

$(g - 2)_\mu$ in the presence of CPV

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arXiv:1712.09613 (to appear in JHEP)

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Planck 2018

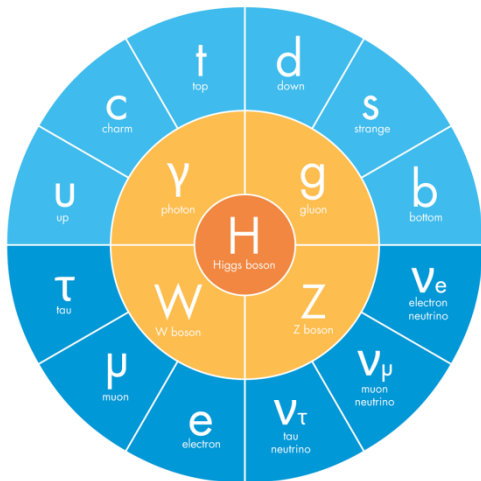
- 1 SM and its scalar extensions
- 2 The 2 Higgs doublet model (2HDM)
 - Heavy scalars
 - Scalars close in mass
 - Light scalars
- 3 Theoretical and experimental bounds
- 4 Summary

In praise of the Standard Model

Current formulation finalised
in the 70's predicted:

- the W & Z (1983)
- the top quark (1995)
- the tau neutrino (2000)
- “a” Higgs boson (2012)

FERMIONS (matter) | BOSONS (force carriers)
● Quarks ● Leptons | ● Gauge bosons ● Higgs boson



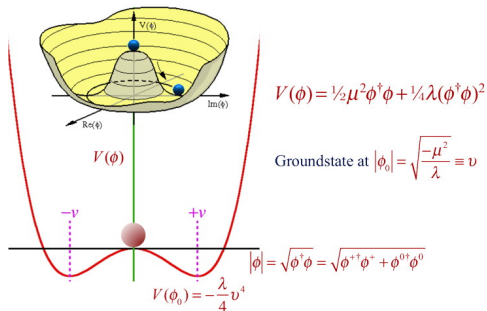
In criticism of the Standard Model

What is missing:

- Fermion mass hierarchy
- EW vacuum stability
- Dark Matter
- CP-Violation
- Muon anomalous magnetic moment $a_\mu = (g - 2)_\mu/2$
- ...

Scalars to the rescue!

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
 & + i \bar{\Psi} \not{D} \Psi + h.c. \\
 & + \bar{\Psi}_i \gamma_{ij} \Psi_j \phi + h.c. \\
 & + \frac{1}{2} \partial_\mu \phi^2 - V(\phi)
 \end{aligned}$$



Two birds with one stone!

- What's up with the muon?

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (287 \pm 80) \times 10^{-11} \quad (3.6\sigma)$$

extra scalars can help!

- Observation $\frac{N(B)}{N(\gamma)} \approx 10^{-9} \gg 10^{-20}$ provided by SM

⇒ new sources of CPV are needed

scalar extensions can help!

Who is stopping them? EDMs

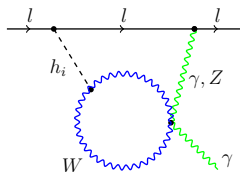
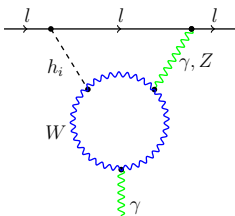
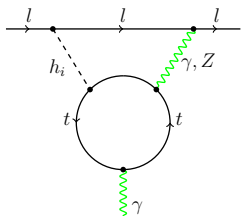
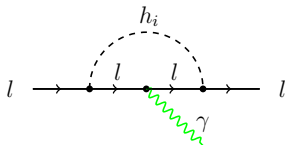
$$d_e < 10.25 \times 10^{-29} \text{ e cm}$$

[Science 343, 269 (2014)], [Phys. Rev. D 73, 072003 (2006)]

Dominant contributions to a_μ and d_e

$$a_\mu = \frac{m_\mu^2}{4\pi^2} \text{Re}(c_L + c_R^*),$$

$$d_e = \frac{e m_e}{4\pi^2} \text{Im}(c_L + c_R^*)$$



The important couplings are: $Y_{\mu\mu}^{h_i}$, $Y_{ee}^{h_i}$, $Y_{WW}^{h_i}$, $Y_{tt}^{h_i}$

The 2 Higgs doublet model (2HDM)

2HDM with explicit CP-violation

Two scalar doublets

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + h_1^0 + ia_1^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + h_2^0 + ia_2^0}{\sqrt{2}} \end{pmatrix},$$

The scalar potential:

$$\begin{aligned} V = & -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \left[\mu_3^2(\Phi_1^\dagger\Phi_2) + \text{h.c.} \right] \\ & + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \text{h.c.} \right]. \end{aligned}$$

Imposing a Z_2 symmetry to avoid FCNCs:

$$\Phi_1 \rightarrow +\Phi_1, \quad \Phi_2 \rightarrow -\Phi_2 \quad \Rightarrow \quad \lambda_6 = \lambda_7 = 0$$

The three CP-mixed mass eigenstates

Rotating to the Higgs basis

$$\begin{pmatrix} \hat{\Phi}_1 \\ \hat{\Phi}_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

with $\tan \beta = v_2/v_1$.

$$\hat{\Phi}_1 = \begin{pmatrix} G^+ \\ \frac{v+\phi_1+iG^0}{\sqrt{2}} \end{pmatrix}, \quad \hat{\Phi}_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+i\phi_3}{\sqrt{2}} \end{pmatrix}$$

Rotation matrix R (assuming all angles are small):

$$\phi_i = R_{ij} h_j, \quad \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 1 & \theta_{12} & \theta_{13} \\ -\theta_{12} & 1 & \theta_{23} \\ -\theta_{13} & -\theta_{23} & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

What constrains the parameter space?

- Bounded from below potential: $\phi_i \rightarrow \infty \Rightarrow V > 0$
- Vacuum stability: $E_{VEW} < E_{V_i}$ or $\tau_{VEW} >$ age of the universe
- Perturbative unitarity: $|\lambda_i| \leq 4\pi$, $|\Lambda_i| \leq 8\pi$
- h_1 being SM-like: $\sin(\theta_{12}) < 0.3$
- Electroweak precision data: $m_{H^\pm} \sim m_{h_3}$

The couplings

The Yukawa couplings

$$-\mathcal{L}_Y = Y_u \bar{Q}'_L i \sigma_2 \Phi_u^* u'_R + Y_d \bar{Q}'_L \Phi_d d'_R + Y_l \bar{L}'_L \Phi_l l'_R + h.c.$$

$$y_u = \bar{u}_L \frac{m_u}{v} u_R \sum_i^3 (R_{1i} + \xi_u (R_{2i} - i R_{3i})) h_i$$

$$y_d = \bar{d}_L \frac{m_d}{v} d_R \sum_i^3 (R_{1i} + \xi_d (R_{2i} + i R_{3i})) h_i$$

$$y_l = \bar{e}_L \frac{m_l}{v} e_R \sum_i^3 (R_{1i} + \xi_l (R_{2i} + i R_{3i})) h_i$$

The gauge couplings

$$\mathcal{L}_{kin} \supset \phi_1 \left(\frac{2m_W^2}{v} W_\mu W^\mu + \frac{m_Z^2}{v} Z_\mu Z^\mu \right) = R_{1i} h_i \left(\frac{2m_W^2}{v} W_\mu W^\mu + \frac{m_Z^2}{v} Z_\mu Z^\mu \right)$$

Different types of 2HDM

	Φ_1	Φ_2	u_R	d_R	e_R	Q_L, L_L	ξ_d	ξ_u	ξ_l
Type-I	+	-	-	-	-	+	$\cot \beta$	$\cot \beta$	$\cot \beta$
Type-II	+	-	-	+	+	+	$-\tan \beta$	$\cot \beta$	$-\tan \beta$
Type-X	+	-	-	-	+	+	$\cot \beta$	$\cot \beta$	$-\tan \beta$
Type-Y	+	-	-	+	-	+	$-\tan \beta$	$\cot \beta$	$\cot \beta$

Recall the couplings contributing to a_μ and d_e :

$$Y_{\mu\mu}^{h_i}, \quad Y_{ee}^{h_i}, \quad Y_{WW}^{h_i}, \quad Y_{tt}^{h_i}$$

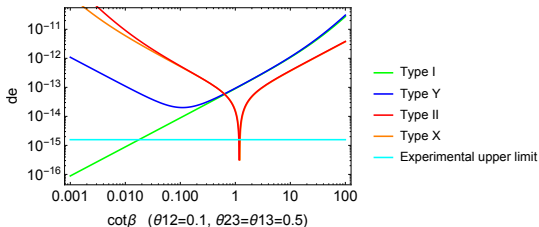
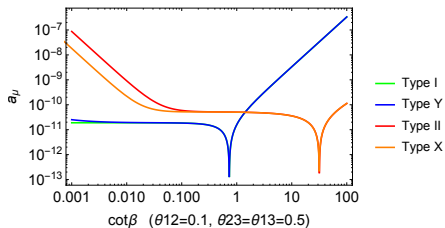
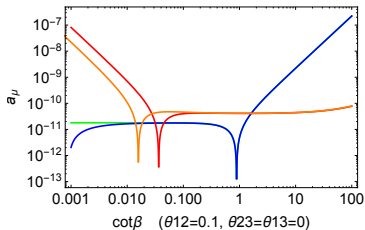
With ξ_d playing a sub-dominant role:

Type-I \cong Type-Y

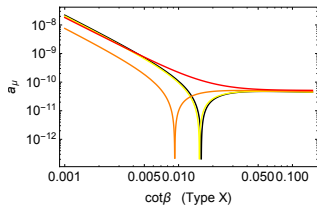
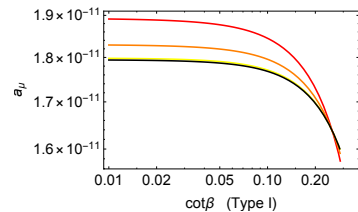
Type-II \cong Type-X

Heavy scalars

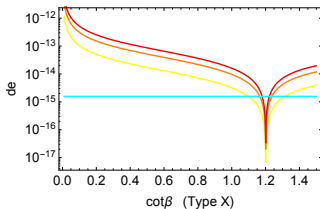
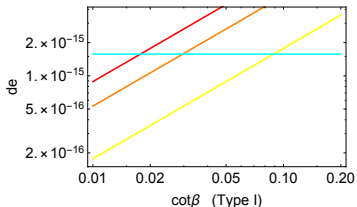
$$m_{h_{2,3}} = 200, 300 \text{ GeV}$$

a_μ with/without CPV and d_e 

A closer look

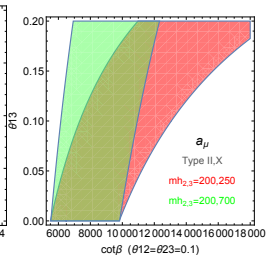
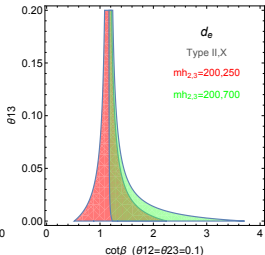
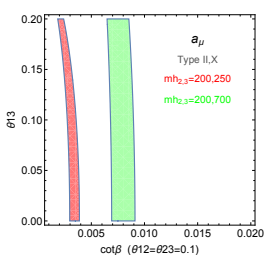
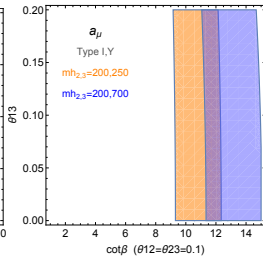
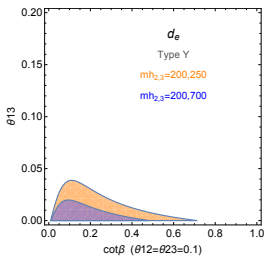
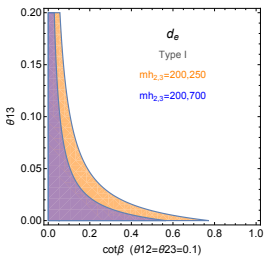


- $\theta_{12}=0.1, \theta_{23}=\theta_{13}=0$
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- $\theta_{12}=0.1, \theta_{23}=\theta_{13}=0.3$
- $\theta_{12}=0.1, \theta_{23}=\theta_{13}=0.5$



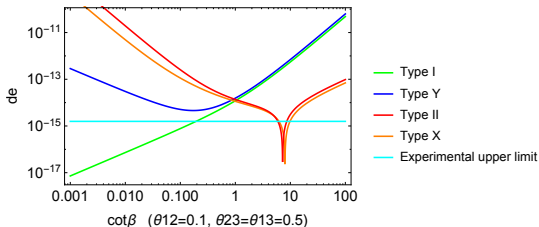
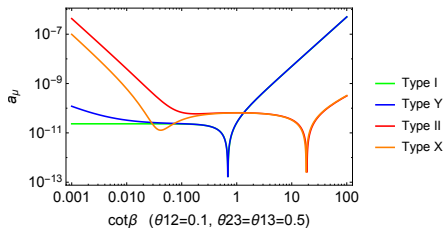
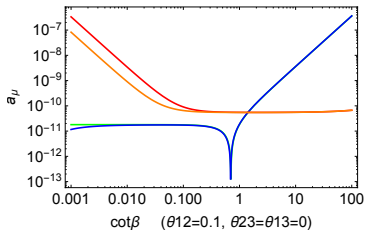
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- $\theta_{12}=0.1, \theta_{23}=\theta_{13}=0.5$
- Experimental upper limit

No overlap between d_e and a_μ regions

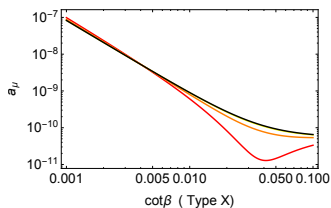
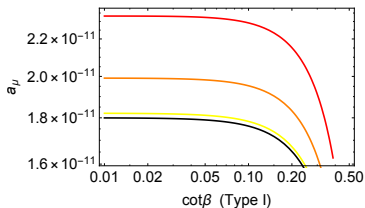


Scalars close in mass

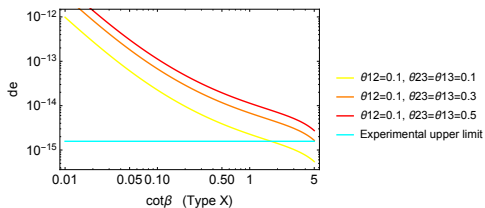
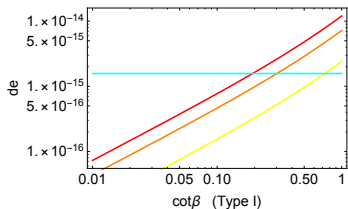
$$m_{h_{2,3}} = 145, 105 \text{ GeV}$$

a_μ with/without CPV and d_e 

A closer look

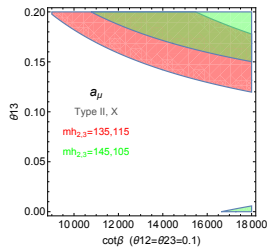
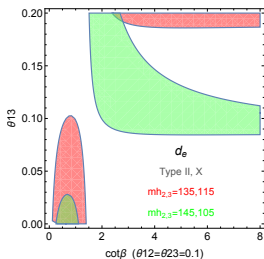
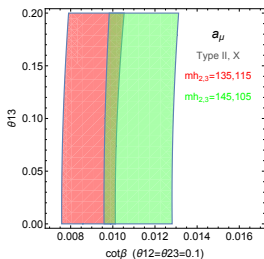
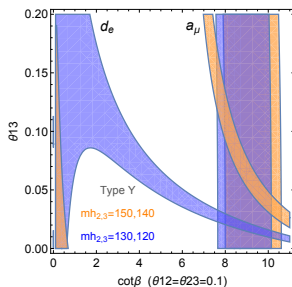
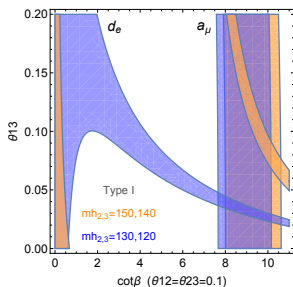


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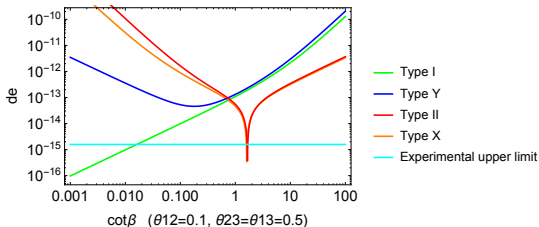
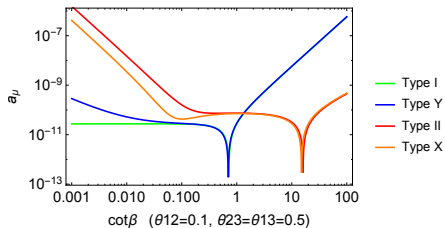
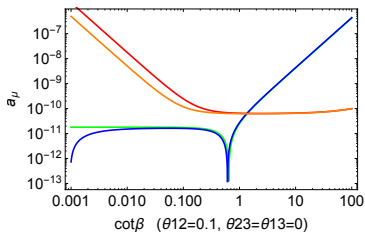
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- Experimental upper limit

Overlap between d_e and a_μ regions

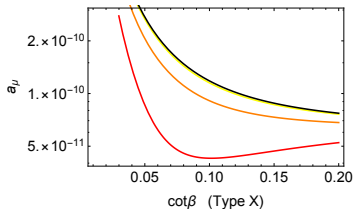
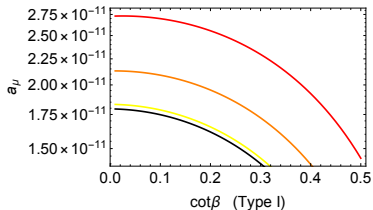


Light scalars

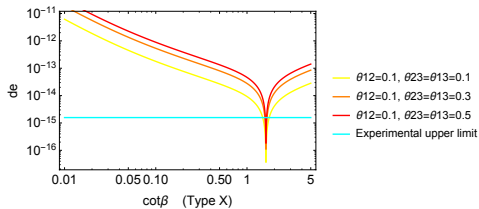
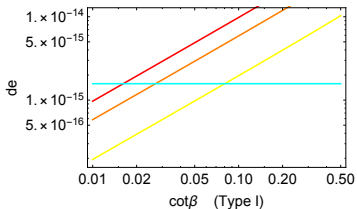
$$m_{h_{2,3}} = 200, 50 \text{ GeV}$$

a_μ with/without CPV and d_e 

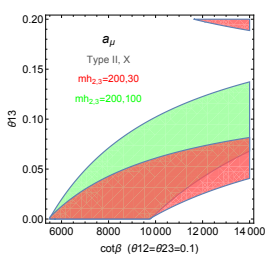
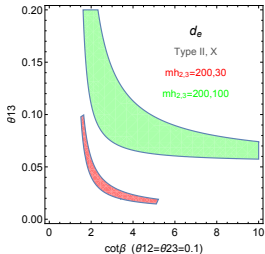
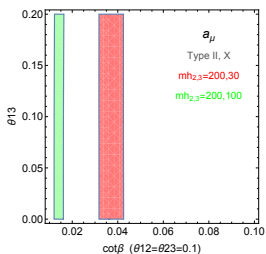
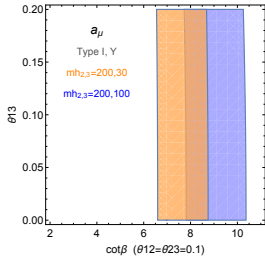
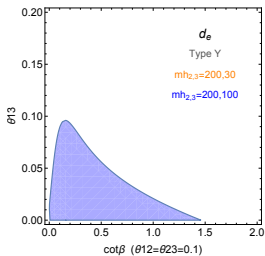
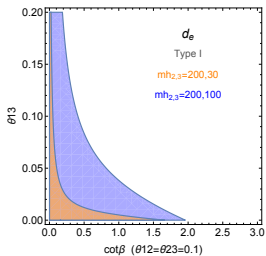
A closer look



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No overlap between d_e and a_μ regions



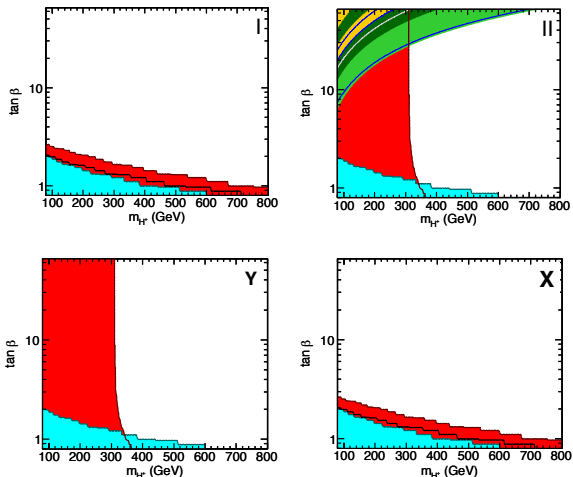
$\tan \beta$ dependant couplings

	Type I	Type II	Type X	Type Y
ξ_h^u	$\cos \theta_{12} / \sin \beta$	$\cos \theta_{12} / \sin \beta$	$\cos \theta_{12} / \sin \beta$	$\cos \theta_{12} / \sin \beta$
ξ_h^d	$\cos \theta_{12} / \sin \beta$	$-\sin \theta_{12} / \cos \beta$	$\cos \theta_{12} / \sin \beta$	$-\sin \theta_{12} / \cos \beta$
ξ_h^ℓ	$\cos \theta_{12} / \sin \beta$	$-\sin \theta_{12} / \cos \beta$	$-\sin \theta_{12} / \cos \beta$	$\cos \theta_{12} / \sin \beta$
ξ_H^u	$\sin \theta_{12} / \sin \beta$	$\sin \theta_{12} / \sin \beta$	$\sin \theta_{12} / \sin \beta$	$\sin \theta_{12} / \sin \beta$
ξ_H^d	$\sin \theta_{12} / \sin \beta$	$\cos \theta_{12} / \cos \beta$	$\sin \theta_{12} / \sin \beta$	$\cos \theta_{12} / \cos \beta$
ξ_H^ℓ	$\sin \theta_{12} / \sin \beta$	$\cos \theta_{12} / \cos \beta$	$\cos \theta_{12} / \cos \beta$	$\sin \theta_{12} / \sin \beta$
ξ_A^u	$\cot \beta$	$\cot \beta$	$\cot \beta$	$\cot \beta$
ξ_A^d	$-\cot \beta$	$\tan \beta$	$-\cot \beta$	$\tan \beta$
ξ_A^ℓ	$-\cot \beta$	$\tan \beta$	$\tan \beta$	$-\cot \beta$

for most H masses: $\sin \theta_{12} < |0.3|$

our analysis: $\sin \theta_{12} = 0.1$

Flavour constraints



Excluded by $BR(B \rightarrow X_s \gamma)$, $B^0 - \bar{B}^0$ mixing, $D_s \rightarrow \tau \nu_\tau$, $D_s \rightarrow \mu \nu_\mu$, $B \rightarrow D \tau \nu_\tau$.

[Phys. Rev. D 81 (2010) 035016]

The general picture!

$\tan \beta$	2HDM-Type	d_e	a_μ	Flavour+Collider Exp.
0.01 – 0.1	I	×	✓	×
	Y	×	✓	×
0.01 – 0.1	II	×	×	×
	X	×	×	×
$\mathcal{O}(1)$	I	✓	✓	×
	Y	✓	✓	×
$\mathcal{O}(1)$	II	✓	×	×
	X	✓	×	×
10 – 100	I	✓	×	✓
	Y	×	×	×
10 – 100	II	×	✓	×
	X	×	✓	✓

Summary

- When it comes to a_μ and d_e calculations:

Type-I \cong Type-Y

Type-II \cong Type-X

- When it comes to flavour and collider bounds:

Type-I \cong Type-X

Type-II \cong Type-Y

- Only when scalars are close in mass, the a_μ and d_e regions overlap in Type I & Y.
- For large a_μ : **Type-X**
- For small d_e : **Type-I**

BACKUP SLIDES

a_μ contribution from scalars

$$\Delta a_\mu = a_\mu^{\text{exp}} - \left(a_\mu^{\text{SM (without scalars)}} + a_\mu^{\text{scalars}} \right) = 0$$
$$\Rightarrow a_\mu^{\text{scalars}} = (2.88 \pm 0.8) \times 10^{-9}$$

SM + singlet scalar

SM + real singlet

SM-Higgs doublet Φ and the scalar singlet S :

$$\Phi = \begin{pmatrix} G^+ \\ \frac{v + \phi_1 + iG^0}{\sqrt{2}} \end{pmatrix}, \quad S = \left(\frac{w + \phi_2}{\sqrt{2}} \right)$$

The scalar potential:

$$V = -\mu_1^2 \Phi^\dagger \Phi - \mu_2^2 S^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 S^4 + \lambda_3 (\Phi^\dagger \Phi) S^2 + \kappa_1 S + \kappa_2 S (\Phi^\dagger \Phi) + \kappa_3 S^3$$

Rotation matrix R :

$$\phi_i = R_{ij} h_j, \quad \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Constrained by experimental and theoretical bounds:

$$|\sin \theta| \lesssim 0.3$$

SM + real singlet

No CPV at the potential level \Rightarrow Higher order operators

$$\mathcal{L}_{CPV} = \frac{\eta}{\Lambda} S \bar{Q}_L \tilde{\phi} t_R + h.c.$$

$$\eta = \text{Re}\eta + i \text{Im}\eta$$

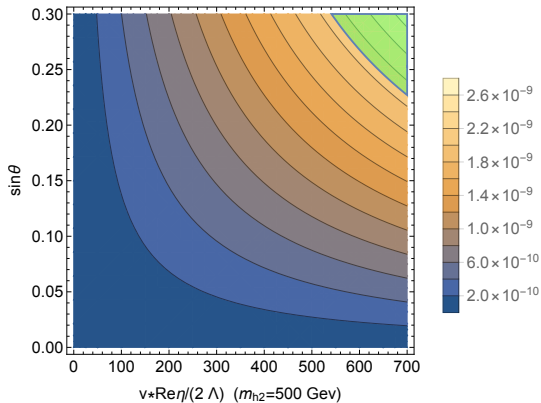
The couplings of h_1, h_2 :

$$Y_{ee}^{h_i} = R_{1i} \left(\frac{m_e}{v} \right)$$

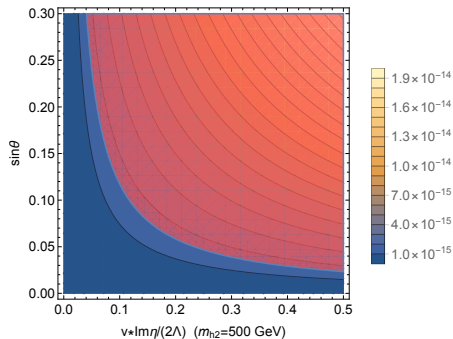
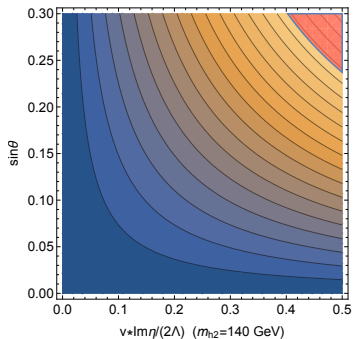
$$Y_{WW}^{h_i} = R_{1i} \left(\frac{2m_W^2}{v} \right)$$

$$Y_{tt}^{h_i} = R_{1i} \left(\frac{m_t}{v} \right) + R_{2i} \left(\frac{v(\text{Re}\eta + i\text{Im}\eta)}{2\Lambda} \right)$$

Contours of a_μ



Contours of d_e



SM + complex singlet

SM-Higgs doublet Φ and the scalar singlet S :

$$\Phi = \left(\begin{array}{c} G^+ \\ \frac{1}{\sqrt{2}} (v + \phi_1 + iG^0) \end{array} \right), \quad S = \left(\frac{w + \phi_2 + i\phi_3}{\sqrt{2}} \right)$$

Rotation matrix R :

$$\phi_i = R_{ij} h_j, \quad \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

SM + complex singlet

No CPV at the potential level \Rightarrow Higher order operators

$$\mathcal{L}_{CPV} = \frac{\eta}{\Lambda} S \bar{Q}_L \tilde{\phi} t_R + h.c.$$

$$\eta = \text{Re}\eta + i \text{Im}\eta$$

The couplings of h_1, h_2, h_3 :

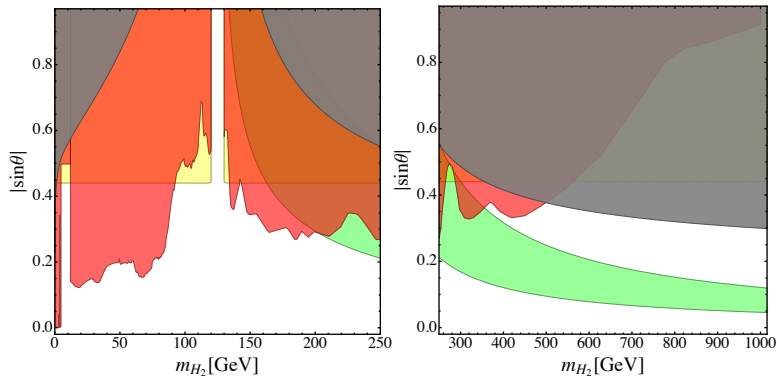
$$Y_{ee}^{h_i} = R_{1i} \left(\frac{m_e}{v} \right)$$

$$Y_{WW}^{h_i} = R_{1i} \left(\frac{2m_W^2}{v} \right)$$

$$Y_{tt}^{h_i} = R_{1i} \left(\frac{m_t}{v} \right) + R_{2i} \left(\frac{v(\text{Re}\eta + i\text{Im}\eta)}{2\Lambda} \right)$$

\Rightarrow Results identical to SM+RS

What constrains the parameter space?



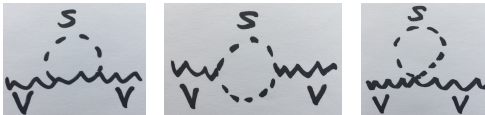
Excluded by **direct searches**, precision tests, H_1 couplings measurements and preferred by **potential stability**.

[JHEP 05, 057 (2015)]

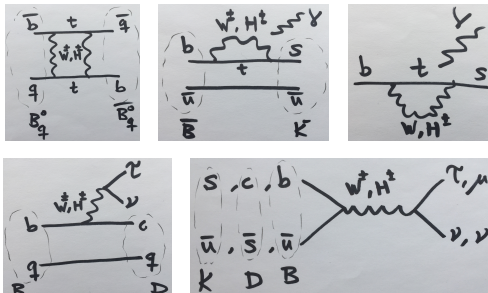
2HDM + singlet scalar

Extra doublets lead to extra constraints

Electroweak precision observables:



Flavour observables:



2HDM + singlet

