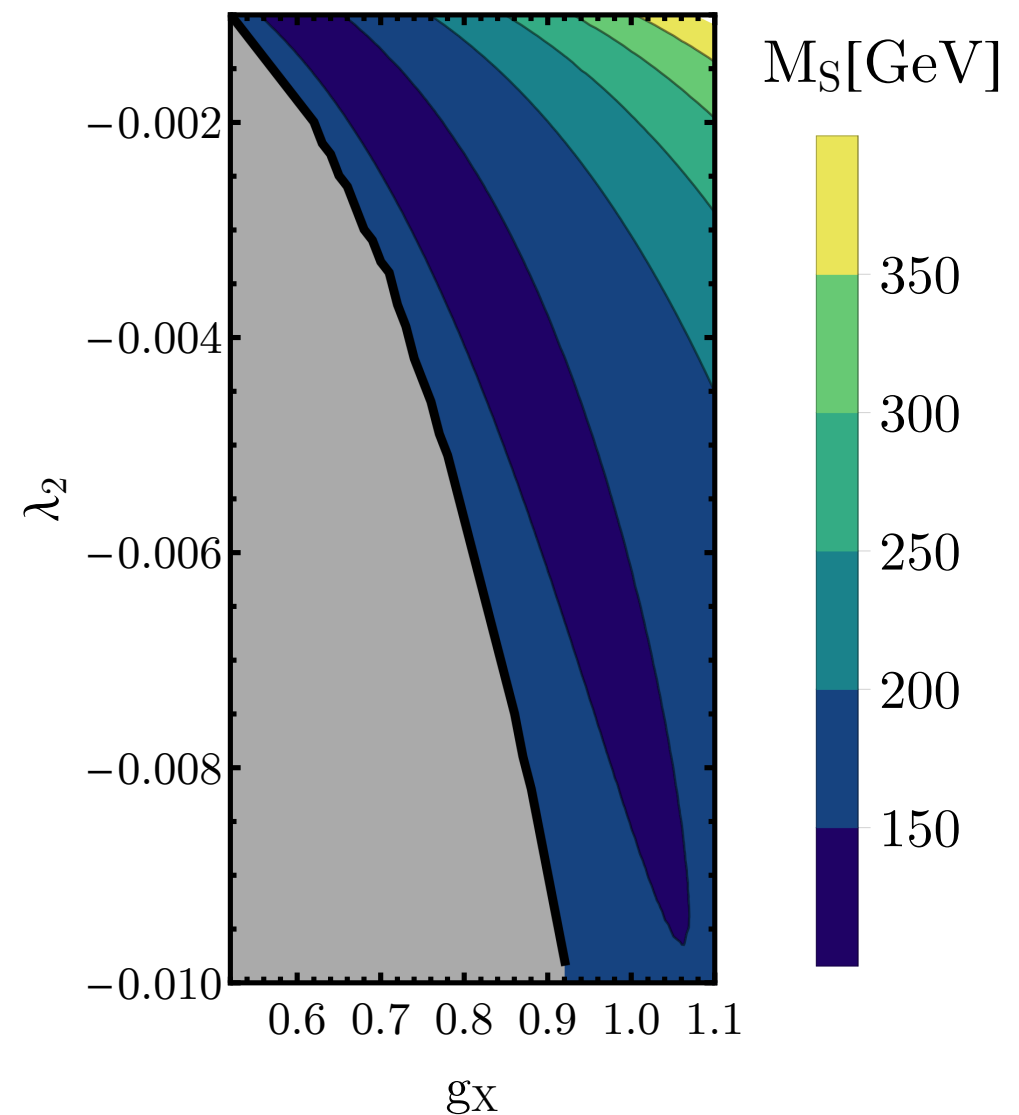
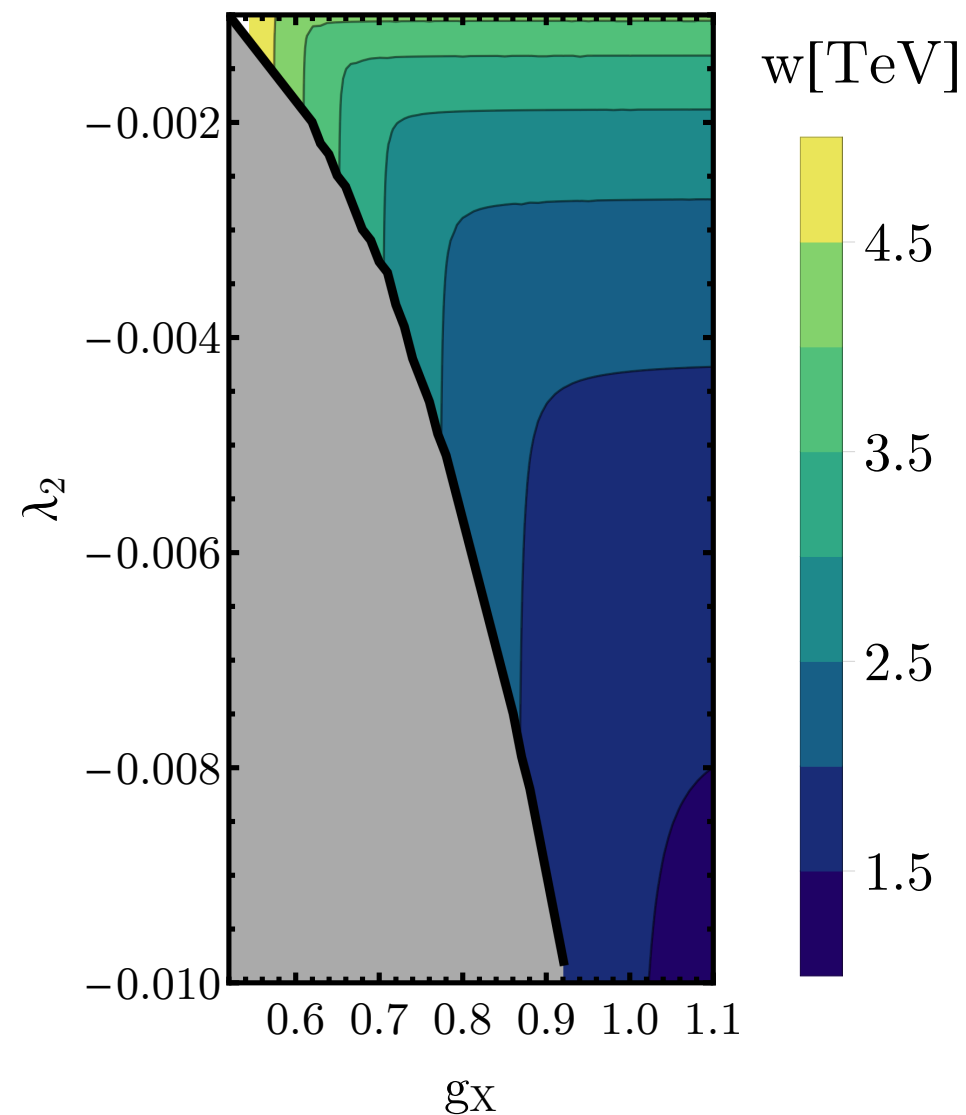
RADIATIVE SYMMETRY BREAKING IN SU(2)CSM

One-loop results



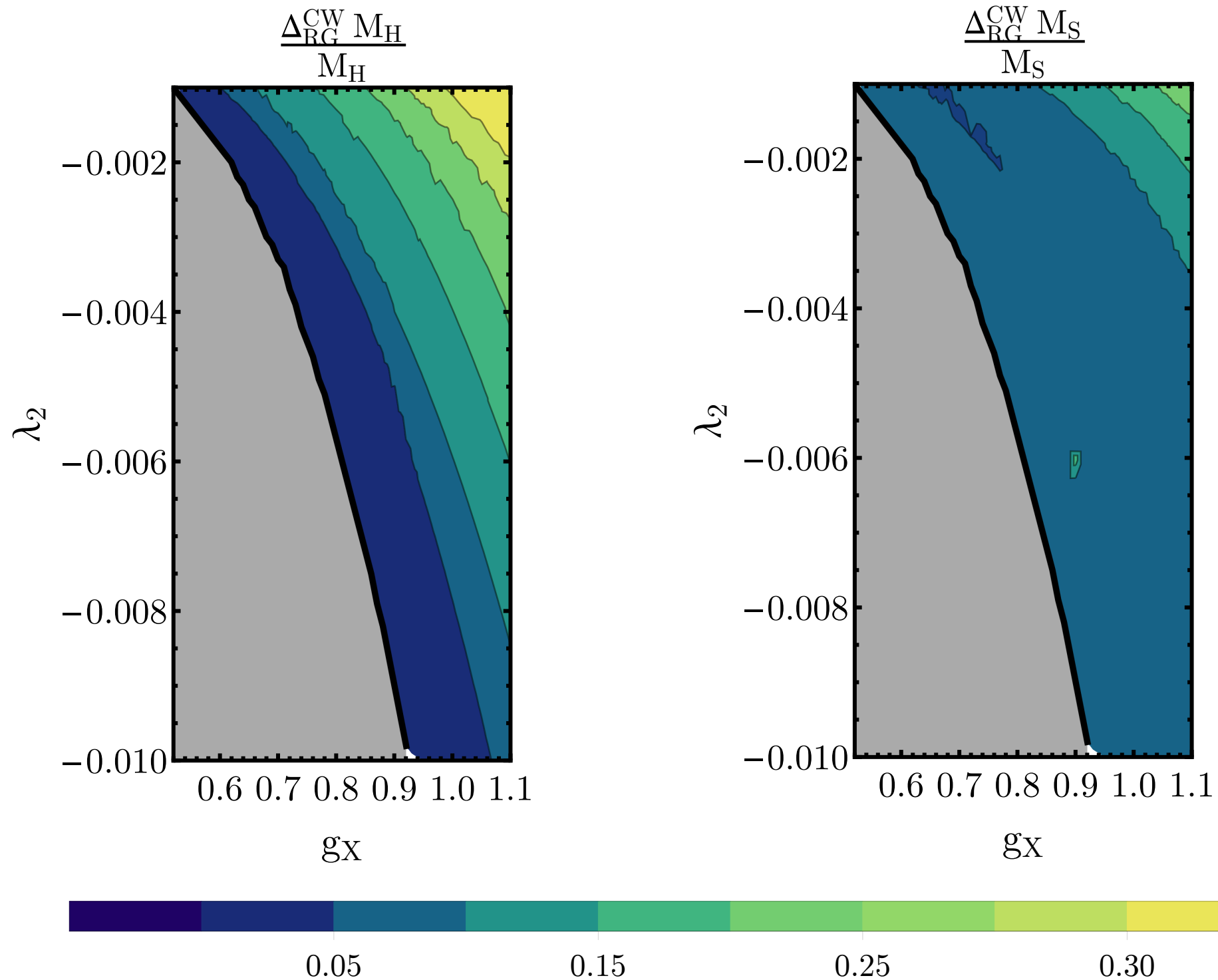
RADIATIVE SYMMETRY BREAKING IN SU(2)CSM

One-loop results



The VEV of the new scalar much greater than 246 GeV

RESULTS FROM RG IMPROVED POTENTIAL



SUMMARY

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

SUMMARY

No explicit large logs

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

SUMMARY

No explicit large logs

Form preserved at higher loops

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

SUMMARY

No explicit large logs

Form preserved at higher loops

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

Applicable to vacuum stability

SUMMARY

No explicit large logs

Form preserved at higher loops

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

Applicable to vacuum stability

Mitigates the running of the VEVs

SUMMARY

No explicit large logs

Form preserved at higher loops

$$V(\mu, \lambda, \phi) = V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$$

Applicable to vacuum stability

Mitigates the running of the VEVs

Implementable and can improve your results

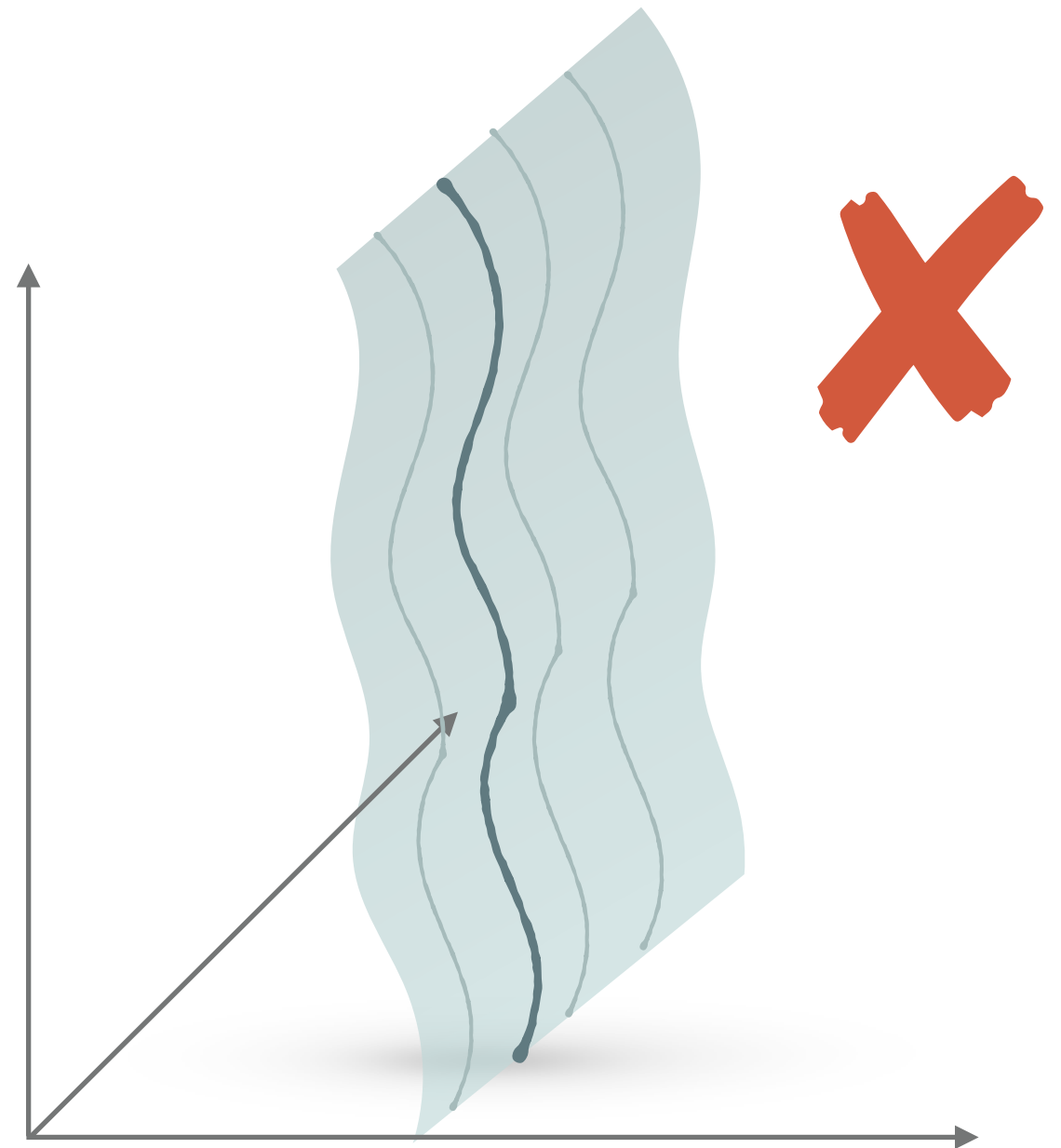
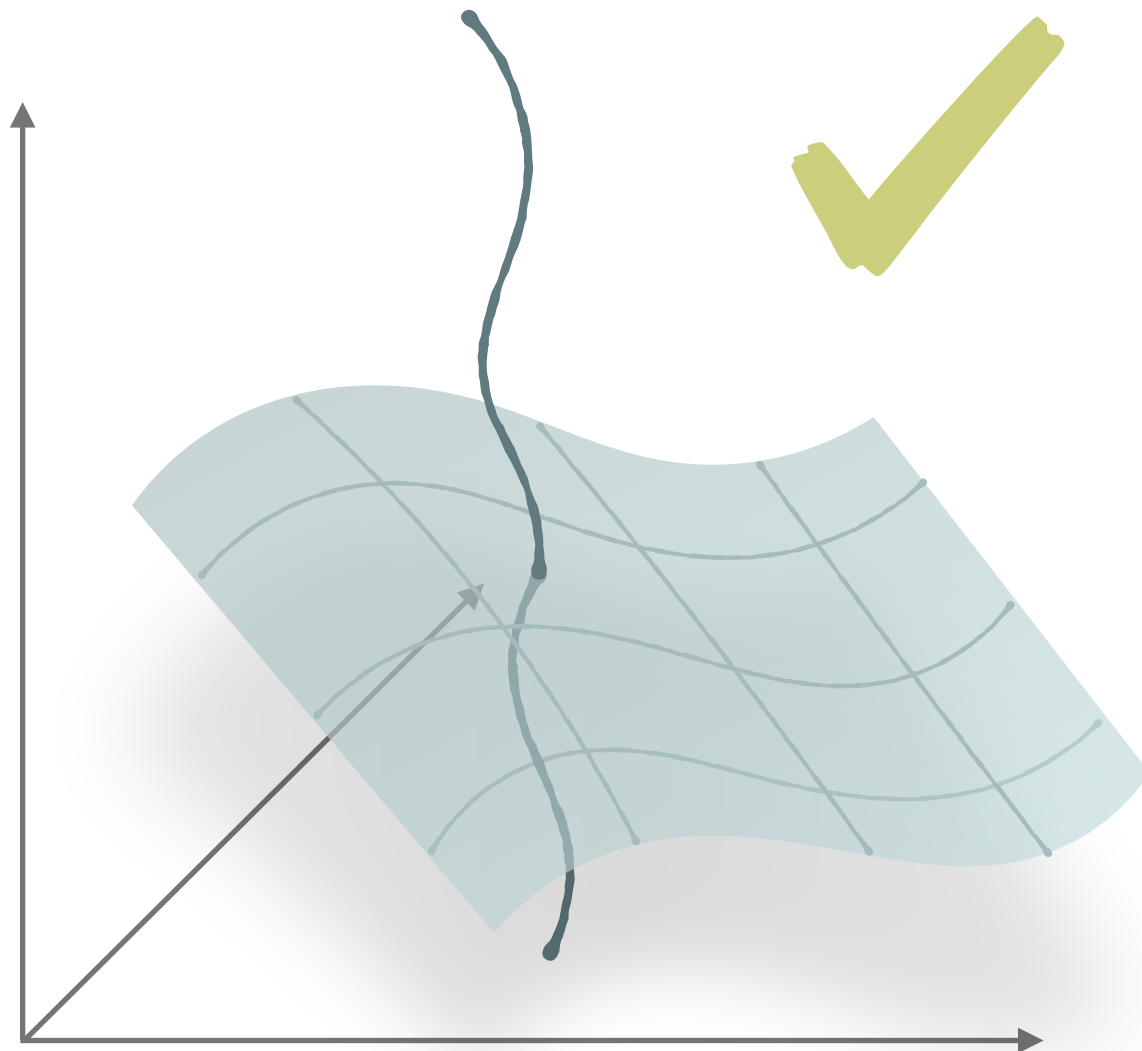
BACKUP SLIDES

MORE TECHNICAL



VALIDITY OF THE METHOD

Boundary surface must be non-characteristic



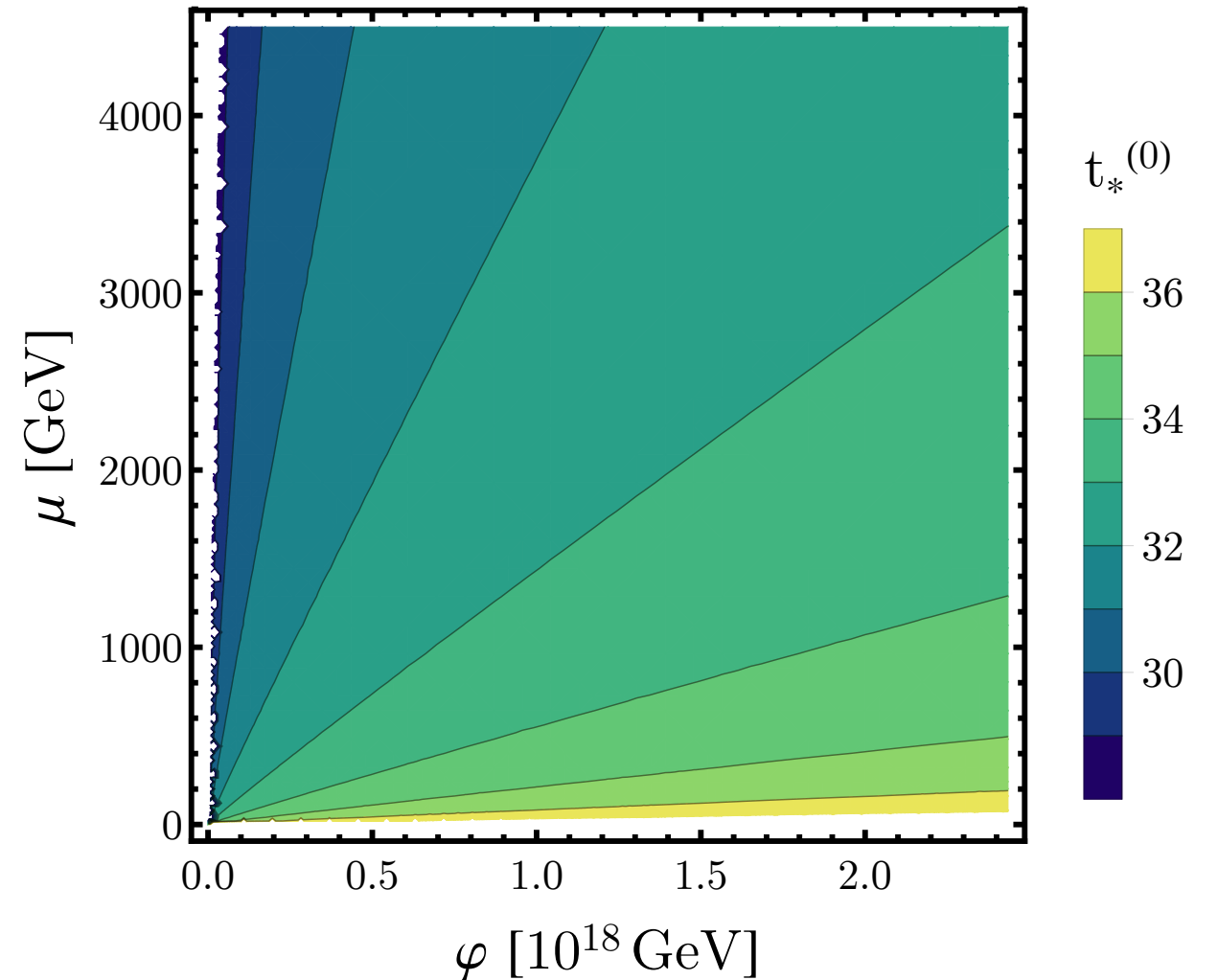
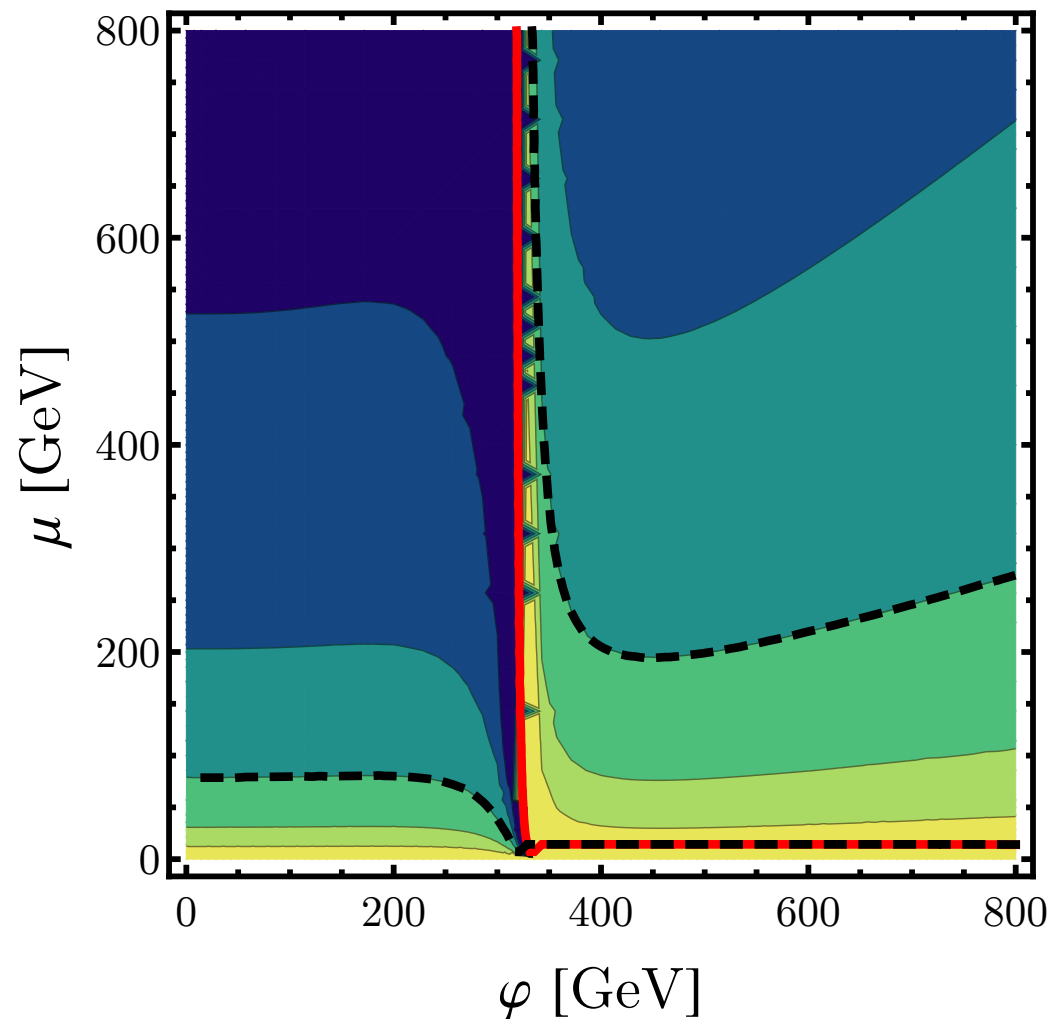
VALIDITY OF THE METHOD

Is there a unique solution for t_* ?

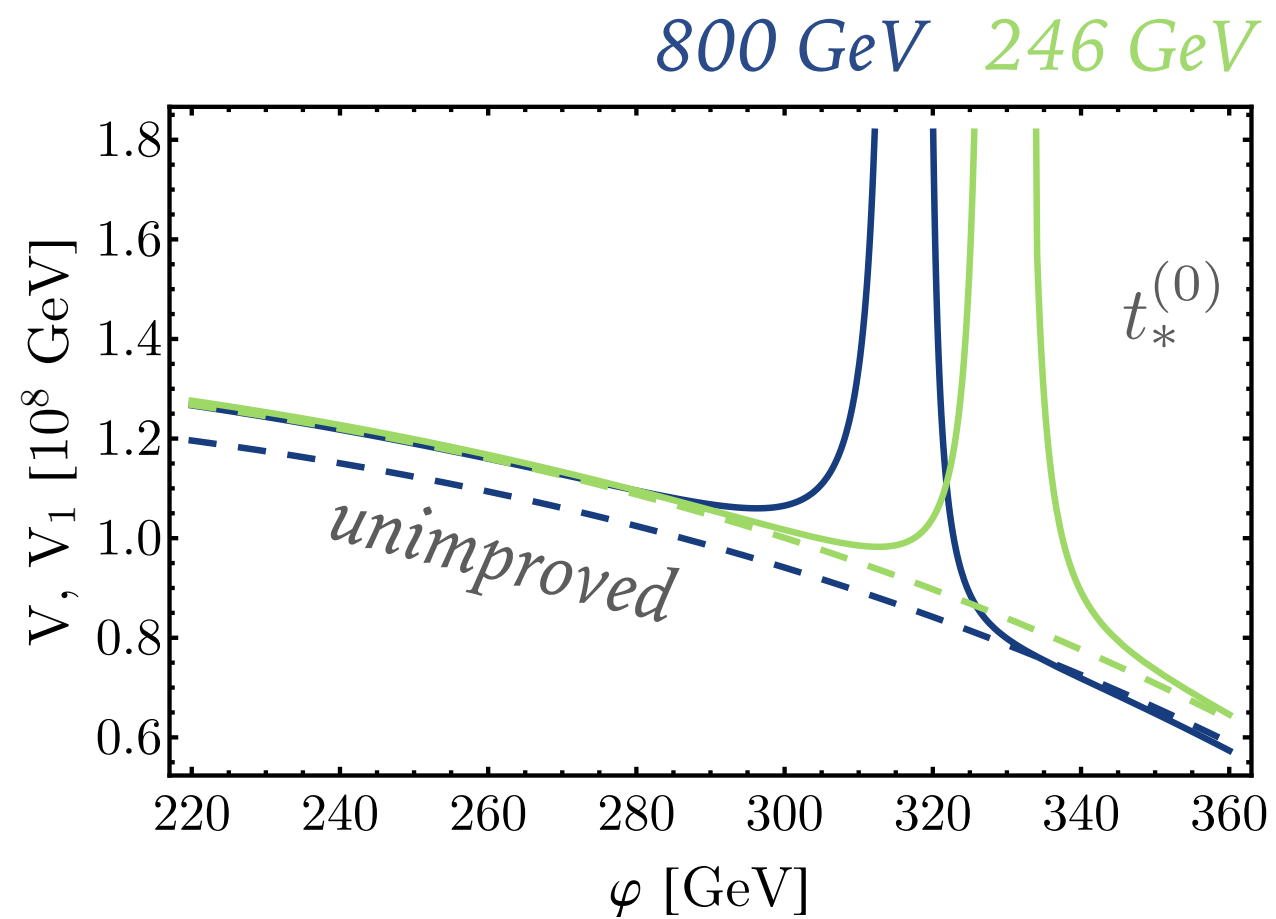
$$t_* = \frac{V^{(1)}(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*))}{2\mathbb{B}(\bar{\lambda}(t_*), \bar{\phi}(t_*))} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

VALIDITY OF THE METHOD

To first approximation $V^{(1)}(\mu; \lambda, \phi) = 0$ is characteristic when $\mathbb{B} = 0$
 $\mathbb{B} = 0$ is characteristic.

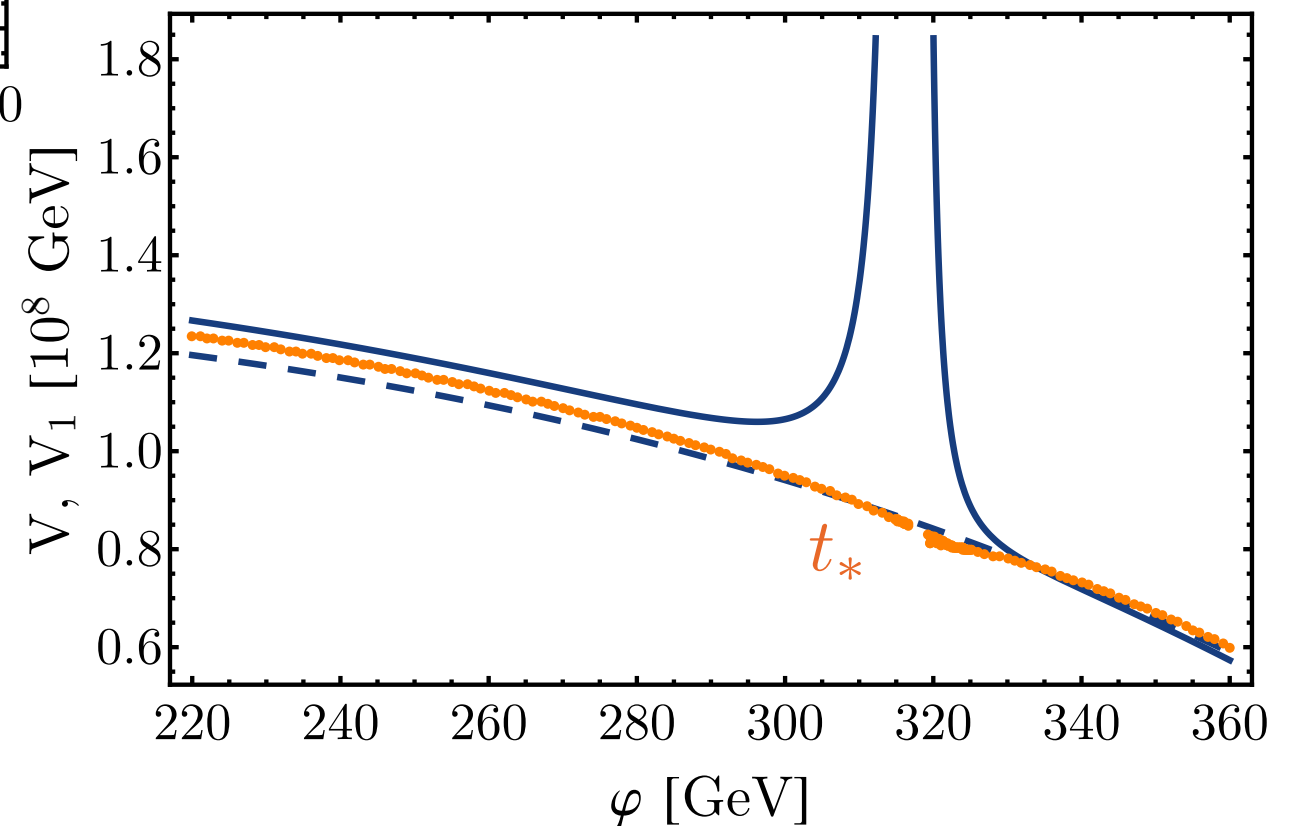
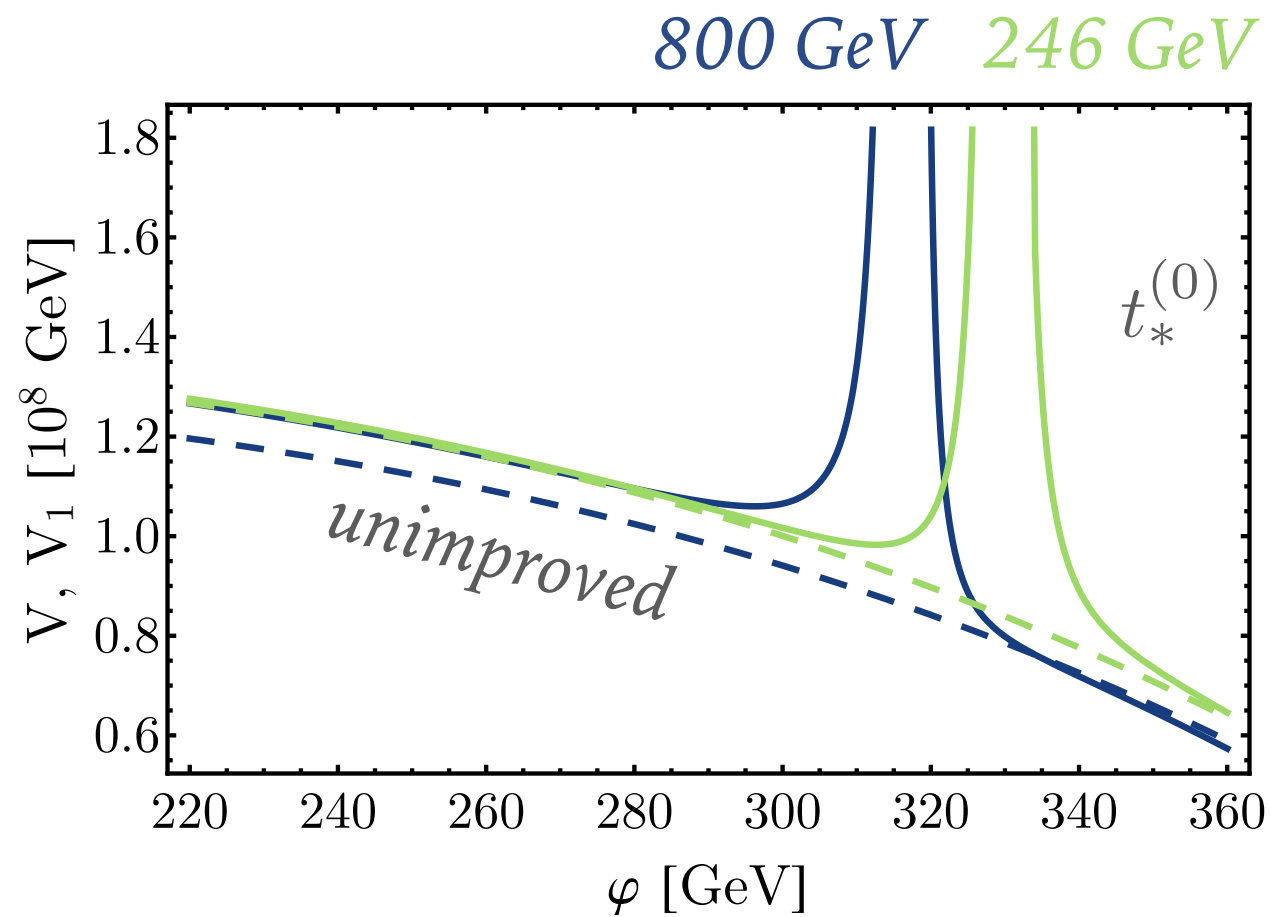


VALIDITY OF THE METHOD

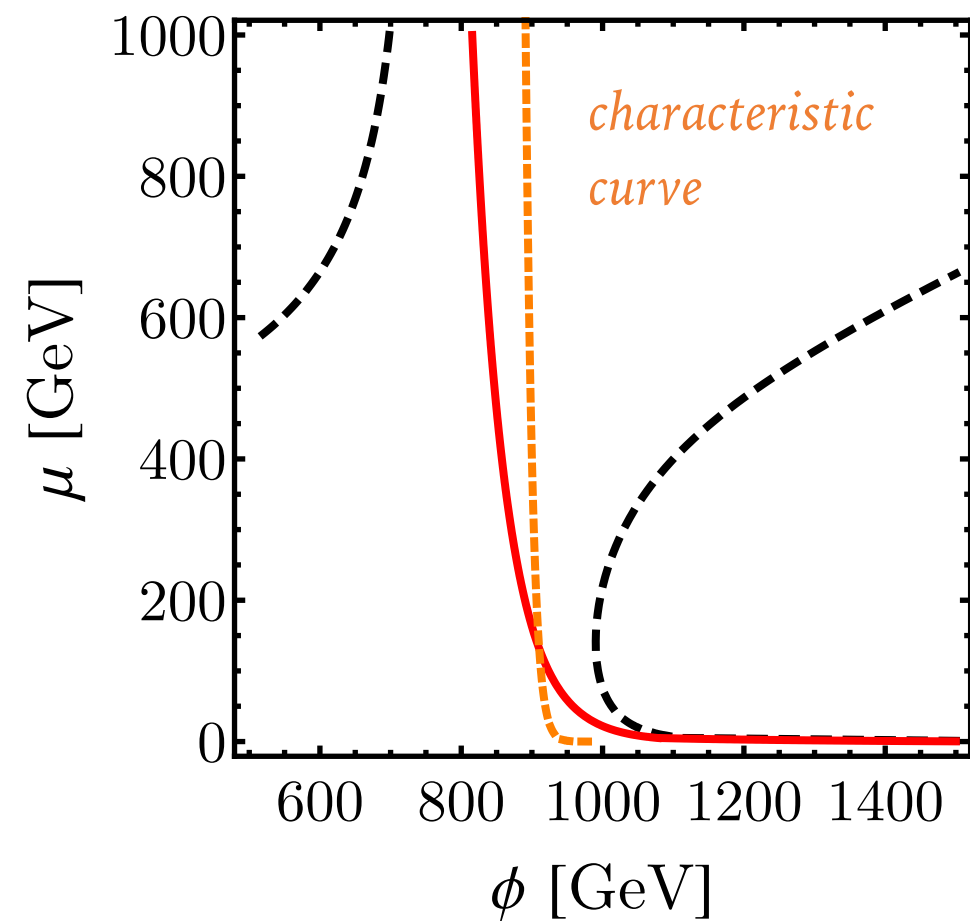
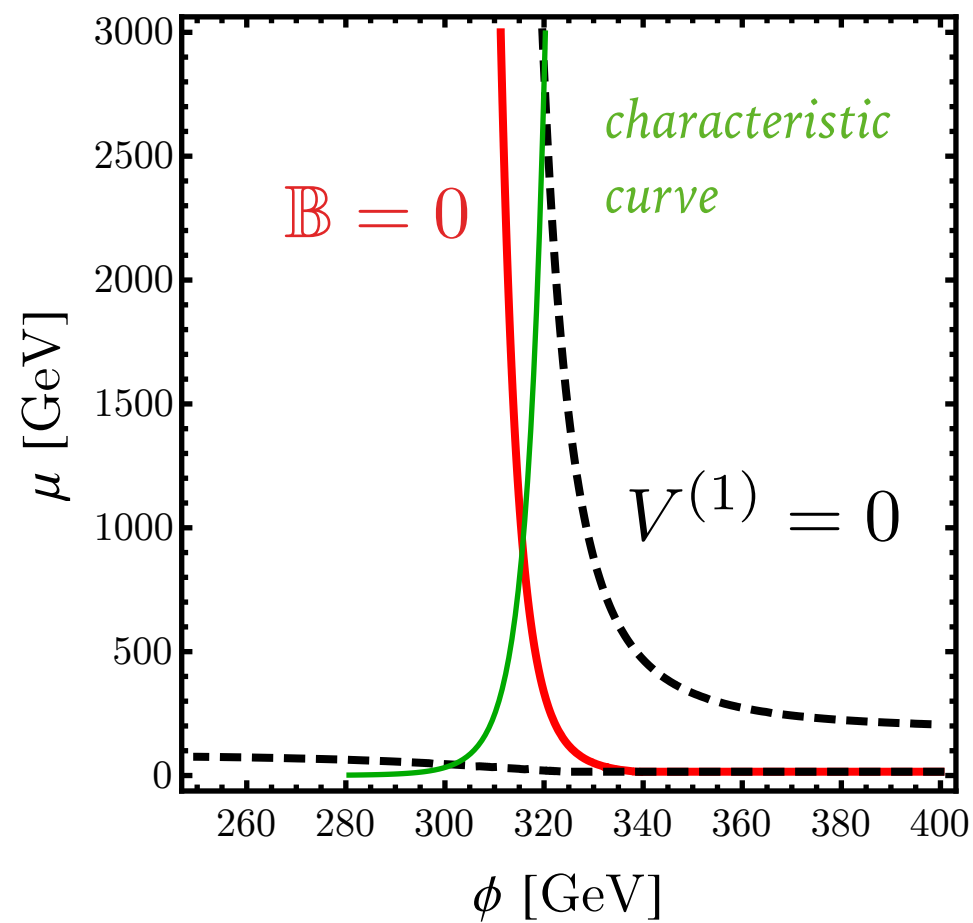


t_*

VALIDITY OF THE METHOD



VALIDITY OF THE METHOD



Higgs-Yukawa model

RESUMMATION

The leading logarithms are not resummed. And what is?

$V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$ resums powers of t_*

$$V(\mu; \lambda, \phi) = \frac{1}{4} \bar{\lambda}(t_*, \lambda) \phi^4 = \frac{1}{4} \phi^4 \sum_{n=0}^{\infty} \lambda^{n+1} \left[\frac{9\hbar}{16\pi^2} \log \frac{3\lambda\phi^2}{\mu^2} \right]^n + \dots$$

one-field case

$$t_* = \frac{V^{(1)}(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*))}{2\mathbb{B}(\bar{\lambda}(t_*), \bar{\phi}(t_*))} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

multi-field case

RESUMMATION

$$t_* = \frac{V^{(1)}(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*))}{2\mathbb{B}(\bar{\lambda}(t_*), \bar{\phi}(t_*))} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

RESUMMATION

$$t_* = \frac{V^{(1)}(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*))}{2\mathbb{B}(\bar{\lambda}(t_*), \bar{\phi}(t_*))} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

$$V^{(1)}(\mu, \lambda, \phi) = \mathbb{A} + \mathbb{B} \log \frac{\mathcal{M}^2}{\mu^2}$$

pivot log

μ -independent



RESUMMATION

$$t_* = \frac{V^{(1)}(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*))}{2\mathbb{B}(\bar{\lambda}(t_*), \bar{\phi}(t_*))} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

$$V^{(1)}(\mu, \lambda, \phi) = \mathbb{A} + \mathbb{B} \log \frac{\mathcal{M}^2}{\mu^2}$$


μ -independent

pivot log

$V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$ resums powers of the pivot log.

RESUMMATION

$$t_* = \frac{V^{(1)}(\mu, \bar{\lambda}(t_*), \bar{\phi}(t_*))}{2\mathbb{B}(\bar{\lambda}(t_*), \bar{\phi}(t_*))} \approx \frac{V^{(1)}(\mu; \lambda, \phi)}{2\mathbb{B}(\lambda, \phi)} + \mathcal{O}(\hbar)$$

$$V^{(1)}(\mu, \lambda, \phi) = \mathbb{A} + \mathbb{B} \log \frac{\mathcal{M}^2}{\mu^2}$$

pivot log

μ -independent

$V^{(0)}(\bar{\lambda}(t_*), \bar{\phi}(t_*))$ resums powers of the pivot log.

If $\left| \log \frac{\mathcal{M}^2}{\mu^2} \right| \gg \max_a \left\{ \left| \log \frac{m_a^2(\lambda, \phi)}{\mathcal{M}^2} \right| \right\}$ these are the dominant terms.

GENERALISATION TO HIGHER ORDERS

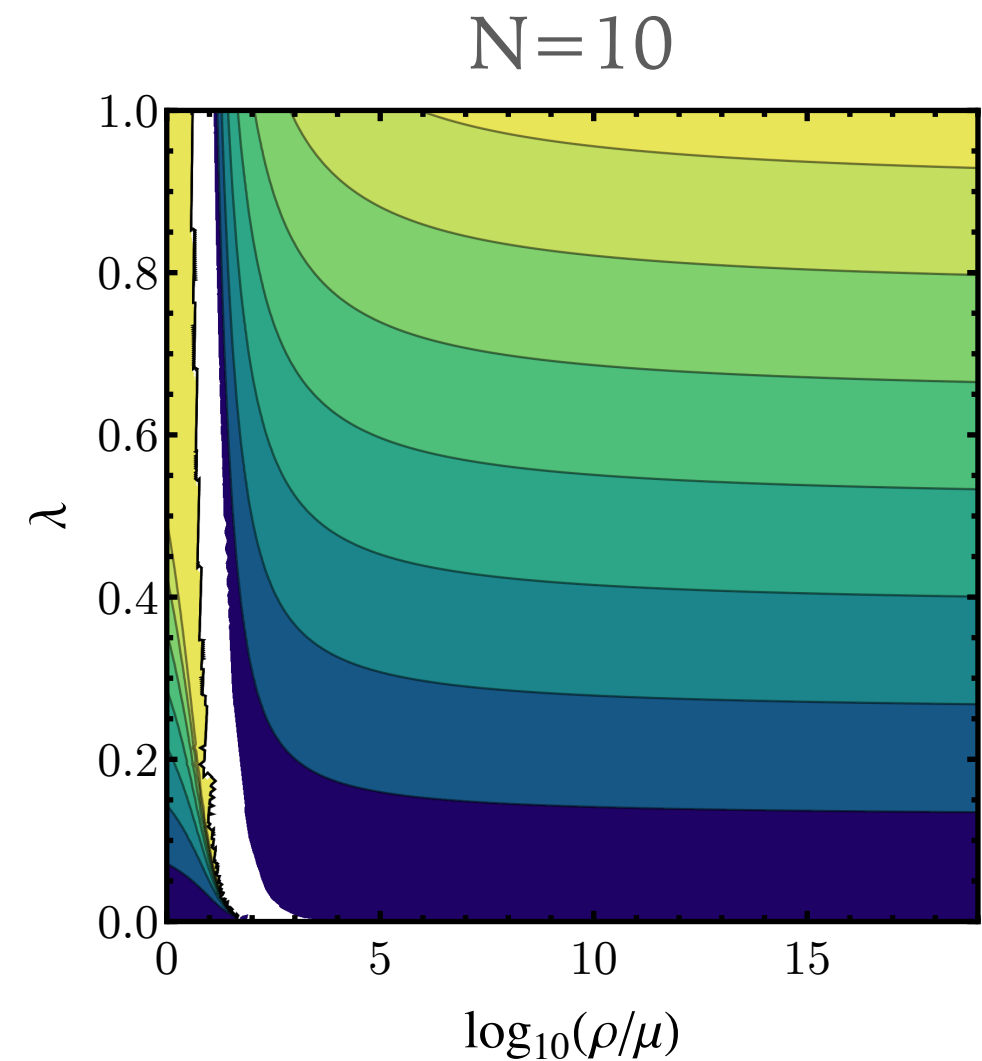
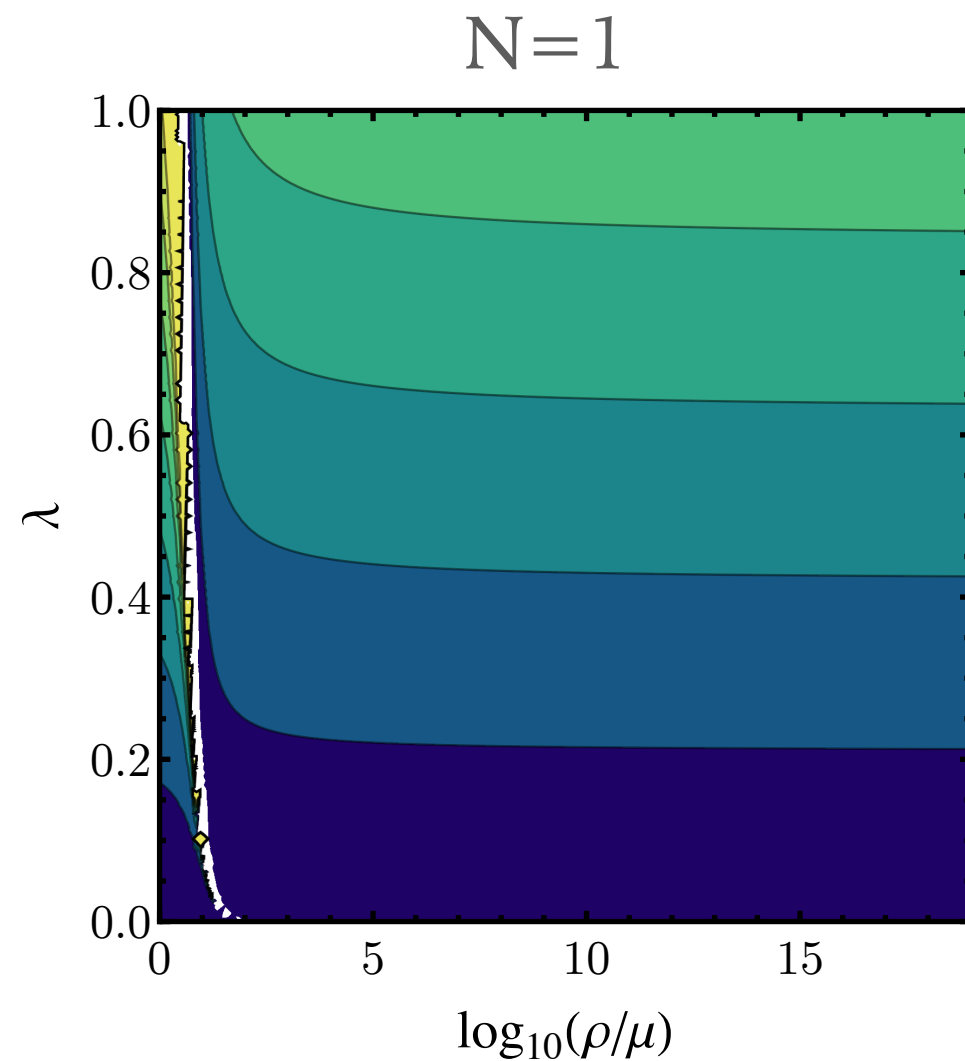
$$V(\mu; \lambda, \phi) = V^{(0)}(\lambda, \phi) + q(\mu, \lambda, \phi) = V^{(0)}(\lambda, \phi) + \sum_{l=1}^{\infty} \hbar^l V^{(l)}(\mu, \lambda, \phi)$$

$$q(\bar{\mu}(t_*), \bar{\lambda}(t_*), \bar{\phi}(t_*)) = 0$$

Solve for t_* perturbatively in $\hbar$

$$t_*^{(1)} = \frac{V^{(2)}(\mu, \lambda, \phi) - [d^{(2)} V^{(0)}]_{t=0} t_*^{(0)} - \frac{1}{2} \left[(d^{(1)})^2 V^{(0)} \right]_{t=0} \left(t_*^{(0)} \right)^2}{[d^{(1)} V^{(0)}]_{t=0}}$$

O(N)-SYMMETRIC MODEL AT TWO-LOOP LEVEL



$$t^{(1)}/t^{(0)}$$

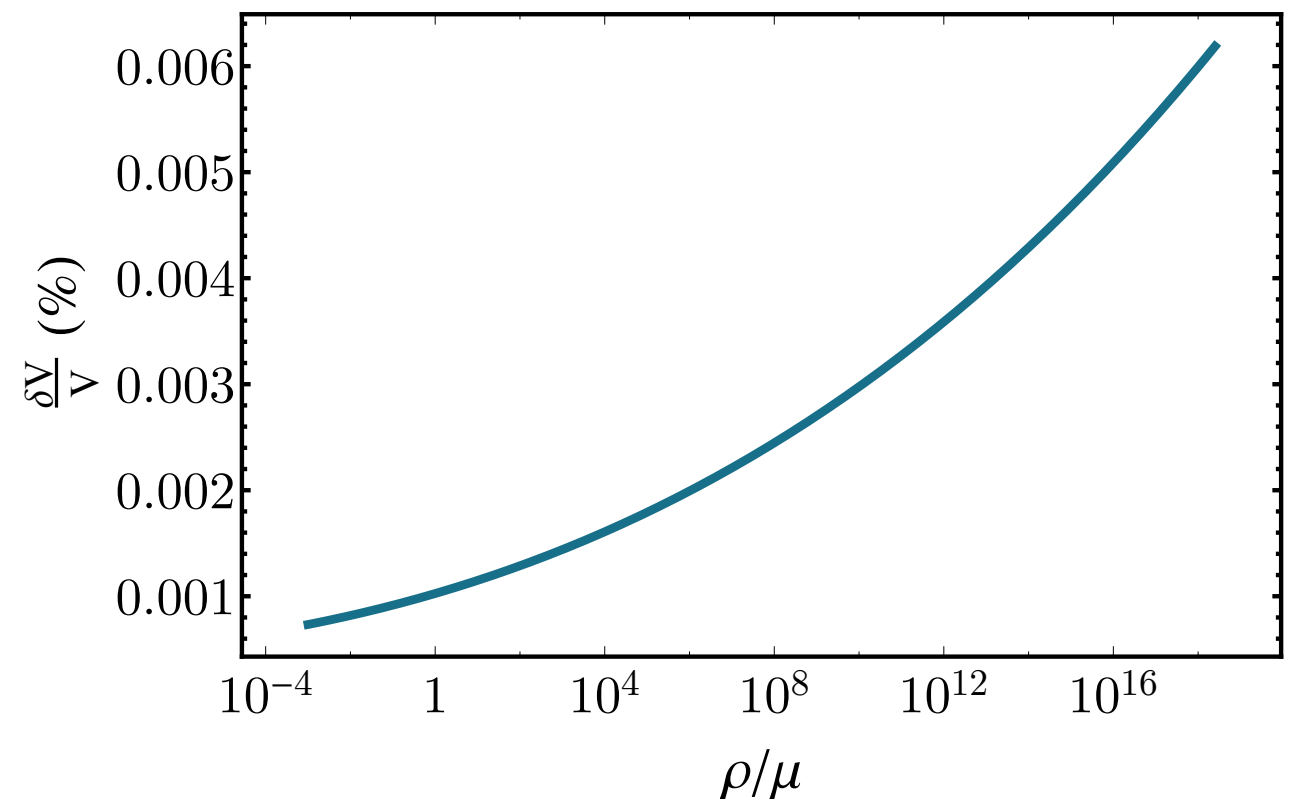
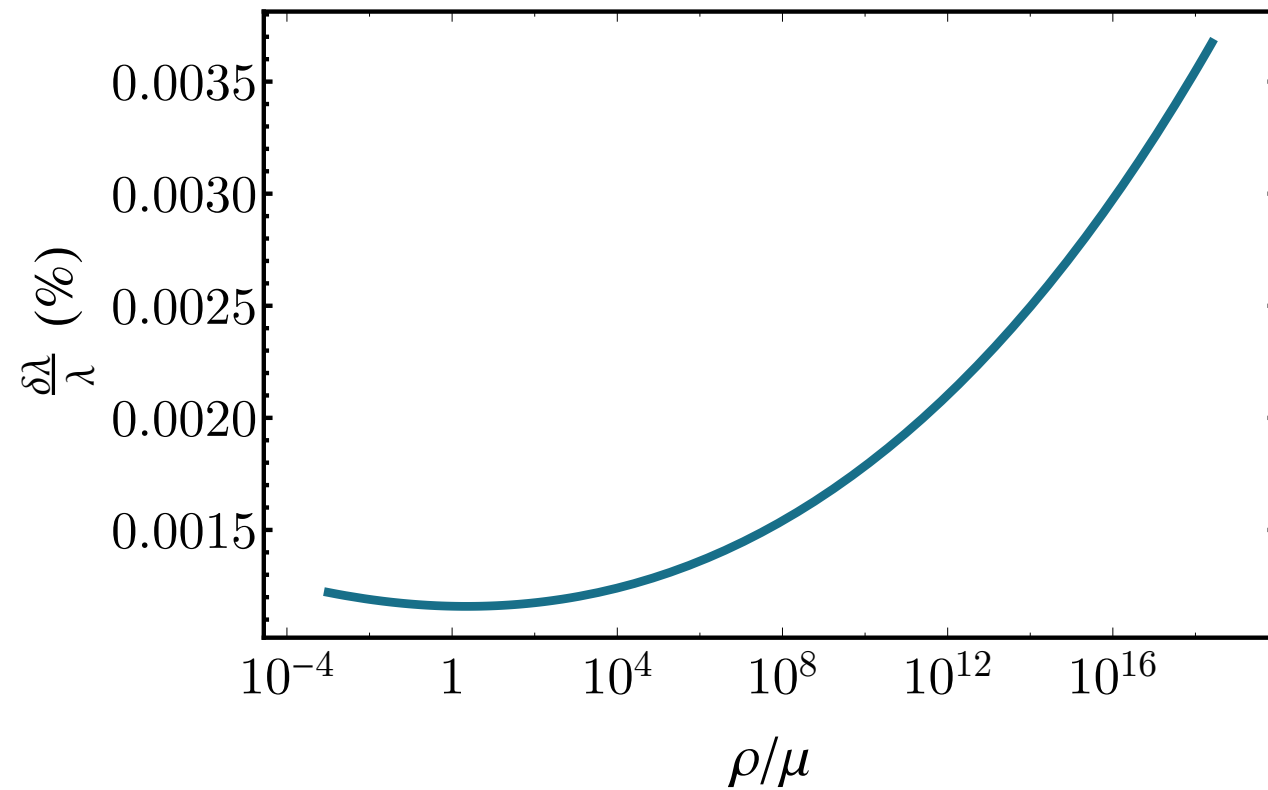
0.002

0.006

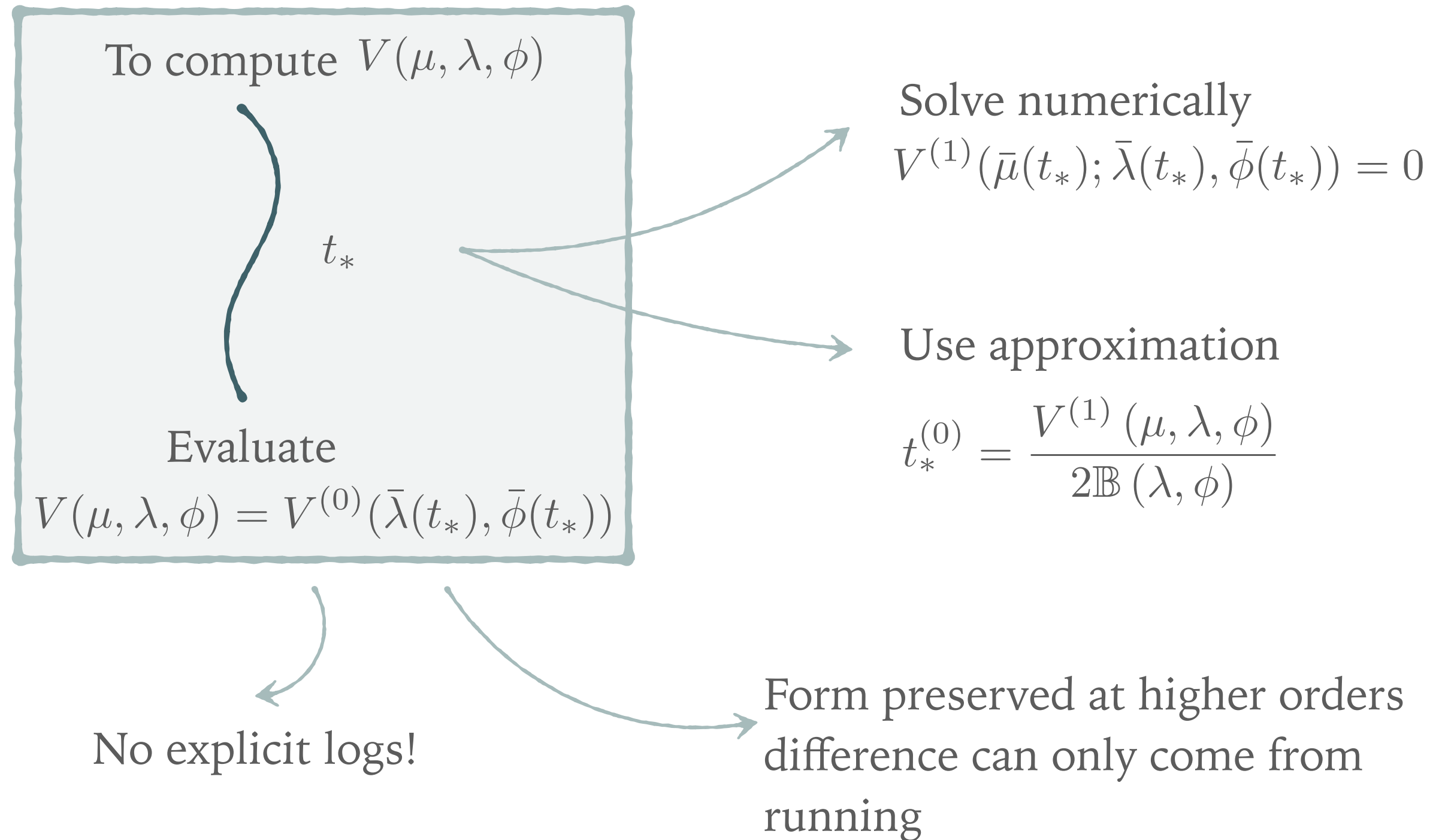
0.010

0.014

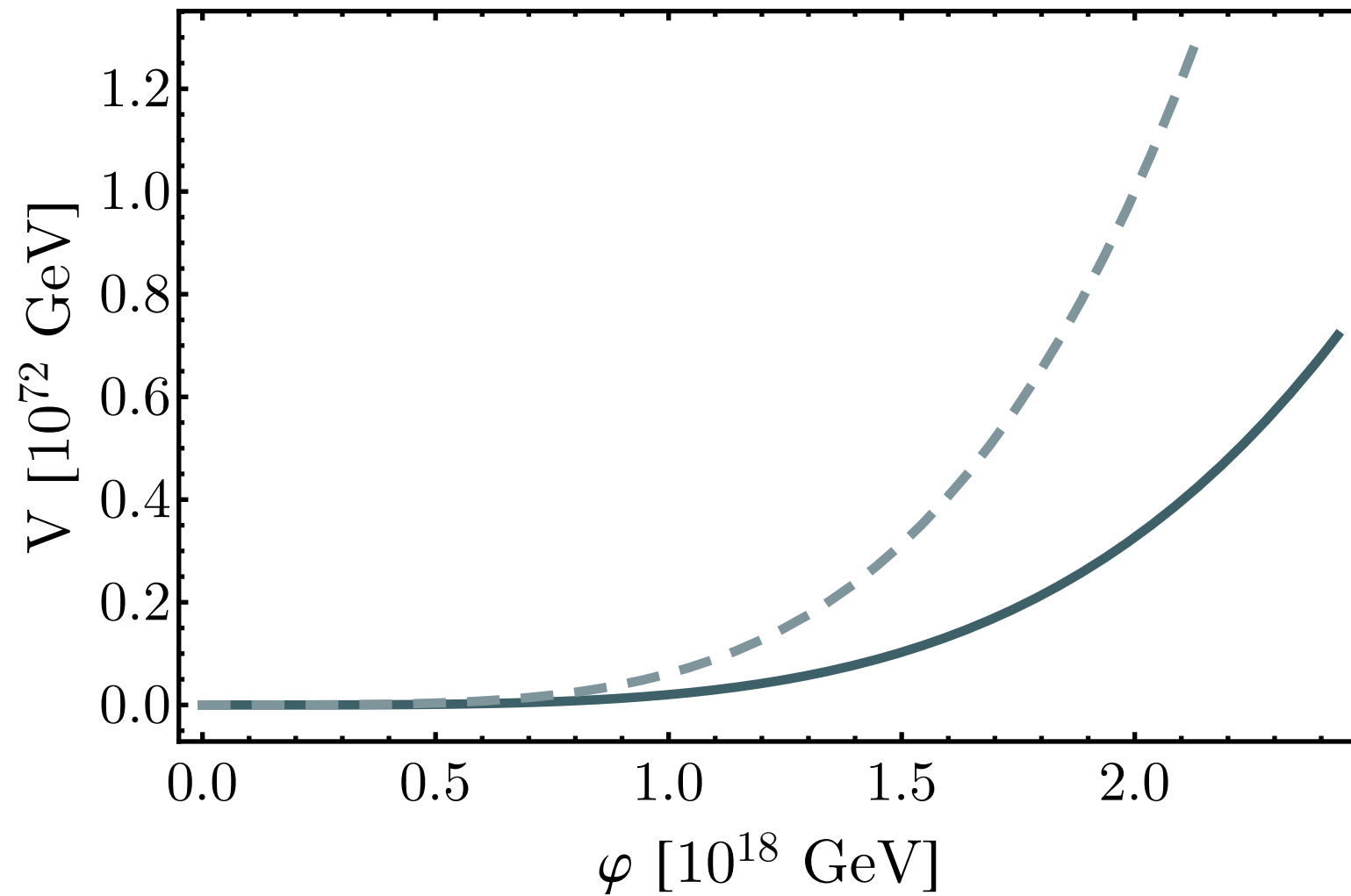
$O(N)$ -SYMMETRIC MODEL AT TWO-LOOP LEVEL



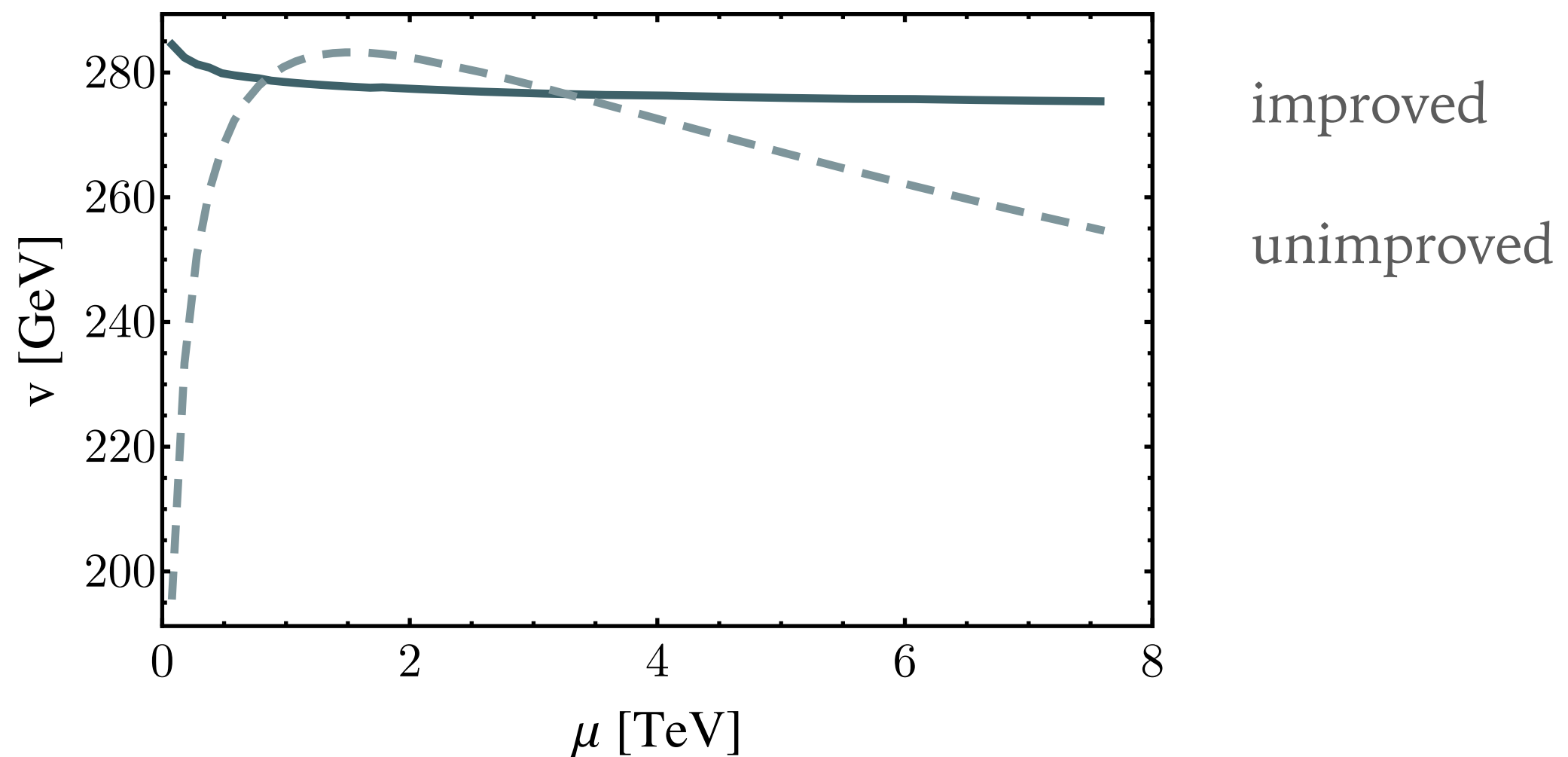
RECAP



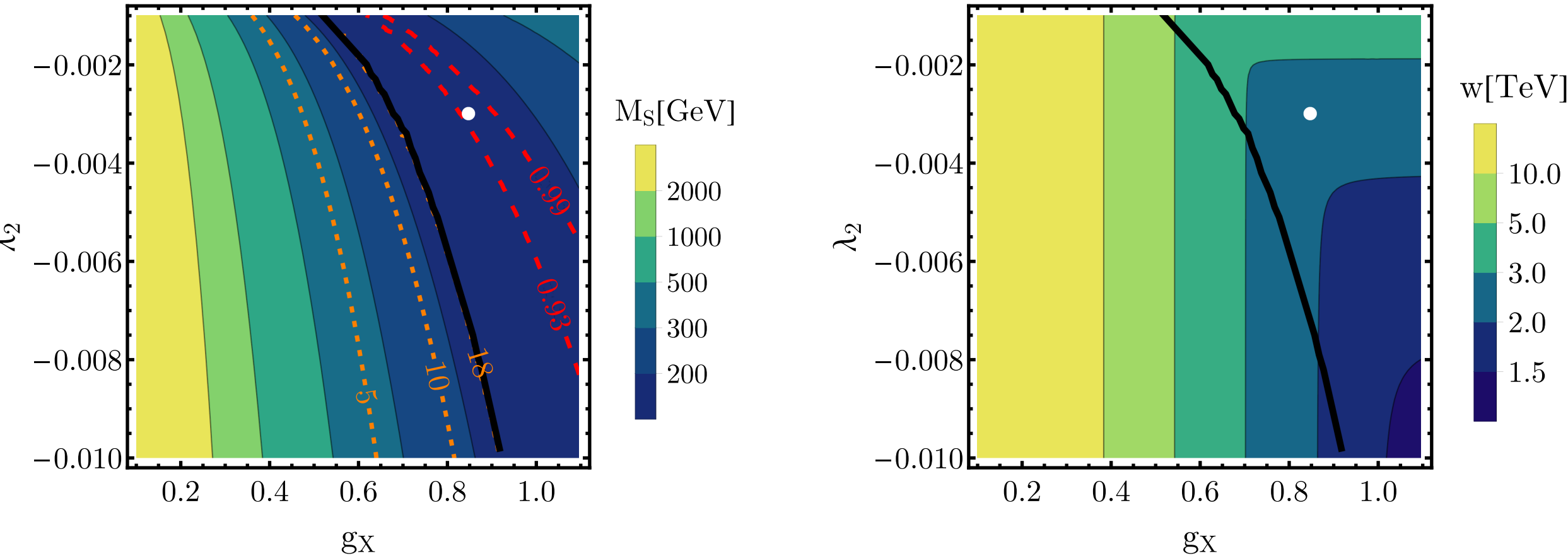
VACUUM STABILITY IN SU(2)CSM



RUNNING VEVs

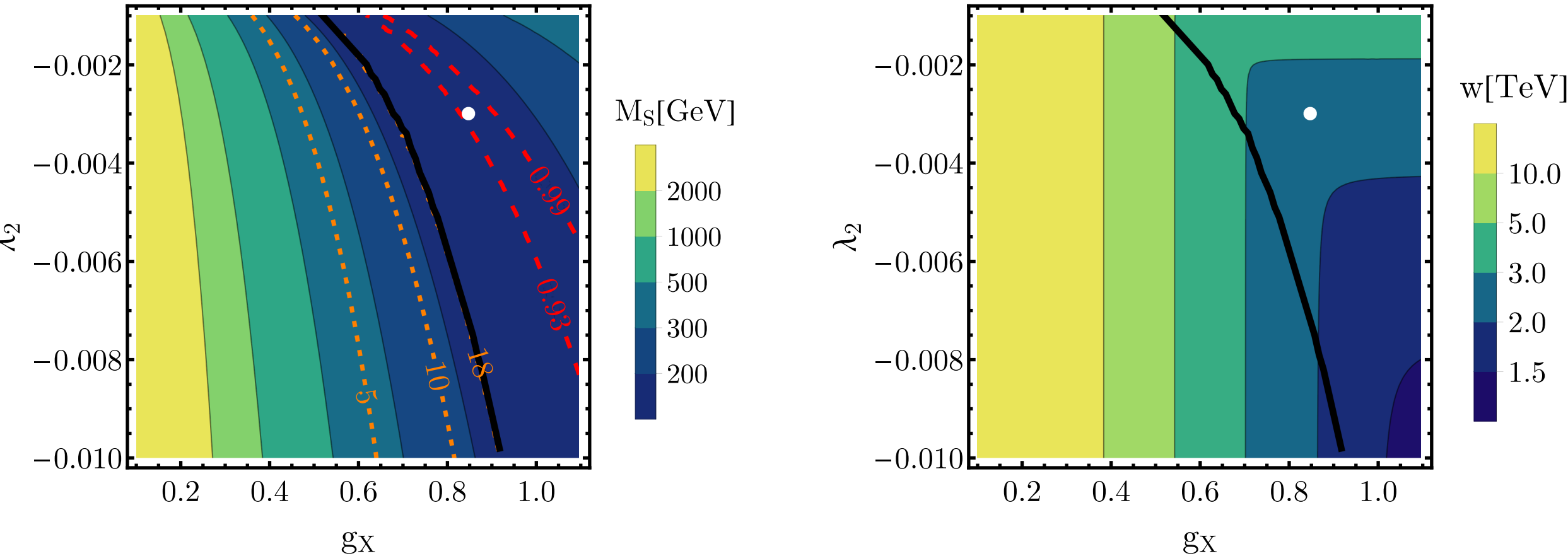


RADIATIVE SYMMETRY BREAKING IN SU(2)CSM



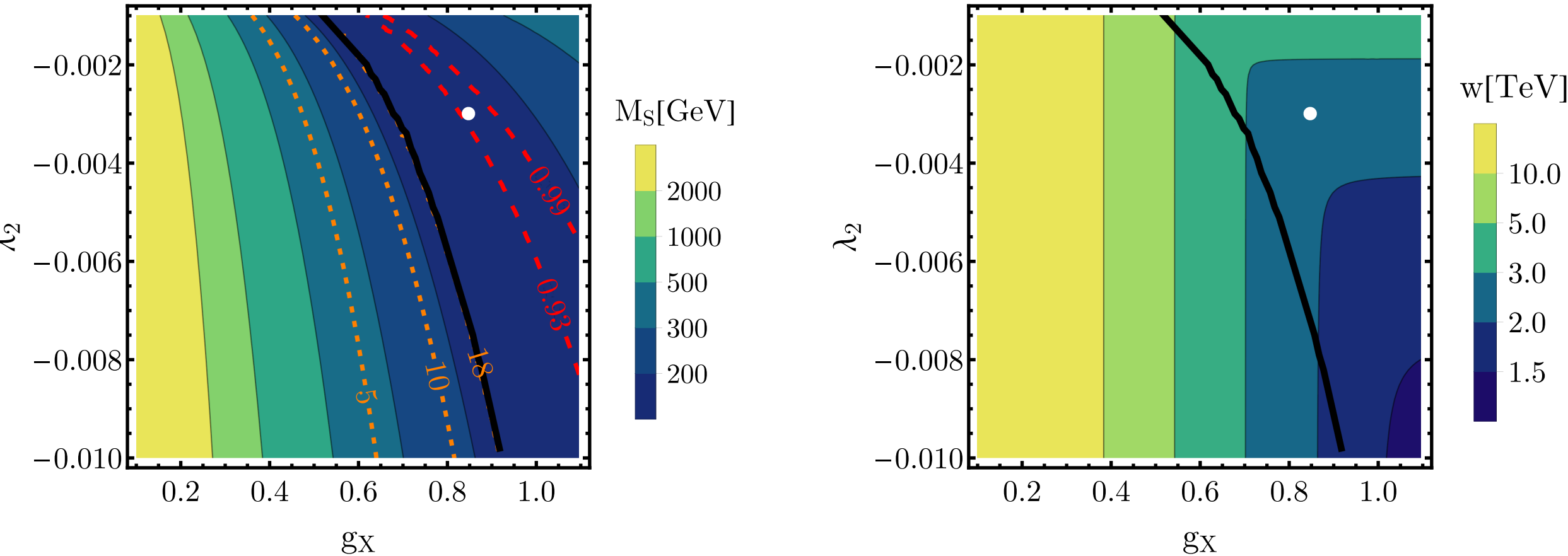
	μ [GeV]	λ_1	λ_2	λ_3	g_X	w [GeV]	$V_{\text{SM}}^{(1)}$ [GeV ⁴]	$V_X^{(1)}$ [GeV ⁴]	$V^{(1)}/V^{(0)}$
CW	246	0.1236	-0.0030	-0.0047	0.8500	2411	$2.38 \cdot 10^7$	$3.18 \cdot 10^{10}$	0.802
GW	940	0.1055	-0.0030	$2 \cdot 10^{-5}$	0.8141	2722	$6.28 \cdot 10^7$	$-1.08 \cdot 10^{10}$	551
RG	738	0.1085	-0.0030	-0.0007	0.8202	2698	$5.75 \cdot 10^7$	$-4.27 \cdot 10^7$	0.002

RADIATIVE SYMMETRY BREAKING IN SU(2)CSM



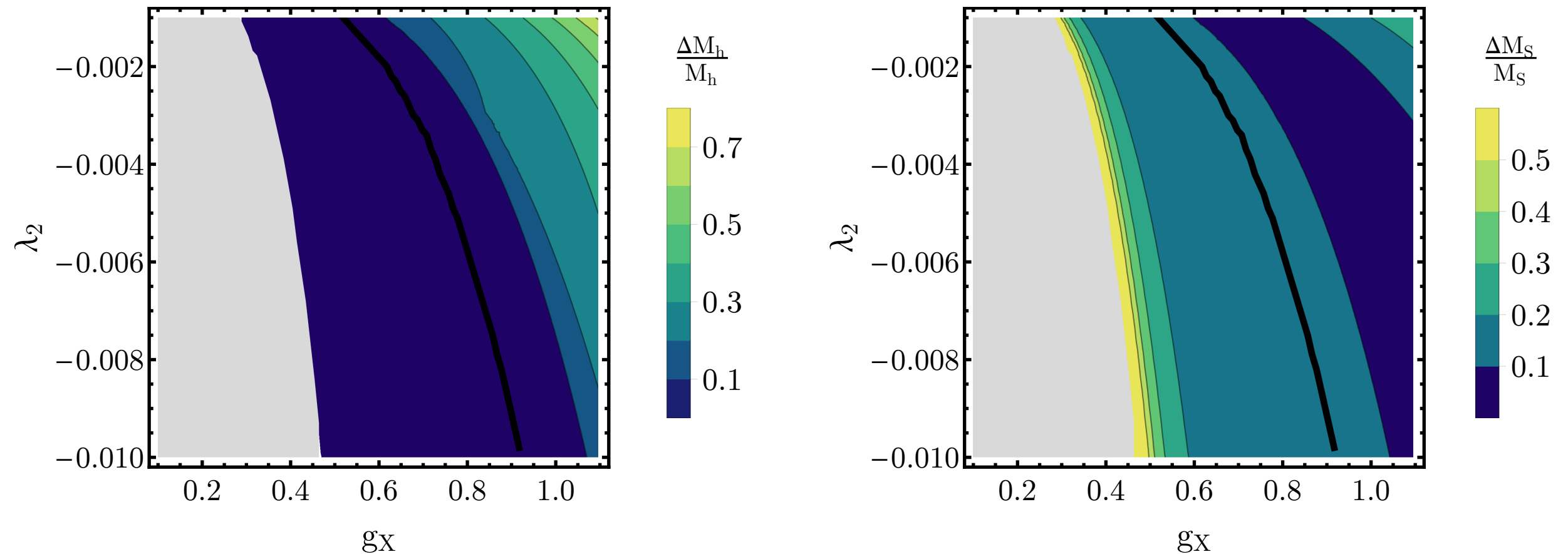
	μ [GeV]	λ_1	λ_2	λ_3	g_X	w [GeV]	$V_{\text{SM}}^{(1)}$ [GeV ⁴]	$V_X^{(1)}$ [GeV ⁴]	$V^{(1)}/V^{(0)}$
CW	246	0.1236	-0.0030	-0.0047	0.8500	2411	$2.38 \cdot 10^7$	$3.18 \cdot 10^{10}$	0.802
GW	940	0.1055	-0.0030	$2 \cdot 10^{-5}$	0.8141	2722	$6.28 \cdot 10^7$	$-1.08 \cdot 10^{10}$	551
RG	738	0.1085	-0.0030	-0.0007	0.8202	2698	$5.75 \cdot 10^7$	$-4.27 \cdot 10^7$	0.002

RADIATIVE SYMMETRY BREAKING IN SU(2)CSM



	μ [GeV]	λ_1	λ_2	λ_3	g_X	w [GeV]	$V_{\text{SM}}^{(1)}$ [GeV ⁴]	$V_X^{(1)}$ [GeV ⁴]	$V^{(1)}/V^{(0)}$
CW	246	0.1236	-0.0030	-0.0047	0.8500	2411	$2.38 \cdot 10^7$	$3.18 \cdot 10^{10}$	0.802
GW	940	0.1055	-0.0030	$2 \cdot 10^{-5}$	0.8141	2722	$6.28 \cdot 10^7$	$-1.08 \cdot 10^{10}$	551
RG	738	0.1085	-0.0030	-0.0007	0.8202	2698	$5.75 \cdot 10^7$	$-4.27 \cdot 10^7$	0.002

RUNNING MASSES



Change induced (mainly) by
 $\mu = 246 \text{ GeV} \rightarrow \mu = 940 \text{ GeV}$