Deviations in **B** Physics and their Implications

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1 Comments on significance

- 2 $b \rightarrow s$ anomalies
- 3 $b \rightarrow c$ anomalies
- 4 Combined explanations

Recap: *B* physics anomalies Talk by U. Nierste

1. $R_{D^{(*)}}$ anomalies: b
ightarrow c au v vs. $b
ightarrow c(e, \mu) v$

$$R_{D^{(*)}} = \frac{\mathsf{BR}(B \to D^{(*)}\tau v)}{\mathsf{BR}(B \to D^{(*)}\ell v)}$$

2. $R_{K^{(*)}}$ anomalies: $b \rightarrow s \mu \mu$ vs. $b \rightarrow s e e$

$$\mathsf{R}_{\mathcal{K}^{(*)}} = \frac{\mathsf{BR}(B \to \mathcal{K}^{(*)}\mu^+\mu^-)}{\mathsf{BR}(B \to \mathcal{K}^{(*)}e^+e^-)}$$

- **3.** $b \rightarrow s \mu \mu$ anomalies
- Angular observables in $B \to K^* \mu^+ \mu^-$
- ▶ Branching ratios in $B \to K^* \mu^+ \mu^-$, $B \to K \mu^+ \mu^-$, $B_s \to \phi \mu^+ \mu^-$

How significant are the "anomalies"?

Statistically sensible questions to ask:

- 1. What is the siginificance of the deviation from the SM prediction in a single observable (pair of observables)?
 - is the uncertainty dominated by statistics, systematics, theory?
- 2. What is the likelihood ratio between a given new physics hypothesis and the SM?
 - model-independent new physics hypotheses using effective field theories

Significance of $R_{D^{(*)}}$



- HFLAV quotes 4.1σ combined
- Theory uncertainties due to form factors Bernlochner et al. 1703.05330, Grinstein and Kobach 1703.08170, Bigi et al. 1707.09509 & QED Boer et al. 1803.05881 under scrutiny, but very small

Significance of $R_{K^{(*)}}$



- 3.1σ combined (only using 1–6 GeV² bins here)
- ► Theory uncertainties due to QED Bordone et al. 1605.07633 completely negligible

Significance of $b \rightarrow s \mu \mu$ anomalies





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Significance of $b \rightarrow s \mu \mu$ anomalies



More complicated

- Several modes, many bins, sizable correlations
- Larger theory uncertainties
- Need new physics hypothesis (EFT) to make statement about significance

Significance of $b \rightarrow s \mu \mu$ anomalies



More complicated

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- Larger theory uncertainties
- Need new physics hypothesis (EFT) to make statement about significance

But: this is not all "dirty" stuff to be swept under the rug!

$B \rightarrow K^{(*)}\mu^+\mu^-$: theoretical challenges



Form factors

- Require non-perturbative calculation, e.g. lattice or light-cone sum rules (LCSR)
 - Good agreement between lattice and LCSR Horgan et al. 1501.00367, Bharucha et al. 1503.05534



"Non-factorisable" hadronic effects

- ► Problematic since operators like $(\bar{c}_L \gamma^\mu b_L)(\bar{s}_L \gamma^\mu c_L)$ generated by *tree-level W* exchange
 - ► analyticity + experimental data on $b \rightarrow sc\bar{c}$ allow to constrain this Khodjamirian et al. 1006.4945, Bobeth et al. 1707.07305, Blake et al. 1709.03921

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Effective theory

NP effects model-independently described by modification of Wilson coefficients of dim.-6 operators

$$\mathcal{H}_{eff} = -\frac{4 \, G_F}{\sqrt{2}} \frac{e^2}{16\pi^2} V_{tb} V_{ts}^* \sum_i C_i O_i + \text{h.c.}$$

$$\mathcal{O}_{7}^{(\prime)} = \underbrace{P_{L(R)}}_{S_{L(R)}} P_{R(L)} P_{S_{10}} = \underbrace{P_{L(R)}}_{S_{L(R)}} P_{L(R)} P_{L(R)} P_{L(R)} P_{L(R)} P_{L(R)} P_{L(R)} P_{R(L)} P_{L(R)} P_{R(L)} P_{L(R)} P_{R(L)} P_{L(R)} P_{R(L)} P_{L(R)} P_{R(L)} P_{L(R)} P_$$

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Model-independent fit to $C_{9,10}^{(\prime)}$ Altmannshofer et al. 1703.09189

Construct a likelihood as function of $C_{9,10}^{(\prime)}$, using measurements of

- ▶ Angular observables in $B^0 \to K^{*0} \mu^+ \mu^-$ (CDF, LHCb, ATLAS, CMS)
- ▶ $B^{0,\pm}
 ightarrow K^{*0,\pm} \mu^+ \mu^-$ branching ratios (CDF, LHCb, CMS)
- ▶ $B^{0,\pm} \to K^{0,\pm} \mu^+ \mu^-$ branching ratios (CDF, LHCb)
- ▶ $B_{s}
 ightarrow arphi \mu^{-}$ branching ratio (CDF, LHCb)
- ▶ $B_s \rightarrow \phi \mu^+ \mu^-$ angular observables (LHCb)
- $B \rightarrow X_s \mu^+ \mu^-$ branching ratio (BaBar)

NB, $R_K \& R_{K^*}$ not used as constraints (yet)!

1D results

Coeff.	best fit	1σ	2σ	pull
$C_9^{\sf NP}$	-1.21	[-1.41, -1.00]	[-1.61, -0.77]	5.2σ
C'9	+0.19	[-0.01, +0.40]	[-0.22, +0.60]	0.9σ
$C_{10}^{\rm NP}$	+0.79	[+0.55, +1.05]	[+0.32, +1.31]	3.4σ
C'_{10}	-0.10	[-0.26, +0.07]	[-0.42, +0.24]	0.6σ
$C_9^{NP}=C_{10}^{NP}$	-0.30	[-0.50, -0.08]	[-0.69, +0.18]	1.3σ
$C_9^{NP} = -C_{10}^{NP}$	-0.67	[-0.83, -0.52]	[-0.99, -0.38]	4.8σ
$C_9^\prime=C_{10}^\prime$	+0.06	[-0.18, +0.30]	[-0.42, +0.55]	0.3σ
$C_{9}' = -C_{10}'$	+0.08	[-0.02, +0.18]	[-0.12, +0.28]	0.8σ

$$pull \equiv \sqrt{x_{SM}^2 - x_{best fit}^2} \qquad (for 1D)$$

Comment on significance

In a pure EFT analysis, discussing number of degrees of freedom or "look-elsewhere effect in theory space" is completely pointless.

Likelihood ratio is well-defined, everything else is neither basis nor scale invariant.

For the same reason, more sophisticated statistical tools like Bayes factors are of little use here.

What you should care about:

- 1. Is the likelihood ratio of the best-fit NP hypothesis in the space of EFT large?
- 2. Is the resulting NP hypothesis credible from a UV (model building) point of view?

2D results



- ▶ best fit $(C_9^{NP}, C_{10}^{NP}) = (-1.15, +0.26)$
- pull 5.0σ

Compatible with fits performed by others:

Geng et al. 1704.05446, Capdevila et al. 1704.05340, Mahmoudi et al. 1611.05060, Ciuchini et al. 1704.05447

Semi-leptonic vs. charming new physics



Operators like

 $O_{VLL}^{bscc} = (\bar{s}_L \gamma^\mu b_L) (\bar{c}_L \gamma_\mu b_L)$

can induce a (LFU!) shift in C_9 (and in addition have a q^2 -dependent matrix element)

Jäger et al. 1701.09183, Talk by S. Jäger

Fit to $b \rightarrow s\mu\mu$ observables (no $R_{K^{(*)}}!$) equally good

Predictions for LFU tests

Using the model-independent fit to $b \to s\mu^+\mu^-$ observables and *assuming* the corresponding $b \to se^+e^-$ observables to be free from NP, can *predict* LFU ratios $R_X = BR(B \to X\mu\mu)/BR(B \to Xee)$



LFU fit Altmannshofer et al. 1704.05435

Now, fit Wilson coefficients of lepton flavour dependent operators:

$$O_{9}^{(\prime)\ell} = (\bar{s}\gamma_{\mu}P_{L(R)}b)(\bar{\ell}\gamma^{\mu}\ell) \qquad O_{10}^{(\prime)\ell} = (\bar{s}\gamma_{\mu}P_{L(R)}b)(\bar{\ell}\gamma^{\mu}\gamma_{5}\ell)$$
(1)

Observables:

- ► R_K (LHCb)
- ▶ *R_{K*}* (LHCb)
- ► $D_{P'_{4,5}} = P'_{4,5}(B \to K^* \mu \mu) P'_{4,5}(B \to K^* ee)$ (Belle)

These observables/measurements are *disjoint* from the ones used in the $b \rightarrow s\mu\mu$ fit!

1D results

Coeff.	best fit	1σ	2σ	pull
C_9^μ	-1.59	[-2.15, -1.13]	[-2.90, -0.73]	4.2σ
C^{μ}_{10}	+1.23	[+0.90, +1.60]	[+0.60, +2.04]	4.3σ
C_9^e	+1.58	[+1.17, +2.03]	[+0.79, +2.53]	4.4σ
C_{10}^{e}	-1.30	[-1.68, -0.95]	[-2.12, -0.64]	4.4σ
$C_9^\mu=-C_{10}^\mu$	-0.64	[-0.81, -0.48]	[-1.00, -0.32]	4.2σ
$C_{9}^{e} = -C_{10}^{e}$	+0.78	[+0.56, +1.02]	[+0.37, +1.31]	4.3σ
$C_{9}^{\prime \mu}$	-0.00	[-0.26, +0.25]	[-0.52, +0.51]	0.0σ
$C_{10}^{\prime \mu}$	+0.02	[-0.22, +0.26]	[-0.45, +0.49]	0.1σ
C' ^e	+0.01	[-0.27, +0.31]	[-0.55, +0.62]	0.0σ
$C_{10}^{\prime e}$	-0.03	[-0.28, +0.22]	[-0.55, +0.46]	0.1σ

$$\mathsf{pull} \equiv \sqrt{x_{\mathsf{SM}}^2 - x_{\mathsf{best fit}}^2} \quad (\mathsf{for 1D})$$

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2D results: $R_{K^{(*)}}$ vs. $b \rightarrow s \mu \mu$



► *Perfect* agreement between best-fit region of $b \rightarrow s\mu\mu$ fit and region preferred by $R_{K^{(*)}}$ fit

$R_{K^{(*)}}$ model degeneracies



• Impossible to distinguish different best-fit scenarios on the basis of $R_{K^{(*)}}$ alone

Predictions for angular observables

$$D_{P'_{4,5}} = P'_{4,5}(B o K^* \mu \mu) - P'_{4,5}(B o K^* ee)$$



► $D_{P'_{4,5}}$ can clearly distinguish hypotheses with C_9 vs. C_{10}

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Effective theory for $b \rightarrow c \tau v$

$$\mathcal{H}_{eff} = rac{4G_F}{\sqrt{2}} V_{cb} \left(O_{V_L} + \sum_i C_i O_i + \mathrm{h.c.} \right)$$

$$\begin{aligned} O_{V_L} &= (\bar{c}_L \gamma^\mu b_L) (\bar{\ell}_L \gamma_\mu v_{\tau L}) & O_{S_R} &= (\bar{c}_L b_R) (\bar{\ell}_R v_{\tau L}) & O_T &= (\bar{c}_R \sigma^{\mu\nu} b_L) (\bar{\ell}_R \sigma_{\mu\nu} v_{\tau L}) \\ O_{V_R} &= (\bar{c}_R \gamma^\mu b_R) (\bar{\ell}_L \gamma_\mu v_{\tau L}) & O_{S_L} &= (\bar{c}_R b_L) (\bar{\ell}_R v_{\tau L}) \end{aligned}$$

- O_{V_R} is LFU at dimension 6 in SMEFT (can only arise from modification of $\bar{c}_R b_R W$ vertex) \Rightarrow ignore
- Ignoring $b \rightarrow c \tau v_{e,\mu}$ for simplicity (contributions relevant in concrete models!)

Constraint from $B_c \rightarrow \tau v$



- Can be strongly enhanced by scalar operators
- sensitive to $C_{S_R} C_{S_L}$
- Even though the decay has not been measured or searched for, theoretical arguments allow to constrain $BR(B_c \rightarrow \tau \nu) \lesssim 0.3$ Li et al. 1605.09308, Alonso et al. 1611.06676
- ► Reinterpreting an old LEP1 search for $B^+ \rightarrow \tau v$ allows to constrain BR($B_c \rightarrow \tau v$) $\lesssim 0.1$ Akeroyd and Chen 1708.04072

Model-independent fit to $b \rightarrow c \tau v$



Fit to R_D , R_{D^*} , $B_c \rightarrow \tau v$

► Not a full fit: $d\Gamma/dq^2$, τ pol., $R_{J/\psi}$ missing

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Combined explanations: SMEFT considerations

- ► Heavy NP must respect SU(2)_L × U(1)_Y gauge invariance ⇒ D = 6 SMEFT (ignoring non-linear HEFT) Alonso et al. 1407.7044, Aebischer et al. 1512.02830, ...
- ▶ Only considering operators that affect $b o s\mu^+\mu^-$, $b o c\tau v$, violate LFU

b
$$ightarrow$$
 s $\mu^+\mu^-$

- ▶ $[C_{lq}^{(1)}]^{2223} \rightarrow C_9 = -C_{10}$
- ▶ $[C_{lq}^{(3)}]^{2223} \to C_9 = -C_{10}$
- ▶ $[C_{ld}]^{2223} \to C_9 = C_{10}$

through RG mixing:

• $[C_{Iu}]^{2233} \rightarrow C_9 = -C_{10}$ Celis et al. 1704.05672

 $m{b}
ightarrow m{c} m{v} m{v}$

- $[C_{lq}^{(3)}]^{33i3} \to C_{V_L}$
- $\blacktriangleright \ [C_{ledq}]^{333i*} \to C_{S_R}$
- $\blacktriangleright \ [C^{(1)}_{lequ}]^{333i} \to C_{S_L}$
- ▶ $[C_{lequ}^{(3)}]^{333i} \to C_T$

through RG mixing: no qualitative change González-Alonso et al. 1706.00410

(Using "Warsaw" operator basis Grzadkowski et al. 1008.4884 + weak basis where $M_{d,l}$ are diagonal)

Tree-level models



Single-mediator tree-level models

Model	$C_{lq}^{(1)}$	$C_{lq}^{(3)}$	C_{qe}	C _{lu}	C _{ledq}	$C_{lequ}^{(1)}$	$C_{lequ}^{(3)}$
Ζ′	×		×	×			
V'		×					
H'					×	×	
S ₁	×	×				×	\times
R_2			×	\times		\times	\times
S ₃	×	×					
U_1	×	×			×		
U_3	\times	×					
V_2			×		×		
ν̃2				\times			

Hiller and Schmaltz 1408.1627, Gripaios et al. 1412.1791, Greljo et al. 1506.01705 Bauer and Neubert 1511.01900, Medeiros Varzielas and Hiller 1503.01084 Bečirević and Sumensari 1704.05835 Barbieri et al. 1512.01560 Fajfer and Košnik 1511.06024, ... many others (sorry)

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Generic constraints: *B*_s mixing



Forces Z' (or vector triplet) models into regime with strong couplings to muons

 $g_{bsZ'}/m_{Z'} \lesssim 0.01/(2.5\,\mathrm{TeV})$

► Upper bound on Z' mass

Altmannshofer and Straub 1308.1501

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Generic constraints: $pp \rightarrow \mu\mu, \tau\tau$

$$\begin{array}{c} q & \mu, \tau \\ \hline \\ q & \mu, \tau \end{array}$$

- Resonances searches
- contact interaction searches

Altmannshofer and Straub 1411.3161, Faroughy et al. 1609.07138, Greljo and Marzocca 1704.09015

Generic constraints: Neutrino trident



Altmannshofer et al. 1403.1269, Altmannshofer et al. 1406.2332

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Generic constraints: $B \rightarrow K v \bar{v}$



- ► S_1 , S_3 , U_3 leptoquarks generate $b \rightarrow sv\bar{v}$ at tree level Buras et al. 1409.4557
- ► *B* factory searches constrain BR $(B \rightarrow K v \bar{v})/BR(B \rightarrow K v \bar{v})_{SM} \lesssim 3$

- $\blacktriangleright S_1: C_L^{\nu_\mu} \gg C_9^{\mu}$
- $S_3: C_L^{\nu_{\mu}} = \frac{1}{2}C_9^{\mu}$
- $U_3: C_L^{v_{\mu}} = 2C_9^{\mu}$

- Particularly problematic if R_{D(*)} should be explained: large contributions to b → sv_τv

 ¯_τ, b → sv_μv

 ¯_τ, b → sv_τv

 ¯_μ
- Possible to suppress using cancellation between S₁ and S₃ contribution Crivellin et al. 1703.09226

Generic constraints: μ -e LFU violation in $b \rightarrow c \ell v$



- Measured precisely at the *B* factories (measures the CKM element V_{cb})
- Recently, generic NP analysis allowing for LFUV

$$\tilde{V}^{e}_{cb}/\tilde{V}^{\mu}_{cb} = 1.01 \pm 0.03$$

► scalar or tensor operators in b → c(e, µ)v very strongly constrained when considering differential distributions

Jung and Straub 1801.01112

How to write a flavour anomaly paper

The tool used for all numerics in this talk is open source:



- Documentation: https://flav-io.github.io/
- Code: https://github.com/flav-io/flavio

Main goal: lower the barrier between model building and flavour pheno

New developments that allow easier combination with other codes, electroweak, precision tests, etc.:

Wilson coefficient exchange format Aebischer et al. 1712.05298

wilson: completely general RG running & matching in SMEFT, QED, QCD Aebischer et al. 1804.05033

Example

- Start with $(C_{lq}^{(1)})_{3323} = (C_{lq}^{(3)})_{3323} = 0.1/\text{TeV}^2$ at $\mu = 1$ TeV (motivated by the scalar LQ scenario)
- Compute R_{D^*} and the constraint from $BR(B \rightarrow Kv\bar{v})$
- SMEFT, QCD, and QED running and matching performed automatically
- SMEFT RG effects spoil cancellation and induce B → Kvv radiatively

```
flavio.np_prediction('Rtaul(B->D*lnu)', w)
flavio.np_prediction('BR(B+->Knunu)', w)
```

Conclusions

- Several significant anomlies in B physics
 - $R_{D^{(*)}}$: τ - ℓ LFUV in $b \rightarrow c\ell v @ 4\sigma_{exp}$
 - $R_{K^{(*)}}$: μ -e LFUV in $b \rightarrow s\ell\ell @ 4\sigma_{exp}$
 - More deviations in $b \rightarrow s \mu \mu$ (could be LFUV or not) @ $5\sigma_{exp+theo}$
- $R_{K^{(*)}}$ and $b\mu\mu$ anomalies fit perfectly together
- Common new physics explanation of b → s and b → c anomalies possible. Single tree-level mediator: leptoquark U₁ preferred
- Time for model building to lend credibility (or not) to new physics explanations
 - Public tools are available!
- Experimental prospects bright: LHCb Run-II data on tape, Belle-II on the horizon



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Backup

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LHCP 2014: *R*_K



$$R_{K} = \frac{\mathsf{BR}(B \to K\mu^{+}\mu^{-})_{[1,6]}}{\mathsf{BR}(B \to Ke^{+}e^{-})_{[1,6]}}$$
$$= 0.745^{+0.090}_{-0.074} \pm 0.036$$

2.4σ





Easter 2017: *R*_{K*}



$$R_{K^*} = rac{{\sf BR}(B o K^*\mu^+\mu^-)}{{\sf BR}(B o K^*e^+e^-)}$$

2.2 & 2.4σ

Cartoon: q^2 dependence of $B \to K^* \ell^+ \ell^-$



2D results



best fit $(C_9^{NP}, C_9') = (-1.25, +0.59)$

pull 5.3σ

Impact of enlarging uncertainties



Doubling form-factor or "non-factorizable" hadronic uncertainties:

- Significance decreases but stays well above 3σ
- best-fit point hardly affected

Impact of enlarging uncertainties



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q^2 dependence of C_9 best-fit



- NP in C_9 would give helicity and q^2 independent effect
- ▶ NP in $b \rightarrow c\bar{c}s$ would give helicity independent but q^2 dependent effect
- hadronic effect could be helicity and q² dependent

$$C_{9}^{\mu}$$
 vs. $C_{9}^{\prime\mu}$



 Right-handed currents not favoured by data

$$C_{9}^{\mu}$$
 vs. C_{9}^{e}



NP in b → se⁺e⁻ not required by data (but not excluded either!)

Right-handed currents (LFU)



• Differential/angular distributions in $B \rightarrow D^* \ell v$ alone allow to exclude large RHC

Scalar operators: endpoint effect

 $\blacktriangleright\,$ At $q^2
ightarrow q^2_{
m max}$, SM & scalar contribution have behave differently:

$$\frac{d\Gamma(B \to D\ell v)}{dq^2} \big|_{\rm SM} \propto f_+^2 \left(q^2 - q_{\rm max}^2\right)^{3/2} \qquad \frac{d\Gamma(B \to D\ell v)}{dq^2} \big|_{\mathcal{C}_{S_{L,R}}} \propto f_0^2 |\mathcal{C}_{S_R} + \mathcal{C}_{S_L}|^2 \left(q^2 - q_{\rm max}^2\right)^{1/2}$$

Last bin is extremely sensitive to scalar operators (much more than total rate!) cf. Nierste et al. 0801.4938, Hiller and Zwicky 1312.1923



Scalar operators

Fit to C_{S_R} and $\tilde{V}_{cb} = V_{cb}(1 + C_{V_L})$ (as e.g. in U_1 and V_2 LQ models)

- Large effects excluded by $B \rightarrow D\ell v$ due to endpoint sensitivity!
- $B \rightarrow D\ell v$ stronger than B_c lifetime constraint



Scalar operators

- C_{S_R} vs. C_{S_L} (e.g. charged Higgs)
- ► slight preference for non-standard values $C_{S_R}^{\mu} \sim -C_{S_L}^{\mu}$ in muons (but large values in conflict with B_c lifetime)



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Tensor operator: endpoint effect

▶ At $q^2 \rightarrow 0$, SM contribution to $B \rightarrow D^* \ell v$ is fully longitudinal, tensor contribution isn't

$$\frac{d\Gamma_{T}(B \to D^{*} \ell v)}{dq^{2}} \propto q^{2} C_{V_{L}}^{2} \left(A_{1}(0)^{2} + V(0)^{2}\right) + 16m_{B}^{2} C_{T}^{2} T_{1}(0)^{2} + O\left(\frac{m_{D^{*}}^{2}}{m_{B}^{2}}\right)$$

First bin of Γ_T is extremely sensitive to C_T (much more than total rate!)



Fit: scalar vs. tensor operator

- Fit to C_{S_l} and C_T
- $C_{S_L} = +4C_T$ predicted at matching scale by R_2 , $C_{S_L} = -4C_T$ by S_1
- ▶ $B \rightarrow D\ell v$ and $B \rightarrow D^* \ell v$ nicely complementary due to endpoint effects



Tree-level models



Tree-level models

Model	C_{V_L}	C_{V_R}	C_{S_R}	C_{S_L}	C_T	$C_{S_L} = 4C_T$	$C_{S_L} = -4C_T$
U, D (v-like fermions)	×						
Q (v-like fermion)		×					
W' (heavy W)	×						
H^{\pm} (charged Higgs)			×	\times			
S ₁ (scalar LQ)	\times						×
R_2 (scalar LQ)						×	
S_3 (scalar LQ)	\times						
U_1 (vector LQ)	\times		×				
V_2 (vector LQ)			×				
U_3 (vector LQ)	×						