Pseudo-Goldstone scalars in the minimal $SO(10)$ Higgs model

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    [Murayama, Pierce: 2001]
    [Bajc, Lavignac, Mede: 2015]
- **Minimal SUSY** $SO(10)$ (without 120_S)
  - Fails to reproduce the absolute neutrino mass scale
    [Bertolini, Schwetz, Malinský: 2006]
    [Aulakh, Garg: 2006]

Matter fields

$$16_F = (1,2,-\tfrac{1}{2}) \oplus (\bar{3},2,\tfrac{1}{3}) \oplus (3,2,\tfrac{1}{6}) \oplus (\bar{3},1,-\tfrac{2}{3}) \oplus (1,1,1) \oplus (1,1,0)$$

$$\underbrace{\overset{\downarrow}{L_L} \quad \overset{\downarrow}{d_L^c}}_{\bar{5}} \quad \underbrace{\overset{\downarrow}{Q_L} \quad \overset{\downarrow}{u_L^c} \quad \overset{\downarrow}{e_L^c}}_{10} \quad \underbrace{\overset{\downarrow}{N_L^c}}_{1}$$

- Automatic anomaly cancellation

Gauge fields

$$45_G = (8,1,0) \oplus (1,3,0) \oplus (1,1,0) \oplus (1,1,0) \oplus (3,1,\tfrac{2}{3}) \oplus (3,2,-\tfrac{5}{6}) \oplus (3,2,\tfrac{1}{6}) \oplus (1,1,1) + h.c.$$

$$(8,1,0) \rightarrow G_\mu^b, \quad (1,3,0) \rightarrow A_\mu^a, \quad (1,1,0) \oplus (1,1,0) \rightarrow B_\mu,\ Y_\mu, \quad (3,2,\tfrac{1}{6}) \oplus (1,1,1) + h.c. \rightarrow X_\mu$$

Scalar sector

- **GUT scale SSB by 45_S**
  - Minimal, preserves rank
  - Robust GUT scale estimates with respect to gravitational effects
  - Two real SM singlets:

$$\langle(1,1,0)_{15}\rangle = \sqrt{3}\omega_{BL} \quad \langle(1,1,0)_{1}\rangle = \sqrt{2}\omega_{R}$$

- **Rank reduction by 126_S**
  - Seesaw scale
  - Renormalizable Yukawa interaction $\leftarrow 16 \times 16 \supset 126$, ~~16 & Witten's loop~~
  - One complex SM singlet

$$\langle(1,1,0)_{\overline{10}}\rangle = \sqrt{2}\sigma$$

  - Phenomenology $\Rightarrow |\sigma| \ll \max(\omega_{BL}, \omega_R) \equiv \omega_{max}$

Tree level scalar spectrum

Scalar fields $45_S \oplus 126_S \leftrightarrow \Phi = (\phi_{ij}, \Sigma_{ijklm}, \Sigma^*_{ijklm})$ form the most general scalar potential

$$V_0(\phi, \Sigma, \Sigma^*) = V_\phi(\phi) + V_\Sigma(\Sigma, \Sigma^*) + V_{\phi,\Sigma}(\phi, \Sigma, \Sigma^*)$$

$$\begin{aligned}
V_\phi =& -\frac{\mu^2}{4}(\phi\phi)_0 + \frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0 + \frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2\,,\\
V_\Sigma =& -\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0 + \frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 + \\
&+ \frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{\lambda_4'}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} + \\
&+ \frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2\,,\\
V_{\phi,\Sigma} =& \frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2 + \frac{\alpha}{2\cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0 + \frac{\beta_4}{4\cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4 + \\
&+ \frac{\beta_4'}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} + \frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2 + \frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2\,.
\end{aligned}$$

Tree level mass matrix contains tachyons

$$\underbrace{\begin{aligned}\mathrm{M}_S^2[(8,1,0)] &= 2a_2(\omega_{BL}-\omega_R)(\omega_R+2\omega_{BL}),\\ \mathrm{M}_S^2[(1,3,0)] &= 2a_2(\omega_R-\omega_{BL})(2\omega_R+\omega_{BL}),\end{aligned}}_{\text{nontachyonic iff near } SU(5)'\times U(1)_{Z'} \text{ SSB chain}}$$

- SM multiplets $(8,1,0),(1,3,0) \leftrightarrow$ pseudo-Goldstone bosons of the broken global symmetry $O(45)$ in the limit $\sigma \to 0, a_2 \to 0$
- If $a_2 << 1$, loop corrections dominant [Bertolini, Di Luzio, Malinský: 2010] [Gráf, Malinský, Mede, Susič: 2017]
- Makes sence at one-loop level [Kolešová, Malinský: 2014]

One more pseudo-Goldstone

SM singlet 2×2 mass submatrix (only 45_S) has an eigenvalue

$$m_{PG}^2 = a_2 \left(-\frac{45\omega_{BL}^4}{3\omega_{BL}^2 + 2\omega_R^2} + 13\omega_{BL}^2 - 2\omega_{BL}\omega_R - 2\omega_R^2 \right) + O\left(a_2^2\right) + O\left(\frac{\sigma^2}{\omega_{max}^2}\right)$$

- $m_{PG}^2 \propto a_2$ up to $O\left(\frac{\sigma^2}{\omega_{max}^2}\right)$ corrections
- Significant one-loop corrections to m_{PG}^2
  - One-loop effective potential calculation

$$
\begin{aligned}
\mathcal{M}_{11}^2 = &\underbrace{-2a_2\ \omega^2[-2,1,1] + 24a_0\omega_{BL}^2 - 12\sigma^2\beta_4'}_{\text{Tree level}} + \frac{g^4}{16\pi^2}\left(\omega^2[40,1,13]+\right. \\
&\left.+13\omega_R^2 - 6\sigma^2\right) + \frac{3g^4}{32\pi^2\omega[1,-1]}\left\{\omega[1,-1]^3 \log\left[\frac{g^2\omega[1,-1]^2}{2\mu_R^2}\right]+\right. \\
&+\left(4\sigma^2\omega[3,-1] + \omega[7,-5]\omega[1,1]^2\right)\log\left[\frac{g^2\left(4\sigma^2+\omega[1,1]^2\right)}{2\mu_R^2}\right] - \\
&-16\left(\sigma^2\omega_R - 2\omega_{BL}^3 + 3\omega_{BL}^2\omega_R\right)\log\left[\frac{2g^2\left(\sigma^2+\omega_{BL}^2\right)}{\mu_R^2}\right]+ \\
&\left.+8\omega_R\left(\sigma^2+\omega_R^2\right)\log\left[\frac{2g^2\left(\sigma^2+\omega_R^2\right)}{\mu_R^2}\right]\right\} + \frac{35\tau^2}{8\pi^2}+ \\
&+\frac{\omega^5[-5,19,-170,1222,-5,19]\omega_R}{16R^2\pi^2}\beta_4^2 + \frac{15\omega[1,1]\omega_R}{4\pi^2}\beta_4\beta_4'+ \\
&+\frac{5\omega_R\omega[-5,11]}{4\pi^2}\beta_4'^2+
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{16\omega[1,-1]\pi^2}F\Big[-9\omega[1,1],-36\omega_{BL}\omega[1,-1],\omega^2[-18,72,24],36\omega_{BL}\omega_R,\\
&\quad 108\omega_{BL}^2\omega[1,-1],144\omega_{BL}^2\omega[1,-1],144\omega_{BL}^2\omega[1,-1],24\omega_{BL}^2\omega[2,-3],\\
&12\omega^3[8,-15,-12,-4],24\omega^3[2,0,3,1]\Big]\log\left[\frac{2\left(\beta_4\omega_{BL}^2+\omega[2,1]\left(2\beta_4'\omega[1,1]-\tau\right)\right)}{\mu_R^2}\right]\\
&+\frac{1}{16\omega[1,-1]\pi^2}F\Big[-3\omega[3,5],-36\omega_{BL}\omega[1,-1],0,24\omega^2[3,-6,-1],36\omega_{BL}^2\omega[1,-1],\\
&\quad 0,-144\omega_{BL}^2\omega[1,-1],0,0,72\omega^3[2,-2,3,1]\Big]\log\left[\frac{2\omega[3,1]\left(2\omega_R\beta_4'-\tau\right)}{\mu_R^2}\right]\\
&+\frac{1}{8\omega[1,1]\pi^2}F\Big[3\omega[3,1],36\omega_{BL}\omega[1,-1],0,-24\omega_R\omega[3,1],108\omega_{BL}^2\omega[1,-1],0,\\
&\quad 144\omega_{BL}^2\omega[1,-1],0,0,24\omega^3[2,0,3,1]\Big]\log\left[\frac{2\omega[1,1]\left(2\beta_4'\omega[2,1]-\tau\right)}{\mu_R^2}\right]+\dots
\end{aligned}
$$

Consistency checks

Particular symmetry breaking limits ($\sigma \to 0$):

- $\mathbf{SU(5)} \times \mathbf{U(1)_Z}$ ($\omega_R \to \pm\omega_{BL}$):

$$(1,1,0)_{PG}, (8,1,0), (1,3,0) \subset (24,0),$$

- $\mathbf{SU(4)_c} \times \mathbf{SU(2)_L} \times \mathbf{U(1)_R}$ ($\omega_{BL} \to 0$):

$$(1,1,0)_{PG}, (8,1,0) \subset (15,1,0),$$

- $\mathbf{SU(3)_c} \times \mathbf{SU(2)_L} \times \mathbf{SU(2)_R} \times \mathbf{U(1)_{BL}}$ ($\omega_R \to 0$):

$$(1,1,0)_{PG} \sim (1,3,0) \quad \longleftarrow \quad \text{D-parity.}$$

Summary and Outlook

- The minimal renormalizable $SO(10)$ Higgs model:
  - gauge bosons 45_G,
  - scalar sector $45_S \oplus 126_S$.
- Potentially realistic scenarios involve tachyonic instabilities.
- Existence of tachyons is relic of the tree-level calculations.
- Scalar spectrum contains pseudo-Goldstones - $(8, 1, 0)$, $(1, 3, 0)$ and newly calculated $(1, 1, 0)$.
- One-loop corrections to the pseudo-Goldstone masses improve the proton lifetime estimates.