# Pendulum Leptogenesis

Neil D. Barrie

Kavli IPMU (WPI)

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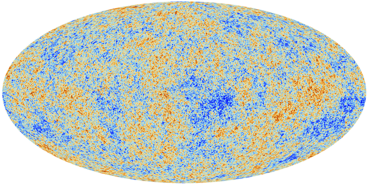
K. Bamba, NDB, A. Sugamoto, T. Takeuchi and K. Yamashita, arxiv:1610.03268, and arxiv:1805.04826.

# Outline

# Matter-Antimatter Asymmetry



The asymmetry is described quantitatively by,

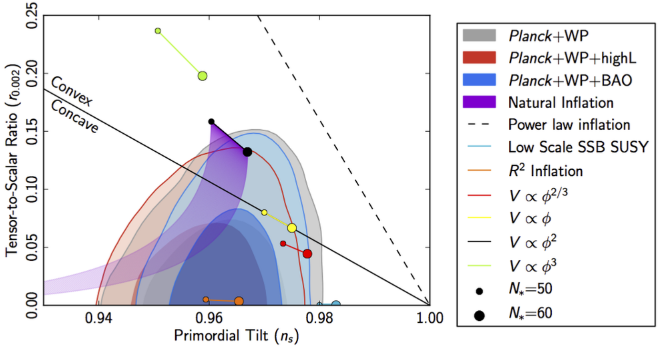
$$\eta = \frac{n_b - n_{\bar{b}}}{s} \simeq 8.5 \times 10^{-11}$$

## The Sakharov Conditions

1. Baryon number violation
2. $\mathcal{C}$ and $\mathcal{CP}$ violation
3. Period of non-equilibrium

# Starobinsky Inflation and Reheating

Inflation solves horizon and flatness problems, and primordial perturbations




Reheating in the Starobinsky model ($\frac{3\mu^2\overline{M}_{Pl}^2}{4}(1 - e^{-\sqrt{2/3}\Phi/\overline{M}_{Pl}})^2$):

- Inflationary potential approaches $\frac{1}{2}\mu^2\Phi^2$ during reheating.
- An approximate matter dominated epoch ($a \propto t^{2/3}$).

# Ratchet Mechanism

Inspired by molecular motors in biological systems, and their ability to generate directed motion.

- Consider an inflaton and complex scalar carrying $L$ charge during reheating.
- A derivative coupling between a complex scalar and inflaton.
- Directed motion in the complex scalar phase gives a non-zero $L$ number density.

# The Model

Interplay between the inflaton and complex scalar during reheating.

$$S = \int dx^4 \sqrt{-g}\, [g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - V_0(\phi,\phi^*)$$
$$+\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - U(\Phi) + \frac{i}{\Lambda}g^{\mu\nu}\left(\phi^*\overleftrightarrow{\partial_\mu}\phi\right)\partial_\nu\Phi\Big],$$

where $V_0(\phi,\phi^*) = \lambda\phi^*\phi(\phi-\phi^*)(\phi^*-\phi)+ ...$

## Satisfying the Sakharov Conditions

1. The complex scalar potential.
2. Derivative coupling interaction.
3. Reheating epoch.

# $U(1)$ Global Symmetry and $L$ Charge

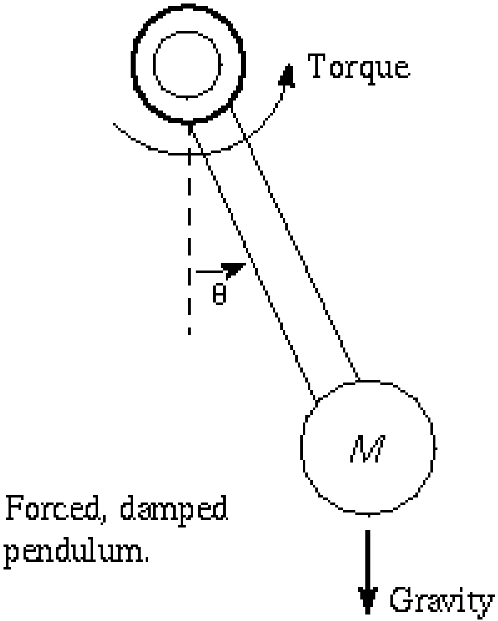
- Identify the $U(1)$ symmetry with $L$ charge, where $\phi$ has charge 2.
- $L$ preserving interactions, $\mathcal{L}_{\rm int} = g_L \phi^* \bar{\nu}_R^c \nu_R + y_H H \bar{L} \nu_R + h.c.$
- Taking $\phi$ in polar form, $\phi = \frac{1}{\sqrt{2}} \phi_r e^{i\theta}$ ,

$$V(\phi_r, \theta) = \lambda \phi_r^4 \sin^2\theta + \cdots$$

and

$$n_L = j^0 = -2\phi_r^2 \left( \dot{\theta} - \frac{\dot{\Phi}}{\Lambda} \right) \ .$$

A non-zero $n_L$ requires driven motion in $\dot{\theta}$ .


Torque
θ
M
Forced, damped pendulum.
Gravity

# Behaviour of the Inflaton

Equation of Motion,

$$\ddot{\Phi} + \left(\frac{2}{t}\,\dot{\Phi} + \Gamma\,\dot{\Phi}\right) + \mu^2\Phi + \frac{\lambda\phi_r^4}{\Lambda}\sin(2\theta) = 0$$

Need the inflaton (Torque) to be unaffected by $\theta$, $(\mu^2\Phi \gg \lambda\phi_r^4/\Lambda)$,

$$\ddot{\Phi} + \left(\frac{2}{t} + \Gamma\right)\dot{\Phi} + \mu^2\Phi \ = \ 0 \, .$$

When $\Gamma \ll \mu$, the approximate solution to this equation is

$$\frac{\Phi(t)}{\Phi_i} \approx \left(\frac{t_i}{t}\right)\cos\big[\mu(t - t_i)\big] \ \ .$$

# Behaviour of $\theta$

Utilising the inflaton EoM, and neglecting $\Phi$ decay term,

$$\ddot{\theta} + (\Gamma_\theta + 3H)\,\dot{\theta} + p\sin(2\theta) \;=\; -q(t)\cos\big[\mu(t - t_i)\big]\;,$$

where

$$p \;=\; \lambda\phi_r^2\;, \qquad q(t) \;=\; \frac{\mu^2\Phi_i}{\Lambda}\left(\frac{t_i}{t}\right)\;.$$

This is analogous to a forced pendulum where

- LHS represents acceleration, damping, and gravitation,
- RHS is a sinusoidal driving torque with amplitude $q$ and frequency $\mu$.

# Possible Cases: $p \ll q(t)$

Consider a large torque ($p \ll q(t)$),

$$\ddot{\theta} + 3H\dot{\theta} = \frac{1}{t^2}\frac{d}{dt}\left(t^2\dot{\theta}\right) = -q(t)\cos\left[\mu(t - t_i)\right] \ ,$$

which gives,

$$\dot{\theta}(t) = \frac{\dot{\Phi}}{\Lambda} \quad \Rightarrow \quad j_0 = 0$$

- $L$ violation has vanished, $\dot{\theta}$ oscillates around zero with $\dot{\Phi}$ .
- Analogous to an effectively massless pendulum.

# Possible Cases: $p \gg q(t)$

Consider a small torque ($p \gg q(t)$),

$$\ddot{\theta} + 3H\dot{\theta} + p\sin(2\theta) = 0$$

Friction term damps $\dot{\theta}$ until $\theta$ settles into a minima $\Rightarrow$ no persistent non-zero $\dot{\theta}$.

- $\mathcal{C}$ and $\mathcal{CP}$ breaking term ignored.
- Analogous to a very massive pendulum, unaffected by input torque.

# Sweet Spot Condition and Driven Motion

We need $p \simeq q(t) \Rightarrow L$, $\mathcal{C}$ and $\mathcal{CP}$ violating terms all contribute.

$$\text{SSC:} \;\; \lambda \phi_r^2 \;\simeq\; \frac{\mu^2 \Phi_i}{\Lambda} \frac{H_d}{H_i}$$

- Equivalent to $F_d \simeq mgl$ ,
- Torque is time dependent so can only satisfy for a finite time.

Thus we want to consider,

$$\ddot{\theta} + (\Gamma_\theta + 3H_d)\dot{\theta} + p\sin(2\theta) \;=\; -q(t_d)\cos\big[\mu(t - t_i)\big] \; .$$

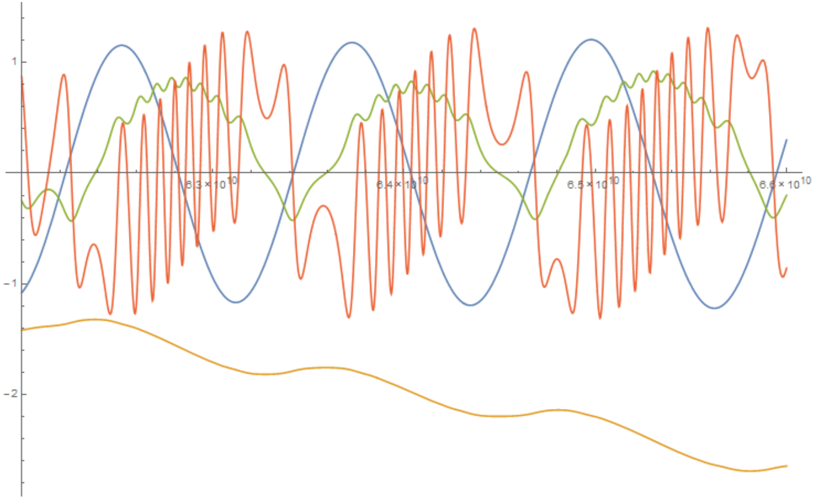
# Phase Locked States

- Solutions increasing monotonously in time with small amplitude modulations.
- Known as “phase-locked states” in the study of forced pendulum.
- Swinging of pendulum matches frequency of torque.

Reparameterise the EoM,

$$\ddot{\Theta} + \frac{1}{Q}\dot{\Theta} + \sin\Theta \; = \; \gamma \cos(\omega\tau) \; ,$$

where, $\gamma \equiv \dfrac{q(t_d)}{p}$.


1
−1
−2
6.3 × 10^10
6.4 × 10^10
6.5 × 10^10
6.6 × 10^10

# Solution and $L$ Number Density

The generic phase-locked state solution,

$$\Theta(\tau) \ = \ \Theta_0 + n\omega\tau - \sum_{m=1}^{\infty} \alpha_m \sin(m\omega\tau - \phi_m) \ \ .$$

Can find the associated $L$ number density,

$$n_L = \left(\mu\phi_r^2\right) n.$$

where $n/2 =$ number of rotations $\theta$ per oscillation of $\Phi$.

Assuming no additional entropy production,

$$\eta^{\text{reh}} \ = \ \frac{n_L}{s} \ \approx \ 0.04 n \times \left(\frac{\mu\phi_r^2}{T_{\text{reh}}^3}\right) \left(\frac{a_d}{a_{rh}}\right)^3 \ .$$

# Estimation of $n$

Reparametrising using $\tau = \mu t$ and $\xi = 2\theta \frac{\mu^2}{2\lambda\phi_r^2}$,

$$\xi'' + \sin\left(\frac{2\lambda\phi_r^2}{\mu^2}\xi\right) + \cos(\tau) = 0 \ ,$$

Assume that directed motion is present and of the form $\xi \propto \tau + ....$

$$\xi = a + b\tau + \frac{\sin(n\tau)}{n^2} + \cos(\tau) \ ,$$

Consistent with our initial assumptions,

$$n \simeq \frac{2\lambda\phi_r^2}{\mu^2} \ ,$$

which agrees well with numerical calculations.

# Dynamics of $\theta$ and Parameter Constraints

- SSC violated $\Rightarrow \phi$ can't produce a non-zero $n$,
- Can approximate the amplitude of oscillations as,

$$\sin 2\theta \approx \frac{H}{H_d} \ ,$$

- After the SSC is violated no simultaneous violation of $\mathcal{C}$, $\mathcal{CP}$ and $L$.

Parameter constraints (e.g. $\Phi$ is unaffected by $\theta$),

$$10^{15}\ \mathrm{GeV} > \phi_r > \mu \ ,$$

$$1 > \lambda > 5 \times 10^{-4} \ ,$$

$$310 > n > 1 \ .$$

# Estimation of the Generated Asymmetry

Generated asymmetry,

$$\eta^{\mathrm{reh}} = \frac{n_L}{s} \approx 0.08 \left( \frac{\lambda \phi_r^4}{\mu T_{\mathrm{reh}}^3} \right) \left( \frac{a_d}{a_{rh}} \right)^3 .$$

where the dilution factor is given by,

$$\left( \frac{a_d}{a_{rh}} \right)^3 = \left( \frac{\pi^2 g_*}{90} \right) \left( \frac{T_{rh}^4}{H_d^2 M_p^2} \right) .$$

Using the SSC, $H_d = \frac{\lambda \phi_r^2 H_0 \Lambda}{\mu^2 \Phi_0} \simeq 2n \times 10^{10}\ \mathrm{GeV}$,

$$\frac{\eta}{\eta_{obs}} = \frac{T_{rh}}{2\lambda \cdot 10^8\ \mathrm{GeV}} .$$

# Neutrino Mass and Seesaw Mechanism

From the $L$ preserving interaction, generate a Majorana mass, $m_{\nu_R} = \frac{g_L \phi_r}{\sqrt{2}}$,

$$10^{14} \ \ \mathrm{GeV} > m_{\nu_R} > 10^{11} \ \ \mathrm{GeV} \ .$$

Via the seesaw mechanism, the mass of the active neutrinos is given by,

$$m_\nu = \frac{y_H^2 v_h^2}{2 m_{\nu_R}} \ ,$$

Decay will occur rapidly, well before the electroweak phase transition suppresses the sphalerons,

$$\Gamma_{\nu_R} = \frac{m_{\nu_L} m_{\nu_R}^2}{4\pi v_h^2} \simeq 10^5 \left( \frac{m_{\nu_R}}{10^{11} \ \ \mathrm{GeV}} \right) \ .$$

# Conclusion and Future Work

- Interplay between inflaton and scalar baryon/lepton during reheating,
- Driven motion can be modelled as a forced pendulum,
- Presence of phase-locked states,
- Asymmetry linearly dependent on the reheating temperature.
- Seesaw mechanism generates active neutrino masses.

## Future work

- Investigation of efficiency required.
- Other cosmological implications