Vacuum Stability.

Constraining BSM Scalar Sectors

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Planck 2018: From the Planck Scale to the Electroweak Scale

24.05.2018

Vacuum Stability in Scalar Potentials

The most general renormalizable scalar potential at tree-level is

$$V(\phi^a) = \lambda_{abcd}\phi^a\phi^b\phi^c\phi^d + A_{abc}\phi^a\phi^b\phi^c + m^2_{ab}\phi^a\phi^b + t_a\phi^a + c\,.$$

V can have many local minima which might be deeper than the EW vacuum. Vacuum stability has to differentiate between:

- absolute stability $\leftrightarrow$ EW vacuum is the global minimum.
- metastability $\leftrightarrow$ Tunneling into any deeper region is long-lived.
- instability $\leftrightarrow$ Tunneling is faster than the age of the universe.

Directions in Fieldspace

$$V(\phi^a) = \lambda_{abcd}\phi^a\phi^b\phi^c\phi^d + A_{abc}\phi^a\phi^b\phi^c + m^2_{ab}\phi^a\phi^b + t_a\phi^a + c\,.$$

Expand around EW vacuum $\vec{\phi} \to \vec{v} + \vec{\varphi}$:

$$V(\varphi^a) = \lambda(\vec{v})_{abcd}\varphi^a\varphi^b\varphi^c\varphi^d + A(\vec{v})_{abc}\varphi^a\varphi^b\varphi^c + m^2(\vec{v})_{ab}\varphi^a\varphi^b\,.$$

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Introduce polar coordinates $\vec{\varphi} \to \varphi\hat{\varphi}$:

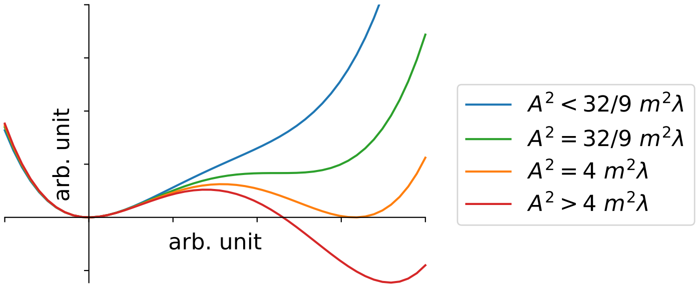
$$V(\varphi) = \lambda(\hat{\varphi})\varphi^4 - A(\hat{\varphi})\varphi^3 + m^2(\hat{\varphi})\varphi^2\,.$$

> $\lambda > 0$ for physical potentials (bounded from below)

> $A > 0$ by choice ($\varphi \leftrightarrow -\varphi$)

> $m^2 > 0$ if the EW-vacuum is a local minimum

Stability of Fieldspace Directions



- at most one additional minimum for each $\hat{\varphi}$
- the additional minimum is deeper if $A(\hat{\varphi})^2 > 4m^2(\hat{\varphi})\lambda(\hat{\varphi})$

Finding Deep Directions in Fieldspace

1. Solve $\vec{\nabla}_\phi V = 0$ to find all stationary points using polynomial homotopy continuation (PHC).
2. Compare the potential values at each stationary point to the value at the EW vacuum.
3. Get $\lambda(\hat{\varphi})$, $A(\hat{\varphi})$ and $m^2(\hat{\varphi})$ for $\hat{\varphi}$ pointing towards each deeper stationary point.

PHC in theory always finds all solutions. In practice it requires the system to be well conditioned to reduce coefficient variability.

Tunneling Times

The decay width per volume for vacuum tunneling is given by

$$\frac{\Gamma}{V} = K e^{-B} \,,$$

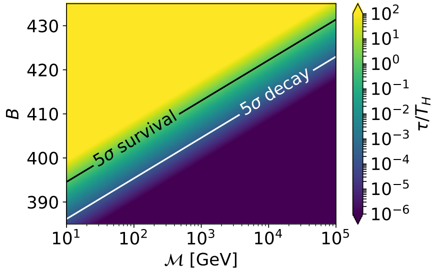
with the bounce action B and a prefactor K with $[K] = \text{GeV}^4$.
For a fixed $\hat{\varphi}$, B is given semi-analytically by [hep-ph/9302321, Adams]

$$B = \frac{\pi^2}{3\lambda}(2-\delta)^{-3}\left(13.8\delta - 10.8\delta^2 + 2.08\delta^3\right)$$

where $\delta = 8\lambda m^2/A^2$.

Estimation of the Bounce Scale

$$K \sim \mathcal{M}^4 \;\Rightarrow \frac{\tau_{\text{decay}}}{T_H} = \frac{1}{T_H^4 \mathcal{M}^4} e^B$$



- survival probability

$$P = e^{-T_H/\tau_{\text{decay}}}$$

- $\mathcal{M} \in [1\,\text{GeV}, 100\,\text{TeV}]$
  $\Leftrightarrow B \in [380, 430] \sim 10\%$

Use $380 < B < 430$ as uncertainty for the stability border.

Precision vs Speed

Vacuum stability as a constraint in BSM parameter scans with $\mathcal{O}(10^{6-7})$ points.

Approximate the tunneling path by a straight line.

- $\mathcal{O}(10\%)$ effect on B. [Masoumi, Olum, Wachter; 1702.00356]
  - Comparable to the uncertainty from unknown K.

Use the tree-level scalar potential.

- The loop-corrected effective potential is not in general a consistent perturbative expansion for B.
  [Andreassen, Farhi, Frost, Schwartz; 1604.06090]
- PHC and the analytic formula for B only work for polynomials.

The scalar sector of the MSSM

In SUSY theories every SM fermion gains a scalar superpartner.
In the MSSM the scalar potential including only the real, neutral Higgs and real $\widetilde{t}_r,\ \widetilde{t}_l$ fields reads

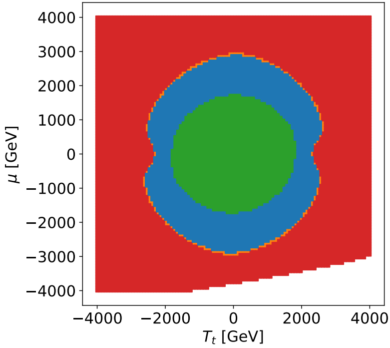
$$V((h_u)^{0,r},(h_d)^{0,r},(\widetilde{t}_r)^r,(\widetilde{t}_l)^r) =$$

$$\frac{g_1^2}{288}\left(3(h_u^2-h_d^2)+\widetilde{t}_l^{\,2}-4\widetilde{t}_r^{\,2}\right)^2+\frac{g_2^2}{32}\left(h_d^2-h_u^2+\widetilde{t}_l^{\,2}\right)^2+\frac{g_3^2}{24}\left(\widetilde{t}_l^{\,2}-\widetilde{t}_r^{\,2}\right)^2$$

$$+\frac{y_t}{4}\left(h_u^2(\widetilde{t}_l^{\,2}+\widetilde{t}_r^{\,2})+\widetilde{t}_l^{\,2}\widetilde{t}_r^{\,2}\right)+\frac{1}{\sqrt{2}}(\mathbf{T_t}h_u-\mu y_t h_d)\widetilde{t}_l\widetilde{t}_r$$

$$+\frac{g_1^2+g_2^2}{16}\left(v_0^2 c_{2\beta}(h_u^2-h_d^2)\right)+\frac{m_A^2}{2}\left(c_\beta h_u-s_\beta h_d\right)^2+\frac{m_{Q_3}^2}{2}\widetilde{t}_l^{\,2}+\frac{m_{U_3}^2}{2}\widetilde{t}_r^{\,2}$$

$\tilde{t}$-Instabilities



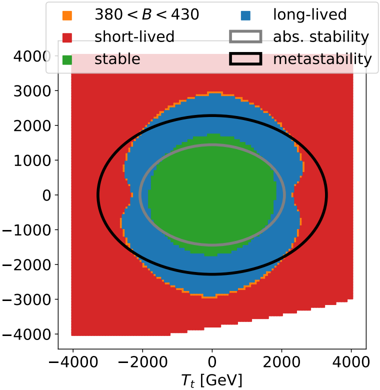
Compare the lifetime τ to the age of the universe T_H:

- stable, EW vacuum is the global minimum
- long-lived
- $380 < B < 430$
- short-lived
- tachyonic masses

Parameters:

$\tan\beta = 20, \quad m_A = 800\,\text{GeV}, \quad T_b = T_\tau = 500\,\text{GeV}, \quad M_{1,2,3} = m_{Q,U,D,L,E} = 1000\,\text{GeV}$

Comparison with Semianalytic studies



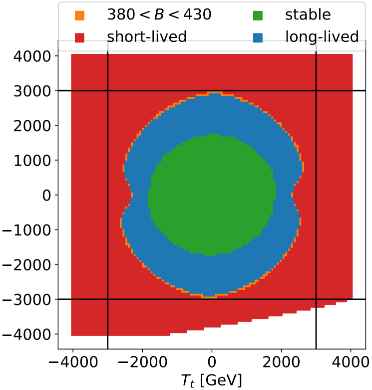
Classical MSSM stability bounds from studying specific (*D-flat*) directions

[Kusenko, Langacker, Segre; hep-ph/9602414]

$$\frac{T_t^2}{Y_t^2} + 3\mu^2 < \left.\begin{matrix} 3 \\ 7.5 \end{matrix}\right\} (m_{\tilde{t}_r}^2 + m_{\tilde{t}_l}^2)$$

for absolute stability and metastability, respectively.

Comparison with Semianalytic studies

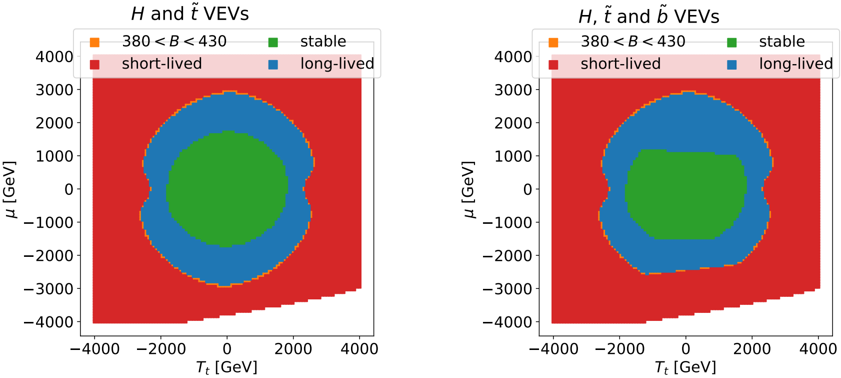


Rule of thumb:

$$\left|\frac{\mu}{M}\right|,\ \left|\frac{T}{M}\right| \lesssim 3$$

$\Rightarrow$ Works pretty well for this plane.

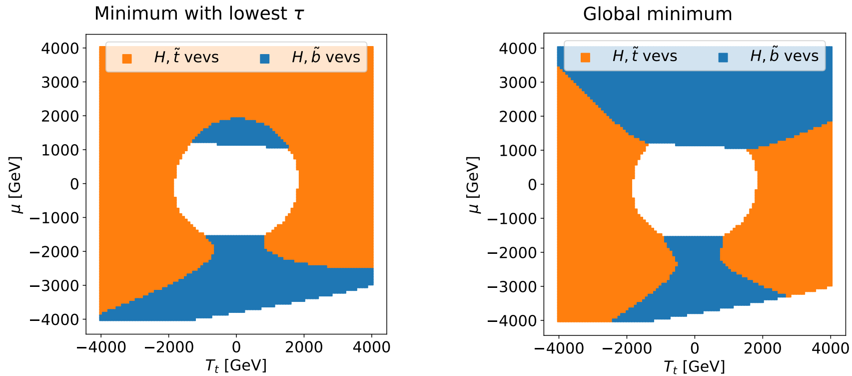
Effects of $\tilde{b}$ VEVs



- $\tilde{t}$ VEVs do not care for the sign of μ or T_t
- Δ_b corrections for $\mu \ll 0$ lead to instabilities in $\tilde{b}$ directions.

The Directions of Instability

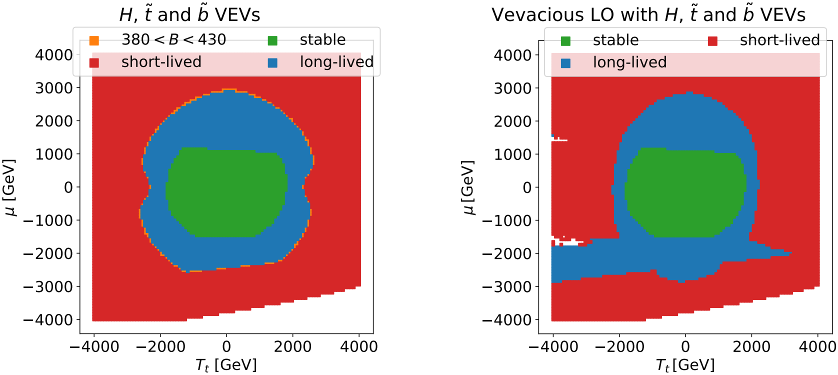
Which fields obtain a VEV at the deeper minimum?



> In general the fastest tunneling does *not* happen in the direction of the global minimum.

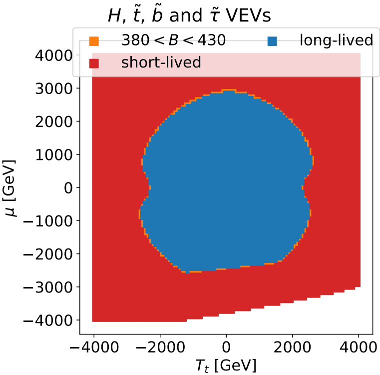
Comparison with the public code Vevacious

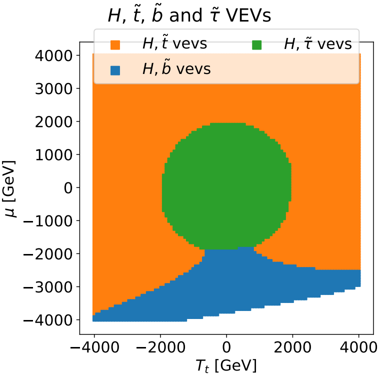
[Camargo-Molina, O'Leary, Porod, Staub; 1307.1477]



- Vevacious misses a minimum for $-1.5\,\text{TeV} < \mu < 3\,\text{Tev}$.
- Slight impact of the more sophisticated tunneling path calculation in Vevacious for $|T_t| \sim 2\,\text{TeV}$.

Stability including all squarks and sleptons





> No more absolute stability in this plane, but no change to the metastability constraints.

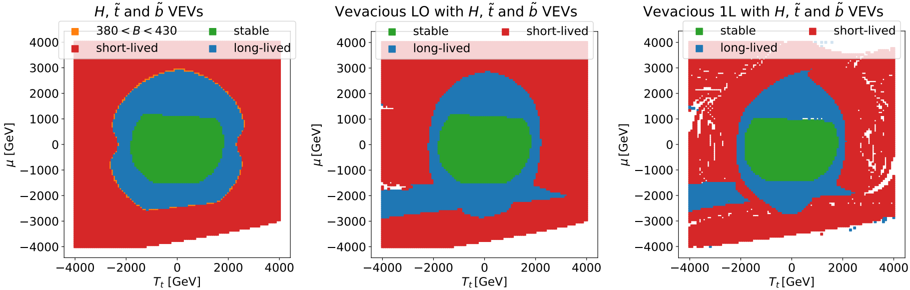
Summary

Vacuum stability provides important constraints on the parameter space of models with large scalar sectors and can help in determining phenomenologically viable regions.

- Different VEVs provide constraints on complementary regions of parameter space.
- The most dangerous minimum for tunneling is not in general the global minimum of the theory.
- The precise value of B has little influence on the allowed parameter space.

We aim to provide efficient and reliable bounds from vacuum stability in any renormalizable model.

Impact of the 1-loop Effective Potential



Perturbative Expansion of the Bounce

$$V(\phi) = \lambda\phi^4 \quad \text{with } \lambda < 0$$

Analytic bounce solution

$$\phi_c(\rho) = \sqrt{-\frac{2}{\lambda}}\frac{R}{R^2+\rho^2} \;\Rightarrow\; B = -\frac{2\pi}{3\lambda}$$

The 1-loop effective action up to two derivatives is

$$S_{\text{eff}} = \underbrace{\lambda\phi^4}_{\text{LO}} + \underbrace{\frac{9\lambda^2}{4\pi^2}\phi^2\left(\ln\frac{12\lambda\phi^2}{\mu^2} - \frac{3}{2}\right)}_{\text{1L eff. potential}} + \underbrace{\frac{(\partial_\mu\phi)^2}{2}\left(1+\frac{\lambda}{4\pi^2}\right)}_{\text{1L } p^2} + \underbrace{\mathcal{O}(\partial^4)}_{\text{1L } p^4}$$

Leading to

$$B = -\frac{2\pi}{3\lambda} + 3\ln\frac{R\mu}{2\sqrt{6}} + \frac{19}{4} + \frac{1}{3} + \mathcal{O}(1)$$

[Andreassen, Farhi, Frost, Schwartz; 1604.06090]