Neutrino masses from Planck-scale lepton number breaking

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Introduction

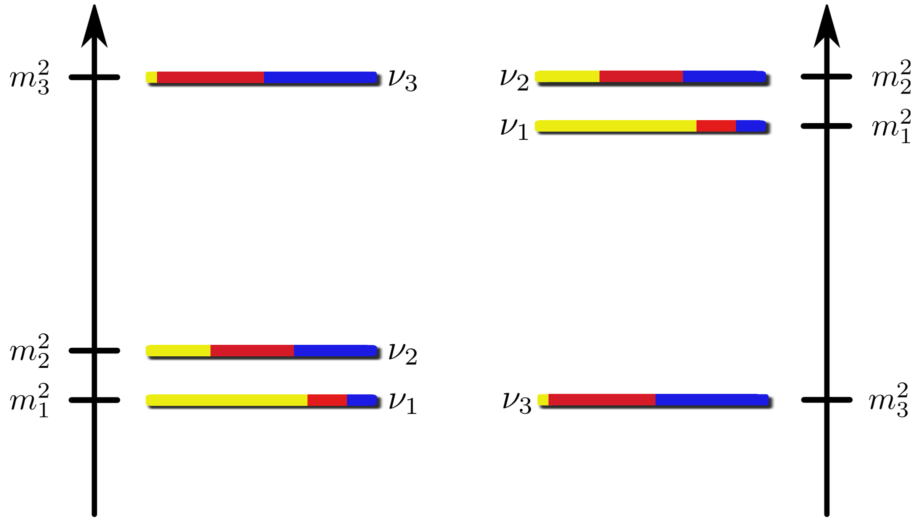
Neutrinos are massive. (massless in the Standard Model)

- Neutrino oscillation data

	NH	IH
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.306^{+0.012}_{-0.012}$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.587^{+0.020}_{-0.024}$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.02179^{+0.00076}_{-0.00076}$
Δm^2_{21} [eV^2]	$7.50^{+0.19}_{-0.17}\times 10^{-5}$	$7.50^{+0.19}_{-0.17}\times 10^{-5}$
$\Delta m^2_{3\ell}$ [eV^2]	$2.524^{+0.039}_{-0.040}\times 10^{-3}$	$-2.514^{+0.038}_{-0.041}\times 10^{-3}$

Esteban et al. JHEP (2017)

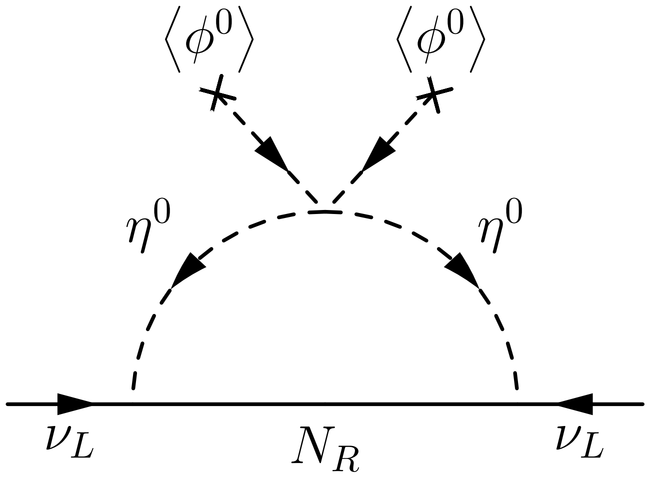
- Very small masses of neutrinos and large mixing angles.
- Mild hierarchy of two heaviest masses ~ 6.
- $\Rightarrow$ different mechanism of mass generation?



Introduction

Neutrino mass generation mechanisms

- Type-I, II, III seesaw mechanism **Minkowski, Yanagida, et al. (1977)**
- Inverse seesaw, Linear seesaw mechanisms, radiative generation of neutrino masses etc
  A. Zee (1980), K.S. Babu (1988), E. Ma, PRD (2006) etc



- Ex. Type-I seesaw: $m_\nu \approx -m_D M^{-1} m_D^T$ diagonalized by $U_{\rm PMNS}$
- We will show that radiative effects are important when right-handed neutrinos are very hierarchical.
  cf: split seesaw. **A. Kusenko, F. Takahashi, T. Yanagida, Phys.Lett. B (2010)**
  $M_1 \sim$ keV, and $M_2, M_3 \sim 10^{12}$ GeV
- No intermediate scale

The Model (Type-I seesaw)

- Add three right-handed neutrinos.

$$\mathcal{L} = \frac{1}{2}\overline{N_i}\partial\!\!\!/ N_i - \frac{M_{ij}}{2}\overline{N_i^c}N_j - (Y_\nu)_{ij}\,\tilde{H}\overline{L_i}N_j + \mathrm{H.c.}$$

- Assumption: rank-1 mass matrix at the Planck scale.

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \qquad M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

at Planck scale   at Electroweak scale

- M_1 and M_2 are generated by radiative effect.
  $\Rightarrow$ Renormalization group equation (RGE) for M.

Renormalization Group Equation for M

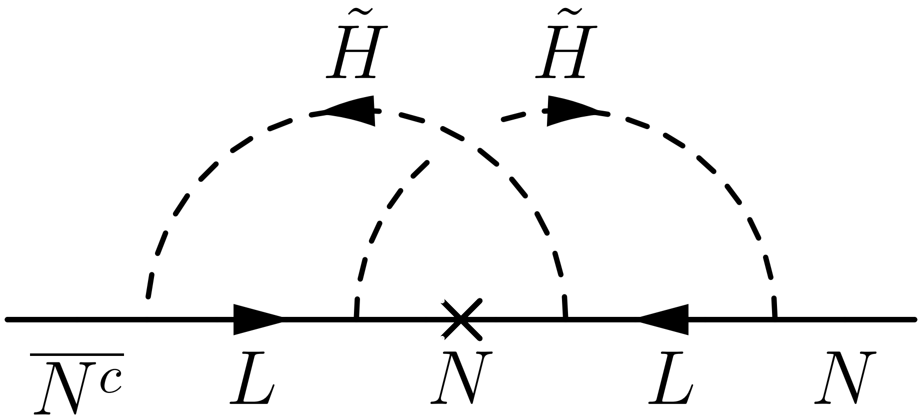
- At 1-loop, only one diagram contributes

$$\beta_M^{\text{1-loop}} = \frac{dM}{dt} = \frac{1}{(4\pi)^2}\left[\left(Y_\nu^\dagger Y_\nu\right)^T M + M\left(Y_\nu^\dagger Y_\nu\right)\right]$$

- At 2-loop, there are many contributions

$$\beta_M^{\text{2-loop}} = \frac{dM}{dt} = \frac{4}{(4\pi)^4}\left(Y_\nu^\dagger Y_\nu\right)^T M\left(Y_\nu^\dagger Y_\nu\right) + \cdots$$

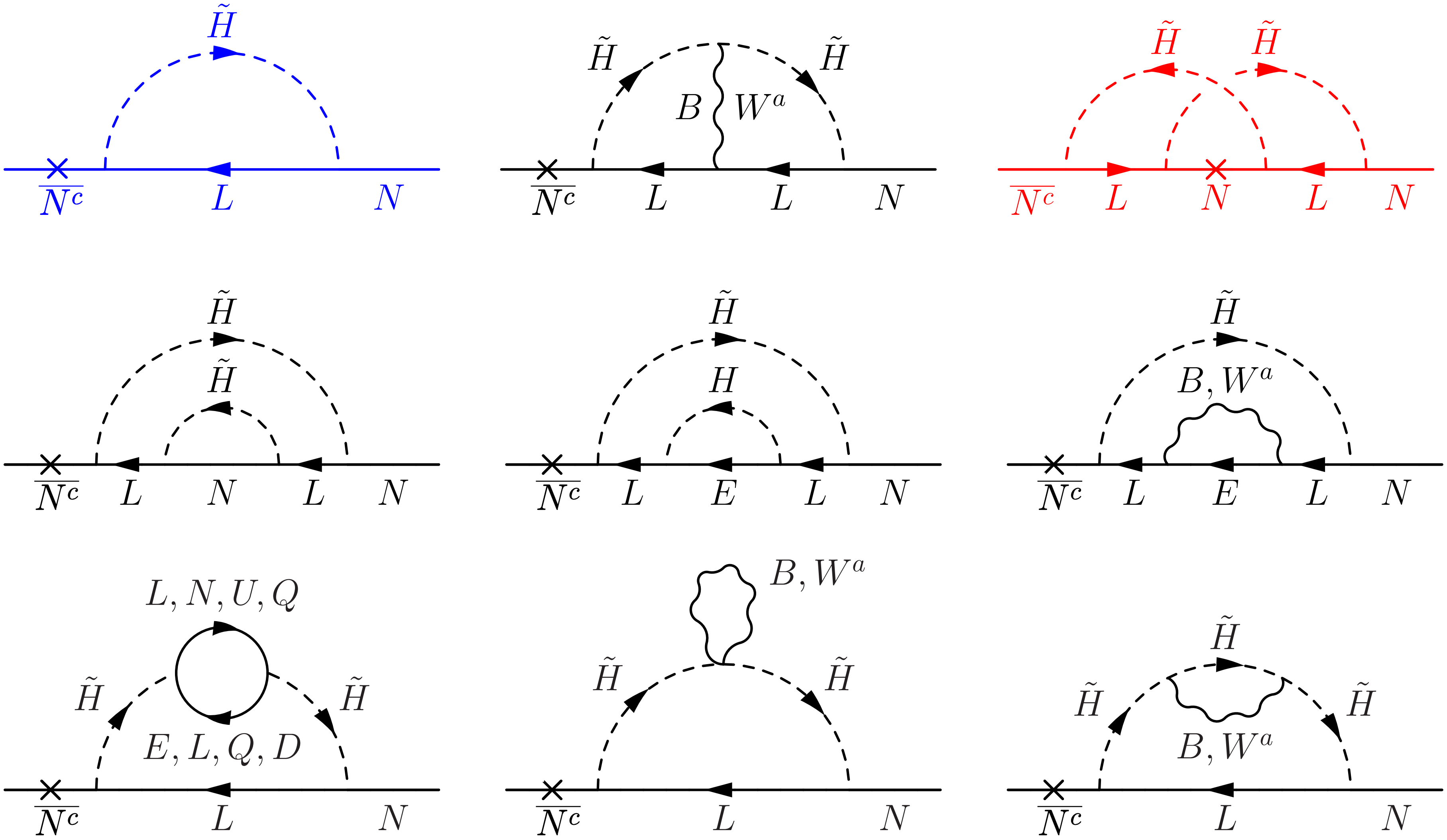
Rank increasing diagram



the other diagrams do not increase rank of M.

Renormalization Group Equation for M

All diagrams



Renormalization Group Equation for M

Full beta function

$$\frac{dM}{dt} = \frac{1}{(4\pi)^2}\left[\left(Y_\nu^\dagger Y_\nu\right)^T M + M\left(Y_\nu^\dagger Y_\nu\right)\right] + \frac{4}{(4\pi)^4}\left(Y_\nu^\dagger Y_\nu\right)^T M\left(Y_\nu^\dagger Y_\nu\right)$$
$$+\frac{1}{(4\pi)^4}\left[\frac{17}{8}\left(g_Y^2+g_2^2\right)\left(Y_\nu^\dagger Y_\nu\right) - \frac{1}{4}Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - \frac{1}{4}Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu\right.$$
$$\left.-\frac{3}{2}\mathrm{Tr}\left(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d\right)\left(Y_\nu^\dagger Y_\nu\right)\right]^T M$$
$$+\frac{1}{(4\pi)^4}M\left[\frac{17}{8}\left(g_Y^2+g_2^2\right)\left(Y_\nu^\dagger Y_\nu\right) - \frac{1}{4}Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - \frac{1}{4}Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu\right.$$
$$\left.-\frac{3}{2}\mathrm{Tr}\left(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d\right)\left(Y_\nu^\dagger Y_\nu\right)\right]$$

- We include only M and Y_ν.

Mass eigenvalues

- General parametrization at two-loop level:

$$\frac{dM}{dt} = P^T M + MP + Q^T MQ$$

where P, Q are matrices given by coupling constants.

Ex: $P = \frac{Y_\nu^\dagger Y_\nu}{(4\pi)^2}$ at 1-loop level, $Q = \frac{2Y_\nu^\dagger Y_\nu}{(4\pi)^2}$ at 2-loop level.

- $\Rightarrow$ In terms of mass eigenvalues

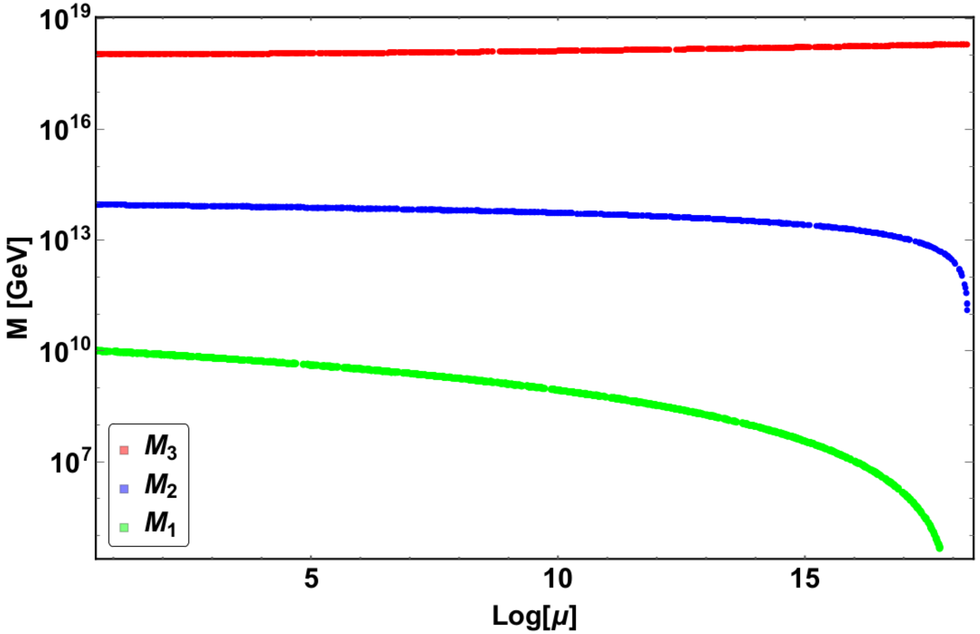
$$\frac{dM_i}{dt} = 2M_i \hat{P}_{ii} + \sum_k M_k \mathrm{Re}\left[\hat{Q}_{ki}^2\right] \quad + \text{RGEs for } U$$

where $U^T MU = \mathrm{diag}\,(M_1, M_2, M_3)$, $\hat{P} = U^\dagger PU$, $\hat{Q} = U^\dagger QU$

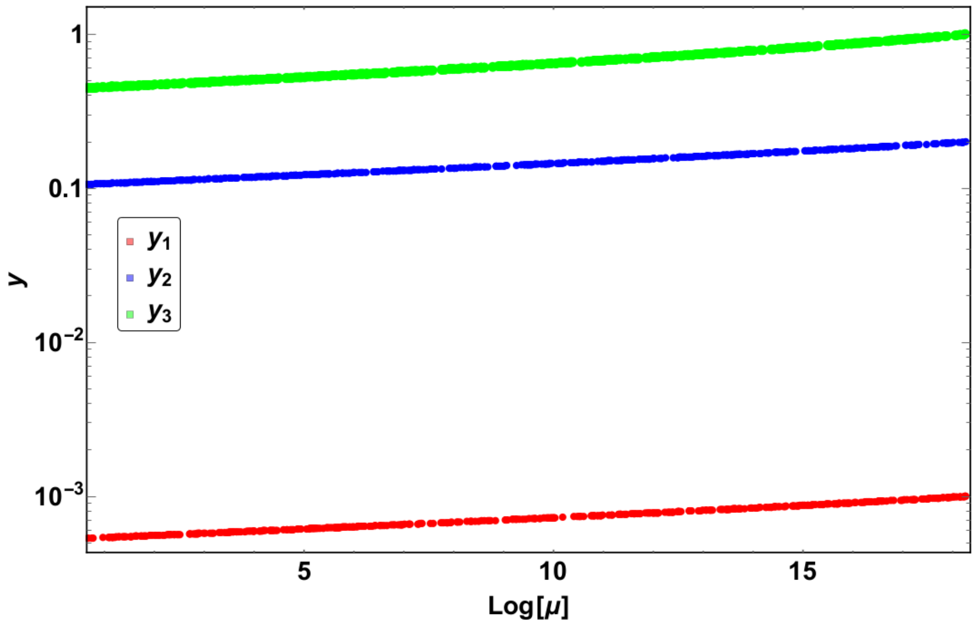
- Only $Q^T MQ$ term increases the rank of M.

Example Plots

RHN mass



Yukawa



- Parametrization: $Y_\nu = V_R \, \mathrm{diag}\,(y_1, y_2, y_3) \, V_L^\dagger$
- Initial values at $\Lambda = M_P$: $\theta_{23} = 0.1$, $\theta_{13} = 0.2$, $\theta_{12} = 0.3$, $y_1 = 0.001$, $y_2 = 0.2$, $y_3 = 1$, $\quad M_1 = M_2 = 0$, $M_3 = M_P$

Analytic results

- Formal RGE: $\dfrac{dM}{dt} = P^T M + MP + Q^T MQ$
- Mass eigenvalues of right-handed neutrinos at scale μ obtained by iterative integration

$$M_1 = \frac{1}{2} M_3(\Lambda) \mathrm{Re}\left(\hat{Q}_{32}^2\right) \mathrm{Re}\left(\hat{Q}_{21}^2\right) \log^2\left(\frac{\mu}{\Lambda}\right) \left[1 + \frac{3}{2}\log\left(\frac{\mu}{\Lambda}\right)\right],$$
$$M_2 = M_3(\Lambda) \mathrm{Re}\left(\hat{Q}_{32}^2\right) \log\left(\frac{\mu}{\Lambda}\right),$$
$$M_3 = M_3(\Lambda)$$

only leading contribution is considered.
$\hat{Q} = U^\dagger Q U$ and U is unitary matrix diagonalizing M.

- M_1 is 4-loop order? $\Rightarrow$ squared logarithm is dominant

Analytic results 2

For light neutrino masses: $m_\nu = -m_D M^{-1} m_D^T \quad (m_D = Y_\nu \langle H \rangle)$

- mass eigenvalues satisfy $m_1 m_2 m_3 = \dfrac{y_1^2 y_2^2 y_3^2 \langle H \rangle^6}{M_1 M_2 M_3}$
  where parametrized as $Y_\nu = V_R Y_D V_L^\dagger$

- One can choose like: $m_3 \sim \dfrac{y_3^2 \langle H \rangle^2}{M_2}, \quad m_2 \sim \dfrac{y_2^2 \langle H \rangle^2}{M_1}, \quad m_1 \sim \dfrac{y_1^2 \langle H \rangle^2}{M_3}$
  Heaviest neutrino mass: $m_3 \sim \mathcal{O}(0.1)\ \mathrm{eV} \left(\dfrac{y_3}{1}\right)^{-2} \left(\dfrac{M_3}{M_P}\right)^{-1}$
  $\Rightarrow$ correct mass scale is obtained with $y_3 = \mathcal{O}\left(1\right)$ because
  $M_2 \sim M_P/(4\pi)^4 \sim 10^{14}$ GeV
  Lightest neutrino mass: $m_1 \lesssim 10^{-5}$ eV (almost massless).
  m_2 and m_3 correspond to solor and atomospheric neutrino masses.

Rank increasement and flavor symmetry

- Number of rank increasement can be understood with global flavor symmetry

$$\mathcal{L} = -\left(Y_e\right)_{ij} H^\dagger \overline{E_i} L_j - \left(Y_\nu\right)_{ij} H \overline{N_i} L_j - \frac{M_{ij}}{2} \overline{N_i^c} N_j + \text{H.c.}$$

- Neutrino masses are protected by $U(3)_L \times U(3)_N$ symmetry (neglecting charged lepton Yukawa)

		rank Y_ν 1	2	3
rank M	1	$U(2)_L \times U(1)_N$	$U(1)_L$	No symmetry
	2	$U(2)_L$	$U(1)_L$	No symmetry
	3	$U(2)_L$	$U(1)_L$	No symmetry

Ex.1 $U(2)_L \times U(1)_N \Rightarrow$ two ν_L and one N are massless
Ex.2 $U(1)_L \Rightarrow$ one ν_L is massless

Number of Parameters

Parametrize $Y_\nu = V_R Y_D V_L^\dagger$

- For $\mathrm{rank}\, Y_\nu = 3$ and $\mathrm{rank}\, M = 3$ (usual case)
  eigenvalues $y_{1,2,3}$, $M_{1,2,3}$ and 6 mixing angles and 6 CP phases
  $\Rightarrow$ 18 parameters

- For $\mathrm{rank}\, Y_\nu = 2$ and $\mathrm{rank}\, M = 1$
  eigenvalues $y_{2,3}$, M_3 and 5 mixing angles and 3 CP phases
  $\Rightarrow$ 11 parameters (minimal number of parameters)
- CP phases are reduced: 6 $\Rightarrow$ 3
  Predictive Leptogenesis

Summary

1. If right-handed neutrino masses are highly hierarchical at the Planck scale, radiative corrections dominate right-handed neutrino masses at low scale. (No any intermediate scale)
2. Small neutrino masses are naturally generated by two lighter RHNs.
3. Number of parameters are reduced and this framework leads predictive phenomenology.

Future Works

1. Application to leptogenesis, sterile neutrino dark matter, the Scotogenic model.