ROLE OF SUSY before/around Big Bang
(Nonlinear-supersymmetric general relativity theory)

Kazunari Shima
Saitama Institute of Technology

OUTLINE

1. Motivation
2. Nonlinear-supersymmetric general relativity theory(NLSUSYGR)
3. Evolutioon of NLSUSYGR and the vacuum structure
4. Cosmology and low energy particle physics of NLSUSYGR
5. Nonlinear vector-spinor SUSYGR(3/2NLSUSYGR)
6. Summary

1. Motivation

@ The success of Two SMs, i.e. GR and GWS model.

However, many unsolved fundamental problems in SMs: e.g.,

- Unification of two SMs.
- Space-time dimension *four*,
- Three generations of quarks and leptons,
- Tiny Neutrino mass M_ν, proton stability in GUT
- Dark Matter, Dark enegy; $\rho_{D.E.} \sim (M_\nu)^4 \Leftrightarrow \Lambda$(cosmological term)?

$\Longrightarrow$ SUGRA!?, Origin of SUSY breaking, $\cdots$ etc.

@ GR describes geometry of space-time.

However, unpleasant differences between GR and SUGRA:

- GR $\Longleftrightarrow$ Geometry of Riemann space-time(Physical:[x^μ], GL(4,R))
- SUGRA $\Longleftrightarrow$ Geometry of superspace (Mathematical:[x^μ, θ_α], sPoicaré)

$\Longrightarrow$ New SUSY paradigm on specific physical space-time!.

@ As for the **three-generations structure**:

Among all SO(N) sP, [$N = 8$ **by M. Gell-Mann]**

SM with just 3 generations emerges from one irreducible rep. of only SO(10) sP.

• **10 supercharges** $Q^I, (I = 1, 2, \cdots .10)$ **are assigned as follows:**

$\underline{\mathbf{10}}_{SO(10)} = \underline{\mathbf{5}}_{SU(5)} + \underline{\mathbf{5}}^*{}_{SU(5)}$ **for** $SO(10) \supset SU(5) \times U(1) \supset SU(3) \times SU(2) \times U(1)$

$\underline{\mathbf{5}}_{SU(5)} = [\ \underline{\mathbf{3}}^{*c}, \underline{\mathbf{1}}^{ew}, (\frac{e}{3}, \frac{e}{3}, \frac{e}{3}) : Q_a(a = 1, 2, 3)\ ] + [\ \underline{\mathbf{1}}^c, \underline{\mathbf{2}}^{ew}, (-e, 0) : Q_m(m = 4, 5)\ ]$.

$\Leftrightarrow$ **Note that supercharge quintet** $\underline{\mathbf{5}}_{SU(5)} = \underline{\mathbf{5}}_{SU(5)GUT}$,

• **Massless helicity states of gravity multiplet of SO(10) sP with CPT conjugation are specified by the helicity** $h = (2 - \frac{n}{2})$ **and the dimension** $\underline{d}_{[n]} = \frac{10!}{n!(10-n)!}$:

$$|h> = Q^n Q^{n-1} \cdots Q^2 Q^1 |2>, \ Q^n \ (n = 0, 1, 2, \cdots, 10)\text{: supercharge}$$

$\lvert h\rvert$	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
$\underline{d}_{[n]}$	$\underline{1}_{[10]}$	$\underline{10}_{[9]}$	$\underline{1}_{[0]}$ $\underline{45}_{[8]}$	$\underline{10}_{[1]}$ $\underline{120}_{[7]}$	$\underline{45}_{[2]}$ $\underline{210}_{[6]}$	$\underline{120}_{[3]}$ $\underline{252}_{[5]}$	$\underline{210}_{[4]}$ $\underline{210}_{[4]}$

@ Spin $\frac{1}{2}$ Dirac particles after superHiggs (*SU(2): L-R symm. case*)

$SU(3)$	Q_e	$SU(2) \otimes U(1)$
1	0 -1 -2	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ (E)
3	$5/3$ $2/3$ $-1/3$ $-4/3$	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} h \\ o \end{pmatrix} \begin{pmatrix} a \\ f \end{pmatrix} \begin{pmatrix} g \\ m \end{pmatrix} \begin{pmatrix} r \\ i \\ n \end{pmatrix}$
6	$4/3$ $1/3$ $-2/3$	$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$
8	0 -1	$\begin{pmatrix} N_1 \\ E_1 \end{pmatrix} \begin{pmatrix} N_2 \\ E_2 \end{pmatrix}$

@ SM Higgs doublet state survives in $h = 0$ state.

• How to construct N=10 SUSY with gravity beyond No-Go theorem in S-matrix ?

• To circumvent the No-Go theorem degeneracy of space-time is considered.

We show in this talk:

• N=10 SUSY with gravity is obtained by the geometric description of General Relativity principle on specific unstable *physical* (Riemann) space-time whose tangent space possesses NLSUSY structure.

$$\Downarrow$$

A quick review of NLSUSY:

- Take flat space-time specified by x^a and ψ_α.
- Consider one form $\omega^a = dx^a + \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$,

κ is an arbitrary constant with the dimension l^{+2}.

- $\delta\omega^a = 0$ under $\delta x^a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$ and $\delta\psi = \zeta$

with a global spinor parameter ζ.

- An invariant acction($\sim$ invariant volume) is obtained:

$S = -\frac{1}{2\kappa^2}\int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA}$,

L_{VA} is N=1 Volkov-Akulov model of NLSUSY given by

$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a - \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right]$,

$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$,

$t^a{}_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \bar{\psi}\gamma^a\partial_b\psi)$,

which is invariant under N=1 NLSUSY transformation:

$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\zeta}\gamma^a\psi)\partial_a\psi$. $\longleftrightarrow$ NG fermioon for SB SUSY

- ψ is Nambu-Goldstone(NG) fermion (the coset space coordinate) of $\frac{superPoincare}{Poincare}$.
- ψ is quantized canonically in compatible with SUSY algebra.

2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

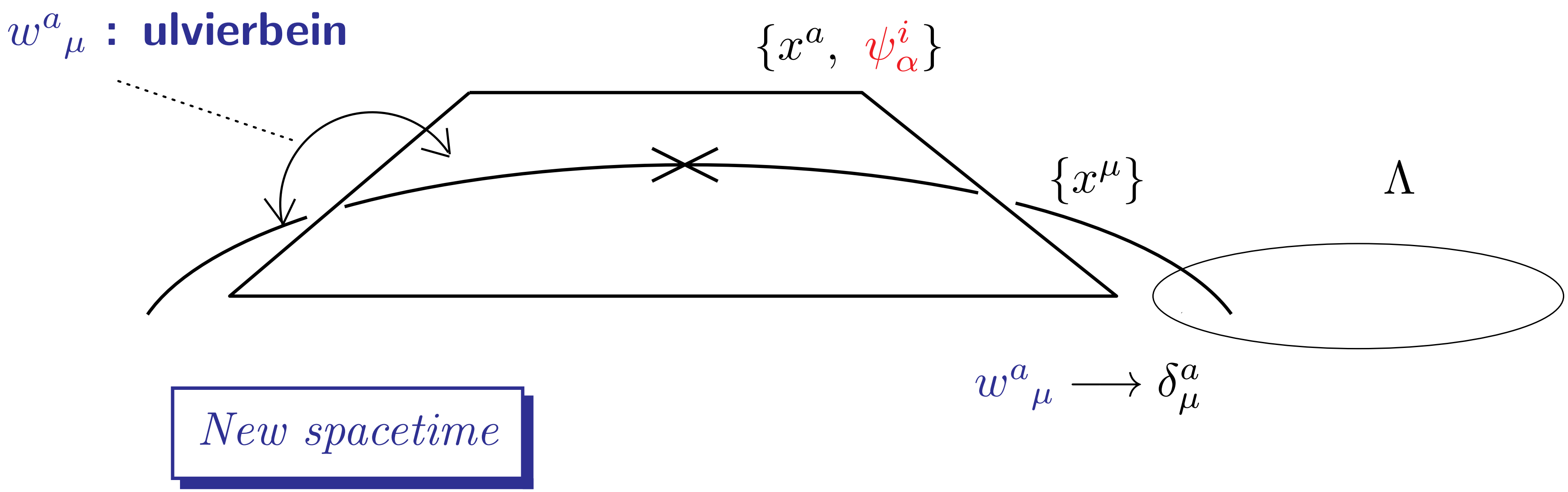
2.1. New Space-time as Ultimate Shape of Nature

We consider new (unstable) *physical* space-time inspired by nonlinear(NL) SUSY:

The tangent space of new space-time is specified by
Grassmann coordinates ψ_α for SL(2,C)
besides the ordinary Minkowski coordinates x^a for SO(1,3),

i.e.,
the coordinate ψ_α of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ turning to the NLSUSY NG fermion (called *superon* hereafter) are attached at every curved space-time point besides x^a.

- **New (empy) unstable space-time:**



(Non-compact groups SO(1,3) and SL(2,C) for space-time symmetry are analogous to compact groups SO(3) and SU(2) for gauge symmetry of 't Hooft-Polyakov monopole, though SL(2,C) is realized nonlinearly.)

- **Note that $SO(1,3) \cong SL(2,C)$ is crucial for NLSUSYGR scenario.**

4 dimensional space-time is singled out.

2.2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

The geometrical arguments of Einstein general relativity(GR) can be extended to new (unstable) space-time.

- **Unified vierbein $w^a{}_\mu(x)$(*ulvierbein*) of new space-time:**

(Note: Grassmann d.o.f. induces the imaginary part of $w^a{}_\mu(x)$.)

$$w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi),$$

$$w^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a,$$

$$w^a{}_\mu(x) w^\mu{}_b(x) = \delta^a{}_b$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), (I = 1, 2, .., N)$$

- **N-extended NLSUSYGR action of Eienstein-Hilbert(EH)-type for new space-time.** $\Longrightarrow$

N-extended NLSUSYGR action:

(Phys.Lett.B501,237(2001), B507,260(2001))

$$L_{\rm NLSUSYGR}(w) = -\frac{c^4}{16\pi G}|w|\{\Omega(w) + \Lambda\}, \quad (1)$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \quad (2)$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I\gamma^a\partial_\mu\psi^I - \partial_\mu\bar{\psi}^I\gamma^a\psi^I), (I = 1, 2, .., N) \quad (3)$$

- $w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi))$: the unified vierbein of new space-time(*ulvierbein*)
- $e^a{}_\mu(x)$: the ordinary vierbein for the local SO(1,3) d.o.f.of GR,
- $t^a{}_\mu(\psi(x))$: the mimic vierbein for the local SL(2,C) d.o.f. composed of the stress-energy-momentum of NG fermion $\psi(x)^I$(called *superons*),
- $\Omega(w)$: Ricci scalar curvature of new space-time computed in terms of $w^a{}_\mu$,
- $s_{\mu\nu} \equiv w^a{}_\mu\eta_{ab}w^b{}_\nu$, $s^{\mu\nu}(x) \equiv w^\mu{}_a(x)\eta^{ab}w^\nu{}_a(x)$: metric tensors of new space-time.
- G : the Newton gravitational constant.
- $\Lambda > 0$: cosmological constant indicating NLSUSY structure of tangent space.

• NLSUSYGR scenario fixes the arbitrary constatnt κ^2 to

$$\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1},$$

with the dimension $(length)^4 \sim (enegy)^{-4}$.

• $\Lambda > 0$ in the action $L_{\rm NLSUSYGR}$ allows
negative dark energy density interpretation of $\frac{\Lambda}{G}$
in the Einstein equation. $\rightarrow$ Sec.4.

• No-go theorem for $\underline{N > 8 \text{ SUSY}}$ has been circumvented
by using NLSUSY, i.e. by the vacuum(flat space) degeneracy.

• Note that $SO(1, D-1) \cong SL(d, C)$, i.e. $\frac{D(D-1)}{2} = 2(d^2-1)$ holds
for only $\underline{D = 4, d = 2}$.

NLSUSYGR scenario predicts 4 dimensional space-time.

2.3. Symmetries of NLSUSYGR(N-extended action)

- **Space-time symmetries** ($\cong sP$):

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \text{R})] \otimes [\text{local Lorentz}] \tag{4}$$

- **Internal symmetries for N-extended NLSUSY GR (N-superons ψ^I $(I = 1, 2, ..N)$):**

$$[\text{global SO(N)}] \otimes [\text{local U}(1)^{\text{N}}] \otimes [\text{chiral}]. \tag{5}$$

For example:

- **Invariance under the new NLSUSY transformation**;

$$\delta_\zeta \psi^I = \frac{1}{\kappa}\zeta^I - i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_\rho \psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_{[\mu} e^a{}_{\rho]}. \tag{6}$$

induce GL(4,R) transformations on $w^a{}_\mu$ and the unified metric $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu \partial_\nu w^a{}_\mu + \partial_\mu \xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\mu\kappa}, \tag{7}$$

where ζ is a constant spinor parameter, $\partial_{[\rho} e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ and
$\xi^\rho = -i\kappa \bar{\zeta}^I \gamma^\rho \psi^I$.

• Commutators of two new NLSUSY transformations (6) on ψ^I and $e^a{}_\mu$
close to GL(4,R),

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^I = \Xi^\mu \partial_\mu \psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a{}_\mu = \Xi^\rho \partial_\rho e^a{}_\mu + e^a{}_\rho \partial_\mu \Xi^\rho, \tag{8}$$

where $\Xi^\mu = 2i\bar{\zeta}^I{}_1 \gamma^\mu \zeta^I{}_2 - \xi_1^\rho \xi_2^\sigma e_a{}^\mu \partial_{[\rho} e^a{}_{\sigma]}$.
$q.e.d.$

- **New NLSUSY (6) is the square-root of GL(4,R)**;

$$[\delta_1, \delta_2] = \delta_{\mathrm{GL(4,R)}}, \quad i.e. \quad \delta \sim \sqrt{\delta_{\mathrm{GL(4,R)}}}.$$

c.f. SUGRA(LSUSY)

$$[\delta_1, \delta_2] = \delta_P \underline{+\delta_L + \delta_g}$$

- The ordinary local GL(4,R) invariance is manifest by the construction.

- **Invariance under new local Lorentz transformation**;

$$\delta_L \psi^I = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^I\gamma_5\gamma_d\psi^I(\partial_\mu\epsilon_{bc}) \tag{9}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$.

(9) induce the familiar local Lorentz transformation on $w^a{}_\mu$:

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{10}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra,
e.g., the new form on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu\beta_{bc}), \tag{11}$$

where $\beta_{ab} = -\beta_{ba}$ is given by $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$.
q.e.d.

3. Evolution of New Space-time:

3.1. Big Collapse of new space-time

@ $\Lambda > 0$ allows $L_{\rm NLSUSYGR}(w)$ breaks down spontaneously**(Big Collapse)** to
ordinary Riemann space-time(graviton) and NG fermion(superon) $L_{\rm SGM}(e,\psi)$.

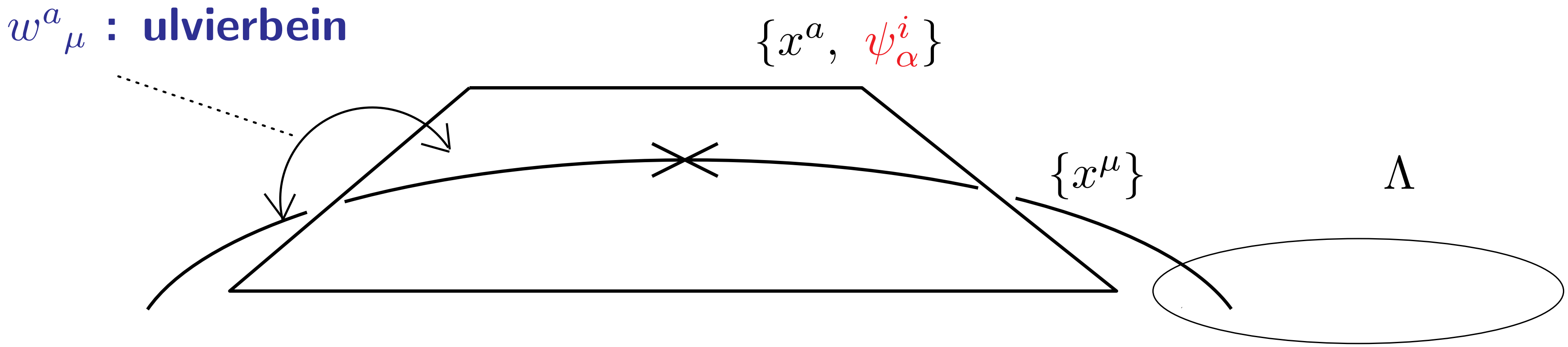
@ **Superon-Graviton Model(SGM) created by Big Collapse:**

$$L_{\rm NLSUSYGR}(w) = L_{\rm SGM}(e,\psi) \equiv -\frac{c^4}{16\pi G}|e|\{R(e) + |w(\psi^I)|\Lambda + \tilde{T}(e,\psi^I)\}. \qquad (12)$$

- $R(e)$: **the Ricci scalar curvature of ordinary Riemann space-time**
- Λ : **the cosmological constant**
- $|w(\psi^I)| = \det w^a{}_b = \det\{\delta^a{}_b + t^a{}_b(\psi^I)\}$: **NLSUSY action for superon**
- $\tilde{T}(e,\psi^I)$: **the gravitational interaction of superon**

@ Big Collapse induces the rapid spacial expamsion of space-time by the Pauli principle.

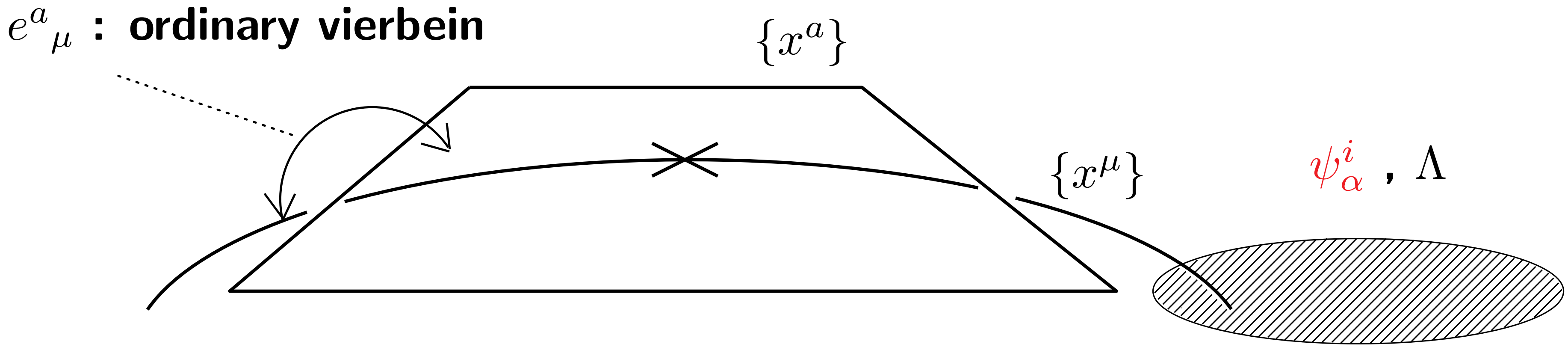
@ $L_{\rm SGM}(e, \psi^I)$ evolves toward the true vacuum by constituting gravitational composite (massless) eigenstates of broken LSUSY SO(N) sP, which is the ignition of the Big Bang SMs scenario. $\Longrightarrow$



$w^a{}_\mu \longrightarrow \delta^a_\mu$

New spacetime

$\Downarrow$ **Big Collapse**



$e^a{}_\mu \longrightarrow \delta^a_\mu$

Riemann spacetime $\oplus$ **matter**

Ignition of Big Bang towards the true vacuum

- **The Noether's theorem gives the conserved supercurrent:**

$$S^{I\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}}{e_a}^{\mu}\gamma^a\psi^I + \cdots . \tag{13}$$

- **The supercurrent couples the graviton and the superon(NG fermion) to the vacuum with the strength $\frac{c^4\Lambda}{8\pi G}$:**

$$< {e_b}^{\nu}{\psi_\beta}^J|{S_\alpha}^{I\mu}|0 >= i\sqrt{\frac{c^4\Lambda}{8\pi G}}\delta^{\mu\nu}\delta^{IJ}(\gamma_b)_{\alpha\beta} \tag{14}$$

with the strength $g_{sv} = \sqrt{\frac{c^4\Lambda}{8\pi G}}$.

3.2. Linearization of NLSUSY and vacuum of $L_{\rm SGM}(e,\psi)$

It is natural to assume that the universal attractive force graviton creates all possible gravitational composites of superons ($(Q^I)^n \sim (\psi^I)^n + \cdots$), which are helicity eigen states of LSUSY sP algebra
c.f. $O(4)$ for rel. H atom

@ The vacuum and the particle configuration of $L_{\rm SGM}(e,\psi)$ is studied by the linearization of NLSUSY to LSUSY based on sP algebra..

• Linearized broken LSUSY theory $L_{\rm LSUSY}({e^a}_\mu, \psi_\nu, v^a, \lambda, \phi, M, N, \cdots)$ is obtained as composites of NG fermions of $L_{\rm SGM}(e,\psi)$:

$$L_{\rm NLSUSYGR}(w) = L_{\rm SGM}(e,\psi) = L_{\rm LSUSY}({e^a}_\mu, \psi_\nu, v^a, \lambda, \phi, M, N, \cdots)$$

$\Longleftrightarrow$ NL/L SUSY relation

• This is the phase transition of NLSUSY $L_{\rm SGM}(e,\psi)$ towards the vacuum achieved by gravitational composite particle states of LSUSY.

3.3. NL/L SUSY relation

We demonstrate NL/L SUSY relation for N=2 SUSY in *flat space*:

$$L_{\rm NLSUSYGR}(w) = L_{\rm SGM}(e,\psi) \to \ L_{\rm NLSUSY}(\psi) = L_{\rm LSUSY}(v^a(\psi),\phi(\psi),\cdots)$$

($N=2$ SGM reduces to $N=2$ NLSUSY (Λ term of NLSUSYGR))

@ N=2, d=2 NLSUSY model:

$$L_{NLSUSY} = -\frac{1}{2\kappa^2}|w_{NLSUSY}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right], \qquad (15)$$

$|w_{NLSUSY}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$, $t^a{}_b = -i\kappa^2(\bar\psi_j\gamma^a\partial_b\psi^j - \bar\psi_j\gamma^a\partial_b\psi^j)$, $(j=1,2)$,

which is invariant under N=2 NLSUSY transformation,

$\delta_\zeta\psi^j = \frac{1}{\kappa}\zeta^j - i\kappa(\bar\zeta_k\gamma^a\psi^k - \bar\zeta_k\gamma^a\psi^k)\partial_a\psi^j, (j=1,2)$.

N=2, d=2 LSUSY Theory (SUSY QED):

- **Helicity states of N=2 vector supermultiplet:**

$$\begin{pmatrix} +1 \\ +\frac{1}{2},\ +\frac{1}{2} \\ 0 \end{pmatrix} + [\mathrm{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell minimal vector supermultiplet: $(v^a, \lambda^i, A, \phi, D;$**i=1,2**$)$**.** **in** ***WZ gauge*****.** **(**A **and** ϕ **are two singlets,** 0^+ **and** 0^-**, scalar fields.)**

- **Helicity states of N=2 scalar supermultiplet:**

$$\begin{pmatrix} +\frac{1}{2} \\ 0,\ 0 \\ -\frac{1}{2} \end{pmatrix} + [\mathrm{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets: **(**χ**,** B^i**,** ν**,** F^i**;** $i = 1, 2$**),** B^i **and** F^i **are complex.**

- **The most genaral $N=2, d=2$ SUSYQED action ($m=0$ case) :**

$$L_{\mathrm{N=2SUSYQED}} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf}, \tag{16}$$

$$\begin{aligned}
L_{V0} &= -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D, \\
L'_{\Phi 0} &= \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}|\partial_a B^i|^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}|F^i|^2, \\
L_e &= e\Big\{ i v_a \bar{\chi}\gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5\nu \\
&\quad + B^i(\bar{\lambda}^i\chi - \epsilon^{ij}\bar{\lambda}^j\nu) - \frac{1}{2}|B^i|^2 D \Big\} + \{h.c.\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)|B^i|^2, \\
L_{Vf} &= f\{A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + (A^2 - \phi^2)D - \epsilon^{ab}A\phi F_{ab}\}
\end{aligned} \tag{17}$$

- **Note that**

$J=0$ **states in the vector multiplet for $N \geq 2$ SUSY induce Yukawa coupling.**

$L_{\mathrm{N=2SUSYQED}}$ is invariant under $N = 2$ LSUSY transformation:

- **For the minimal vector off-shell supermultiplet:**

$$\delta_\zeta v^a = -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j,$$
$$\delta_\zeta \lambda^i = (D - i\partial\!\!\!/ A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\partial\!\!\!/\phi\zeta^j,$$
$$\delta_\zeta A = \bar{\zeta}^i\lambda^i,$$
$$\delta_\zeta \phi = -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j,$$
$$\delta_\zeta D = -i\bar{\zeta}^i\partial\!\!\!/\lambda^i. \tag{18}$$

$$[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{19}$$

where $\zeta^i, i = 1, 2$ are constant spinors and $\delta_g(\theta)$ is the $U(1)$ gauge transformation for only v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i\ v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- **For the two scalar off-shell supermultiplets:**

$$
\begin{aligned}
\delta_\zeta \chi &= (F^i - i\partial\!\!\!/ B^i)\zeta^i - e\epsilon^{ij} V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij} \bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\partial\!\!\!/ B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \partial\!\!\!/ \chi - i\epsilon^{ij}\bar{\zeta}^j \partial\!\!\!/ \nu \\
&\quad -e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i) B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\chi = \Xi^a \partial_a \chi - e\theta\nu, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]B^i = \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\nu = \Xi^a \partial_a \nu + e\theta\chi, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]F^i = \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{20}
$$

with $V^i = iv_a\gamma^a\zeta^i - \epsilon^{ij}A\zeta^j - \phi\gamma_5\zeta^i$ **and the U(1) gauge parameter** θ**.**

$N=2$ NL/L SUSY relation:

$$L_{\rm N=2SUSYQED} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} = L_{\rm N=2NLSUSY} + [\text{surface terms}], \quad (21)$$

is established by
(I) commutator-based(heuristic) linearization
or
(II) superfield-based(systematic) linearization procedures.

(I) Commutator-based linearization:

@The product of Lorentz tensors composed of ψ^i play a basic role.

- **For the product of Lorentz tensors (of current) of ψ^i**

$$b^i{}_A{}^{jk}{}_B{}^{l\cdots m}{}_C{}^n\left((\psi^i)^{2(n-1)}|w|\right) = \kappa^{2n-3}\bar\psi^i\gamma_A\psi^j\bar\psi^k\gamma_B\psi^l\cdots\bar\psi^m\gamma_C\psi^n|w|, \quad (22)$$

$$f^{ij}{}_A{}^{kl}{}_B{}^{m\cdots n}{}_C{}^p\left((\psi^i)^{2n-1}|w|\right) = \kappa^{2(n-1)}\psi^i\bar\psi^j\gamma_A\psi^k\bar\psi^l\gamma_B\psi^m\cdots\bar\psi^n\gamma_C\psi^p|w|, \quad (23)$$

which mean

$$b=\kappa^{-1}|w|,\ \ b^i{}_A{}^j=\kappa\bar\psi^i\gamma_A\psi^j|w|,\ \ b^i{}_A{}^{jk}{}_B{}^l=\kappa^3\bar\psi^i\gamma_A\psi^j\bar\psi^k\gamma_B\psi^l|w|,\ \cdots, \tag{24}$$
$$f^i=\psi^i|w|,\ \ f^{ij}{}_A{}^k=\kappa^2\psi^i\bar\psi^j\gamma_A\psi^k|w|,\ \cdots, \tag{25}$$

the variations under the NLSUSY transformations become

$$\begin{aligned}\delta_\zeta b^i{}_A{}^{jk}{}_B{}^{l\cdots m}{}_C{}^n &= \kappa^{2(n-1)}\left[\left\{\left(\bar\zeta^i\gamma_A\psi^j+\bar\psi^i\gamma_A\zeta^j\right)\bar\psi^k\gamma_B\psi^l\cdots\bar\psi^m\gamma_C\psi^n+\cdots\right\}|w|\right.\\ &\left.\quad+\kappa\partial_a\left(\xi^a\bar\psi^i\gamma_A\psi^j\bar\psi^k\gamma_B\psi^l\cdots\bar\psi^m\gamma_C\psi^n|w|\right)\right],\end{aligned} \tag{26}$$

$$\begin{aligned}\delta_\zeta f^{ij}{}_A{}^{kl}{}_B{}^{ml\cdots n}{}_C{}^p &= \kappa^{2n-1}\left[\left\{\zeta^i\bar\psi^j\gamma_A\psi^k\bar\psi^l\gamma_B\psi^m\cdots\bar\psi^n\gamma_C\psi^p\right.\right.\\ &\quad\left.+\psi^i\left(\bar\zeta^j\gamma_A\psi^k+\bar\psi^j\gamma_A\zeta^k\right)\bar\psi^l\gamma_B\psi^m\cdots\bar\psi^n\gamma_C\psi^p+\cdots\right\}|w|\\ &\left.\quad+\kappa\partial_a\left(\xi^a\psi^i\bar\psi^j\gamma_A\psi^k\bar\psi^l\gamma_B\psi^m\cdots\bar\psi^n\gamma_C\psi^p|w|\right)\right],\end{aligned} \tag{27}$$

which indicate that the bosonic and fermionic Lorentz tensor products in Eqs.(22) and (23) are linearly transformed to each other.

- **They satisfy the commutator**

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \tag{28}$$

where $\delta_P(v)$ **is a translation with a parameter** $v^a = 2i(\bar{\zeta}^i_{1L}\gamma^a\zeta^i_{2L} - \bar{\zeta}^i_{1R}\gamma^a\zeta^i_{2R})$

- **These results show that**

the commutator-based linearization closes on the all possible Lorentz tensors composed of ψ^i**.**

- **Identify each composite Lorentz tensor with the component field of the LSUSY supermultiplet including the auxiliary field(***susy compositenenss***), which reproduces the familiar LSUSY transformation among the supermultiplet.**

- **Substituting SUSY compositeness relations into** $L_{\rm N=2LSUSYQED}$**,**

we obtain $L_{\rm N=2NLSUSY}$**, i .e. the NL/L SUSY relation(equivalence).**

• **SUSY compositeness for the vector off-shell minimal supermultiplet:**

$$\begin{aligned} v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|, \\ \lambda^i &= \xi\psi^i|w|, \\ A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|, \\ \phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|, \\ D &= \frac{\xi}{\kappa}|w|, \end{aligned} \tag{29}$$

where ξ is a VEV factor of the auxiliary field D.

• **Note that ψ^i is the leading term of the supercharge Q^i.**

• **SUSY compositeness for scalar off-shell minimal supermultiplets:**

$$\chi = \xi^i \left[\psi^i |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j |w| \} \right]$$

$$B^i = -\kappa \left(\frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|,$$

$$\nu = \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k |w| \} \right],$$

$$F^i = \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8} \kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\} - i \kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|)$$

$$\underline{- \frac{1}{4} e \kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|}. \tag{30}$$

• **The quartic fermion self-interaction term in F^i is the origin of the local $U(1)$ gauge symmetry of LSUSY.**

• **ξ^i is the VEV factor of the auxiliary field F^i.**

• **SUSY compositeness** **produces** **under NLSUSY transformation**

a new off-shell commutator algebra which closes on only a translation:

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \tag{31}$$

where $\delta_P(v)$ **is a translation with a parameter**

$$v^a = 2i(\bar{\zeta}^i_{1L}\gamma^a\zeta^i_{2L} - \bar{\zeta}^i_{1R}\gamma^a\zeta^i_{2R}) \tag{32}$$

• **Note that the commutator does not induce the U(1) gauge transformation, which is different from the ordinary LSUSY.**

• **Substituting SUSY copositeness into** $L_{\rm N=2LSUSYQED}$,
we find NL/L SUSY relation for the minimal supermultiplet:

$$L_{\rm N=2LSUSYQED} = f(\xi, \xi^i) L_{\rm N=2NLSUSY} + [\text{suface terms}], \tag{33}$$

$$f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1. \tag{34}$$

$\Rightarrow$ **LSUSY may be regarded as composite eigenstates of (space-time) symmetries.**

• **NL/L SUSY relation bridges naturally**
the cosmology and the low energy particle physics in NLSUSYGR. ($\Rightarrow$ Sec. 4).

• **The direct linearization of highly nonlinear SGM action (12)**
in curved space remains to be carried out.
$\rightarrow$

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the cosmological term Λ of SGM and materializes the true vacuum of SGM
(as gravitational composites of NG fermion)

SM can be a low energy effective theory of SGM/NLSUSYGR.

(II) Linearization of NLSUSY by the superfield formulation($d = 2$)

- **General superfields** are given for the $N = 2$ vector supermultiplet by

$$\mathcal{V}(x,\theta^i) = C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2}\bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2}\bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar{\theta}^i \gamma_5 \theta^j \phi(x)$$
$$- \frac{i}{4}\epsilon^{ij}\bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2}\bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8}\bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x), \quad (35)$$

and for the $N = 2$ scalar supermultiplet by

$$\Phi^i(x,\theta^i) = B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij}\bar{\theta}^j \nu(x) - \frac{1}{2}\bar{\theta}^j \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i\bar{\theta}^i \partial\!\!\!/ B^j(x)\theta^j$$
$$+ \frac{i}{2}\bar{\theta}^j \theta^j (\bar{\theta}^i \partial\!\!\!/ \chi(x) - \epsilon^{ik}\bar{\theta}^k \partial\!\!\!/ \nu(x)) + \frac{1}{8}\bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \partial_a \partial^a B^i(x). \quad (36)$$

- **Consider the following generajized superspace with $-\kappa\psi(x)$,**

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \tag{37}$$

and denote the resulting superfields on (x'^a, θ'^i) and their θ-epansions as

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \tag{38}$$

- **Genealized global SUSY transformations $\tilde{\delta} = \delta^L(x.\theta) + \delta^{NL}(\psi)$ on (x'^a, θ'^i) give:**

$$\tilde{\delta}\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \tilde{\delta}\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \tag{39}$$

- **Therefore, the following conditions, i.e. SUSY invariant constraints:**

$$\tilde{\varphi}^I_{\mathcal{V}}(x) = \xi^I_{\mathcal{V}}(\text{constant}) \quad \tilde{\varphi}^I_{\Phi}(x) = \xi^I_{\Phi}(\text{constant}), \tag{40}$$

are invariant (conserved quantities) under generalized supertrasformations, which provide SUSY compositeness.

• **Putting in general constants as follows:**

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi_\Lambda^i, \quad \tilde{M}^{ij} = \xi_M^{ij}, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi_v^a, \quad \tilde{\lambda}^i = \xi_\lambda^i, \quad \tilde{D} = \frac{\xi}{\kappa}, \tag{41}$$

$$\tilde{B}^i = \xi_B^i, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{42}$$

where mass dimensions of constants (or constant spinors) in $d = 2$ are defined by (-1, $\frac{1}{2}$, 0, 0, 0, $-\frac{1}{2}$) for (ξ_c, ξ_Λ^i, ξ_M^{ij}, ξ_ϕ, ξ_v^a, ξ_λ^i), (0, $-\frac{1}{2}$, $-\frac{1}{2}$) for (ξ_B^i, ξ_χ, ξ_ν) and 0 for ξ^i for convenience.

• **We obtain straightforwardly SUSY compositeness $\varphi_\mathcal{V}^I = \varphi_\mathcal{V}^I(\psi)$ for the vector supermultiplet**

$$\begin{aligned} C &= \xi_c + \kappa \bar{\psi}^i \xi_\Lambda^i + \frac{1}{2}\kappa^2 (\xi_M^{ij} \bar{\psi}^i \psi^j - \xi_M^{ii} \bar{\psi}^j \psi^j) + \frac{1}{4}\xi_\phi \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - \frac{i}{4}\xi_v^a \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \\ &\quad -\frac{1}{2}\kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \xi_\lambda^j - \frac{1}{8}\xi \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j, \\ \Lambda^i &= \xi_\Lambda^i + \kappa (\xi_M^{ij} \psi^j - \xi_M^{jj} \psi^i) + \frac{1}{2}\xi_\phi \kappa \epsilon^{ij} \gamma_5 \psi^j - \frac{i}{2}\xi_v^a \kappa \epsilon^{ij} \gamma_a \psi^j \end{aligned}$$

$$
\begin{aligned}
& -\frac{1}{2}\xi_\lambda^i \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2}\kappa^2(\psi^j \bar{\psi}^i \xi_\lambda^j - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi_\lambda^j - \gamma_a \psi^j \bar{\psi}^i \gamma^a \xi_\lambda^j) \\
& -\frac{1}{2}\xi \kappa^2 \psi^i \bar{\psi}^j \psi^j - i\kappa \not{\partial} C(\psi)\psi^i, \\
M^{ij} \;=\; & \xi_M^{ij} + \kappa \bar{\psi}^{(i} \xi_\lambda^{j)} + \frac{1}{2}\xi\kappa \bar{\psi}^i \psi^j + i\kappa \epsilon^{(i|k|}\epsilon^{j)l} \bar{\psi}^k \not{\partial} \Lambda^l(\psi) - \frac{1}{2}\kappa^2 \epsilon^{ik}\epsilon^{jl} \bar{\psi}^k \psi^l \partial^2 C(\psi), \\
\phi \;=\; & \xi_\phi - \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \xi_\lambda^j - \frac{1}{2}\xi\kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - i\kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \not{\partial} \Lambda^j(\psi) + \frac{1}{2}\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 C(\psi), \\
v^a \;=\; & \xi_v^a - i\kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \xi_\lambda^j - \frac{i}{2}\xi\kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j - \kappa \epsilon^{ij} \bar{\psi}^i \not{\partial} \gamma^a \Lambda^j(\psi) + \frac{i}{2}\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j \partial^2 C(\psi) \\
& -i\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^b \psi^j \partial^a \partial_b C(\psi), \\
\lambda^i \;=\; & \xi_\lambda^i + \xi \psi^i - i\kappa \not{\partial} M^{ij}(\psi)\psi^j + \frac{i}{2}\kappa \epsilon^{ab}\epsilon^{ij} \gamma_a \psi^j \partial_b \phi(\psi) \\
& -\frac{1}{2}\kappa \epsilon^{ij} \left\{ \psi^j \partial_a v^a(\psi) - \frac{1}{2}\epsilon^{ab} \gamma_5 \psi^j F_{ab}(\psi) \right\} \\
& -\frac{1}{2}\kappa^2 \{ \partial^2 \Lambda^i(\psi) \bar{\psi}^j \psi^j - \partial^2 \Lambda^j(\psi) \bar{\psi}^i \psi^j - \gamma_5 \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma_5 \psi^j
\end{aligned}
$$

$$-\gamma_a \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma^a \psi^j + 2\partial\!\!\!/ \partial_a \Lambda^j(\psi) \bar{\psi}^i \gamma^a \psi^j \} - \frac{i}{2}\kappa^3 \partial\!\!\!/ \partial^2 C(\psi) \psi^i \bar{\psi}^j \psi^j,$$

$$\begin{aligned} D \;=\; & \frac{\xi}{\kappa} - i\kappa \bar{\psi}^i \partial\!\!\!/ \lambda^i(\psi) \\ & + \frac{1}{2}\kappa^2 \left\{ \bar{\psi}^i \psi^j \partial^2 M^{ij}(\psi) - \frac{1}{2}\epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 \phi(\psi) \right. \\ & \left. + \frac{i}{2}\epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial^2 v^a(\psi) - i\epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial_a \partial_b v^b(\psi) \right\} \\ & - \frac{i}{2}\kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \partial\!\!\!/ \partial^2 \Lambda^j(\psi) + \frac{1}{8}\kappa^4 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j \partial^4 C(\psi), \end{aligned} \tag{43}$$

and SUSY compositeness for the scalar multiplet $\varphi_\Phi^I = \varphi_\Phi^I(\psi)$:

$$\begin{aligned} B^i \;=\; & \xi_B^i + \kappa(\bar{\psi}^i \xi_\chi - \epsilon^{ij} \bar{\psi}^j \xi_\nu) - \frac{1}{2}\kappa^2 \{ \bar{\psi}^j \psi^j F^i(\psi) - 2\bar{\psi}^i \psi^j F^j(\psi) + 2i \bar{\psi}^i \partial\!\!\!/ B^j(\psi) \psi^j \} \\ & - i\kappa^3 \bar{\psi}^j \psi^j \{ \bar{\psi}^i \partial\!\!\!/ \chi(\psi) - \epsilon^{ik} \bar{\psi}^k \partial\!\!\!/ \nu(\psi) \} + \frac{3}{8}\kappa^4 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \partial^2 B^i(\psi), \\ \chi \;=\; & \xi_\chi + \kappa \{ \psi^i F^i(\psi) - i\partial\!\!\!/ B^i(\psi) \psi^i \} \end{aligned}$$

$$
\begin{aligned}
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\chi(\psi)\bar{\psi}^i\psi^i - \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\nu(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\nu(\psi)\}] \\
& +\frac{1}{2}\kappa^3\psi^i\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \frac{i}{2}\kappa^3\partial\!\!\!/F^i(\psi)\psi^i\bar{\psi}^j\psi^j + \frac{1}{8}\kappa^4\partial^2\chi(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
\nu \;=\; & \xi_\nu - \kappa\epsilon^{ij}\{\psi^i F^j(\psi) - i\partial\!\!\!/B^i(\psi)\psi^j\} \\
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\nu(\psi)\bar{\psi}^i\psi^i + \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\chi(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\chi(\psi)\}] \\
& +\frac{1}{2}\kappa^3\epsilon^{ij}\psi^i\bar{\psi}^k\psi^k\partial^2 B^j(\psi) + \frac{i}{2}\kappa^3\epsilon^{ij}\partial\!\!\!/F^i(\psi)\psi^j\bar{\psi}^k\psi^k + \frac{1}{8}\kappa^4\partial^2\nu(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
F^i \;=\; & \frac{\xi^i}{\kappa} - i\kappa\{\bar{\psi}^i\partial\!\!\!/\chi(\psi) + \epsilon^{ij}\bar{\psi}^j\partial\!\!\!/\nu(\psi)\} \\
& -\frac{1}{2}\kappa^2\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \kappa^2\bar{\psi}^i\psi^j\partial^2 B^j(\psi) + i\kappa^2\bar{\psi}^i\partial\!\!\!/F^j(\psi)\psi^j \\
& +\frac{1}{2}\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial^2\chi(\psi) + \epsilon^{ik}\bar{\psi}^k\partial^2\nu(\psi)\} - \frac{1}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 F^i(\psi). \qquad (44)
\end{aligned}
$$

• **Choosing the following Lorentz invariant and SUSY invariant constraints of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,**

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \qquad (45)$$

give previous SUSY compositeness for the minimal supermultiplet.

Actions in the $d = 2,\ N = 2$ NL/L SUSY relation

By changing the integration variables $(x^a, \theta^i) \to (x'^a, \theta'^i)$, we can confirm systematically that LSUSY actions reduce to NLSUSY representation.

(a) The kinetic (free) action with the Fayet-Iliopoulos (FI) D term for the $N = 2$ vector supermultiplet $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$S_{\mathcal{V}\mathrm{free}} = \int d^2x \left\{ \int d^2\theta^i \ \frac{1}{32} (\overline{D^i \mathcal{W}^{jk}} D^i \mathcal{W}^{jk} + \overline{D^i \mathcal{W}_5^{jk}} D^i \mathcal{W}_5^{jk}) + \int d^4\theta^i \ \frac{\xi}{2\kappa} \mathcal{V} \right\}_{\theta^i=0}$$
$$= \xi^2 S_{N=2\mathrm{NLSUSY}}, \qquad (46)$$

where

$$\mathcal{W}^{ij} = \bar{D}^i D^j \mathcal{V}, \quad \mathcal{W}_5^{ij} = \bar{D}^i \gamma_5 D^j \mathcal{V}. \qquad (47)$$

(Note) The FI D term gives the correct sign of the NLSUSY action.

(b) Yukawa interaction terms for $\mathcal{V}$ vanish,
i.e.

$$S_{\mathcal{V}f} = \frac{1}{8}\int d^2x \ f \left[\int d^2\theta^i \ \mathcal{W}^{jk}(\mathcal{W}^{jl}\mathcal{W}^{kl} + \mathcal{W}_5^{jl}\mathcal{W}_5^{kl}) + \int d\bar{\theta}^i d\theta^j \ 2\{\mathcal{W}^{ij}(\mathcal{W}^{kl}\mathcal{W}^{kl} + \mathcal{W}_5^{kl}\mathcal{W}_5^{kl}) + \mathcal{W}^{ik}(\mathcal{W}^{jl}\mathcal{W}^{kl} + \mathcal{W}_5^{jl}\mathcal{W}_5^{kl})\}\right]_{\theta^i=0} = 0, \quad (48)$$

by means of cancellations among four NG-fermion self-interaction terms.

[Note]

- **General mass terms for $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$ vanish as well. $\rightarrow$ Chirality is encoded in the vacuum.**

(c) The *most general* **gauge invariant action for Φ^i coupled with $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;**

$$S_{\rm gauge} = -\frac{1}{16}\int d^2x \int d^4\theta^i e^{-4e\mathcal{V}}(\Phi^j)^2$$
$$= -(\xi^i)^2 S_{N=2\mathrm{NLSUSY}}. \qquad (49)$$

• **Here $U(1)$ gauge interaction terms with the gauge coupling constant e produce**

four NG-fermion self-interaction terms as

$$S_e(\text{for the minimal off shell multiplet}) = \int d^2x \left\{\frac{1}{4}e\kappa\xi(\xi^i)^2\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\right\}, \qquad (50)$$

which are absorbed in the SUSY invariant relation of the auxiliary field $F^i = F^i(\psi)$ by adding four NG-fermion self-interaction terms as (30):

$$F^i(\psi) \longrightarrow F^i(\psi) - \frac{1}{4}e\kappa^2\xi\xi^i\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k|w_{VA}|. \qquad (51)$$

Therefore,

- **under SUSY invariant relations,**

the $N = 2$ NLSUSY action $S_{N=2\text{NLSUSY}}$ is related to $N = 2$ SUSY QED action:

$$f(\xi, \xi^i) S_{N=2\text{NLSUSY}} = S_{N=2\text{SUSYQED}} \equiv S_{\mathcal{V}\text{free}} + S_{\mathcal{V}f} + S_{\text{gauge}} \qquad (52)$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$.

- **NL/L SUSY relation bridges the cosmology and the low energy particle physics in NLSUSYGR scenario $\Longrightarrow$ Sec. 4.**

• **SGM scenario predicts the magnitude of the bare gauge coupling constant.**

For more general SUSY invariant constraints, i.e. foe NLSUSY vev of 0^+ auxiliary fiel

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \tag{53}$$

NL/L SUSY relation gives

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \; e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c}, \tag{54}$$

where e is the bare gauge coupling constant.

• **This mechanism is natural and favorable for SGM scenario as a theory of everything.**

Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.

3.3. $N = 3$ NL/L SUSY relation and SUSY Yang-MIlls theory

- **Physical helicity states of $N = 3$ LSUSY vector supermultiplet:**

$$\left[\ \underline{1}(+1), \underline{3}\left(+\frac{1}{2}\right), \underline{3}(0), \underline{1}\left(-\frac{1}{2}\right)\ \right] + [\text{CPT conjugate}], \tag{55}$$

where $\underline{n}(\lambda)$ means the dimension $\underline{n}$ and the helicity λ, are accomodated in $N = 3$ off-shell vector supermultiplet($d = 2$):

- $N = 3$ **superYang-Mills(SUSYYM) minimal off-shell gauge multiplet,**

$$\{v^{aI}(x), \lambda^{iI}(x), A^{iI}(x), {\chi_\alpha}^I(x), \phi^I(x), D^{iI}(x)\}, \quad (I = 1, 2, \cdots, dim.G) \tag{56}$$

Each component field belongs to the adjoint representation of the YM gauge group G:
$[T^I, T^J] = if^{IJK}T^K$ **and denoted as $\varphi^i = \varphi^{iI}T^I$, etc..**

- **$N=3$ (pure) SUSYYM action:**

$$S_{\rm SYM} = \int d^2x\ {\rm tr}\Big\{-\frac{1}{4}(F_{ab})^2+\frac{i}{2}\bar{\lambda}^i \not{D}\lambda^i+\frac{1}{2}(D_aA^i)^2+\frac{i}{2}\bar{\chi}\not{D}\chi+\frac{1}{2}(D_a\phi)^2+\frac{1}{2}(D^i)^2$$
$$-ig\{\epsilon^{ijk}A^i\bar{\lambda}^j\lambda^k-[A^i,\bar{\lambda}^i]\chi+\phi(\bar{\lambda}^i\gamma_5\lambda^i+\bar{\chi}\gamma_5\chi)\}$$
$$+\frac{1}{4}g^2([A^i,A^j]^2+2[A^i,\phi]^2)\Big\}, \tag{57}$$

where g is the gauge coupling constant, D_a and F_{ab} are the covariant derivative and the YM gauge field strength defined as

$$D_a\varphi = \partial_a\varphi - ig[v_a,\varphi],$$
$$F_{ab} = \partial_a v_b - \partial_b v_a - ig[v_a,v_b]. \tag{58}$$

• **SUSYYM action is invariant under $N=3$ LSUSY transformations:**

$$\begin{aligned}
\delta_\zeta v^a &= i\bar{\zeta}^i \gamma^a \lambda^i, \\
\delta_\zeta \lambda^i &= \epsilon^{ijk}(D^j - i\partial\!\!\!/ A^j)\zeta^k + \frac{1}{2}\epsilon^{ab}F_{ab}\gamma_5\zeta^i - i\gamma_5 \partial\!\!\!/ \phi \zeta^i \\
&\quad +ig([A^i, A^j]\zeta^j + \epsilon^{ijk}[A^j, \phi]\gamma_5\zeta^k), \\
\delta_\zeta A^i &= \epsilon^{ijk}\bar{\zeta}^j \lambda^k - \bar{\zeta}^i \chi, \\
\delta_\zeta \chi &= (D^i + i\partial\!\!\!/ A^i)\zeta^i + ig(\epsilon^{ijk}A^i A^j \zeta^k - [A^i, \phi]\gamma_5 \zeta^i), \\
\delta_\zeta \phi &= \bar{\zeta}^i \gamma_5 \lambda^i, \\
\delta_\zeta D^i &= -i\epsilon^{ijk}\bar{\zeta}^j \partial\!\!\!/ \lambda^k - i\bar{\zeta}^i \partial\!\!\!/ \chi + ig(\bar{\zeta}^i[\lambda^j, A^j] + \bar{\zeta}^j[\lambda^i, A^j] - \bar{\zeta}^j[\lambda^j, A^i] \\
&\quad -\epsilon^{ijk}\bar{\zeta}^j[\chi, A^k] + \epsilon^{ijk}\bar{\zeta}^j\gamma_5[\lambda^k, \phi] + \bar{\zeta}^i\gamma_5[\chi, \phi]),
\end{aligned} \tag{59}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P(\Xi^a) + \delta_G(\theta) + \delta_g(\theta), \tag{60}$$

where $\delta_G(\theta)$ means $\delta_G(\theta)\varphi = ig[\theta, \varphi]$ and $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i v_a - \epsilon^{ijk}\bar{\zeta}_1^i\zeta_2^j A^k - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- **SUSY invariant(composite) relations for $N = 3$ YM off-shell gauge supermultiplet**

$$
\begin{aligned}
v^{aI} &= -\frac{i}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma^a\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) + \frac{1}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_b(\bar{\psi}^j\gamma_5\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
\lambda^{iI} &= \epsilon^{ijk}\xi^{jI}\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) \\
&\quad +\frac{i}{2}\kappa^2\xi^{jI}\partial_a\{\epsilon^{ijk}\gamma^a\psi^k\bar{\psi}^l\psi^l + \epsilon^{ab}\epsilon^{jkl}(\gamma_b\psi^i\bar{\psi}^k\gamma_5\psi^l - \gamma_5\psi^i\bar{\psi}^k\gamma_b\psi^l)\} + \mathcal{O}(\kappa^4), \\
A^{iI} &= \kappa\left(\frac{1}{2}\xi^{iI}\bar{\psi}^j\psi^j - \xi^{jI}\bar{\psi}^i\psi^j\right)(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k) - \frac{i}{2}\kappa^3\xi^{iI}\partial_a(\bar{\psi}^i\gamma^a\psi^j\bar{\psi}^k\psi^k) + \mathcal{O}(\kappa^5), \\
\chi^I &= \xi^{iI}\psi^i(1 - i\kappa^2\bar{\psi}^j\partial\!\!\!/\psi^j) + \frac{i}{2}\kappa^2\xi^{iI}\partial_a(\gamma^a\psi^i\bar{\psi}^j\psi^j) + \mathcal{O}(\kappa^4), \\
\phi^I &= -\frac{1}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma_5\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\!\!\!/\psi^l) - \frac{i}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_a(\bar{\psi}^j\gamma_b\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
D^{iI} &= \frac{1}{\kappa}\xi^{iI}|w| - i\kappa\xi^{jI}\partial_a\{\bar{\psi}^i\gamma^a\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k)\} \\
&\quad -\frac{1}{8}\kappa^3\partial_a\partial^a\{(\xi^{iI}\bar{\psi}^j\psi^j - 4\xi^{jI}\bar{\psi}^i\psi^j)\bar{\psi}^k\psi^k\} + \mathcal{O}(\kappa^5), \qquad (61)
\end{aligned}
$$

• **Arbitrary real constants ξ^{iI} of auxirially fields D^{iI} bridge $N=3$ SUSY and the YM gauge group G.**

• **Substituting (61) into the SYM action (57), we can show the NL/L SUSY relation for $N=3$ SUSY:**

$$S_{\rm SUSYYM}(\psi) = -(\xi^{iI})^2 S_{\rm NLSUSY} + [\text{surface terms}]. \tag{62}$$

4. Low energy particle physics and Cosmology of NLSUSYGR

4.1. Low Energy Particle Physics of NLSUSYGR :

@ As we have seen that

$N=2$ SGM is essentially N=2 NLSUSY action in tangent(flat)) space-time,
we focus on N=2 NLSUSY action for extracting physical implications of SGM.

- **The low energy theorem for NLSUSY gives**

the following superon(massless NG fermion)-vacuum coupling

$$< \psi^j{}_\alpha(x)|J^{k\mu}{}_\beta|0 >= i\sqrt{\frac{c^4\Lambda}{8\pi G}}(\gamma^\mu)_{\alpha\beta}\delta^{jk} + \cdots, \tag{63}$$

where $J^{k\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}}\gamma^\mu\psi^k + \cdots$ is the conserved supercurrent.

$\sqrt{\frac{c^4\Lambda}{8\pi G}} = \frac{1}{\sqrt{2}\kappa}$ **is the coupling constant (g_{sv}) of superon with the vacuum.**

• **How to extract the vacuum configuration of SGM.**

In Riemann-flat space-time, NL/L SUSY relation(equivalence) gives:

$$L_{N=2\mathrm{SGM}} \longrightarrow L_{N=2\mathrm{NLSUSY}} + [\text{suface terms}] = L_{N=2\mathrm{SUSYQED}}. \tag{64}$$

• **We study vacuum structures of $N = 2$ LSUSY QED action** **in stead of $N = 2$ SGM.**

The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\mathrm{LSUSYQED}}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{\frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e|B^i|^2\right\} D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2. \tag{65}$$

• **Substituting the solution of the equation of motion for the auxiliary field D we obtain**

$$V(A,\phi,B^i)=\frac{1}{2}f^2\left\{A^2-\phi^2-\frac{e}{2f}|B^i|^2-\frac{\xi}{f\kappa}\right\}^2+\frac{1}{2}e^2(A^2+\phi^2)|B^i|^2\geq 0. \tag{66}$$

• **Two different types of vacua $V=0$ exist in (A,ϕ,B^i)-space:**

$$(I)\quad A=\phi=0,\quad |\tilde{B}^i|^2=-k^2\quad\left(\tilde{B}^i=\sqrt{\frac{e}{2f}}B^i,\quad k^2=\frac{\xi}{f\kappa}\right) \tag{67}$$

and

$$(II)\quad |\tilde{B}^i|^2=0,\quad A^2-\phi^2=k^2.\quad\left(\quad k^2=\frac{\xi}{f\kappa}\right) \tag{68}$$

• **Expansions of $A,\phi,\tilde{B}^i$ around vacuum values give**
low energy particles $\hat{A},\hat{\phi},\hat{B}^i$ in the true vacuum.

- **For the type (I) vacuum with $SO(2)$ symmetry for $(\tilde{B}^1, \tilde{B}^2)$,** $\quad e\xi < 0$**,**

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} \;=\; & \frac{1}{2}\{(\partial_a \rho)^2 - 2(-ef)k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a \hat{A})^2 + (\partial_a \hat{\phi})^2 - 2(-ef)k^2(\hat{A}^2 + \hat{\phi}^2)\} \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2 v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) + \cdots,
\end{aligned}
\tag{69}
$$

and following mass spectra

$$m_\rho^2 = m_{\hat{A}}^2 = m_{\hat{\phi}}^2 = m_{v_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa},$$
$$m_{\lambda^i} = m_\chi = m_\nu = 0. \qquad (70)$$

- The vacuum breaks both SUSY and the local $U(1)(O(2))$ spontaneously and Higgs-Kibble mechanism works.
- All bosons have the same mass and all fermions remain massless.
- λ^i are NG fermions.

• **For the type (II) vacuum with $SO(1,1)$ symmetry for (A,ϕ), e.g. $f\xi > 0$,**

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{(\partial_a \hat{A})^2 - 4f^2k^2\hat{A}^2\} \\
& +\frac{1}{2}\{|\partial_a \hat{B}^1|^2 + |\partial_a \hat{B}^2|^2 - e^2k^2(|\hat{B}^1|^2 + |\hat{B}^2|^2)\} \\
& +\frac{1}{2}(\partial_a \hat{\phi})^2 \\
& -\frac{1}{4}(F_{ab})^2 \\
& +\frac{1}{2}(i\bar{\lambda}^i \partial\!\!\!/ \lambda^i - 2fk\bar{\lambda}^i\lambda^i) \\
& +\frac{1}{2}\{i(\bar{\chi}\partial\!\!\!/ \chi + \bar{\nu}\partial\!\!\!/ \nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots . \qquad (71)
\end{aligned}
$$

and following mass spectra:

$$
\begin{aligned}
m^2_{\hat{A}} &= m^2_{\lambda^i} = 4f^2k^2 = \frac{4\xi f}{\kappa}, \\
m^2_{\hat{B}^1} &= m^2_{\hat{B}^2} = m^2_{\chi} = m^2_{\nu} = e^2k^2 = \frac{\xi e^2}{\kappa f}, \\
m_{v_a} &= m_{\hat{\phi}} = 0,
\end{aligned} \tag{72}
$$

which produces mass hierarchy by the factor $\frac{e}{f}$ independent of κ. $(\kappa^{-2} = \frac{c^4\Lambda}{16\pi G})$

- The vacuum breaks both **SUSY** and $SO(1,1)$ for (A,ϕ)

and restores(maintains) $SO(2)(U(1))$ for $(\tilde{B}^1, \tilde{B}^2)$, spontaneously,

which produces NG-Boson $\hat{\phi}$ and massless photon v_a
and gives soft masses $< A >$ to λ^i.

- **We have shown explicitly that**

N=2 LSUSY QED, i.e. the matter sector(Λ term) of $N = 2$ SGM (in flat-space), possesses a true vacuum type (II).

- **The resulting model describes**

lepton-Higgs-U(1) sector analogue of SM:

one massive charged Dirac fermion ($\psi_D{}^c \sim \chi + i\nu$),
one massive neutral Dirac fermion ($\lambda_D{}^0 \sim \lambda^1 - i\lambda^2$),
one massless vector (a photon) (v_a),
one charged scalar ($\hat{B}^1 + i\hat{B}^2$),
one neutral complex scalar ($\hat{A} + i\hat{\phi}$),

which are composites of superons.

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
as composites of NG fermion
from
the NLSUSY cosmological term of SGM.

4.2 Cosmological implications of SGM scenario

The variation of SGM action $L_{\rm SGM}(e,\psi)$ with respect to $e^a{}_\mu$ yields
Einstein equation equipping with matter and cosmological term:

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4}\{\tilde{T}_{\mu\nu}(e,\psi) - g_{\mu\nu}\frac{c^4\Lambda}{8\pi G}\}. \tag{73}$$

where $\tilde{T}_{\mu\nu}(e,\psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

- **Note that the cosmological term $-\frac{c^4\Lambda}{8\pi G}$ can be interpreted as the negative energy density of space-time, i.e. the dark energy density ρ_D.**

• **Big collapse may induce 3 dimensional expansion of space-time by Pauli principle:**

$$ds^2 = s_{\mu\nu}(x)dx^\mu dx^\nu = \{g_{\mu\nu} + \Phi_{\mu\nu}(e,\psi)\}dx^\mu dx^\nu.$$
$$\{\psi(x), \bar{\psi}(y)\} = 0 \;\Rightarrow\; \{\psi(x), \bar{\psi}(y)\} = \delta^{(3)}(\mathbf{x}-\mathbf{y})$$

• **Big Collapse produces composite (massless) eigen states of SO(N) sP angebra due to the universal gravitational force,**

which is the ignition of Big Bang(BB) SM scenario.

• **In the composite SGM view of $N=2$ LSUSY QED, the vacuum (II) may explain naturally observed mysterious (numerical) relations:**

$$\text{縲}\; dark\ energy\ density\ \rho_D \sim O(\kappa^{-2}) \sim {m_\nu}^4 \sim (10^{-12}GeV)^4 \sim {g_{sv}}^2,$$

provided ${\lambda_D}^0$ is identified with neutrino and $f\xi \sim O(1)$.

• **In this case, one neutral scalar new particle with mass $\sim O(m_\nu)$ exists.**
A candidate of dark matter.

5. Nonlinear vector-spinor SUSY GR

- **New SUSY algebra containing spinor-vector generators Q^μ_α:**

$$\{Q^\mu_\alpha, Q^\nu_\beta\} = \varepsilon^{\mu\nu\lambda\rho} P_\lambda (\gamma_\rho \gamma_5 C)_{\alpha\beta}, \tag{74}$$

$$[Q^\mu_\alpha, P^\nu] = 0, \tag{75}$$

$$[Q^\mu_\alpha, J^{\lambda\rho}] = \frac{1}{2}(\sigma^{\lambda\rho} Q^\mu)_\alpha + i\eta^{\lambda\mu} Q^\rho_\alpha - i\eta^{\rho\mu} Q^\lambda_\alpha, \tag{76}$$

where Q^μ_α are vector-spinor generators satisfying Majorana condition $Q^\mu_\alpha = C_{\alpha\beta}\overline{Q}^\mu_\alpha$.

- **Consider the following global (3/2 super)translations:**

$$\psi^a_\alpha \longrightarrow \psi^a_\alpha + \zeta^a_\alpha. \tag{77}$$

$$x_a \longrightarrow x_a + i\kappa\varepsilon_{abcd}\bar{\psi}^b \gamma^c \gamma_5 \zeta^d, \tag{78}$$

where ζ^a_α is a constant Majorana tensor-spinor parameter.

- **The invariant differential forms become:**

$$\omega_a = dx_a + i\kappa\varepsilon_{abcd}\bar{\psi}^b \gamma^c \gamma_5 d\psi^d. \tag{79}$$

- **Invariant action of nonlinear representation of vector-spinor SUSY:**

$$S = \frac{1}{\kappa}\int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \frac{1}{\kappa}\int \det w_{ab} d^4x, \tag{80}$$

$$w_{ab} = \delta_{ab} + t_{ab}, \quad t_{ab} = i\kappa\varepsilon_{acde}\bar{\psi}^c\gamma^d\gamma_5\partial_b\psi^e, \tag{81}$$

- **By similar geometrical arguments to SGM we obtain vector-spinor NLSUSY GR:**

$$L_{vsNLSUSYGR} = -\frac{c^3}{16\pi G}|w|\{\Omega(w^a{}_\mu) + \Lambda\}, \tag{82}$$

$$|w| = det w^a{}_\mu = det(e^a{}_\mu + t^a{}_\mu), \tag{83}$$

- **Unified vierbein become:**

$$w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x), \quad t^a{}_\mu(x) = i\kappa\varepsilon^{abcd}\bar{\psi}_b\gamma_c\gamma_5\partial_\mu\psi_d, \tag{84}$$

- $L_{vsNLSUSYGR}$ **possesses similar symmetry properties as SGM.**

6. Summary

NLSUSYGR(SGM) scenario for unity of nature:

• **Ultimate entity; New unstable $d = 4$ space-time U:$[x^a, \psi_\alpha{}^N; x^\mu]$ described by** $[L_{\rm NLSUSYGR}(w^a{}_\mu)]$ **: NLSUSYGR on New space-time with** $\Lambda > 0$

• **Mach principle is encoded geometrically**

$\Longrightarrow$ **Big Collapse (due to false vacuum $V_{\rm P.E.} = \Lambda > 0$) to** $[L_{\rm SGM}(e.\psi)]$**;**

• **The creation of Riemann space-time $[x^a; x^\mu]$ and massless fermionic matter** $[\psi_\alpha{}^{\mathbf{N}}]$

$[L_{\rm SGM} = L_{\rm EH}(e) - \Lambda + T(\psi.e)]$ **: Einstein GR with $V_{\rm P.E.} = \Lambda > 0$ and N superon**

• **Phase transition towards the true vacuum $V_{\rm P.E} = 0$, achieved by forming composite massless LSUSY and subsequent oscilations around the true vacuum.**

$\Longrightarrow$ **Ignition of Big Bang Universe** $\Longrightarrow$ **(MS)SM**

• **In flat space-time, broken N-LSUSY theory emerges from the N-NLSUSY cosmological term of $L_{\rm SGM}(e, \psi)$ [NL/L SUSY relation].** $\longleftrightarrow$ **BCS vs GL**

The cosmological constant is the origin of everything!

Predictions and Speculations:

@SO(10) sP algebra with $\underline{10} = \underline{5}_{\rm SU(5)GUT} + \underline{5}^*_{\rm SU(5)GUT}$ SGM:

- **New $1^{\rm C}$ state besides SM particles:**

One neutral massive vector boson S.

spin 1/2 double-charge fermion $E^{2\pm}$

- **Proton decay diagrams of SU(5) GUT in SGM view are forbidden by superon selection rule. $\Rightarrow$ stable proton**

@Field theory via Linearization:

- **NLSUSYGR(SGM) scenario predicts 4 dimensional space-time.**
- **The bare gauge coupling constant is determined.**
- **N-LSUSY from N-NLSUSY $\Longleftrightarrow$ superon-quintet hypothesis for all particles**

cosmological term $\leftrightarrow$ dark energy density $\leftrightarrow$ SUSY Br. $\rightarrow m_\nu$

Many Open Questions ! e.g.,

- Mathematical and physical analysis of $L_{NLSUSY}(w) = L_{SGM}(e, \psi)$

- Direct linearization of SGM action, i.e.
NL/L SUSY relation in curved space-time.
(Find the broken SUGRA-like(?) equivalent theory?)
- Superfield systematics of NL/L SUSY relation for SGM action.

- Revisit unsolved problems of SMs and GUT from SQM composite viewpoints.
e.g.,
(e, ν_e): $\delta^{ab} Q_a {Q^*}_b Q_m$, (μ, ν_μ):$\delta^{ab} Q_a {Q^*}_b \epsilon^{lm} Q_l Q_m Q^*_n$, (τ, ν_τ): $\epsilon^{abc} Q_b Q_c \epsilon_a{}^{bc}\ {Q^*}_b Q^*_c Q_m$
(u, d):$\epsilon^{abc} Q_b Q_c Q_m$, (c, s): $\epsilon^{abc} Q_a Q_b Q_c {Q^*}_d Q_m$ (t, b) $\epsilon^{lm} Q_l Q_m \epsilon^{abc} Q_b Q_c {Q^*}_n, \cdots$

- Physical consequences of spin $\frac{3}{2}$ NLSUSYGR.

@ [Ref.] K. Shima, Plenary talk at Conference on Cosmology, Gravitational Waves and Particles, 6-10, January, 2017, NTU, Singapore. Proceeding of CCGWP, ed. Harald Fritzsch, (World Scientific, Singapore, 2017), 301.